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## **URBAN SPATIAL PATTERNS IN A MONOCENTRIC CITY WITH CAR-OWNING AND NO-CAR-OWNING RESIDENTS**

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### **ABSTRACT**

The existing monocentric city models mostly assume that residents are completely rational when making residential location and mode choice, and their direct utilities only depend on the consumption of two normal goods, a housing service and a composite non-housing good. This paper develops a new model by explicitly integrating the preference for solo-driving into car owners' direct utility functions. It is found that, an upward sloping rent-distance relationship exists nearby the Central Business District (CBD) when all residents are car owners, which is significantly different from the conclusion reported in literature. Considering the impact of car owners' preference for solo-driving, we differentiate two possible urban spatial patterns in a monocentric city with car-owning and no-car-owning residents. Numerical results show that an increase of car ownership does not always lead to an expansion in the city size.

Keywords: monocentric city model, urban spatial equilibrium, household residential choice, mode choice, car ownership

### **1. INTRODUCTION**

Most of countries in the world have experienced substantial increases in private car ownership over the recent decades. Taking China for example, according to a report stated by the Ministry of Public Security, China, 23.85 million new cars were registered in China, taking car ownership up to 172 million in 2015 ([http://news.xinhuanet.com/english/2016-01/25/c\\_135043964.htm](http://news.xinhuanet.com/english/2016-01/25/c_135043964.htm)). For every 100 households there are 31 private cars, the statement said, adding that in big cities such as Beijing and Shenzhen, the number may be up to 60. This rapid development on car ownership raises an important question: how does the urban spatial structure changes accordingly? The answer to this question may have significant implication in the sustainable urban development.

Classical monocentric city models adopt such assumptions that all residents in a monocentric city are homogeneous in all aspects and there is only one transport mode, i.e., car, to provide commuting service (Alonso, 1964; Muth, 1969; Mills, 1972). They found a downward sloping rent-distance relationship to achieve locational equilibrium due to the trade-off between accessibility and space. In reality, various transport modes, e.g., private car, bus and subway, compete with each other for serving residents who live in different places in a city. To our knowledge, some researchers investigated the joint choices of travel mode and residential location in a city model with user homogeneity or by dividing residents into different income groups (e.g., LeRoy and Sonstelie, 1983; Sasaki, 1989, 1990; Brueckner, 2005; Borck, and Wrede, 2008; Su and DeSalvo, 2008; Creutzig, 2014; Buyukeren and Hiramatsu, 2015; Xu et al., 2016). In these models, all residents are assumed to be completely rational and have similar consuming decisions, i.e., their direct utilities only depend on the consumption of two normal goods, a housing service and a composite non-housing good. However, car owners can behave completely differently from non-car owners in reality. Non-car owners have no options but to

commute by public transport. In contrast, car owners can choose to commute by either car or public transport (PT), but they usually have a higher preference for solo-driving. It can be predicted that integrating the preference for solo-driving into car owners' direct utility functions has important impacts on car owners' residential location choices and housing bid rents. Furthermore, The bi-rent competition between car-owners and non-car owners in the housing (land) market to a large extent determines the spatial structure of a city.

Considering a continuous space monocentric city where all jobs are located in the Central Business District (CBD), we develop an urban spatial equilibrium model for the integrated analysis of car-owning and no-car-owning residents' residential location and mode choice. This model contributes to the literature by explicitly integrating the preference for solo-driving into car owners' direct utility functions. Based on the developed model, we firstly analyze the case that all residents are car owners and find that incorporating the preference for solo-driving make an upward sloping rent-distance relationship nearby the CBD possible. Secondly, we differentiate two possible urban spatial patterns under car-owners and non-car owners' joint choice decisions of residential location and travel mode, and numerically examine the impact of car ownership on the urban spatial structure.

## 2. THE MODEL

In this section, we introduce two important components of the developed monocentric city model with car ownership heterogeneity, i.e., car owners and non-car owners' choice decisions, respectively. Non-car owners are assumed to only commute by public transport to the CBD and make residential location decisions on the consumption of two normal goods, a housing service and a composite non-housing good. In contrast, car owners are assumed to choose to commute by car or by public transport using a logit form. Furthermore, besides the consumption of two normal goods, car owners' direct utilities at location  $x$  (its distance to the CBD is  $x$ ) are also related to the preference for solo-driving, i.e., the probability of choosing car mode at location  $x$ , which is computed according to the relative difference between the actual daily commute costs of using the two modes.

### 2.1 Car owner's choice decisions

#### 2.1.1 Logit-based mode choice

Suppose that there are two modes of transportation providing commuting service from the city boundary to the CBD, car or public transport mode. The actual daily commute cost of using the two modes at location  $x$  are both congestion-free and increase with commute distance  $x$ , given as

$$C_i(x) = 2(f_i + t_i x), \quad (1)$$

where  $i = 1, 2$  represent the car and public transport modes, respectively;  $f_i$  is the one-way fixed cost of using mode  $i$ , which includes all one-way travel costs independent of commute distance;  $t_i$  is the marginal (mainly time) cost of commute per unit distance, so  $t_i x$  is the one-way variable cost of using mode  $i$ , which includes all costs that vary with the commute distance. Compared with solo drivers, PT users generally have the smaller fixed cost and the larger marginal variable cost for completing a trip. Hence, we assume  $f_1 > f_2$  and  $t_1 < t_2$  hold here for simplicity.

Car owners are assumed to make mode-choice decisions in terms of the perceived daily commute costs of using the two modes, which is characterized as follows:

$$PC_i(x) = C_i(x) + \varepsilon_i(x), \quad (2)$$

where  $C_i(x)$  is defined in Eq. (1);  $\varepsilon_i(x)$  is the random error term. Suppose that all random terms  $\varepsilon_i(x)$  are identically and independently distributed (IID) Gumbel variables with mean zero and

variance  $\sigma$ , then the probability of choosing car mode at location  $x$  can be governed by the following logit formulation:

$$P(x) = \left(1 + \exp(\theta(C_1(x) - C_2(x)))\right)^{-1}, \quad (3)$$

where  $\theta = \pi/\sqrt{6\sigma}$ . Differentiating Eq. (3) with respect to  $x$  produces

$$\frac{dP(x)}{dx} = 2\theta \exp(\theta(C_1(x) - C_2(x))) (t_2 - t_1) \left( \sum_{j=1,2} \exp(-\theta C_j(x)) \right)^{-2} > 0. \quad (4)$$

Since  $t_1 < t_2$  according to the assumption, Eq. (4) shows that the probability of choosing car mode increases with the commute distance.

Accordingly, the expected daily commute cost for car owners located at  $x$  is (Oppenheim, 1995)

$$C_c(x) = -\frac{1}{\theta} \ln \left( \sum_{j=1,2} \exp(-\theta C_j(x)) \right). \quad (5)$$

Differentiating Eq. (5) with respect to  $x$ , getting

$$\frac{dC_c(x)}{dx} = \frac{2 \exp(-\theta C_1(x)) t_1 + 2 \exp(-\theta C_2(x)) t_2}{\sum_{j=1,2} \exp(-\theta C_j(x))} > 0. \quad (6)$$

Eq. (6) implies that the expected daily commute cost for car owners increases with the commute distance.

### 2.1.2 Household consumption and residential location choice

For a car owner at location  $x$ , her optimal decision is to resolve the following direct utility maximization problem under her budget constraint:

$$U_1(x) = \max_{z_1(x), q_1(x), P(x)} V_1(z_1(x), q_1(x), P(x)), \quad (7)$$

subject to the budget constraint

$$z_1(x) + R_1(x)q_1(x) = Y_c - \varphi C_c(x) - F. \quad (8)$$

Here,  $V_1(z_1(x), q_1(x), P(x))$  is car owners' direct utility function, where  $z_1(x)$  is the location-dependent consumption of a composite non-housing good,  $q_1(x)$  is the location-dependent consumption of housing (also called the lot size), and  $P(x)$  is the location-dependent probability of choosing car mode as defined before, representing the preference for solo-driving;  $U_1(x)$  is car owners' location-dependent indirect utility function;  $R_1(x)$  is the location-dependent housing rental price per unit and the price of non-housing good is taken to be unity for simplicity;  $C_c(x)$  is the expected daily commute cost defined in Eq. (5);  $\varphi$  and  $Y_c$ , respectively, are the frequency of commuting and the wage revenue of car owners in a consumption period; and  $F$  is the fixed cost of purchasing a car allocated to each consumption period. For convenience of further analysis, the following Cobb–Douglas form of direct utility function is adopted in this paper:

$$V_1(z_1(x), q_1(x), P(x)) = z_1(x)^\alpha q_1(x)^\beta P(x), \quad \alpha, \beta > 0, \quad \alpha + \beta = 1. \quad (9)$$

By solving the budget constraint in  $z_1(x)$  and substituting it into Eq. (9), the first order condition of Eq. (9) with respect to  $q_1(x)$  gives a unique demand for the lot size, which is implicitly defined as

$$-\frac{\partial V}{\partial z_1} R_1(x) + \frac{\partial V}{\partial q_1} = 0. \quad (10)$$

Then we obtain

$$q_1(x) = \frac{\beta(Y_c - \varphi C_c(x) - F)}{R_1(x)}, U_1(x) = [Y_c - \varphi C_c(x) - F - R_1(x)q_1(x)]^\alpha q_1(x)^\beta P(x). \quad (11)$$

All car owners are assumed to be homogenous, thus the urban spatial equilibrium must yield an identical utility level for all car owners, denoted as  $u_1$ . So, we have  $U_1(x) = u_1$  for all  $x$ . Combining this with Eq. (11), we derive the  $R_1(\cdot)$  and  $q_1(\cdot)$ , which are functions of utility level  $u_1$ , as follows:

$$R_1(x, u_1) = \alpha^{\frac{\alpha}{\beta}} \beta (Y_c - \varphi C_c(x) - F)^{\frac{1}{\beta}} u_1^{-\frac{1}{\beta}} P(x)^{\frac{1}{\beta}}, \quad (12)$$

$$q_1(x, u_1) = \alpha^{-\frac{\alpha}{\beta}} (Y_c - \varphi C_c(x) - F)^{-\frac{\alpha}{\beta}} u_1^{\frac{1}{\beta}} P(x)^{-\frac{1}{\beta}}. \quad (13)$$

It can easily be observed from Eqs. (12) and (13) that, the housing rental price  $R_1(x, u_1)$  will decrease with the expected daily commuting cost and increase with the probability of choosing car mode if utility level  $u_1$  is fixed. But, the lot size  $q_1(x, u_1)$  has the opposite property.

Differentiating Eq. (12) with respect to  $x$ , getting

$$\frac{dR_1}{dx} = \alpha^{\frac{\alpha}{\beta}} \beta u_1^{-\frac{1}{\beta}} (Y_c - \varphi C_c(x) - F)^{\frac{\alpha}{\beta}} P(x)^{\frac{\alpha}{\beta}} \left( -\varphi P(x) \frac{dC_c}{dx} + (Y_c - \varphi C_c(x) - F) \frac{dP}{dx} \right). \quad (14)$$

Then, we have

$$\frac{dR_1}{dx} \begin{matrix} > \\ = \\ < \end{matrix} 0 \text{ if } (Y_c - \varphi C_c(x) - F) \frac{dP}{dx} \begin{matrix} > \\ = \\ < \end{matrix} \varphi P(x) \frac{dC_c}{dx}. \quad (15)$$

Hence, for the occurrence of an upward sloping rent-distance relationship, the change of the probability of choosing car mode must dominate the change of the expected daily commuting cost as the commute distance increases. If the latter dominates, the housing rental price will decrease with the commute distance.

## 2.2 Non-car owner's choice decisions

For a non-car owner, she chooses the consumption of a composite non-housing good and a housing service to maximize his/her utility under her budget constraint. The utility maximization problem for non-car owners can be written as (Li et al., 2013; Gubins and Verhoef, 2014; Xu et al., 2016)

$$U_2(x) = \max_{z_2(x), q_2(x)} V_2(z_2(x), q_2(x)) = z_2(x)^\alpha q_2(x)^\beta, \alpha, \beta > 0, \alpha + \beta = 1, \quad (16)$$

subject to the budget constraint

$$z_2(x) + R_2(x)q_2(x) = Y_{nc} - \varphi C_{nc}(x). \quad (17)$$

Here,  $V_2(z_2(x), q_2(x))$  is non-car owners' direct utility function, where  $z_2(x)$  is the location-dependent consumption of a composite non-housing good and  $q_2(x)$  is the location-dependent consumption of housing;  $U_2(x)$  is non-car owners' location-dependent indirect utility function;  $R_2(x)$  is the location-dependent housing rental price per unit and the price of non-housing good is taken to be unity for simplicity;  $C_{nc}(x)$  is non-car owners' daily commute cost by PT; and  $Y_{nc}$  is the wage revenue of car owners in a consumption period.

All non-car owners are assumed to be homogenous, thus the urban spatial equilibrium must yield an identical utility level for all non-car owners, denoted as  $u_2$ . So, we have  $U_2(x) = u_2$  for all  $x$ . At equilibrium, we derive the  $R_2(\cdot)$  and  $q_2(\cdot)$ , which are functions of utility level  $u_2$ , as follows:

$$R_2(x, u_2) = \alpha^{\frac{\alpha}{\beta}} \beta (Y_{nc} - \varphi C_{nc}(x))^{\frac{1}{\beta}} u_2^{-\frac{1}{\beta}}, \quad (18)$$

$$q_2(x, u_2) = \alpha^{-\frac{\alpha}{\beta}} (Y_{nc} - \phi C_{nc}(x))^{-\frac{\alpha}{\beta}} u_2^{\frac{1}{\beta}}. \quad (19)$$

Obviously, under a given level of utility, the housing rental price decreases and the lot size per household increases with the distance from the CBD.

### 3. POSSIBLE URBAN SPATIAL PATTERNS

On the basis of all residents' choice decisions, the equilibrium land rent curve coincides with the upper envelope of the bid-rent curves associated with car owners and non-car owners. Thus, the equilibrium rent  $R(x, u_1, u_2)$  can be expressed as

$$R(x, u_1, u_2) = \max \{R_1(x, u_1), R_2(x, u_2)\}. \quad (20)$$

At equilibrium, there might be various urban spatial patterns with respect to the city system parameters. Hereafter, we only consider two possible patterns, where at least part of non-car owners live near the CBD.

**Case (I).** All non-car owners live near the CBD, while all car owners live farther. The urban structure of this pattern is depicted in Fig.1(I). The spatial equilibrium conditions are given as

$$R_1(x_0, u_1) = R_2(x_0, u_2), \quad R_1(B, u_1) = R_a, \quad (21)$$

$$\int_0^{x_0} 1/q_2(x, u_2) dx = N_{nc}, \quad \int_{x_0}^B 1/q_1(x, u_1) dx = N_c, \quad (22)$$

where  $B$  is the endogenous city boundary,  $R_a$  is the exogenously fixed agricultural rent,  $x_0$  is the residential separation point between car owners and non-car owners in the city, and  $N_c$  and  $N_{nc}$  are the number of car owners and non-car owners, respectively. Eq. (21) states that the bid-rents of the two groups are identical at the residential separation point, and the bid-rent of car owners who live in the city boundary is equal to the agricultural rent. Eq. (22) is the population conservation condition, which shows all non-car owners living between the CBD and location  $x_0$  and all car owners reside between location  $x_0$  and the city boundary.

**Case (II).** Part of non-car owners live near the CBD and some non-car owners live close to the city boundary, while all car owners live in the middle region. This urban spatial structure is depicted in Fig.1 (II). The spatial equilibrium conditions are given as

$$R_1(x_1, u_1) = R_2(x_1, u_2), \quad R_1(x_2, u_1) = R_2(x_2, u_2), \quad R_2(B, u_2) = R_a, \quad (23)$$

$$\int_0^{x_1} 1/q_2(x, u_2) dx + \int_{x_2}^B 1/q_2(x, u_2) dx = N_{nc}, \quad \int_{x_1}^{x_2} 1/q_1(x, u_1) dx = N_c, \quad (24)$$

where  $x_1$  and  $x_2$  are the residential separation points between car owners and non-car owners in the city. Eq. (23) states the Land-market equilibrium condition and Eq. (24) is the population conservation condition.

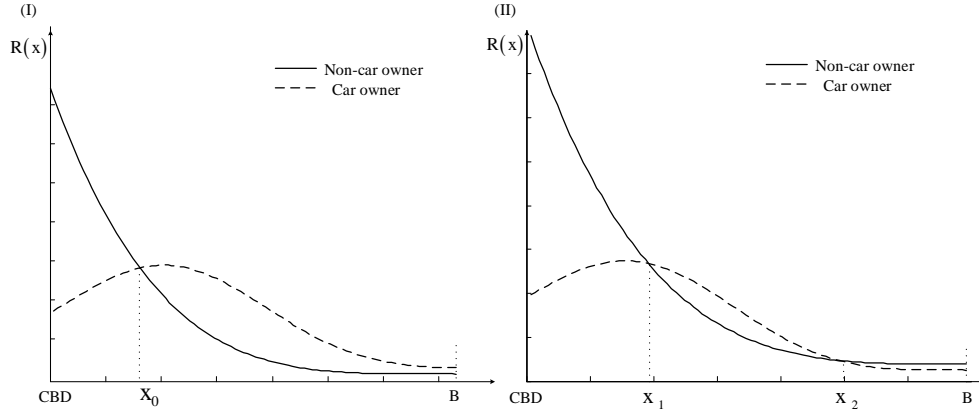


Fig 1. Urban spatial structures

#### 4. NUMERICAL EXAMPLES

To facilitate the presentation of the essential idea and contributions of this paper, in this section we employ some numerical examples to illustrate the application of the proposed model. The first scenario aims to show the effect of the driving preference on the urban spatial structure when all residents are car owners. The second scenario is used to explore the effect of the bid-rent competition between car owners and non-car owners on the urban spatial structure. The values of parameters associated with the two groups are summarized in Table 1.

Table 1. Summary of the values of parameters

	Car owners	Non-car owners
Number of population	$N_c = 8000 / 8500$	$N_{nc} = 22000 / 21500$
Annual income (RMB)	$Y_c = 8 \times 10^4$	$Y_{nc} = 6 \times 10^4$
Parameters in utility function	$\alpha = 0.75$ and $\beta = 0.25$	
Agricultural rent (RMB/km <sup>2</sup> )	$R_a = 3.0 \times 10^5$ (Section 4.1 and Section 4.2 case I) $R_a = 3.0 \times 10^3$ (Section 4.2 case II)	
Daily commuting cost (RMB)	By car: $C_1(x) = 2 \times (10 + 24/90x)$ By PT: $C_2(x) = 2 \times (6 + 24/70x)$	$C_{nc}(x) = 2 \times (6 + 18/70x)$
Fixed cost of purchasing a car (RMB)	$F = 25000$	
Parameter in Logit formulation	$\theta = 0.05$	
Frequency of commuting	$\varphi = 300$	

##### 4.1 The case that all residents are car owners

Firstly, we look at the case that all residents are car owners to explore the effect of driving preference. Fig. 2 (a) and (b), respectively, show equilibrium patterns of rents and housing land consumption over the distance from the CBD. It is found that the land rent first increases and then decreases with distance from the CBD, which is different from the downward sloping rent function obtained in most of the existing monocentric city models. Furthermore, the consumption of housing land first decreases and then increases over space. In our setting, car owners' preference for solo-driving makes an upward sloping rent-distance relationship nearby the CBD possible.

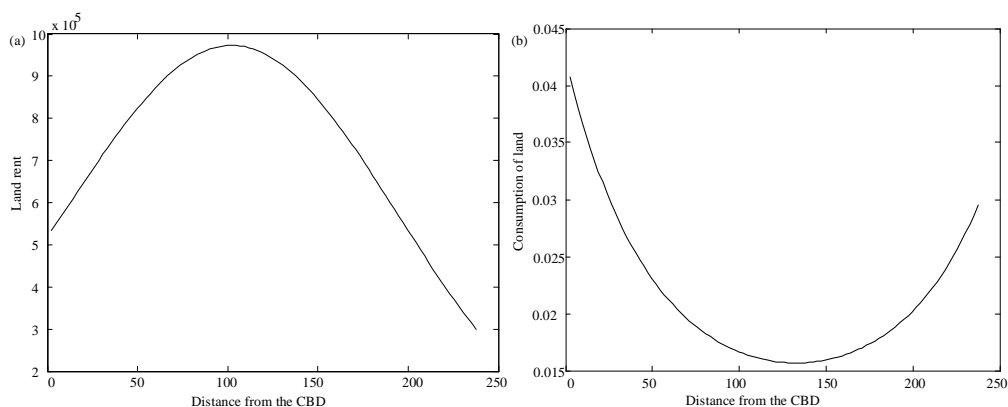


Fig 2. Rents and consumption of land by residential location

## 4.2 The cases with car-owning and no-car-owning residents

Next, we present two specific cases of a city system with car-owning and no-car-owning residents for further insights, which correspond to different urban spatial patterns analyzed in Section 3, respectively.

### (1) Case I

Fig. 3 shows the spatial patterns of rents and land consumption after the car ownership increases (with thicker line), in the case where all non-car owners live near the CBD, while all car owners live farther. The most striking result is that an increase of the car ownership generates an increased demand for housing land, i.e., the city boundary moves from 295.84 km to 296.11 km. It is also found that the residential separation point between the population decreases and the residential area for car owners expands. However, due to the competition between car owners, their bid rents increase, the consumption of housing land decreases, and the utility level of car owners reduces from 378.42 to 377.09. For non-car owners, the decrease of land rent and the increase of housing land consumption lead to the improvement of utility level from 705.36 to 706.72.

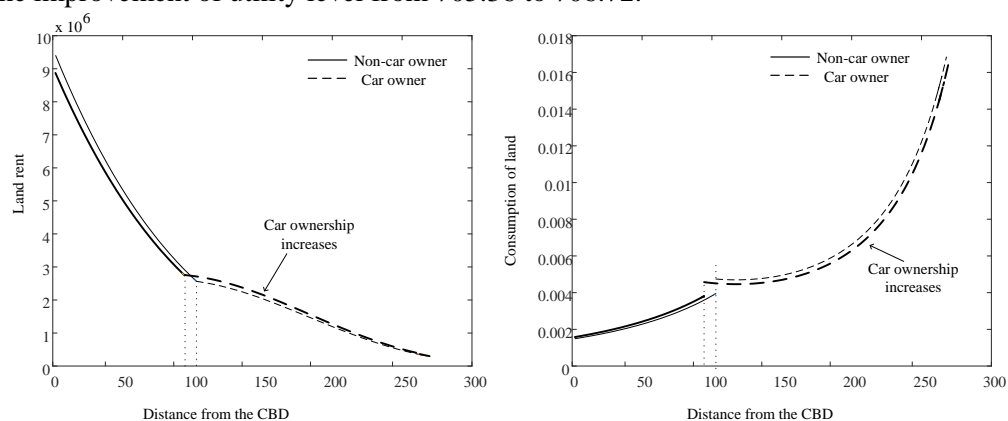


Fig 3. Increase of the car ownership in Case (I)

### (2) Case II

In this case, the agricultural land rent is  $R_a = 3.0 \times 10^3$  RMB/km<sup>2</sup>. Due to the low land rent near the city boundary, some non-car owners will choose to reside in this area. Fig. 4 displays the change of the housing land rent and consumption with the increase of the car ownership. It can be observed that the city boundary decreases from 306.04 km to 305.92 km, which is different from the results in Case I. Furthermore, the utility level of non-car owners increases from 707.10 to 708.44, while the utility level of car owners decreases from 376.43 to 375.26.



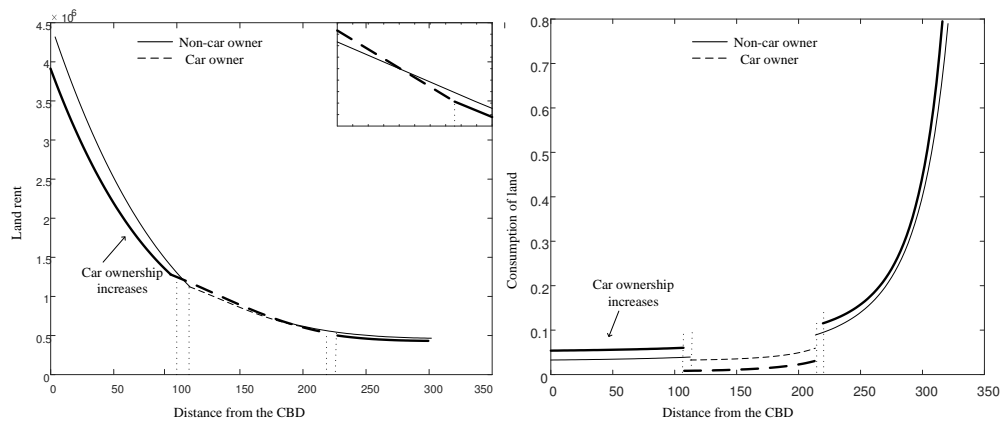


Fig 4. Increase of car ownership in Case (II)

## 5. CONCLUSIONS AND FURTHER STUDIES

This paper investigated the effects of car owners' preference for solo-driving and car ownership changes on the urban spatial structure. For car owners, they can choose to commute by either car or public transport using a logit form, and the probability of choosing car mode influences their direct utility levels. The developed monocentric city model with car-owning and no-car-owning residents provides some important findings and new insights for urban economics and transportation research. Firstly, when all residents are car owners, an upward sloping rent-distance relationship becomes possible. Second, as far as the two possible urban spatial patterns explored are concerned, the increase of car ownership not always lead to the urban expansion. It may produce a contraction in the city size if some non-car owners live close to the city boundary.

Although the findings and insights might be valuable, there are some limitations of the study on the in-depth interactions between transportation systems and urban systems. Both the car and public transit modes are assumed to be congestion-free for simplicity. This assumption is widely adopted in the urban economics literature, but it is to a large extent unrealistic in most of cities around the world. Accordingly, it is necessary to endogenize residents' daily commuting costs by taking traffic congestion or crowding effects into account. Furthermore, a monocentric urban structure is assumed in this paper. It is meaningful to extend to investigate a ploycentric urban structure.

## 6. ACKNOWLEDGEMENTS

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