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Limited capacity identity processing of multiple integers

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Abstract

The architecture of the numerical cognition system is currently not well understood, but at a general level, assumptions are made about two core components: a *quantity processor* and an *identity processor*. The quantity processor is concerned with accessing and using the stored magnitude denoted by a given digit. The identity processor is concerned with the recovery of the corresponding digit's identity. Blanc-Goldhammer and Cohen (2014) established that the recovery and use of quantity information operates in an unlimited capacity fashion. Here, we assess whether the identity processor operates in a similar fashion. We present two experiments that are digit identity variations of Blanc-Goldhammer and Cohen's (2014) magnitude estimation paradigm. The data across both experiments reveal a limited capacity identity processor whose operation reflects cross-talk with the quantity processor. Such findings provide useful evidence that is used to adjudicate between competing models of the human number processing system.

Keywords: Numerical Cognition; Numerical Architecture; Parallel Processing; Capacity Limits; Numerical Distance Effect

Limited Capacity Identity Processing of Multiple Integers

Although interest in human numerical cognition has grown considerably over the last twenty five years (Cohen Kadosh & Dowker, 2015), many of the key issues remain under-researched. Here we address two fundamental issues concerning the architecture of the numerical cognition system: (i) whether there exist capacity limits on the processes dedicated to the identification of digits, and, (ii) whether these processes interact with those dedicated to quantity information (i.e., semantic information). We begin with a brief review of extant models of the architecture of the number processing system, focusing on Arabic digit encoding and identification. We then describe potential configurations of the major components of these models with particular respect to processing capacity limits. Finally, we present two experiments that adjudicate between these models.

Models of Numerical Architecture

McCloskey's *abstract code model* (1992; Sokol, McCloskey, Cohen, & Aliminosa, 1991) is one of the earliest models of cognitive architecture underlying number processing. The *abstract code model* posits separate visual encoding modules for Arabic digits and number words that feed into a single semantic module that stores quantity information in an abstract code. Together these modules constitute 'the number-comprehension system' (McCloskey et al., 1985). For a multidigit number such as 4137, the number comprehension system operates by parsing the number into its constituent digits, retrieving their corresponding quantities, and assigning them to place values. A 'number-production system' comprises separate systems for generating responses containing visual digits and number words, respectively. The model is based on a single route from input (characterised by the operation of the number-comprehension system) to output (characterised by the number-production system) via the semantic store of quantity information. Access to semantics was assumed to operate only once a digit has been encoded and identified.

In contrast to this single route model, various multiple route models have been considered. For example, arguments have been put forward that posit the existence of an *asemantic* route, as well as the more traditional *semantic* route (Cohen & Dehaene, 1991; Deloche & Seron, 1982). The asemantic route accommodates evidence that some number processing tasks are accomplished in the absence of accessing quantity information, such as transcoding (e.g., Deloche & Seron, 1982).

Such ideas were explored further by Cipolotti (1995) in terms of a dual route architecture dedicated to processing Arabic numerals. The first stage of processing was captured by an *Arabic numeral input system* and was said to receive "information from early visual processing mechanisms and allows the identification of single digits" (p. 332). The model was heavily influenced by the then current models of visual word processing (Morton, 1969; Warrington & Shallice, 1980) whereby letter identity is captured by the recovery of the letter's visual form. Critically, at this stage of processing, neither phonological nor sematic information is accessed; this happens only subsequently via the operations of the semantic and asemantic routes. Asemantic processing proceeds as a means of recovering spoken forms that then supports spoken number name output. In a later paper, Cipolotti and Butterworth (1995) explored a model that posited separate asemantic routes for written Arabic numbers, written named numbers and spoken numbers. According to Cipolotti (1995), asemantic processing is preferentially activated in order to perform transcoding tasks.

Asemantic processing is also discussed in the context of a variant of the triple code model of Dehaene (1992). In the variant of this model (see Dehaene & Akhavein, 1995), there is an initial stage of early visual processing in which object segmentation and feature extraction take place. Subsequent to this are two further modules. One is dedicated to *Arabic comprehension* and operates to identify visual Arabic digits regardless of font, size or colour – this seems to be essentially the same as the Arabic numeral input system as discussed by

Cipolotti (1995). The lexical entries (digit detectors) here are akin to the kinds of letter detectors posited in the interactive activation and competition models discussed by McClelland and Rumelhart (1981) in their account of visual word recognition. The second module is designated *verbal comprehension* and operates to identify number words regardless of font, size or colour. This is akin to a lexicon in which the entries are the orthographic forms for number words. Outputs from each of these modules feed forwards to the semantic magnitude representation and, a final module, labelled Phonological representation. This module takes the input from the digit and number word lexica and converts these to a spoken form. In allowing interconnections between the two lexica and the phonological representation, the model allows for asemantic transcoding between Arabic and verbal numbers via phonology.

The current theoretical approach

This early work provided the foundations of much of the numerical cognition research performed today. Here, we examine in detail a central theme of this work: the separation between early visual encoding processes and a later stage of visual identification in which the input stimulus is assigned to a digit identity (Cipolotti, 1995; Dehaene & Akhavein, 1995). We focus solely on the processing of visually presented Arabic digits in a bid to provide a relatively fine-grained account of how key encoding and identification processes relate to accessing number semantics. As such, this work is akin to that undertaken by researchers in their quest to understand how processes concerning visual featural and letter encoding inform more general theories concerning the mental architecture underpinning word recognition and reading (see e.g., McClelland & Rumelhart, 1981; Rumelhart & Siple, 1974). Although historically these 'low-level' processes have not featured heavily in traditional accounts, there is now a growing body of evidence that these need to be considered if adequate models of numerical cognition are to be developed (see Cohen, 2009; 2010; Cohen & Quinlan, 2016;

Cohen, Warren & Blanc-Goldhammer, 2013; Defever, Sasangie, Vandewaetere & Reynvoet, 2012; García-Orza, Perea, Mallouh, & Carreiras, 2012; Wong & Szücs, 2013; Zhang, Xin, Feng, Chen, & Szücs, 2018).

We have chosen to focus on input and decision processes. Figure 1 provides a schematic representation of different accounts of how these input and identification components may be arranged, and how these underpin number processing once a digit is presented visually[1]. Because each component is generally understood in relation to the types of errors it is responsible for, we briefly describe each component with respect to their predicted errors. All of the models assume that the first stage of processing is constituted by visual encoding. The encoding system does not identify or interpret the stimulus. It operates on the physical structure of the stimulus in order to recover shape. As such, the similarity in the physical structure between different stimuli is the predicted source of confusions when the encoding stage is operating. Importantly, the specific pattern of these confusions is not task-specific. That is, the physical structure of every digit may be confused with the physical structure of every other digit, and the probability of this confusion is a function of the physical similarity of each digit to every other digit (similar assumptions are central to the interactive activation and competition model of letter processing put forward by McClelland and Rumelhart, 1981). Cohen and Quinlan (2016) modeled this process in detail. Encoding is characterized as a continuously updating process whereby information is continuously fed to the other processing stages (see Cohen & Quinlan, 2016).

The other two components are labelled, respectively, the *Quantity processor* and an *Identify Digit processor* (henceforth, simply, the *identity* processor). The operations associated with the quantity processor bring about access to the underlying quantity representations linked to each digit. As a consequence, the performance of the quantity processor reflects characteristics of the manner in which quantities are stored and accessed

We align the quantity processor with the types of semantic systems discussed by Cipolotti (1995) and Dehaene and Akhavein (1995) and note that it is sometimes referred to as the *approximate number system* (Feigenson, Dehaene & Spelke, 2004).

The quantity processor operates on the quantity denoted by the symbol and therefore similarity in denoted quantity is the predicted source of confusions. Furthermore, the specific pattern of these confusions is a function of the quantities being compared. The quantities being compared, in turn, are task-specific. So, if the task is to determine whether a quantity is greater than or less than a given standard digit such as “5” (or, for example, 5 is used as a prime), then the pattern of confusions is predicted to be a function of the *numerical distance* between the presented quantity and 5 (i.e., the comparison quantity). If a different target digit is chosen, then the predictions concerning confusions will change accordingly. The operation of the quantity processor is revealed by confusions regarding numerical distance.

In contrast, the identity processor comprises operations concerned with recovering the corresponding stored identity of a given input digit[2]. Similar to Cipolotti’s (1995) Arabic numeral input system and Dehaene and Akhavein (1995) “Arabic comprehension[3],” the identity processor is entrained once early visual encoding processes have run their course, for example, as in the case of taking the digit “5” and deriving a unique code (cf. Posner, 1969) that signifies the digit’s identity. Such a code underpins being able to make an appropriate discriminative response to the digit. The identity processor operates on the physical structure of the symbol. Therefore, similarity in *physical structure* is the predicted source of confusions when the identity processor is operating, but now the specific pattern of these confusions is a function of the digits being compared. Again, the identity processor may be activated in a variety of tasks (e.g., identify a numeral, discriminate between numerals, etc.). The digit comparison process is, in turn, a function of the task. For example, if the task is to determine whether a digit is a “5,” then the pattern of confusions is predicted to be a function of the physical similarity between the presented digit and a "5" (i.e., the comparison digit). If a different target digit is chosen, then the predictions concerning structural confusions will change accordingly. In this way, the operation of the identity processor is revealed by these kinds of physical similarity relations. Finally, the operation of the identity processor is disambiguated from the operation of the encoding system because the confusions of the encoding system are common to all tasks (i.e., the presented digit is confused with all other digits), whereas the confusions of the identity processor are task-specific (i.e., the presented digit is confused with the comparison digit).

It is important to note that the identify processor is tuned to differentiate digits from one another, rather than being a generalized shape categorizer. We hypothesize this because Cohen (2009) demonstrated that the physical similarity of digits is a function of the *symbol structure*, rather than visual form. Cohen's (2009) physical similarity function first simplified the digits into the 8-line structure as incorporated in a digital clock. Then a ratio is calculated of the number of lines that the two digits share divided by the number of lines that are required to make up the two digits, that they do not share. Because the physical similarity function of digits is derived from the underlying structure of the digits, it predicts errors regardless of the font in which the digit is displayed (see Cohen, 2009). A basic assumption of our approach is that there are principled differences between early visual encoding and digit identification. Such differences made are explicit in the models of Cipolotti (1995) and Dehaene and Akhavein (1995), and analogical differences have been explored most thoroughly in the context of visual letter processing (see e.g., Coltheart, 1972). Furthermore, there is neurophysiological evidence of the separation between encoding and digit identification (e.g., Pinel, Le Clec'H, Van de Moortele, Naccache, Le Bihan, & Dehaene, 1999).

In distinguishing between the identity and the quantity processors we acknowledge that the respective processes are separable. We also note, at the outset, that the successful operation of these processors is predicated on the successful encoding of the input digit. In a recent paper, Cohen and Quinlan (2016) provided a detailed model and computer simulations of the encoding and quantity comparison processes. Cohen and Quinlan (2016) showed, via computer simulations, that it is computationally feasible for quantity information to be accessed from a numerical symbol without this being dependent on accessing the identity of that symbol. Nevertheless, Cohen and Quinlan (2016) were agnostic as to how and when symbol identification occurs. In the following, we broaden the discussion to include the three components shown in Figure 1 and we are particularly interested how digits are identified. We explore the processing capacity of the identity processor and the degree to which the identity and quantity processors are functionally independent of one another. In the following, we sketch out various possible cognitive architectures that we take to be plausible structures for basic number processing and our primary intention is to adjudicate between these. Figure 1 provides schematic representations of the particular architectures that we will consider in detail.

The current empirical approach

We start by addressing the issue of processing capacity limits. In this respect, a seminal reference is to Shiffrin and Gardner (1972) who were interested in the degree to which stimulus identification (*qua* letter identification) is limited in capacity. In their paradigm, performance was examined across two conditions: a Simultaneous condition (here termed SIM) and a Successive condition (here termed SUCC). In the SIM condition, participants were presented with a brief display containing four characters, which were masked upon removal. In the SUCC condition, participants were briefly presented with two displays in succession – each containing two characters, which were masked upon removal.

In both the SIM and SUCC conditions the character displays were presented for the same brief amount of time (i.e., 50 ms). In this way, the amount of time available to extract information from a two-character display was the same as that available to extract information from a four-character display. On the assumption that character identification is a limited capacity process, it was predicted that participants would be more accurate in the SUCC condition than the SIM condition. Critically, however, the results showed that there was, essentially, no difference in accuracy across the two conditions: consequently, it was concluded that, at least in the conditions tested, letter identification is unlimited in capacity. In summarizing the relevant findings and ensuing literature, Pashler (1998; p. 123) concluded that when a small number of characters have to be processed, there are no capacity limitations. This is tempered by the fact that evidence of capacity limitations only emerges once the difficulty of target discrimination is considerable (see e.g., Kleiss & Lane, 1986). Furthermore, after a careful review of the relevant word processing literature, Lachter, Forster and Ruthruff (2004) argued strongly that there is no identification without attention: An implication being that stimulus identification is capacity limited (see, Pashler, 1998, Chapter 5).

In an effort to better understand the architecture of the numerical cognition system, Blanc-Goldhammer and Cohen (2014) addressed whether the quantity processor is capacity limited. Blanc-Goldhammer and Cohen (2014) used a pared down version of the Shiffrin and Gardner (1972) task. All of the characters were digits and the task was to identify the diagonal in which the largest digit occurred. The rationale was the same as before, namely, if the act of deriving relative quantity information is capacity limited, then performance ought to be better in the SUCC than the SIM condition. Contra to this prediction, however, a key finding was that there was no overall difference in performance across these two conditions.

Blanc-Goldhammer and Cohen (2014) concluded that the process of comparing quantities conveyed by integer digits is unlimited in capacity.

Figure 1 presents alternative models of the numerical cognition system. To interpret the figure, a gray box with a solid border indicates unlimited capacity processing, whereas a white box with a dashed border indicates limited capacity processing. Figure 1assumes that all processing stages prior to the quantity processor occur in an unlimited capacity fashion. This assumption is necessary because in a stage model, a limited capacity processor acts as a bottleneck. This bottleneck reduces the information flow to the later stages, causing the later stages to mimic limited capacity processing as well (regardless of the later stages processing capacity). Given that Blanc-Goldhammer and Cohen (2014) found no evidence of limited capacity processing in their task, in all models in Figure 1, we assume the quantity processor operates with unlimited capacity. Furthermore, given that encoding operates prior to accessing quantities, we conclude that encoding also operates in an unlimited capacity fashion. Finally, we rule out the possibility of a limited capacity identity processor that operates prior to the unlimited capacity quantity processor. With these limitations, Figure 1 outlines the remaining possible models. Figure 1*a* and 1*b* set out sequential systems wherein the operations related to the quantity processor precede the operations of the identity processor. In Figure 1*a* the identity processor is unlimited in capacity whereas in Figure 1*b* identity processor is limited in capacity.

Immediately, however, we can rule out all models in which the quantity processor is located before the identity processor (as in Figure 1*a* and 1*b*). Such models predict that quantity must be processed before digit identification can take place. If this were the case, then digit identification would be influenced by the same factors that influence quantity comparison. Evidence of accessing quantity information exists when participants' reaction times (RTs) for correct responses decreases monotonically as the numerical distance between the two digits increases (e.g., Moyer & Landauer, 1967) . This is termed the *numerical distance effect* and is taken to indicate the operation of the quantity processor (Banks & Flora, 1977; Besner & Coltheart; 1979; Dehaene & Akhavein, 1995; Duncan & McFarland, 1980; Schwartz & Heinze, 1998).

Critically, Cohen (2009) demonstrated that digit identification can take place without activation of the quantity processor (for additional supporting evidence of this see Ratinckx, Brysbaert & Fias, 2005; and, Pansky & Algom, 2002). Cohen (2009) conducted a digit identification task where, on every trial, a single visual digit was presented and the participant simply had to identify whether the digit was a “5.” Primary interest was performance on ‘different trials’ where a digit other than “5” was presented. If performance was driven primarily by digits accessing their corresponding quantity representations then RTs on the different trials should scale according to the numerical distance between the presented digit and 5. In contrast, the performance on different trials revealed the importance of physical similarity. That is, RTs varied according to how visually similar the actual presented digit was to the digit “5” (termed a *physical similarity* effect). Here, we assume that physical similarity effects reflect, in part, operations concerned with digit identification (see also Cohen, et al., 2013). As a consequence, we assert that when participants engage in a digit identification task, performance will, to large measure, reflect effects of physical similarity that are key to that particular identification judgement.

Having ruled out models 1a and b, different accounts need to be considered. Figure 1*c* – *g* show various architectures based on the premise that the quantity processor operates in an unlimited capacity fashion, but the individual cases differ in other critical regards. For instance, Figure 1*c* shows the case where the two processors operate sequentially with the identity processor situated before the quantity processor. This model captures key features of the models discussed by McCloskey (1992), Cipolotti (1995) and Dehaene and Akhavein

(1995). Following visual encoding, a digit is first identified and then semantics are accessed. In the particular model shown, the quantity processor is of unlimited capacity (see Blanc-Goldhammer & Cohen, 2014). The identity processor is also unlimited capacity and therefore does not create a bottleneck prior to the unlimited capacity quantity processor. If it is assumed that both processors must run to completion prior to a response, then we can rule out this model for the same reasons that models 1*a* and 1*b* have been ruled out: performance in simple digit identification tasks do not reveal effects of numerical distance (see Cohen, 2009). The reasoning is that if a response can only be emitted once the corresponding quantity has been accessed then effects of numerical distance should obtain. However, if a response can be initiated once the identity processor completes then model 1*c* remains viable.

Models 1*d* - *g* are radically different from the sequential accounts because, in all cases, the two processors are seen to operate in parallel. Models 1*d* and 1*e* assume that both processors operate in an unlimited capacity way. Whereas the processors in model 1*d* are functionally independent, in model 1*e* cross-talk is possible between the processors. Model 1*e* allows for information exchange across the processors (see Mordkoff & Yantis, 1991). By one reading of the model, the suggestion is that the processors function in a mutually dependent way, but this form of dependency has already been questioned by the data that show that digit identification can proceed quite independently of mechanisms concerning the accessing of quantity information. A more flexible reading is where cross-talk between the processors is possible but is not necessary. Further discussion of cross-talk is included as the material unfolds. The final models shown in the Figure 1*f* and 1*g* are similar to models 1*d* and 1*e*, respectively, with the exception that models 1*f* and 1*g* are based on the assumption that the identity processor operates in a limited capacity way.

To adjudicate amongst the viable alternative accounts in Figure 1, we modified Blanc-Goldhammer and Cohen (2014) paradigm so that the task was simply to decide whether a single target digit (a “5”) was presented. Our primary concern was whether the identity processor is limited capacity as would be revealed by the finding that performance in SUCC condition was better than in the SIM condition, or, unlimited capacity as supported by the finding of equivalent levels of performance in the two conditions. Models 1c-1e assume the identity processor is unlimited in capacity. Models 1*f* and 1*g* assume the identity processor is limited in capacity.

In the right-most column in Figure 1 we map out what each model predicts in terms of the physical similarity function and the numerical distance function. A detailed description of physical similarity of the digits used can be found in Cohen (2009) and numerical distance standardly refers to the degree to which the quantities referenced by the digits differ. As described above, we assume that the physical similarity function (here, the task-specific physical similarity to the digit “5”) will be observed when the identity processor is activated, and the numerical distance function (here, the task-specific numerical similarity to the quantity “5”) will be observed when the quantity processor is activated. Because we assume the identity processor is necessary to complete the identification task, we should always observe the physical similarity function in the data. The numerical distance function should be observed only when the quantity processor is active. This will happen if the quantity processing is a prerequisite to the identity process, or if the two processes interact. Therefore, Models 1*c*, *d*, and *f* all predict an effect of physical similarity and not numerical distance. Models 1*e* and *g* predict an effect of both physical similarity and numerical distance. Not considered here, in any detail, are models in which digit identity can be completed solely through the quantity processor. These models would predict an effect of numerical distance but not physical similarity and given the emerging evidence against this pattern, this outcome is extremely unlikely and not considered further (Cohen, 2009; Cohen et al., 2013; Defever, Sasanguie et al., 2012; García-Orza et al., 2012; Wong, & Szücs, 2013; Zhang et al., 2018).

Because the current task necessitates the activation of the identity processor, all models predict effects of physical similarity. However, only a subset of the models predict effects of numerical distance. On these grounds, any putative numerical distance effect is key in being able to adjudicate between the different models. Moreover, the most revealing finding would be where numerical distance effects emerged only after the influence of physical similarity had been removed from the data. As Cohen (2009) has shown, physical similarity and numerical distance are positively correlated and typically effects of numerical distance may be abolished once effects of physical similarity have been accounted for.

Experiment 1

Methods

Participants

One hundred and twenty-two naïve participants volunteered for class credit. All participants were undergraduates at the University of North Carolina at Wilmington (UNCW). Although it would have been preferable to record demographic data, at UNCW students average age is 22, there are 62% females, and 83% of students are white, 6% are African American, 6% are of one or more other races, and 4% are unknown. About 7% of students are Hispanic.

To determine sample size, we started with Kleiss and Lane (1986), who conducted a similar experiment with letters from which we estimated effect sizes ranging from about d = 0.5 to d = 1.3 (depending on the physical similarity of the letters). However, we temper this with the conclusion of Brysbaert and Stevens (2018), "… it is bad practice to use effect sizes of published article as an estimate for power analysis, because they tend to be exaggerated. Much better is to assume effect sizes of d = .4 or d = .3 (the typical effect sizes in psychology) …" (p. 16). With this in mind, for the key analysis (a paired sample t-test) we assumed a conservative $d = .3$, power = .8, then the required $n = 93$. We therefore set the minimum number of participants to 100.

Apparatus and Stimuli

All stimuli were presented on a 24-inch LED color monitor with a 72-Hz refresh rate controlled by a Macintosh Mini running OS X. The resolution of the monitor was 1920 X 1200 pixels.

Figure 2 provides a schematic representation of the sequence of events on a trial in, respectively, the SIM and SUCC conditions. (To be clear, the figure incorporates all conditions across two experiments and in Experiment 1the SUCC condition is labelled SUCC|0.) Prior to each trial four digits were randomly chosen (with replacement) from the digits "1" - "9" excluding "5." On half the trials, the target (i.e., "5") randomly replaced one of the four distractor digits. The characters were presented as Arabic digits in Arial font subtending about 0.68^{o} visual angle vertically.

Procedure and Design

The display timings are as shown in Figure 2. In all major respects, the methods were the same as those used by Blanc-Goldhammer and Cohen (2014). However, in this case, participants were instructed that, on each trial they were to determine whether the target (i.e., "5") was present in the display. A single target was used to reduce any interference that might result from switching targets between trials. Furthermore, a target of "5" is optimal because it allows for the greatest number of single digit distractors that are symmetrically distributed above and below the target. Half of the participants were instructed to press "k" if the target (i.e., "5") was present, and "d" if it was not. The other half of the participants used the reverse key assignment. To facilitate task familiarity, displays were presented for 75 ms in the practice trials: On experimental trials, this was reduced to 66 ms.

Within a given testing session, the SIM and SUCC conditions were blocked. Each block contained 75 trials. The presentation conditions alternated for a total of four blocks. Twelve practice trials were presented prior to the start of each of the first two blocks. The participants were not told that there were different presentation conditions in the different blocks. Half of the participants were presented the SUCC condition first and the other half were presented the SIM condition first. Participants were allowed to take self-timed breaks between blocks.

Results and Discussion

Thirteen participants experienced computer problems and their data were removed prior to analysis. We calculated *d*' for each of the remaining participants for each condition ($M = 1.29$, $SD = 0.56$) and we removed participants who had a *d*' < 0.2 (at or near chance) in at least one condition. We applied this exclusion criterion because chance performance indicates that the participant is hitting floor in that condition. If a participant hits floor in at least one condition, then the difference between the SIM and SUCC conditions will be biased low for that individual. This will shift the overall results toward identifying no difference between the SIM and SUCC conditions. The unlimited capacity model predicts equal sensitivity in these conditions and to include the data from chance performers in the eventual dataset compromises our ability to offer a reasonable test of this prediction. This criterion excluded three participants (average performance after removing the three participants: $M = 1.32$, $SD = 0.54$)[4].

A paired *t*-test revealed that participants were more sensitive to the target in the SUCC condition ($M = 1.37$, $SD = 0.495$) than in the SIM condition ($M = 1.27$, $SD = 0.577$), $t(105) = 2.15$, $p = 0.03$. This result suggests that identifying a specific digit is a capacity limited process. Clearly the effect is small, $d = 0.2$, therefore, we designed Experiment 2 in a bid to set it on more a more solid basis.

We also assessed the influence of the physical similarity and the numerical distance on performance on the *non-target* trials. Clear predictions can be made regarding the relations between false alarms (i.e., errors on non-target trials) and physical similarity, and, respectively, numerical distance. Accordingly, participants should be more inaccurate as the physical similarity between the non-targets and the target increases, whereas they should be less inaccurate as the numerical distance between the non-targets and the target increases. In both cases, the participant is presumably mis-perceiving the non-target as the target. If processing capacity is limited, the identities of some but not all of the digits on a trial are processed. A default assumption is that sampling of the digits is random hence over trials the expected interference from the non-target digits will be indexed by the average physical similarity and numerical distance of those digits. To test these predictions, we calculated the average physical similarity and average numerical distance for each four-digit, non-target combination. We calculated these averages by doing the following:

(1) For each digit in a four-digit, non-target combination, we calculated the physical similarity relative to the digit “5” (see Cohen, 2009, for a complete description of the physical similarity measures).
(2) We then averaged across these four physical similarities to get the average physical similarity for that specific four-digit, non-target combination.
(3) We repeated this process for each unique, four-digit, non-target combination.
(4) To get the average numerical distance, we repeated steps 1-3 substituting the calculation of numerical distance for physical similarity.

At this point, each unique four-digit, non-target combination was summarized by its average physical similarity and average numerical distance relative to the target “5.” The average physical similarity and average numerical distance of these four-digit, non-target combinations are correlated: SIM r = -.41; SUCC r = -.38. To obtain stable estimated false alarm rates, we collapsed trials across (a) participants, and (b) the four-digit, non-target combinations with the same average physical similarity statistic and the same average numerical distance statistics (e.g., the data for all the four-digit, non-target combinations in which the average PS = 3 and average ND = 2 would be combined, collapsing over participants). These false alarm rates were used as the DV in the regression models described below. Figure 3 presents these data for the SIM and SUCC conditions.

All models in Figure 1 assume that the physical nature of the digits influences performance in the tasks[5]. Therefore, showing an effect of physical similarity on performance is of no consequence in being able to adjudicate between the different models under consideration. What is germane is whether, having taken account of the effects of physical similarity, effects of numerical distance still emerge. To test this line of reasoning we adopted the following analytic approach. To assess the influence of numerical distance after removing the influence of physical similarity, we calculated two linear models for each condition. First, we computed a linear regression with false alarm rate (described above) as the criterion variable and physical similarity as the predictor variable. We then extracted the residuals of this model and used them as the criterion in the second model in which numerical distance was the predictor variable. If the regression coefficient of the second model were to be statistically significant, then we may conclude that there is an influence of numerical distance after the removal of the influence of physical similarity. As such, our analysis is designed to disambiguate the models, rather than disambiguate the shared variance.

For the SIM condition, physical similarity alone accounted best for the data, $r^2 = .15$, $F(1, 195) = 35.48$, $p < .001$, slope = 0.05. Numerical distance added no predictive benefit, $F(1, 195) = 0.68$, *ns*. These results reveal that as the physical similarity of the display digits to the target increased, the participants' false alarms increased.

For SUCC condition, both physical similarity, $r^2 = .07$, $F(1, 200) = 15.97$, $p < .001$, slope = 0.03, and numerical distance, $r^2 = .04$, $F(1, 200) = 7.42$, $p < .001$, slope = -0.03, were significant predictors. These results show that as the physical similarity of the other display digits to the target increased, the participants' false alarms increased. In addition, as the numerical distance of the other display digits to the target increased, the participants' false alarms also decreased.

In sum, the observed differences in sensitivity between the sequential and simultaneous conditions support the conclusion that digit identification is a capacity limited process. Moreover, from the more detailed analyses of the data, both physical similarity and numerical distance affected performance. Following on from the previous work by Cohen (2009), the effect of physical similarity is as predicted given that visual encoding plays a critical role in completing the task. Performance, in part, reflects the degree to which the different digits can be distinguished from the digit "5." In addition, performance (in the SUCC task at least) was also modulated by the numerical distance between the target and the non-target digits in non-target displays. This finding shows that numerical quantities were being accessed despite the fact that successful responding did not necessitate this.

Overall therefore the data sit most comfortably with model 1*g*. The evidence is consistent with assuming the operation of a limited capacity identity processor together with cross-talk between the quantity and identity processors. Nonetheless, the numerical distance effect is surprising. In both the Cohen (2009) study and the present case, digit identification was key and yet, whereas in the earlier study, no effect of numerical distance was found, here such an effect did emerge.

To examine this contrasting pattern of findings we carried out a second experiment. Now, the testing conditions were adapted so that now display exposure was manipulated in a particular and systematic way. The standard SUCC condition (i.e., SUCC|0) was repeated, and two further versions of the SUCC condition were added. In the SUCC|-33condition, the presentation of the second pair of characters began 33 ms before the removal of the first pair. The first pair of characters was presented for 33 ms, next all four characters were presented for 33 ms and then the first pair of characters on one diagonal were removed leaving only the second diagonal characters visible for 33 ms. In the SUCC|+33 condition, a masker display was interpolated between the two-character displays for 33ms. That is, there was an ISI of 33 ms between the removal of the first pair of characters on one diagonal and presentation of the second pair on the other diagonal. All four masks remained visible during this ISI.

The conditions were motivated by the following reasoning. The intention behind the SUCC|-33 condition was to attempt to overload the system by introducing the intermediate display between the two two-item displays. Here, the system has to process fewer stimuli per ms than the SIM condition but more stimuli per ms than the SUCC|0 condition. In contrast, the intervening masking display in the SUCC|+33 was intended to allow additional time for the system to first process the initial pair of digits prior to the onset of the second pair. Here, the system has to process fewer stimuli per ms than the SUCC|0 condition. If processing capacity is unlimited, then SIM = SUCC|-33 = SUCC|0 = SUCC|+33. If, however, processing capacity is limited, as suggested by the results of Experiment 1, then we expect performance in the SIM condition to be poorer than the SUCC conditions, SIM < (SUCC|-33 & SUCC|0 & SUCC|+33). More subtly, varying the timing of the successive condition provides a way to assess the severity of the capacity limits. If capacity limits are large relative to the information being processed, then every incremental addition of time will provide a performance advantage: SIM < SUCC|-33 < SUCC|0 < SUCC|+33. If, however, the processing capacity limits are small relative to the information being processed, then only the first incremental addition of time will provide a performance advantage: SIM < SUCC|-33 = SUCC|0 = SUCC|+33. This is the case because the additional time increment in the

SUCC|-33 will nullify the relatively small influence of processing capacity limits. Because the influence of processing capacity limits was already nullified, the additional time increments in the SUCC|0 and SUCC|+33 conditions cannot produce further performance increases.

Finally, assuming the sample mean and standard deviation equal the population mean and standard deviation, respectively, a post-hoc power analysis revealed that the Experiment 1 had a power = .57. We therefore increased the number of participants to increase the power of the experiment. The increased power can help determine whether the lack of an influence of numerical distance in the SIM condition was undetected because it was not present, or, was too small given the power of the first experiment.

Experiment 2

Methods

Participants

Two hundred and twenty-nine naïve participants (from the same university sample) volunteered for class credit. Sample size was determined in the same way as Experiment 1. The key analysis of Experiment 2 is with respect to a mixed effects model because of the increased power that this model provides. An a priori power analysis on a mixed effects model, however, is a relatively uncertain process (Brysbaert & Stevens, 2018). Therefore, we calculated power assuming a paired *t*-test with the knowledge that the mixed effects model analysis will provide greater power than that estimated for the *t*-test. The power analysis, assuming an effect size equal to that obtained in Experiment 1, indicated a required *n* of 182. We set the minimum number of participants to 200 to ensure enough power to identify effects, if they exist.

Apparatus and Stimuli

The apparatus and stimuli in Experiment 2 were identical to those of Experiment 1.

Procedure and Design

The general procedure in Experiment 2 was the same as that in Experiment 1. However, now the four conditions, SIM, SUCC|0, SUCC|-33 and SUCC|+33 were administered in separate blocks of trials. Each block contained 146 trials. The order of the presentation conditions was randomized over four blocks. Twelve practice trials were presented prior to the start of each block. The participants were simply told to identify the target number "5" and were not told that there were different presentation conditions. Participants were allowed to take self-timed breaks between blocks.

Results and Discussion

Sixteen participants experienced computer problems and were removed from the dataset prior to the analysis. As in Experiment 1, we calculated *d'* for each of the remaining participants for each condition (M = 1.35, SD = 0.66). Eighteen participants were removed who had a $d' < 0.2$ (at or near chance) in at least one condition (performance after removing the 18 participants: M = 1.44, SD = 0.61)[6].

A mixed-model ANOVA, with participants as a random factor and the four presentation conditions as a fixed factor, revealed a significant effect of presentation condition, $F(3, 582) = 83.3$, $p < .001$. Calculating effect sizes for mixed effects models is not straightforward, but Nakagawa and Schielzeth (2013) have shown that useful information can be gained by calculating the marginal and condition r^2s of these models ($r^2_{marginal} = .1$; $r^2_{conditional} = .70$). A post-hoc power analysis using the simr package in r, revealed power = 1.0 (95% confidence intervals: .98-1.0). A Tukeys HSD test revealed that the participants were less sensitive to the target in the simultaneous condition than in the other three successive conditions ($p < .001$, see Table 1). The three successive conditions were not significantly different from one another ($p > .05$). This result replicates the central finding of Experiment 1: *digit identification is a capacity limited process*.

We again assessed the influence of the physical similarity and the numerical distance on performance in the same manner as Experiment 1. Because participants were equally sensitive to the three SUCC conditions, we combined the data for them in this analysis. Again, physical similarity and numerical distance are correlated in the 4-digit displays: SIM $r = -.39$; combined SUCC $r = -.38$. Figure 4 presents false alarms plotted as a function of physical similarity and numerical distance for the SIM and combined SUCC conditions. For the SIM condition, both the physical similarity, $r^2 = .09$, $F(1, 201) = 19.38$, $p < .001$, slope = 0.03, and the numerical distance, $r^2 = .08$, $F(1, 201) = 17.69$, $p < .001$, slope = -0.03, were significant predictors. For the combined data for the SUCC conditions, both physical similarity, $r^2 = .13$, $F(1, 609) = 86.9$, $p < .001$, slope = 0.03, and numerical distance, $r^2 = .02$, $F(1, 609) = 86.9$, $p < .001$, slope = -0.02, were significant predictors. As in Experiment 1, these results reveal that as the physical similarity of the digits to the target increased, the participants' false alarms increased. In addition, as the size of the numerical distance of the digits to the target increased, the participants' false alarms decreased. These final analyses have established that both physical similarity and numerical distance influenced performance in the same way as shown in Experiment 1. More particularly, the initially unpredicted effect of numerical distance found in the SUCC condition in Experiment 1 has been found in the data for both the SIM and SUCC conditions in Experiment 2. Thus, the lack of an effect of numerical distance in the SIM condition in Experiment 1 was likely the result of a small effect that required increased power to be detected. In sum the data fit most comfortably with the parallel, interactive processors account shown in Figure 1*g*. Responses reveal effects of both physical similarity and to a lesser extent numerical distance and we therefore conclude the evidence is more in line with Model *g* than Model *f*.

General Discussion

The central findings, replicated across two experiments, support the parallel interactive processors account of how the human number system operates (i.e., as shown in Figure 1*g*). The present data show that digit identification is a limited capacity process. In addition, effects of both physical similarity and numerical distance are present in the data. The effects of numerical distance, though small, have been found in three for the four general cases tested (the exception is in the SIM data for Experiment 1) and these will be considered in more detail shortly.

The primary finding of the present research is that the digit identity processor has a limited capacity. This conclusion is supported by the data from both Experiments 1 and 2, which showed that participants were more accurate in the SUCC conditions than the SIM condition. Furthermore, Experiment 2 showed that participants were equally sensitive in all three SUCC conditions. This finding suggests that the limited capacity is small relative to the amount of information being processed. That is, although the addition of a small increment of time in the SUCC|-33 condition improved performance over that of the SIM condition, additional increments of time in the SUCC|0 and SUCC|+33 conditions provided no increase in sensitivity. This suggests that capacity was not the critical constraint limiting performance in the SUCC|0 and SUCC|+33 conditions. Therefore, the digit identity processor has a limited capacity, but that limit is not severe relative to the amount of information being processed.

The finding that the digit identity processor has a limited capacity is consistent with Lachter, et al. (2004) who claimed that there is no identification without attention. Nevertheless, a limited capacity digit identity processor may be considered an unintuitive finding. This is because Blanc-Goldhammer and Cohen (2014) determined that the quantity processor has unlimited capacity. The data suggest that that within the numerical cognition system the processes concerning digit identity do not precede those concerned with the extraction of quantity. More likely, these separate processors operate in parallel.

The evidence that the identity and quantity processors are functionally separate provides some insight into how quantities are represented by numerical symbols. Blanc-Goldhammer and Cohen (2014) demonstrated that participants could locate the symbol, in a field of four numerical symbols, that denoted the greatest quantity in parallel. However, the present experiment demonstrated that people could not identify those same symbols concurrently. This suggests that extracting a quantity from a numerical symbol has some similarity to extracting a quantity from, say, a field of dots. This is because one can judge which of two fields of dots has a greater quantity, without knowing the exact identity of the quantity associated each field of dots (see e.g., Burgess & Barlow, 1983). So, numerical symbols, when processed for quantity rather than identity, appear to activate a noisy representation of quantity (see Cohen & Quinlan, 2016) and no precise identity information. In contrast, when numerical symbols are processed for precise identity rather than quantity, they appear to sometimes activate identity information and no quantity information (see Cohen, 2009) and other times activate both identity and quantity information (Experiments 1 and 2). We discuss this further below.

Effects of physical similarity are present in the data for both the SIM and SUCC conditions in both experiments. Moreover, these effects are expressed consistently across all cases. Participants were less accurate when processing displays that contained digits that were highly physically similar to “5” than when the digits were highly physically dissimilar to “5.” This pattern of responding fits comfortably with the notion that digit identification reflects processes concerning the physical nature of the digits and more particularly errors that arise due to the similarity relations that exist between the input digits and the target digit. Different errors are assumed to arise when the identification task is changed accordingly. The nature of the identification processes is, nonetheless, seen to be capacity limited in nature and are distinct from those concerning the recovery of quantity information. This account applies equally to the case where on each trial a single digit is presented (i.e., as in Cohen, 2009) and here where four digits are presented and where the same target digit is being probed.

Participants were also less accurate in their responses on target absent trials when the non-target digits were numerically close to 5 than when the digits were numerically distant from 5. This pattern of performance arose in all cases except in the SIM condition in Experiment 1. Nevertheless, the increased power in Experiment 2 likely made it possible to detect the small effect of numerical distance in the SIM condition that was difficult to detect in Experiment 1. Such effects are as predicted if it is assumed that digit identification relies on quantity representations being accessed but this was not a necessary precursor to successful task completion: the task was essentially simple pattern classification that did not necessitate accessing any semantic information. The general pattern of findings leads to the conclusion that the identity and quantity processors exhibit functional cross-talk. Critically the fact that the data reveal effects of both physical similarity and numerical distance shifts the focus away from Model *f* onto Model *g*. Model f suggests that a response could be based on the operation of either processor with no influence of the other but the data show that this is not so. Responses revealed the influence of both processors and therefore the evidence is more in line with Model *g* than Model *f*.

We take such cross-talk to be a desirable feature (cf. Mordkoff & Yantis, 1991). That is, if one is presented with a visual quantity (e.g., dots), it is possible to understand quantity and verbally label the corresponding number (e.g., say 'fīv'). Similarly, if one is presented with sound 'fīv,' it is possible to access the corresponding quantity as demanded by a mental arithmetic problem. However, cases are beginning to emerge where task performance does not depend on such cross-talk (Cohen, 2009; García-Orza, et al., 2012; Zhang et al., 2018), and cases where the operation of a processor is revealed even though successful task completion does not demand it (as in the experiments here). Such demonstrations should be considered against other cases in which effects are clearly context dependent and emerge only as a consequence of the task constraints (see for example, Campbell, 2011; Cohen & Dehaene, 1995). Clearly future work needs to address such contrasting cases with a view to better understanding which functional components are influential in a given task and why. Nonetheless, the context effects that do occur as a consequence of particular task constraints, can perhaps best be understood by allowing flexible cross-talk between the quantity and identity processors.

Although the Model *g* can accommodate the present data, we may ask why it is that effects of numerical distance have arisen here, when similar effects are not present in other digit identity tasks when physical similarity is taken into account Cohen (2009; see also García-Orza, et al., 2012; and, Zhang et al., 2018). This appears surprising given that digit identification was the basic task in Cohen's earlier study (2009) as it was here. Nonetheless, at the level of methods, the two cases are strikingly different. In the earlier study, on every trial, a single digit was presented until response, whereas here on every trial four digits were presented very briefly and were pattern masked. As a consequence, both the timing constraints and the number of stimuli present on a trial differs across the two cases. Either or both of these may be crucial in understanding the contrasting patterns of performance in the two cases. We simply take it that the different paradigms challenge the number processing system in different ways.

In the current model, we make the following stipulations:

(1) The identity processor operates in limited capacity fashion. This is evidenced by the results of Experiments 1 and 2.

(2) The identity processor is most efficient when there is only a single imperative stimulus present (Cohen, 2009). This is evidenced by the decrements in performance that are observed in cases where there is more than one target present (see e.g., Duncan, 1980).

(3) Recovery of digit identity from the identity processor is more accurate than recovery of digit identity from the quantity processor. Because the quantity processor recovers noisy quantity representations (e.g., Cohen & Quinlan, 2016), any identity information recovered from the quantity processor will also be noisy.

(4) Recovery of digit identity from the quantity processor is faster for multiple digits (as in Blanc-Goldhammer & Cohen, 2014) than recovery of digit identity from the identity processor for multiple digits. This is because the quantity processor has unlimited capacity whereas the identity processor has a limited capacity.

When identifying a digit, the identity processor is preferred because of (3). Furthermore, on single stimulus trials where the task is to identify the digit, performance reflects the efficient operation of the identity processor. However, in cases where more than one imperative stimulus is presented and displays are brief, then the different stimuli compete for access to the identity processor. Competition for the identity processor acts to impede it. Here, the unlimited capacity quantity processor may influence performance by processing some of the digits for identification. When this occurs, accuracy will decrease and the numerical distance effect will be present.

We note that in accepting the interactive model proposed in Figure 1*g* that this stands in contrast to the conclusions regarding processing drawn by Dehaene (1996). Dehaene (1996) reported performance in a number comparison task in which, on every trial, the participant made a speeded judgement as to whether the quantity conveyed by the stimulus was greater or less than "5." The stimulus was either a number name (e.g., FOUR) or an

Arabic digit. Dehaene (1996) discussed the work in the context of a variant of the triple code model based around a *sequential stage account of processing*. Stage 1 is labelled 'Identification,' Stage 2 is labelled 'Comparison' and Stage 3 is labelled 'Response.' In adopting additive factors logic (Sternberg, 1975), Dehaene argued that that the notation of the stimulus would tap into the identification component, the signified quantity of the stimulus would tap into the comparison component, and the response side would tap into the response component. Generally speaking, the results reveal patterns of additivity across these different variables suggesting that the different components are also additive and not interactive in their operations. Clearly this contrasts with our conclusion that stimulus identification and the accessing of semantics can operate in an interactive way.

There are several reasons why the Dehaene (1996) results do not directly contradict those reported here. First, the paradigms in the respective cases are very different. Here the multi-digit arrays were briefly presented and masked whereas in Dehaene's study a single stimulus was presented briefly but was unmasked. As discussed above, we suggest that the interactions between the identification stage and the quantity stage are only manifest when the identification stage is overwhelmed (as occurs here but not in the case reported by Dehaene). Second, Dehaene's sequential stage account assumed that the stimulus must be "identified" prior to accessing quantity. Without this assumption, the additive factors' logic is not valid. We believe that identification is not necessary for quantity to be accessed. Rather, the shape of the digit (or word) alone can activate stored quantity information (see Cohen & Quinlan, 2016). As such, Dehaene (1996) may not be an adequate test of independence between the identity and quantity processors. In this regards the patterns of additivity reported by Dehaene (1996) are tangential to the effects we report.

Limitations

Finally, we note some limitations of the present research. First, the predicted physical similarity effect and the numerical distance effect are inherently correlated (*r*~0.4). Because one cannot statistically disambiguate the shared variance, we analyzed the data consistent with the theoretical testing of models. That is, all of our models predict a physical similarity effect. Therefore, the critical feature for disambiguating the models is whether a numerical distance effect is present after removal of the physical similarity effect. Our analyses were directed at answering the following question, "Is numerical distance necessary to explain some of the data, or can it all be explained parsimoniously with the physical similarity effect alone?" It is, nonetheless, possible that future models may be proposed that lead to alternative accounts, but, at this stage, speculating about what these might be is not warranted or helpful.

Second, some of the statistically reliable effects of numerical distance reported here are small. For instance, the effect of numerical distance in the SUCC condition in Experiment 2 accounts for only 2% of the total variance. Indeed, in no case does the effect of numerical distance exceed 8% of the total variance. As in any such case, there is an issue over whether very small effects are of any psychological significance (see for instance, Prentice & Miller, 1992). We have chosen to take the effects as being of some theoretical import although others may choose not to be convinced of this. We do so because the very small effects reflect the influence of a key variable that has been taken by many to be the critical factor that determines performance in simple numerical tasks (see e.g., Dehaene & Akhavein, 1995).

Conclusions

We have made some progress in mapping out the basic number processing architecture. We claim that the parallel interactive account as shown in Figure 1*g* provides the current best account of performance across a range of simple numerical cognition tasks. Following encoding, separate quantity and identity processors concurrently operate to recover, respectively, a digit's stored quantity representation, and, to generate a structural code that supports the digit's conventional identity. The current data has been interpreted as showing that these processors operate in parallel in an interactive fashion. Whereas accessing quantity information operates rapidly and effortlessly, digit identification is a more effortful endeavor that reflects interpretive operations that depend on the visual decoding of digits. Whereas certain first-order quantity comparisons can be carried out with unlimited capacity, digit identification is capacity limited.

Open Practices Statement

The data for all experiments are deposited at UNC Dataverse. Neither of the experiments were pre-registered.

https://dataverse.unc.edu/dataverse/limitedCapacity2019

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Table 1

The mean d', SD, and n for each condition of Experiment 2.

Condition	Mean	SD	n
SIM	1.11	0.50	195
SUCC|-33	1.57	0.56	195
SUCC|0	1.50	0.61	195
SUCC|+33	1.57	0.65	195

Figure Captions

Figure 1. Schematic representations of the alternative accounts of the human number processing system considered in the text. PS and ND stand for the experimental variables *physical similarity* and *numerical distance*, respectively. A tick in the corresponding column signifies that the particular model predicts an effect of the variable whereas a cross signifies a case where the model makes no such prediction. The left-most column specifies whether or not the model predicts capacity limitations.

Figure 2: Schematic representation of the key trial events for the experimental conditions used in the experiments. Each row in the figure corresponds to a trial with time running from left to right. SIM and SUCC|0 were common to both experiments. All four conditions were tested in Experiment 2 only. The # indicates a mask as described in the text. Although in all the SUCC cases the left diagonal is shown as being presented before the right diagonal, the order of presentation of these diagonals was randomized.

Figure 3: Experiment 1's false alarm rates as a function of Physical Similarity (top) and Numerical Distance (bottom) for the SIM (left) and SUCC (right) conditions. These are the data used in the regression analysis, with each 4-digit display having a PS-ND value. For each PS-ND combination, we averaged false alarm rate over participants.

Figure 4: Experiment 2's false alarm rates as a function of Physical Similarity (top) and Numerical Distance (bottom) for the SIM (left) and combined SUCC (right) conditions. These are the data used in the regression analysis, with each 4-digit display having a PS-ND value. For each PS-ND combination, we averaged false alarm rate over participants.

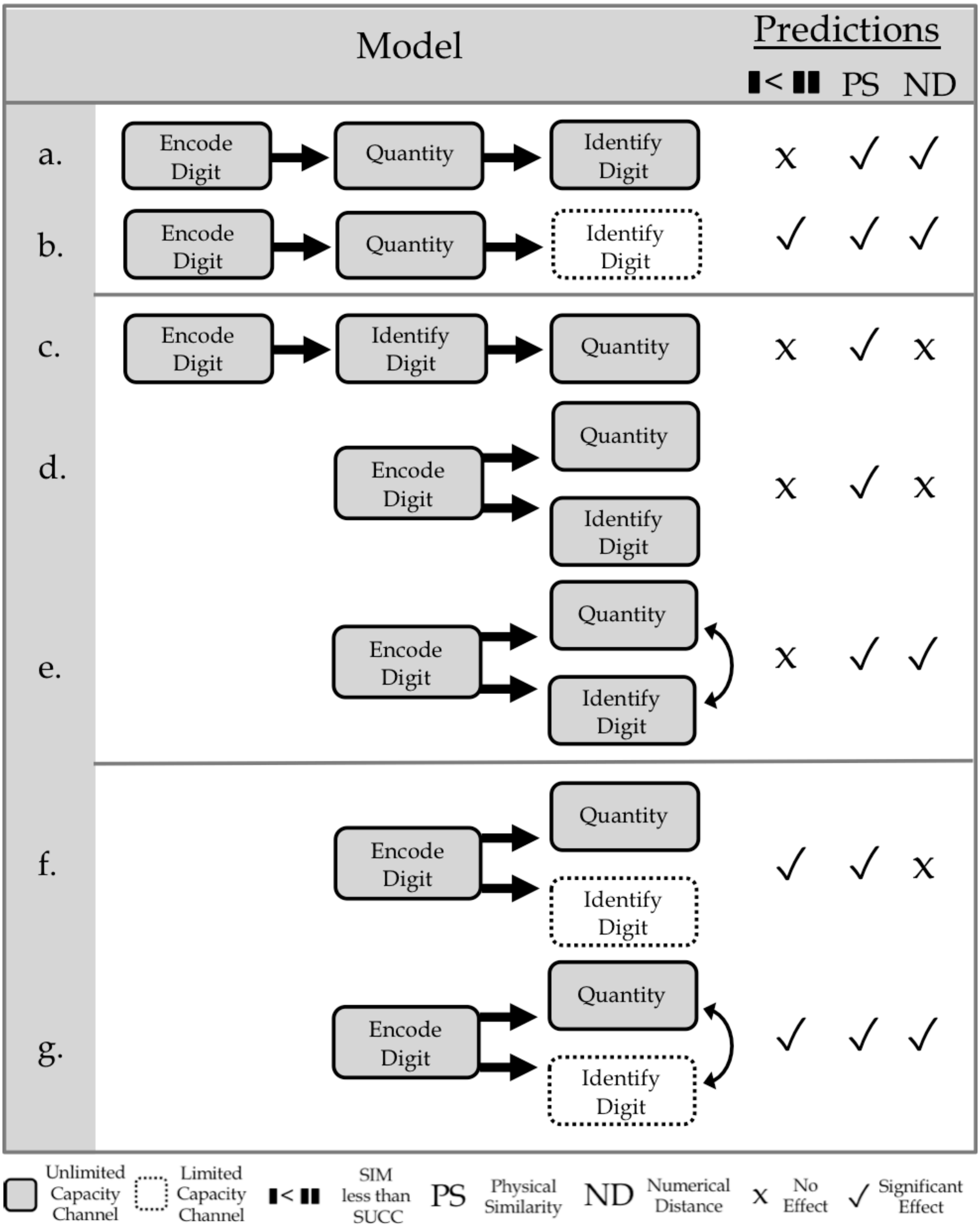
Model
Predictions
PS
ND
a.
Encode Digit
Quantity
Identify Digit
b.
c.
d.
e.
f.
g.
Unlimited Capacity Channel
Limited Capacity Channel
SIM less than SUCC
PS Physical Similarity
ND Numerical Distance
x No Effect
√ Significant Effect

Figure 1

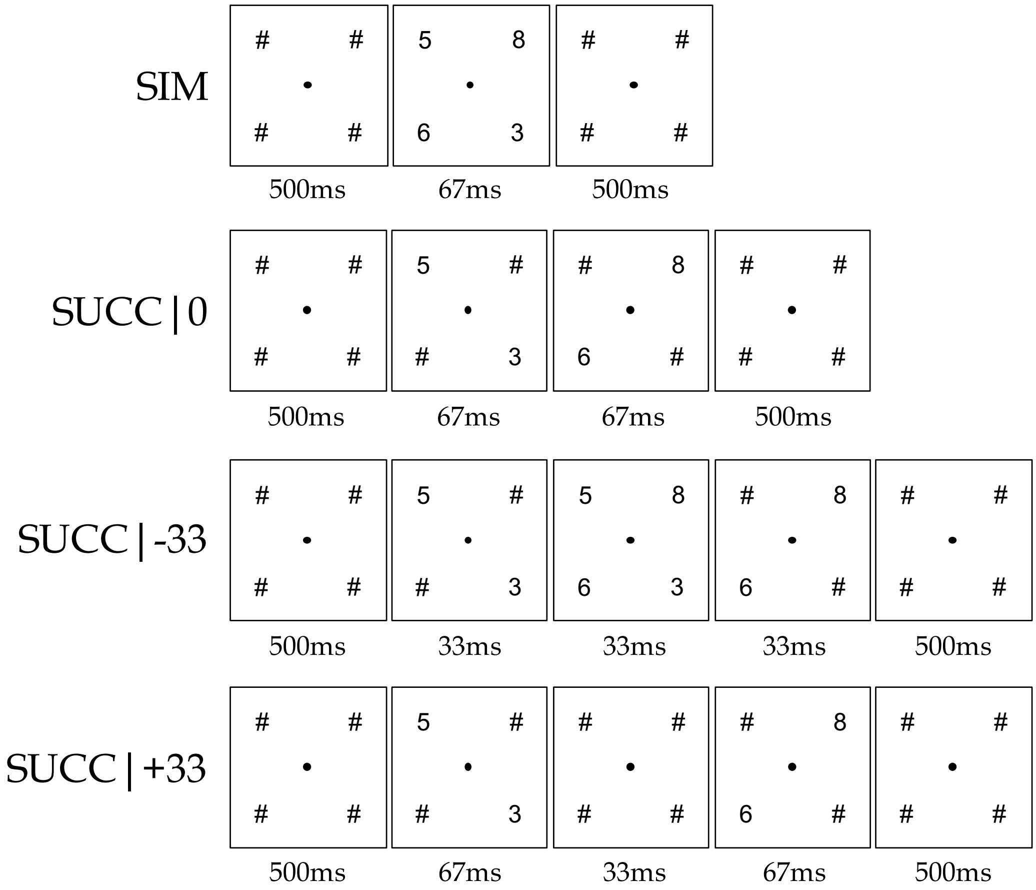
SIM
# # # #
5 8 6 3
# # # #
500ms
67ms
500ms
SUCC|0
# # # #
5 # # 3
8 6
# # # #
500ms
67ms
67ms
500ms
SUCC|-33
# # # #
5 # # 3
5 8 6 3
8 6
# # # #
500ms
33ms
33ms
33ms
500ms
SUCC|+33
# # # #
5 # # 3
# # # #
8 6
# # # #
500ms
67ms
33ms
67ms
500ms

Figure 2.

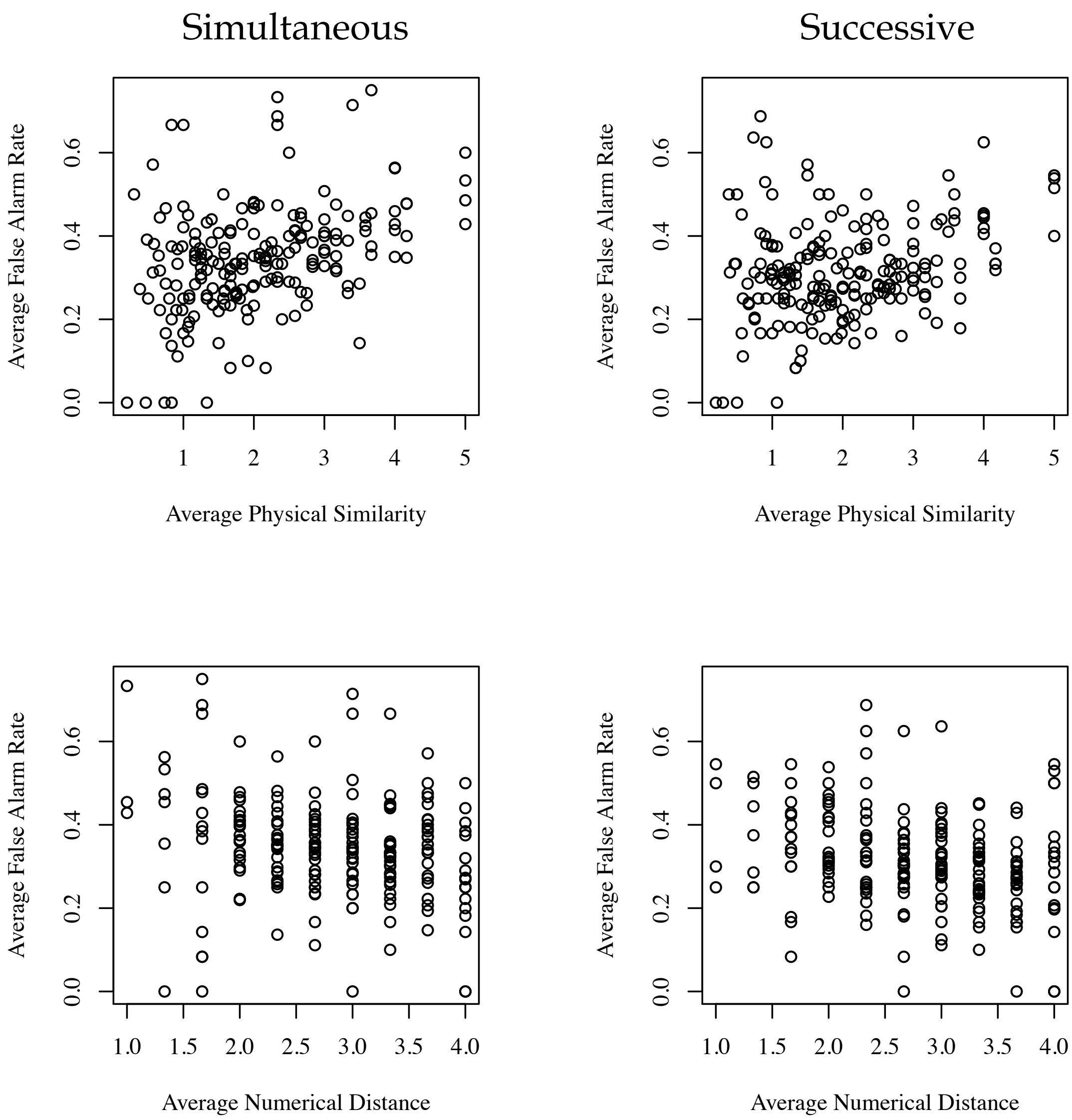
Simultaneous
Successive
Average False Alarm Rate
0.0
0.2
0.4
0.6
1
2
3
4
5
Average Physical Similarity
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Average Numerical Distance

Figure 3.

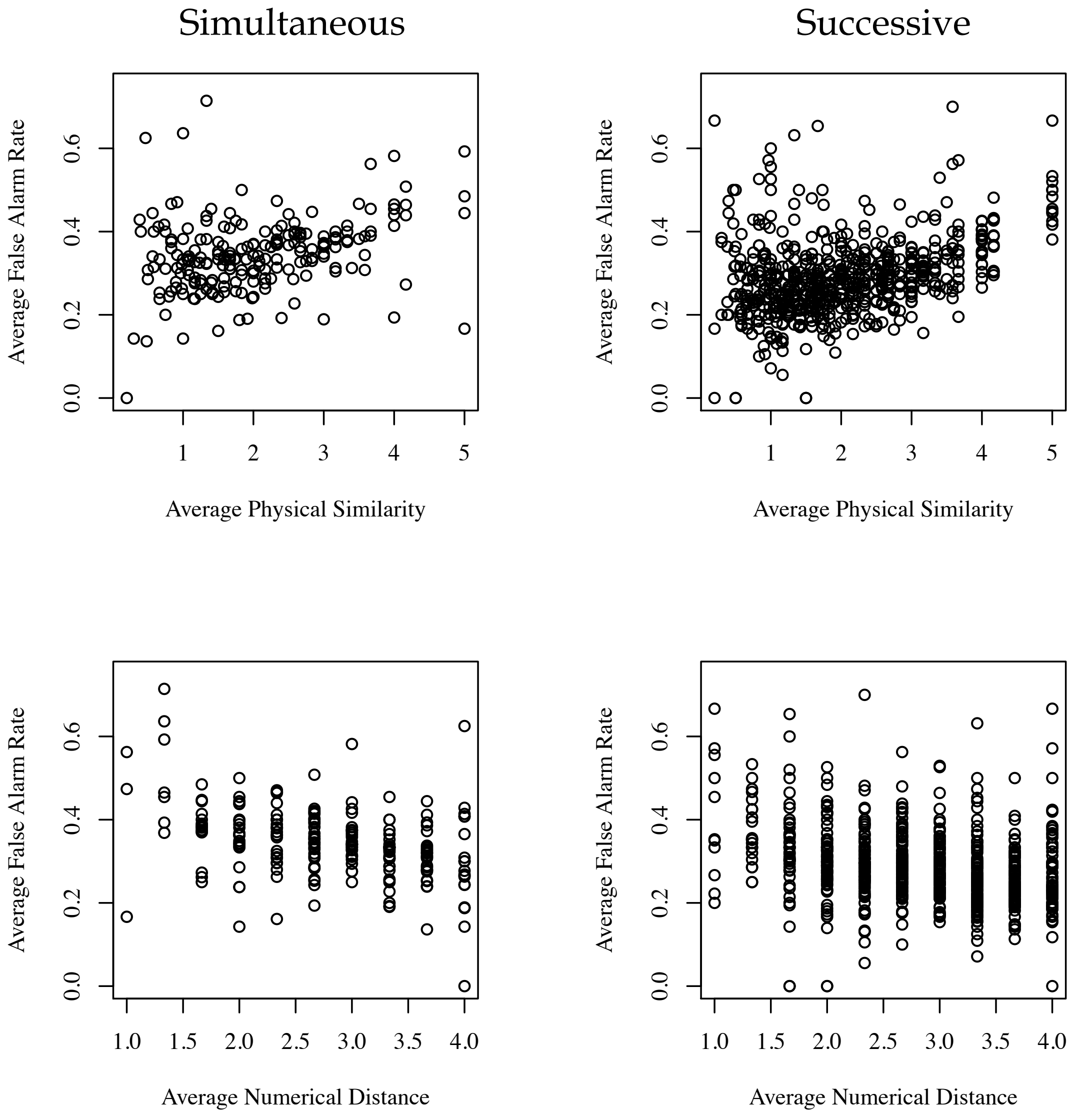
Simultaneous
Successive
Average False Alarm Rate
0.0
0.2
0.4
0.6
1
2
3
4
5
Average Physical Similarity
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Average Numerical Distance

Figure 4.

Footnotes

[1] We do not doubt that in certain contexts the recovery of a digit's name is important, such as in digit naming tasks (see e.g., Reynvoet, Brysbaert & Fias, 2002), but here we limit the discussion to cases that do not necessitate phonological processing. On these grounds, we limit discussion to putative mechanisms that support visual processing.

[2] In discussing the identity processor, we are limiting ourselves to discussion of visual processes that underpin the ability to link the visual structure of a digit with its visual identity, for example, the ability to identify the input digit for five as "5." We assume that there are analogical processes in the other senses so that, for instance, an utterance can be identified as the spoken form /fiv/. We are agnostic as to whether corresponding spoken form of a visual digit is automatically accessed when the visual digit is presented (though see Damian, 2004), but we do assume that that processing phonological forms plays no *obligatory* role in visual identification. The empirical consequences of this assumption however are not tested here. Nonetheless, we do test the statistical significance of a model of processing that takes into account the physical similarity of the input digits defined in visual terms.

[3] We are agnostic as to whether the identify processor completes tasks beyond recovering the corresponding stored identity of a given input digit (e.g., multidigit operations, parity, etc. claimed by Dehaene, 1992).

[4] The patterns of results remain the same with the no *d*' exclusion criteria.

[5] The physical structure of the numerical symbol must be encoded, at least in part, to provide meaningful information to the quantity process (see Cohen & Quinlan, 2016).

[6] The patterns of results remain the same with the no *d*' exclusion criteria.