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# Optimal Combining and Performance Analysis for Two-Way EH Relay Systems with TDBC Protocol 

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#### Abstract

In this paper, we investigate a simultaneous wireless information and power transfer (SWIPT) based two-way decode-and-forward (DF) relay network, where time switching (TS) is employed for SWIPT and time division broadcast (TDBC) is employed for two-way relaying. We focus on the design of a combining scheme that decides how the relay combines the signals received from two terminals through a power allocation ratio at the relay. We formulate an optimization problem to minimize the system outage probability and obtain the optimal power allocation ratio in closed form. For the proposed optimal combining scheme, we derive the expression for the system outage probability. Simulation results verify our derived expressions and show that the proposed scheme achieves a lower system outage probability than the existing schemes.


Index Terms-Simultaneous wireless information and power transfer, two-way decode-and-forward relay, optimal combining, system outage probability.

## I. Introduction

0WING to its high spectrum efficiency, two-way relaying has been deemed an integral part of Internet of Things [1]. It allows two terminals to exchange their information through an intermediate relay using either the multiple access broadcast (MABC) or time division broadcast (TDBC) protocol. However, in an energy-constrained wireless sensor network, the intermediate relay is likely to have a limited battery capacity and would be unwilling to assist the terminals [1]. To solve this problem, simultaneous wireless information and power transfer (SWIPT) based two-way relaying, where the intermediate relay splits or switches the received radio frequency (RF) signal in the power or time domain through powersplitting (PS) or time-switching (TS), has been proposed. SWIPT based two-way relaying with MABC has been widely studied (see [2]-[4] and the references therein). For example, the authors in [2] investigated the system outage probability of an analog network coding based two-way amplify-and-forward (AF) relay system with multiple-antenna source terminals and single-antenna energy harvesting (EH) relay. Authors in [4] proposed an energy efficient precoding design for SWIPT enabled MIMO two-way AF relay networks.

Since the TDBC protocol can utilize the direct link between the terminals even when they operate in a half-duplex mode and the operational complexity of TDBC is lower than that of MABC, the study of TDBC in SWIPT based two-way relaying has recently received a lot of attention [5]-[12]. In [5], the

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authors studied the achievable throughput of SWIPT based additive AF relaying with TBDC under three wireless power transfer policies. For SWIPT based multiplicative AF relaying with TBDC, optimal symmetric PS [6] and asymmetric PS [7] schemes were proposed to minimize the system outage probability. The authors in [8] and [9] investigated the outage probability of PS SWIPT based two-way decode-and-forward (DF) relaying with TDBC under both linear and non-linear energy harvesting models. The authors in [10] studied the outage performance of a decode-amplify-forward protocol in TS SWIPT based two-way relaying. The secrecy performance, e.g., the intercept probability, of PS SWIPT based two-way DF relaying with TDBC was investigated in [11]. The combination of PS SWIPT based two-way DF relaying with TDBC and cognitive radio was studied in [12] with a focus on the outage probabilities of the primary user and the secondary user. It has been shown that for TDBC, the combining scheme that combines the two received signals at the relay can significantly improve the outage performance [13]. However, the optimization of combining scheme has not been studied for SWIPT based two-way relaying with TDBC.

In this paper, we propose an optimal combining scheme for a TS SWIPT based two-way DF $^{1}$ relay network, where TDBC is employed. Our contributions are summarized as follows. Firstly, we propose an optimal combining scheme, where the power allocation ratio at the relay is adjustable. Specifically, we formulate an optimization problem to minimize the system outage probability and derive the optimal power allocation ratio in closed form. Secondly, with the optimal combining scheme, we derive the expression of the system outage probability considering the EH circuit sensitivity. Simulation results are presented to verify the accuracy of the derived expressions and demonstrate the superiority of the proposed scheme in terms of system outage performance.

## II. System model

We consider a TDBC based two-way DF relay network in the presence of a direct link between the terminal node A and the terminal node B , where the "harvest-then-forward" strategy is adopted to incentivize the energy-constrained relay R to help with the information transmission between node A and B. All participating nodes are equipped with a single antenna ${ }^{2}$ and operate in the half-duplex mode. All channels

[^0]are assumed to be reciprocal and undergo independent and identically distributed (i.i.d.) Rayleigh fading [2], [3], [5], [8]. Let $h_{i}(i \in\{\mathrm{~A}, \mathrm{~B}\})$ denote the fading coefficient between $i$ and R , where $\left|h_{i}\right|^{2} \sim \exp \left(\frac{1}{\lambda_{i}}\right)$. The path loss of the link between $i$ and R is given by $d_{i}^{-\alpha_{s}}$, where $d_{i}$ and $\alpha_{s}$ are the distance and the path loss exponent of the $i-\mathrm{R}$ link, respectively. For the direct link between A and B, the channel model is denoted by $|g|^{2} d_{t}^{-\alpha_{t}}$, where $g$ is the fading coefficient, $d_{t}$ is the distance between A and B , and $\alpha_{t}$ is the corresponding path loss exponent.

Relay R adopts TS SWIPT, where each transmission block $T$ is divided into four time slots. During the first time slot of duration $\beta T, \beta \in(0,1)$, the relay harvests energy from the RF signals transmitted by both A and B. The total harvested energy is given by

$$
E_{\mathrm{total}}= \begin{cases}0, & P_{\mathrm{in}}<P_{\mathrm{th}}  \tag{1}\\ \beta T \eta P_{\mathrm{in}}, & P_{\mathrm{in}} \geq P_{\mathrm{th}}\end{cases}
$$

where $P_{\text {in }}=P\left(\left|h_{\mathrm{A}}\right|^{2} d_{\mathrm{A}}^{-\alpha_{s}}+\left|h_{\mathrm{B}}\right|^{2} d_{\mathrm{B}}^{-\alpha_{s}}\right)$ is the received RF power at the relay; $P$ is the transmit power used by A and $\mathrm{B} ; P_{\mathrm{th}}$ is the circuit sensitivity of the energy harvester; and $\eta \in(0,1]$ is the energy conversion efficiency of the energy harvester.

In the second (third) time slot of duration $\frac{1-\beta}{3} T, \mathrm{~A}(\mathrm{~B})$ transmits its signal $s_{\mathrm{A}}\left(s_{\mathrm{B}}\right)$. The received signals from $i(i \in$ $\{\mathrm{A}, \mathrm{B}\})$ at relay R and terminal $\bar{i}$ are given respectively by

$$
\begin{equation*}
y_{i \mathrm{R}}=h_{i} \sqrt{P d_{i}^{-\alpha_{s}}} s_{i}+n_{\mathrm{R}}, \quad y_{i \bar{i}}=g \sqrt{P d_{t}^{-\alpha_{t}}} s_{i}+n_{\bar{i}} \tag{2}
\end{equation*}
$$

where $\bar{i}=\left\{\begin{array}{l}\mathrm{A}, \text { if } i=\mathrm{B} \\ \mathrm{B}, \text { if } i=\mathrm{A}\end{array} ; \mathbb{E}\left\{\left|s_{i}\right|^{2}\right\}=1 ; n_{\mathrm{R}} \sim \mathcal{C N}\left(0, \sigma_{\mathrm{R}}^{2}\right)\right.$ and $n_{\bar{i}} \sim \mathcal{C N}\left(0, \sigma_{\bar{i}}^{2}\right)$ are the additive white Gaussian noise (AWGN) at R and $\bar{i}$, respectively. Thus, the received SNR at R and $\bar{i}$ are given by

$$
\begin{equation*}
\gamma_{i \mathrm{R}}=P\left|h_{i}\right|^{2} d_{i}^{-\alpha_{s}} / \sigma_{\mathrm{R}}^{2}, \quad \gamma_{i \bar{i}}=P|g|^{2} d_{t}^{-\alpha_{t}} / \sigma_{\bar{i}}^{2} \tag{3}
\end{equation*}
$$

In the fourth time slot of duration $\frac{1-\beta}{3} T, \mathrm{R}$ will combine the decoded signals $\widetilde{s}_{\mathrm{A}}$ and $\widetilde{s}_{\mathrm{B}}$ with a power allocation ratio $\theta \in$ $(0,1)$ as $s_{\mathrm{R}}=\frac{\theta \widetilde{s}_{\mathrm{A}}+(1-\theta) \widetilde{s}_{\mathrm{B}}}{\sqrt{\theta^{2}+(1-\theta)^{2}}}$ and broadcast $s_{\mathrm{R}}$ to both A and B with the harvested energy $E_{\text {total }}$. Note that the value of $\theta$ decides how relay R combines the signals received from A and B. Then the received signal at $i$ is $y_{\mathrm{R} i}=h_{i} \sqrt{P_{\mathrm{R}} d_{i}^{-\alpha_{s}}} s_{\mathrm{R}}+n_{i}$, where $P_{\mathrm{R}}=\frac{3 E_{\text {total }}}{(1-\beta) T}$ is the transmit power at R .

For analytical simplicity, we assume $\sigma_{\mathrm{R}}^{2}=\sigma_{\mathrm{A}}^{2}=\sigma_{\mathrm{B}}^{2}=\sigma^{2}$ [8]. After using successive interference cancellation (SIC) ${ }^{3}$ at node $i$, the received SNR of the R-i link is given by

$$
\gamma_{\mathrm{R} i}=\left\{\begin{array}{cc}
0, & P_{\mathrm{in}}<P_{\mathrm{th}}  \tag{4}\\
X_{i}\left(\left|h_{\mathrm{A}}\right|^{2} d_{\mathrm{A}}^{-\alpha_{s}}+\left|h_{\mathrm{B}}\right|^{2} d_{\mathrm{B}}^{-\alpha_{s}}\right), & P_{\mathrm{in}} \geq P_{\mathrm{th}}
\end{array}\right.
$$

[^1]where $X_{i}=\left\{\begin{array}{c}\frac{3 \rho \beta \eta(1-\theta)^{2}\left|h_{\mathrm{A}}\right|^{2} d_{\mathrm{A}}^{-\alpha_{s}}}{(1-\beta)\left[\theta^{2}+(1-\theta)^{2}\right]}, \text { if } i=\mathrm{A} \\ \frac{3 \rho \beta \eta \theta^{2}\left|h_{\mathrm{B}}\right|^{2} d_{\mathrm{B}}^{-\alpha_{s}}}{(1-\beta)\left[\theta^{2}+(1-\theta)^{2}\right]}, \text { if } i=\mathrm{B}\end{array} \quad\right.$ and $\rho=\frac{P}{\sigma^{2}}$ is the transmit SNR. By implementing the maximal ratio combining (MRC) of the signals received from nodes $\bar{i}$ and R , the final SNR at node $i(i, \bar{i} \in\{\mathrm{~A}, \mathrm{~B}\})$ is given by

$$
\begin{equation*}
\gamma_{i}=\gamma_{\mathrm{R} i}+\gamma_{\bar{i} i} \cdot \mathbf{1}_{\gamma_{\bar{i} \mathrm{R}} \geq \gamma_{\mathrm{th}}} \tag{5}
\end{equation*}
$$

where $\gamma_{\text {th }}$ is the predefined SNR threshold for both the relay-to-node links and the direct link and $\mathbf{1}_{D}$ is the indicator function that equals 1 only when $D$ is true and 0 otherwise.

## III. Optimal Combining and Performance Analysis

## A. Optimal Combining Scheme

Let $P_{\text {out }}^{s}$ denote the system outage probability for the considered TS SWIPT based two-way DF relay network with TDBC. Then $P_{\text {out }}^{s}$ can be expressed as

$$
\begin{equation*}
P_{\mathrm{out}}^{s}=1-P_{1}-P_{2} \tag{6}
\end{equation*}
$$

where $P_{1}=\mathbb{P}\left(\gamma_{\mathrm{AB}} \geq \gamma_{\mathrm{th}}\right)$ is the probability that the direct link achieves the given $\operatorname{SNR}$ threshold, $P_{2}=\mathbb{P}\left(\gamma_{\mathrm{AB}}<\right.$ $\left.\gamma_{\mathrm{th}}, \min \left(\gamma_{\mathrm{AR}}, \gamma_{\mathrm{BR}}\right) \geq \gamma_{\mathrm{th}}, \min \left(\gamma_{\mathrm{A}}, \gamma_{\mathrm{B}}\right) \geq \gamma_{\mathrm{th}}, P_{\mathrm{in}} \geq P_{\mathrm{th}}\right)$ is the probability of successful two-way relaying when the direct link is not available, and $\mathbb{P}(\cdot)$ denotes the probability. Based on (6), we propose an optimal combining scheme to minimize the system outage probability by optimizing the power allocation ratio $\theta$. It can be seen that there are only two SNRs, $\gamma_{\mathrm{RA}}$ and $\gamma_{\mathrm{RB}}$, related with $\theta$. Thus, minimizing $P_{\text {out }}^{s}$ is equivalent to maximizing the lower SNR between $\gamma_{\mathrm{RA}}$ and $\gamma_{\mathrm{RB}}$, and the optimization problem is formulated as

$$
\begin{align*}
& \underset{\theta}{\operatorname{maximize}} \min \left(\gamma_{\mathrm{RA}}, \gamma_{\mathrm{RB}}\right)  \tag{7}\\
& \text { s.t. }: 0<\theta<1
\end{align*}
$$

By listing cases of $\gamma_{\mathrm{RA}} \geq \gamma_{\mathrm{RB}}$ and $\gamma_{\mathrm{RA}} \leq \gamma_{\mathrm{RB}}$, it is easy to show that the optimal power allocation ratio $\theta^{*}$ can be obtained by letting $\gamma_{\mathrm{RA}}=\gamma_{\mathrm{RB}}$. Thus, the optimal power allocation ratio $\theta^{*}$ is given by

$$
\begin{equation*}
\theta^{*}=\frac{\left|h_{\mathrm{A}}\right| d_{\mathrm{A}}^{-\alpha_{s} / 2}}{\left|h_{\mathrm{A}}\right| d_{\mathrm{A}}^{-\alpha_{s} / 2}+\left|h_{\mathrm{B}}\right| d_{\mathrm{B}}^{-\alpha_{s} / 2}} \tag{8}
\end{equation*}
$$

Note that the optimal combining scheme at the relay can also be extended into the case with multiple antennas. For example, for the case with multiple-antenna source terminals and single-antenna EH relay, similar to the derivation of (8), the optimal power allocation ratio is $\frac{\sqrt{\operatorname{Tr}\left(\mathrm{H}_{\mathrm{RA}} \mathrm{F}_{\mathrm{R}} \mathrm{F}_{\mathrm{R}}^{H} \mathrm{H}_{\mathrm{RA}}^{H}\right)}}{\sqrt{\operatorname{Tr}\left(\mathrm{H}_{\mathrm{RA}} \mathrm{F}_{\mathrm{R}} \mathrm{F}_{\mathrm{R}}^{H} \mathrm{H}_{\mathrm{RA}}^{H}\right)}+\sqrt{\operatorname{Tr}\left(\mathrm{H}_{\mathrm{RB}} \mathrm{F}_{\mathrm{R}} \mathrm{F}_{\mathrm{R}}^{H} \mathrm{H}_{\mathrm{RB}}^{H}\right)}}$, where $\mathrm{F}_{\mathrm{R}}$ is the beamformer at the relay and $\mathrm{H}_{\mathrm{R} i}$ denotes the channel matric from R to $i$.

## B. Performance Analysis

1) Derivation of $P_{\text {out }}^{s}$ : In the following, we calculate $P_{1}$ and $P_{2}$ and obtain the expression of $P_{\text {out }}^{s}$.

Firstly, based on the expression of $\gamma_{\mathrm{AB}}$ in (3), $P_{1}$ can be computed as

$$
\begin{equation*}
P_{1}=\mathbb{P}\left(\gamma_{\mathrm{AB}} \geq \gamma_{\mathrm{th}}\right) \stackrel{(\mathrm{a})}{=} \exp \left(-\frac{\gamma_{\mathrm{th}} a_{t}}{\rho}\right) \tag{9}
\end{equation*}
$$

where step (a) follows by $|g|^{2} \sim \exp \left(\frac{1}{\lambda_{t}}\right)$ and $a_{t}=\frac{d_{t}^{\alpha}}{\lambda_{t}}$. Then, by substituting $\theta^{*}$ into $\gamma_{\mathrm{RA}}$ and $\gamma_{\mathrm{RB}}$ in (4), $P_{2}$ is obtained as

$$
\begin{align*}
& P_{2}=\mathbb{P}\left(Y_{\mathrm{A}} \geq \frac{\gamma_{\mathrm{th}}}{\rho}, Y_{\mathrm{B}} \geq \frac{\gamma_{\mathrm{th}}}{\rho},\right. \\
& \left.Z<\frac{\gamma_{\mathrm{th}}}{\rho}, Y_{\mathrm{A}}+Y_{\mathrm{B}} \geq \frac{P_{\mathrm{th}}}{P}, Z+\Omega Y_{\mathrm{A}} Y_{\mathrm{B}} \geq \frac{\gamma_{\mathrm{th}}}{\rho}\right) \tag{10}
\end{align*}
$$

where $Y_{i}=\left|h_{i}\right|^{2} d_{i}^{-\alpha_{s}}(i \in\{\mathrm{~A}, \mathrm{~B}\}), Z=|g|^{2} d_{t}^{-\alpha_{t}}$, and $\Omega=$ $\frac{3 \beta \eta}{1-\beta}$. Since the case with $Y_{\mathrm{A}} \geq \frac{\gamma_{\text {th }}}{\rho}$ and $Y_{\mathrm{B}} \geq \frac{\gamma_{\text {th }}}{\rho}$ is equivalent to the case with $\left(Y_{\mathrm{A}}-\frac{\gamma_{\mathrm{th}}}{\rho}\right)\left(Y_{\mathrm{B}}^{-}-\frac{\rho_{\mathrm{th}}}{\rho}\right) \geq 0$ and $Y_{\mathrm{A}}+Y_{\mathrm{B}} \geq 2 \frac{\gamma_{\mathrm{th}}}{\rho}$, $P_{2}$ can be rewritten as

$$
\begin{align*}
& P_{2}=\mathbb{P}\left(X \geq t_{\min }, \frac{\gamma_{\mathrm{th}}}{\rho}-\frac{\Omega}{Y} \leq Z<\frac{\gamma_{\mathrm{th}}}{\rho}, X \leq \frac{\rho}{\gamma_{\mathrm{th}} Y}+\frac{\gamma_{\mathrm{th}}}{\rho}\right)= \\
& \mathbb{P}\left(t_{\min } \leq X \leq \frac{\rho}{Y \gamma_{\mathrm{th}}}+\frac{\gamma_{\mathrm{th}}}{\rho}, \frac{\gamma_{\mathrm{th}}}{\rho}-\frac{\Omega}{Y} \leq Z<\frac{\gamma_{\mathrm{th}}}{\rho}, Y \leq \Delta_{\max }\right) \tag{11}
\end{align*}
$$

where $X=Y_{\mathrm{A}}+Y_{\mathrm{B}}, Y=\frac{1}{Y_{\mathrm{A}} Y_{\mathrm{B}}}, t_{\mathrm{min}}=\max \left(\frac{P_{\mathrm{th}}}{P}, 2 \frac{\gamma_{\mathrm{th}}}{\rho}\right)$ and $\Delta_{\max }=\frac{1}{\left(t_{\text {min }}-\frac{\gamma_{\text {th }}}{\rho}\right) \frac{\gamma_{\text {th }}}{\rho}}$.

Let $F_{Z}(\Delta), F_{Y}(\Delta)$ and $F_{X}(\Delta)$ denote the cumulative distribution functions (CDFs) of $Z, Y$ and $X$, respectively. Then the expressions of $F_{Z}(\Delta), F_{Y}(\Delta)$ and $F_{X}(\Delta)$ are obtained in Lemma 1.

Lemma 1 The CDFs of $Z, Y$ and $X$ are given by

$$
\begin{align*}
& F_{X}(\Delta)= \\
& \left\{\begin{array}{l}
1-e^{-a_{\mathrm{B}} \Delta}-\frac{a_{\mathrm{B}}}{a_{\mathrm{A}}-a_{\mathrm{B}}}\left(e^{-a_{\mathrm{B}} \Delta}-e^{-a_{\mathrm{A}} \Delta}\right), \text { if } a_{\mathrm{A}} \neq a_{\mathrm{B}} \\
1-e^{-a_{\mathrm{B}} \Delta}-a_{\mathrm{B}} \Delta e^{-a_{\mathrm{B}} \Delta}, \text { if } a_{\mathrm{A}}=a_{\mathrm{B}}
\end{array}\right.  \tag{12}\\
& F_{Y}(\Delta)=\frac{1}{\lambda_{\mathrm{B}}} \sqrt{\frac{4 \lambda_{\mathrm{B}} d_{\mathrm{A}}^{\alpha_{s}} d_{\mathrm{B}}^{\alpha_{s}}}{\lambda_{\mathrm{A}} \Delta}} K_{1}\left(\sqrt{\frac{4 d_{\mathrm{A}}^{\alpha_{s}} d_{\mathrm{B}}^{\alpha_{s}}}{\lambda_{\mathrm{A}} \lambda_{\mathrm{B}} \Delta}}\right) \\
& F_{Z}(\Delta)=1-\exp \left(-a_{t} \Delta\right) \tag{13}
\end{align*}
$$

where $a_{i}=\frac{d_{i}^{\alpha_{s}}}{\lambda_{i}}$ and $K_{1}(\cdot)$ is the modified Bessel function of the second kind.

Proof: See the Appendix.
Based on Lemma 1, the probability density functions (PDFs) of $X, Y$ and $Z$, i.e., $f_{X}(X), f_{Y}(Y)$ and $f_{Z}(Z)$, is obtained.

Since $Z=|g|^{2} d_{t}^{-\alpha_{t}} \geq 0$, we have $P_{2}=P_{21}+P_{22}$, where $P_{21}$ is the probability of $\frac{\gamma_{\mathrm{th}}}{\rho}-\frac{\Omega}{Y}>0$ and $P_{22}$ is the probability of $\frac{\gamma_{\text {th }}}{\rho}-\frac{\Omega}{Y} \leq 0$. For the case with $\frac{\gamma_{\text {th }}}{\rho}-\frac{\Omega}{Y}>0$, we have $Y>\frac{\rho \Omega}{\gamma_{\mathrm{th}}}$. Thus, $P_{21}$ is given by
$P_{21}=\mathbb{P}\left(t_{\min } \leq X \leq \frac{\rho}{Y \gamma_{\mathrm{th}}}+\frac{\gamma_{\mathrm{th}}}{\rho}, \frac{\gamma_{\mathrm{th}}}{\rho}-\frac{\Omega}{Y} \leq Z<\frac{\gamma_{\mathrm{th}}}{\rho}, \Delta_{\min } \leq Y \leq \Delta_{\max }\right)$
$=\int_{\Delta_{\text {min }}}^{\Delta_{\text {max }}} f_{Y}(Y) \int_{t_{\text {min }}}^{\frac{\rho}{Y \gamma_{\text {th }}}+\frac{\gamma_{\text {th }}}{\rho}} f_{X}(X) \int_{\frac{\gamma_{\text {th }}-\frac{\Omega}{\rho}}{\frac{\gamma_{\text {th }}}{\rho}}}^{\rho_{Z}}(Z) d Z d X d Y$
$=P_{1}[\left.F_{Y}(Y) \chi_{1}(Y)\right|_{\Delta_{\min }} ^{\Delta_{\max }}-\underbrace{\int_{\Delta_{\min }}^{\Delta_{\max }} F_{Y}(Y) \chi_{1}^{\prime}(Y) d Y}_{\Xi}]$,
where $\Delta_{\text {min }}=\min \left(\frac{\rho \Omega}{\gamma_{\mathrm{th}}}, \Delta_{\max }\right), \quad \chi_{1}(Y)=$ $\left[\exp \left(\frac{a_{t} \Omega}{Y}\right)-1\right]\left[F_{X}\left(\frac{\rho}{Y \gamma_{\mathrm{th}}}+\frac{\gamma_{\mathrm{th}}}{\rho}\right)-F_{X}\left(t_{\mathrm{min}}\right)\right]$, $\chi_{1}^{\prime}(Y)=\frac{\partial \chi_{1}(Y)}{\partial Y}$.

By using ${ }^{2}$ Gaussian-Chebyshev quadrature approximation [9], $\Xi$ can be approximated as follows,

$$
\begin{equation*}
\Xi \approx \frac{\pi\left(\Delta_{\max }-\Delta_{\min }\right)}{2 M} \sum_{m=1}^{M} \sqrt{1-\nu_{m}^{2}} F_{Y}\left(\kappa_{m}^{(1)}\right) \chi_{1}^{\prime}\left(\kappa_{m}^{(1)}\right), \tag{16}
\end{equation*}
$$

where $M$ is a parameter that determines the tradeoff between complexity and accuracy, $\nu_{m}=\cos \frac{2 m-1}{2 M} \pi$, and $\kappa_{m}^{(1)}=$ $\frac{\left(\Delta_{\max }-\Delta_{\min }\right)}{2} \nu_{m}+\frac{\left(\Delta_{\max }+\Delta_{\min }\right)}{2}$.
For the case with $\frac{\gamma_{\text {th }}{ }^{2}}{\rho}-\frac{\Omega}{Y} \leq 0$, we have $Y \leq \frac{\rho \Omega}{\gamma_{\text {th }}}$. Thus, $P_{22}$ is given by

$$
\begin{align*}
P_{22} & =\mathbb{P}\left(t_{\min } \leq X \leq \frac{\rho}{Y \gamma_{\mathrm{th}}}+\frac{\gamma_{\mathrm{th}}}{\rho}, 0 \leq Z<\frac{\gamma_{\mathrm{th}}}{\rho}, 0 \leq Y \leq \Delta_{\min }\right) \\
& \approx\left(1-P_{1}\right)\left[F_{Y}\left(\Delta_{\min }\right) \chi_{2}\left(\Delta_{\min }\right)\right. \\
& \left.-\frac{\pi \Delta_{\min }}{2 M} \sum_{m=1}^{M} \sqrt{1-\nu_{m}^{2}} F_{Y}\left(\kappa_{m}^{(2)}\right) \chi_{2}^{\prime}\left(\kappa_{m}^{(2)}\right)\right] \tag{17}
\end{align*}
$$

where $\chi_{2}(Y)=F_{X}\left(\frac{\rho}{Y \gamma_{\mathrm{th}}}+\frac{\gamma_{\mathrm{th}}}{\rho}\right)-F_{X}\left(t_{\min }\right), \chi_{2}^{\prime}(Y)=$ $\frac{\partial \chi_{2}(Y)}{\partial Y}$, and $\kappa_{m}^{(2)}=\frac{\Delta_{\text {min }}}{2} \nu_{m}+\frac{\Delta_{\text {min }}}{2}$.
Thus, the system outage probability with the optimal combining strategy is approximated by

$$
\begin{align*}
& P_{\mathrm{out}}^{s} \approx 1-P_{1}-\Theta+\frac{\pi}{2 M} \sum_{m=1}^{M} \sqrt{1-\nu_{m}^{2}}\left[P_{1}\left(\Delta_{\max }-\Delta_{\min }\right) F_{Y}\left(\kappa_{m}^{(1)}\right)\right. \\
& \left.\times \chi^{\prime}{ }_{1}\left(\kappa_{m}^{(1)}\right)+\left(1-P_{1}\right) \Delta_{\min } F_{Y}\left(\kappa_{m}^{(2)}\right) \chi_{2}^{\prime}\left(\kappa_{m}^{(2)}\right)\right] \tag{18}
\end{align*}
$$

where $\Theta=P_{1}\left[F_{Y}\left(\Delta_{\max }\right) \chi_{1}\left(\Delta_{\max }\right)-F_{Y}\left(\Delta_{\min }\right) \chi_{1}\left(\Delta_{\min }\right)\right]$ $+\left(1-P_{1}\right) F_{Y}\left(\Delta_{\min }\right) \chi_{2}\left(\Delta_{\min }\right)$.
2) High SNR Approximation of $P_{\text {out }}^{s}$ : Let $\widehat{P}_{\text {out }}^{s}$ be the high SNR approximation of $P_{\text {out }}^{s}$. When $\rho \rightarrow \infty$, we have $\lim _{\rho \rightarrow \infty} \frac{1}{\rho}=0$. In this case, we have $P_{21} \simeq 0$ due to $\Delta_{\min } \simeq$ $\Delta_{\max }$. Further, by letting $\left.\lim _{\rho \rightarrow \infty} \chi_{2}(Y) \approx \chi_{2}(Y)\right|_{1 / \rho=0}=1-$ $F_{X}\left(t_{\min }\right), \widehat{P}_{\text {out }}^{s}$ is given by

$$
\begin{align*}
\widehat{P}_{\mathrm{out}}^{s} & =1-P_{1}-\left(1-P_{1}\right) \int_{0}^{\Delta_{\min }} f_{Y}(Y)\left[1-F_{X}\left(t_{\min }\right)\right] d Y \\
& =\left(1-P_{1}\right)\left[1-F_{Y}\left(\Delta_{\min }\right)+F_{Y}\left(\Delta_{\min }\right) F_{X}\left(t_{\min }\right)\right] . \tag{19}
\end{align*}
$$

## IV. Simulations

In this section, we validate the effectiveness of our proposed combining scheme and verify the accuracy of the derived expressions via $1 \times 10^{8}$ Monte-Carlo simulations. According to [2], the simulation parameters are set as follows: $d_{\mathrm{A}}=5 \mathrm{~m}$, $d_{t}=20 \mathrm{~m}, d_{\mathrm{B}}=d_{t}-d_{\mathrm{A}}=15 \mathrm{~m}$ and $\sigma^{2}=-70 \mathrm{dBm}$. The EH circuit sensitivity $P_{\mathrm{th}}$ is given by -30 dBm and the energy conversion efficiency is $\eta=0.6$. The transmission rate is assumed as $U=3 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$ resulting in $\gamma_{\mathrm{th}}=2^{U}-1$. Unless otherwise specified, we set $\alpha_{s}=\alpha_{t}=4$ and $\beta=0.25$.
Fig. 1 plots the system outage probability versus the transmit power, where three cases are considered: (1) relay with direct link, (2) relay without direct link, and (3) direct link. Note that the case of relay with/without direct link, the power allocation ratio at relay is determined by (8). For the case of relay with direct link, we use the Gaussian-Chebyshev quadrature approximation with $M=4$ and the high SNR approximation to obtain the system outage probability. It can be observed that the result achieved by Gaussian-Chebyshev quadrature approximation matches the simulation result well, which verifies the correctness of the derived analytical expression in (18). For the high SNR approximation, with the increasing of the transmit power, the difference between the result based on the high


Fig. 1. System outage probability versus $P$ with $\lambda_{\mathrm{A}}=\lambda_{\mathrm{B}}=1$ and $\lambda_{t}=2$.


Fig. 2. System outage probability versus $P$ with $\lambda_{\mathrm{A}}=\lambda_{\mathrm{B}}=1$ and $\lambda_{t}=2$.
SNR approximation and the simulation result becomes smaller. Thus, the derived approximation in (19) is also accurate for the high SNR regions. It can also be seen that the case of relay with direct link can achieve a lower system outage probability than the cases of relay without direct link and direct link. This is because that using a relay to help the information transmission can achieve a higher diversity gain.

Fig. 2 plots the system outage probability versus $d_{\mathrm{A}}$ under above three cases. For the case of relay with direct link, two schemes are considered, which are the proposed scheme and baseline scheme in which the power allocation ratio $\theta$ is fixed as $0.3,0.5$ and 0.7 , respectively. We set $P=10 \mathrm{~mW}$ and $d_{\mathrm{B}}=20-d_{\mathrm{A}}$. It can be observed that with the increase of $d_{\mathrm{A}}$, the system outage probability increases, reaches the maximum value and then decreases. This is because that the total harvested energy is higher when the relay is closer to either of the nodes. By comparison, we can see that the proposed scheme outperforms the baseline scheme in terms of outage performance. In addition, it can also be seen that the optimal relay location is closer to either of the terminal nodes.

## V. Conclusions

In this paper, we have proposed an optimal combing scheme to minimize the overall system outage probability and have derived the closed-form expression for the optimal power allocation ratio $\theta$. For the proposed optimal combining scheme, we have obtained expression of the system outage probability considering EH circuit sensitivity. We have demonstrated that both the relay location and combining scheme are critical to
achieving good outage performance and our proposed combining scheme outperforms the existing combining scheme.

## APPENDIX

According to the definition of $F_{X}(\Delta)$, we have

$$
\begin{align*}
& F_{X}(\Delta)=\mathbb{P}(X \leq \Delta)=\mathbb{P}\left[x \leq\left(\Delta-y d_{\mathrm{B}}^{-\alpha_{s}}\right) d_{\mathrm{A}}^{\alpha_{s}}, y \leq \Delta d_{\mathrm{B}}^{\alpha_{s}}\right] \\
& =\int_{0}^{\Delta d_{\mathrm{B}}^{\alpha_{s}}}\left[1-\exp \left(-a_{\mathrm{A}}\left(\Delta-y d_{\mathrm{B}}^{-\alpha_{s}}\right)\right)\right] \frac{\exp \left(-y / \lambda_{\mathrm{B}}\right)}{\lambda_{\mathrm{B}}} d y \\
& =1-e^{-a_{\mathrm{B}} \Delta}-\frac{e^{-a_{\mathrm{B}} \Delta}}{\lambda_{\mathrm{B}}} \int_{0}^{\Delta d_{\mathrm{B}}^{\alpha_{s}}} \exp \left(\frac{a_{\mathrm{A}}-a_{\mathrm{B}}}{a_{\mathrm{B}} \lambda_{\mathrm{B}}} y\right) d y \tag{20}
\end{align*}
$$

where $x=\left|h_{\mathrm{A}}\right|^{2}, y=\left|h_{\mathrm{B}}\right|^{2}$ and $a_{i}=\frac{d_{i}^{\alpha_{s}}}{\lambda_{i}}$. When $a_{\mathrm{A}}=a_{\mathrm{B}}$, (21) can be computed as $F_{X}(\Delta)=1-e^{-a_{\mathrm{B}} \Delta}-a_{\mathrm{B}} \Delta e^{-a_{\mathrm{B}} \Delta}$. For the case with $a_{\mathrm{A}} \neq a_{\mathrm{B}}$, (21) is given by $F_{X}(\Delta)=$ $1-e^{-a_{\mathrm{B}} \Delta}-\frac{a_{\mathrm{B}}}{a_{\mathrm{A}}-a_{\mathrm{B}}}\left(e^{-a_{\mathrm{B}} \Delta}-e^{-a_{\mathrm{A}} \Delta}\right)$. Thus, $F_{X}(\Delta)$ can be rewritten as (12).

Similarly, $F_{Y}(\Delta)$ and $F_{Z}(\Delta)$ are given by

$$
\begin{align*}
& F_{Y}(\Delta)=\mathbb{P}\left(x \geq \frac{d_{\mathrm{A}}^{\alpha_{s}} d_{\mathrm{B}}^{\alpha_{s}}}{y \Delta}\right)=\frac{1}{\lambda_{\mathrm{B}}} \int_{0}^{+\infty} \exp \left(-\frac{d_{\mathrm{A}}^{\alpha_{s}} d_{\mathrm{B}}^{\alpha_{s}}}{\lambda_{\mathrm{A}} \Delta y}-\frac{y}{\lambda_{\mathrm{B}}}\right) d y \\
& =\frac{1}{\lambda_{\mathrm{B}}} \sqrt{\frac{4 \lambda_{\mathrm{B}} d_{\mathrm{A}}^{\alpha_{s}} d_{\mathrm{B}}^{\alpha_{s}}}{\lambda_{\mathrm{A}} \Delta}} K_{1}\left(\sqrt{\frac{4 d_{\mathrm{A}}^{\alpha_{s}} d_{\mathrm{B}}^{\alpha_{s}}}{\lambda_{\mathrm{A}} \lambda_{\mathrm{B}} \Delta}}\right)  \tag{21}\\
& F_{Z}(\Delta)=\mathbb{P}(Z \leq \Delta)=\mathbb{P}\left(|g|^{2} \leq \Delta d_{t}^{\alpha_{t}}\right)=1-\exp \left(-a_{t} \Delta\right) \tag{22}
\end{align*}
$$

where $K_{1}(\cdot)$ is the modified Bessel function of the second kind. The proof is completed.

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[^0]:    ${ }^{1}$ Although AF relaying is simpler than DF relaying, the major drawback of AF relaying is noise amplification at the relay, which may degrade the received signal-to-noise ratio (SNR) at the destination node. DF relaying is a commonly used relaying protocol for eliminating the noise amplification effect.
    ${ }^{2}$ Note that energy harvesting is particularly applicable to wireless sensor networks [1], where it may be difficult for low-cost, small wireless sensor nodes to have multiple antennas. Meanwhile, energy harvesting with a single antenna at each node has also been assumed in many related recent works, e.g., [5]-[12].

[^1]:    ${ }^{3}$ Since the self-signal is known at each destination node, a destination node can obtain the information from the other node by cancelling the effect of its self-signal from the received combined signal. This process requires the channel state information (CSI), which can be obtained following [14].

