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Enhanced Receding Horizon Optimal Performance for On-line Tuning of PID Controller Parameters

Yongling Wu

Department of Automation,
Shanghai Jiao Tong University,
Shanghai, 200240, China
E-mail: ylwu318@sjtu.edu.cn

Shaoyuan Li

Department of Automation,
Shanghai Jiao Tong University,
Shanghai, 200240, China
E-mail: syli@sjtu.edu.cn

Kang Li

School of Electronics, Electrical Engineering and Computer Science,
Queens University Belfast,
Belfast BT9 5AH, UK
E-mail: k.li@qub.ac.uk

Abstract: In this paper, a new on-line Proportional-Integral-Derivative (PID) controller parameter optimization method is proposed by incorporating the philosophy of the model predictive control (MPC) algorithm. The future system predictive output and control sequence are first written as a function of the controller parameters. Then the PID controller design is realized through optimizing the cost function under the constraints on the system input and output. The MPC based PID on-line tuning is easy to handle the constraints and time delay. Simulation results in three situations, changing the control weight, adding constraints on the overshoot and control signal and changing the reference value, confirm that the proposed method is capable of producing good tracking performance with low energy consumption and short settling time.

Keywords: On-line parameter optimization; PID controller; model predictive control; tracking performance; control energy.

Biographical notes: Yongling Wu received her B.Sc. degree in Automation from the School of Control Science and Engineering, Shandong University in 2008 and M.Sc. degree in Control Theory and Control Engineering, from the School of Electronic Information and Electrical Engineering, Shanghai Jiao Tong University in 2011. She joined the Energy, Power and Intelligent Control Cluster at Queen's University Belfast as a joint Ph.D. student in 2012-2014. Her research interests include controller parameters design based on energy saving and model predictive control.

Shaoyuan Li was born in Hebei, China, in 1965. He received the B.Sc. and M.Sc. degrees in automation from Hebei University of Technology, Tianjin, China, in 1987 and 1992, respectively, and the Ph.D. degree from the Department of Computer and System Science, Nankai University, Tianjin, in 1997. He is currently a professor with the Department of Automation, Shanghai Jiao Tong University, Shanghai, China. His research interests include fuzzy systems, model predictive control, dynamic system optimization, and system identification. Professor Li is a vice president of Chinese Association of Automation and a senior member of IEEE.

Kang Li received the B.Sc. degree from Xiangtan University in 1989, the M.Sc. degree from Harbin Institute of Technology in 1992, and the Ph.D. degree from Shanghai Jiao Tong University in 1995, all in China. He is currently a Professor of Intelligent Systems and Control with the School of Electronics, Electrical Engineering and Computer Science, Queens University, Belfast, UK. His research interests are in nonlinear system modeling and control, and bio-inspired computational intelligence, with recent applications to decarbonizing the whole energy system, from integration of renewable energy, smart grid, and electrical vehicles, to low cost energy monitoring and energy saving in industrial processes, in particular polymer processing. He has published over 200 papers and edited 12 conference proceedings (Springer).

1 Introduction

With the rapid development of science and technology in the past century, many advanced control strategies have been successfully applied across many sectors, like MPC (Mayne (2014)), linear quadratic regulator (LQR), fuzzy control and neural network control methods, etc. Nevertheless, PID control, though has been developed for over a century and has various limitations, is still the most widely used method, particularly in power and energy system where reliability is a primary concern. In some cases, the PID controller is used in the low-level regulation loop while some advanced control methods are used at a higher-level control loop.

Further, the combination of the PID with advanced control strategies, such as the fuzzy control (Patel and Mohan (2002); Kim et al. (2016)), adaptive control (Pomerleau et al. (1996); Mahmoodabadi et al. (2014)), and MPC control (Liu et al. (2014); Singh et al. (2013)), etc, are proposed in many literatures. All these references show a well-tuned PID controller is required to realize the successful implementation of advanced control methods and the on-line parameter tuning is one of the most important popular approaches to improve its performance in real-time applications.

Ziegler and Nichols method (Ziegler and Nichols (1942)) is one of the early and well known PID tuning methods, which is based on the time domain or frequency domain response characteristics of the process. However, it may produce a quite large overshoot, which could not satisfy the system requirements. After that, various methods such as Cohen and Coon method (Cohen and Coon (1953)), feedback relay method, parametric or non-parametric (Boiko (2013)) method, model-free or model-based method, and optimal or non-optimal method have been developed. Tuning the parameters with the genetic algorithm (GA), particle swarm optimization (PSO) and artificial bee colony (ABC) algorithms (Elkhateeb and Badr (2014); Taeib and Chaari (2015)), etc, may cause the convergence and computation complexity problems. The strategies of matching the specified gain and phase margins (Keyu (2013); Wang and Tian (2013)) are not very clear and intuitional on the requirements of the control performance and energy.

A multi-objective optimal method for PID controller design with almost every performance parameter in the criterion, like steady-state error, overshoot, settling time, etc, is proposed in Zamani et al. (2009), which increases the difficulty on the selection of the weightings. Another two references (Jin et al. (2014); Sánchez et al. (2017)) propose a tuning rule by minimizing the integral of the time weighted absolute error (ITAE) and the integrated absolute error (IAE) of the system output respectively, which however, do not take the control energy into account. And no constraints on the maximum system response and control signal are considered in the PID controller design in these references.

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Over the years, modified PID algorithms have been proposed combining with the strategies of modern control methods. A receding horizon method for auto-tuning PID controller parameters under controlled auto-regressive integrated moving average (CARIMA) model is proposed in Xu et al. (2005) based on generalized predictive control (GPC) method. Another MPC-based method using a neural model is given in Majdabadi-Farahani et al. (2014). In real-time applications, more effective on-line parameter tuning methods are often important though challenging when both the control energy and system control performance are considered simultaneously.

In our previous research, we have shown that the choice of controller parameters should be an elegant trade-off between tracking performance and control energy use (Wu et al. (2016)). The aim of this paper is to extend the previous off-line numerical experimental results in continuous system to the on-line PID controller parameter optimization in a discrete domain, considering both the tracking performance and energy consumption in the cost function using the philosophy of MPC strategy. The linear quadratic optimal form is chosen as the objective index subject to the constrains on the overshoot and control signal.

For many systems one or two terms, like P, PI or PD control are enough to satisfy the system requirements. To facilitate analytic results, a simple two-layer voltage source converters (VSCs) system model, a first-order plus dead time (FOPDT) model, which can sufficiently describe many industrial processes (Bagheri and Sedigh (2013)) and a second-order plus dead time (SOPDT) system are used to verify the P, PI and PID controller parameter on-line tuning in three situations. The simulation results, comparing the control effects by changing the control weight, adding constraints and observing the change of controller parameters and tracking performance by changing the reference respectively, show the MPC-PID tuning can provide a good tracking performance with low control energy.

The remainder of the paper is outlined as follows. In section 2, the PID and MPC control algorithm are introduced briefly and the MPC-PID on-line tuning strategy is represented. In section 3, the theoretical derivations of the on-line PID parameter optimization are proposed. The simulation results are presented in section 4. Finally, section 5 concludes the paper.

2 Problem formulation

In this section, the basic theory of the PID and MPC algorithm are first introduced with some variables defined. And then the idea of MPC based PID controller parameter on-line optimization strategy is proposed.

2.1 PID algorithm

PID control is a complete data-driven control method without knowing any priori knowledge of the controlled plant. The control effect totally depends on the user defined controller parameters. However to obtain a better control effect, the system model, which represents the mathematical description of the plant, should be known first. Especially with the increasing of the system complexity, and to deal with both the constraints and large time delay, PID controller design becomes more difficult.

The standard discrete PID control law is given by

$$u_t = K_p e_t + K_i \sum_{k=1}^{t} e_k T_s + K_d \frac{e_t - e_{t-1}}{T_s} \tag{1}$$

where T_s is the sampling time and the discrete transfer function of the PID controller is

$$G_c(z) = \frac{U(z)}{E(z)} = \frac{d_0 z^2 + d_1 z + d_2}{c_0 z^2 + c_1 z} \tag{2}$$

The parameters are related to the controller parameters K_p, K_i, K_d and the sampling time T_s.

$$\begin{aligned} c_0 &= T_s \\ c_1 &= -T_s \\ d_0 &= K_p T_s + K_i T_s^2 + K_d \\ d_1 &= -K_p T_s - 2K_d \\ d_2 &= K_d \end{aligned} \tag{3}$$

Define two vectors of the coefficients in the transfer function.

$$\begin{aligned} \boldsymbol{v_c} &= \begin{pmatrix} c_0 \, c_1 \end{pmatrix} \\ \boldsymbol{v_d} &= \begin{pmatrix} d_0 \, d_1 \, d_2 \end{pmatrix} \end{aligned} \tag{4}$$

Also, $\boldsymbol{v_d}$ could be transformed into the product of a coefficient matrix and a vector of the controller parameters, which are more intuitive. That is

$$\boldsymbol{v_d}^T = \boldsymbol{T_s^{pid}} \boldsymbol{k_{pid}} = \begin{pmatrix} T_s & T_s^2 & 1 \\ -T_s & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} K_p \\ K_i \\ K_d \end{pmatrix} \tag{5}$$

$\boldsymbol{k_{pid}}$ is the variable to be optimized.

2.2 MPC algorithm

Model predictive control (MPC) is an advanced control method that was first used in the process industries since 1970s. It relies on the dynamic model of the process obtained by system identification. MPC takes control actions according to the error between the predictive future events and reference trajectory in a prediction horizon. PID however does not have the predictive ability, it depends on the current and historical tracking errors to take the control actions. The model used in the MPC usually represents a relationship between the behaviour of the dynamic system output and manipulated variables. In this paper, the relationship is between the system output and the controller parameters.

$$\boldsymbol{\tilde{y}_f} = f(\boldsymbol{k_{pid}}) \tag{6}$$

The structure of a discrete MPC-PID control system is shown in Figure 1. All the reference value, historical tracking error, historical system input and output are sent to the MPC module, and then the optimized controller parameters are sent to the PID controller module.

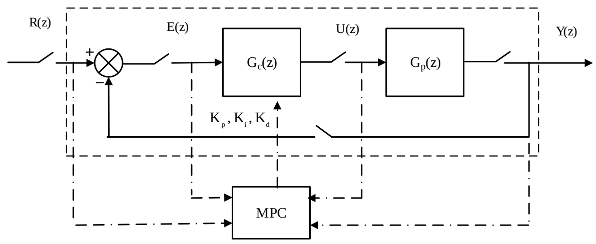

Figure 1 Discrete MPC-PID control System.

3 MPC based PID controller synthesis

System model

The general form of a discrete model is

$$G_p(z) = \frac{Y(z)}{U(z)} = \frac{b_0 z^m + b_1 z^{m-1} + \cdots + b_m}{a_0 z^n + a_1 z^{n-1} + \cdots + a_n} \tag{7}$$

where $n \geq m$. Similarly to the definition of vectors $\boldsymbol{v_c}$ and $\boldsymbol{v_d}$, the parameters of the plant are also written in a vector form.

$$\begin{aligned} \boldsymbol{v_a} &= \begin{pmatrix} a_0 \, a_1 \, c_2 \cdots \, a_n \end{pmatrix} \\ \boldsymbol{v_b} &= \begin{pmatrix} b_0 \, b_1 \, b_2 \cdots \, b_m \end{pmatrix} \end{aligned} \tag{8}$$

Define time delay $t_d = n - m$, then Equation (7) can be rewritten as the following form.

$$A(z^{-1}) y_t = B(z^{-1}) u_t \tag{9}$$

where $A(z^{-1}) = a_0 z^{t_d} + a_1 z^{t_d - 1} + \cdots + a_n z^{-m}$, $B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}$. The relationship between input and output of the plant is given below, which is shown in Figure 2.

$$\begin{aligned} & a_0 y_{t+t_d} + a_1 y_{t+t_d-1} + \cdots + a_n y_{t-m} \\ =& b_0 u_t + b_1 u_{t-1} + \cdots + b_m u_{t-m} \end{aligned} \tag{10}$$

In this paper, not only the constraints are considered in the PID controller parameter on-line tuning procedure, but also the time delay is fully utilized in the proposed method. The tracking errors from the current time t to $t + N_2 - 1$ are used to construct the error matrix $\boldsymbol{E_r}$, where N_2 is the prediction horizon and $N_2 \leq t_d$.

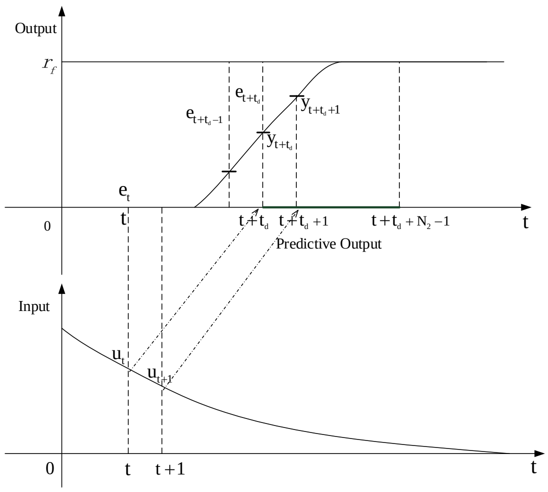

Figure 2 Relationship between output and input with time delay.

Derivation of the predictive outputs

In most cases, the characteristics of the closed-loop system transfer function are analysed, which gives the relation between $Y(z)$ and $R(z)$. However, in this paper the relationship between the system output and the tracking error is produced, which is the product of the controller and controlled plant transfer function.

$$\frac{Y(z)}{E(z)} = G_c(z)G_p(z) \tag{11}$$

Define another two matrices of the system parameters.

$$\begin{aligned} \boldsymbol{M_a} &= \begin{pmatrix} \boldsymbol{v_{a1}} \\ \boldsymbol{v_{a2}} \end{pmatrix} \\ \boldsymbol{M_b} &= \begin{pmatrix} \boldsymbol{v_{b1}} \\ \boldsymbol{v_{b2}} \\ \boldsymbol{v_{b3}} \end{pmatrix} \end{aligned} \tag{12}$$

where $\boldsymbol{v_{a1}} = \begin{pmatrix} \boldsymbol{v_a}\, 0 \end{pmatrix}$, $\boldsymbol{v_{a2}} = \begin{pmatrix} 0\, \boldsymbol{v_a} \end{pmatrix}$, $\boldsymbol{v_{b1}} = \begin{pmatrix} \boldsymbol{v_b}\, 0\, 0 \end{pmatrix}$, $\boldsymbol{v_{b2}} = \begin{pmatrix} 0\, \boldsymbol{v_b}\, 0 \end{pmatrix}$ and $\boldsymbol{v_{b3}} = \begin{pmatrix} 0\, 0\, \boldsymbol{v_b} \end{pmatrix}$. Then the following equation can be derived from Equation (11).

$$\boldsymbol{v_c M_a} \begin{pmatrix} y_{t+t_d} \\ y_{t+t_d-1} \\ \vdots \\ y_{t-m-1} \end{pmatrix} = \begin{pmatrix} e_t\, e_{t-1} \cdots e_{t-m-2} \end{pmatrix} \boldsymbol{M_b^T v_d^T} \tag{13}$$

Define $\boldsymbol{c_A} = \boldsymbol{v_c M_a}$ and $\boldsymbol{B_d^T} = \boldsymbol{M_b^T v_d^T}$. Suppose the current time is t, by placing the time subscript of the output in Equation (13) from $t + t_d$ to $t + t_d + N_2 - 1$, the following equation group of the relationship between the system output and tracking error can be derived.

The algebraic equations in (14) can be written in a vector-matrix form, that is, the future system predictive output $\boldsymbol{\tilde{y}_f}$ as a function of the controller parameter $\boldsymbol{k_{pid}}$.

$$\boldsymbol{\tilde{y}_f} = \boldsymbol{\Xi_y k_{pid}} + \boldsymbol{\Gamma_y y_0} \tag{15}$$

where $\boldsymbol{\Xi_y} = \boldsymbol{C_A^{y}}^{-1} \boldsymbol{E_r^y}$, $\boldsymbol{\Gamma_y} = \boldsymbol{C_A^{y}}^{-1} \boldsymbol{C_A^0}$ and $\boldsymbol{E_r^y} = \boldsymbol{E_r M_b^T T_s^{pid}}$. The definitions of all symbols are given below.

$$\boldsymbol{C_A^y} = \begin{pmatrix} \boldsymbol{c_A}(1) & 0 & & \cdots & 0 \\ \boldsymbol{c_A}(2) & \boldsymbol{c_A}(1) & & & \vdots \\ \vdots & & \ddots & & \\ 0 & \cdots & \boldsymbol{c_A}(n+2) & \cdots & \boldsymbol{c_A}(1) \end{pmatrix} \tag{16}$$

$$\boldsymbol{E_r} = \begin{pmatrix} e_t & e_{t-1} & \cdots & e_{t-m-2} \\ e_{t+1} & e_t & & e_{t-m-1} \\ \vdots & & \ddots & \vdots \\ e_{t+N_2-1} & e_{t+N_2-2} & \cdots & e_{t+N_2-m-3} \end{pmatrix} \tag{17}$$

and

$$\boldsymbol{C_A^0} = \begin{pmatrix} -\boldsymbol{c_A}(n+2) & \cdots & -\boldsymbol{c_A}(3) & -\boldsymbol{c_A}(2) \\ & \ddots & -\boldsymbol{c_A}(4) & -\boldsymbol{c_A}(3) \\ \vdots & & & \vdots \\ 0 & & \cdots & 0 \end{pmatrix} \tag{18}$$

The vectors of the future and historical system output are

$$\boldsymbol{\tilde{y}_f} = \begin{pmatrix} y_{t+t_d}\, y_{t+t_d+1} \cdots y_{t+t_d+N_2-1} \end{pmatrix}^T \tag{19}$$

$$\boldsymbol{y_0} = \begin{pmatrix} y_{t-m-1} \cdots y_{t+t_d-2}\, y_{t+t_d-1} \end{pmatrix}^T \tag{20}$$

As we know, the cost function in MPC algorithm is composed of two or more items. In the next subsection, the future system input in N_u control horizon, where $1 \leq N_u \leq N_2$, is derived.

Future control sequence

From Equation (2), it is easy to obtain the relationship between the future system input and tracking error. Writing it in a vector-matrix form with the same procedure of the derivation of system future predictive output,

$$\boldsymbol{u_f} = \boldsymbol{\Xi_u k_{pid}} + \boldsymbol{\Gamma_u u_0} \tag{21}$$

where $\boldsymbol{\Xi_u} = \boldsymbol{C^u}^{-1} \boldsymbol{E_r^u}$, $\boldsymbol{\Gamma_u} = -\boldsymbol{C^u}^{-1} c_1$ and $\boldsymbol{E_r^u} = \boldsymbol{E_r(1:N_u, 1:3) T_s^{pid}}$. Matrix $\boldsymbol{E_r(1:N_u, 1:3)}$ means its elements are extracted from the first N_u rows and three columns of the matrix $\boldsymbol{E_r}$.

$$\boldsymbol{C^u} = \begin{pmatrix} c_0 & 0 & \cdots & & 0 \\ c_1 & c_0 & 0 & \cdots & \vdots \\ \vdots & & \ddots & & \\ 0 & \cdots & 0 & c_1 & c_0 \end{pmatrix} \tag{22}$$

The vectors of the future and historical system input are

$$\boldsymbol{u_f} = \begin{pmatrix} u_t\, u_{t+1} \cdots u_{t+N_u-1} \end{pmatrix}^T \tag{23}$$

$$\boldsymbol{u_0} = \begin{pmatrix} u_{t-1}\, 0 \cdots 0 \end{pmatrix}^T \tag{24}$$

$$\begin{aligned}\boldsymbol{c_A}(1)y_{t+t_d}+\boldsymbol{c_A}(2)y_{t+t_d-1}+\cdots+\boldsymbol{c_A}(n+2)y_{t-m-1}&=\boldsymbol{B_d}(1)e_t+\cdots+\boldsymbol{B_d}(m+3)e_{t-m-2}\\ \boldsymbol{c_A}(1)y_{t+t_d+1}+\boldsymbol{c_A}(2)y_{t+t_d}+\cdots+\boldsymbol{c_A}(n+2)y_{t-m}&=\boldsymbol{B_d}(1)e_{t+1}+\cdots+\boldsymbol{B_d}(m+3)e_{t-m-1}\\ &\vdots\\ \boldsymbol{c_A}(1)y_{t+t_d+N_2-1}+\boldsymbol{c_A}(2)y_{t+t_d+N_2-2}+\cdots+\boldsymbol{c_A}(n+2)y_{t+N_2-m-2}&=\boldsymbol{B_d}(1)e_{t+N_2-1}+\cdots+\boldsymbol{B_d}(m+3)e_{t+N_2-m-3}\end{aligned} \tag{14}$$

Constraints and the cost function

Almost every practical controlled plant considers the constraints in the output, input and the change of the input. Take the limitation of the maximum output overshoot $\boldsymbol{y_{ub}}$ and the operating limits $\boldsymbol{u_{ub}}$ for example.

$$\begin{aligned}&\tilde{\boldsymbol{y}}_{\boldsymbol{f}}\le \boldsymbol{y_{ub}}\\ &\boldsymbol{u_f}\le \boldsymbol{u_{ub}}\end{aligned} \tag{25}$$

Substituting Equation (15) and Equation (21) into the above equation, it can be written in the following regular form

$$\boldsymbol{A_{con}}\boldsymbol{k_{pid}}\le\boldsymbol{b_{con}} \tag{26}$$

where

$$\begin{aligned}\boldsymbol{A_{con}}&=\begin{pmatrix}\boldsymbol{\Xi_y}\\ \boldsymbol{\Xi_u}\end{pmatrix}\\ \boldsymbol{b_{con}}&=\begin{pmatrix}\boldsymbol{y_{ub}}-\boldsymbol{\Gamma_y}\boldsymbol{y_0}\\ \boldsymbol{u_{ub}}-\boldsymbol{\Gamma_u}\boldsymbol{u_0}\end{pmatrix}\end{aligned} \tag{27}$$

The on-line tuning problem is to minimize the cost function constructed by the tracking performance and the control energy. And Equation (26) forms a linear constraint of the cost function.

$$\begin{aligned}\min_{\boldsymbol{k_{pid}}}\quad & J=\|\boldsymbol{r_f}-\tilde{\boldsymbol{y}}_{\boldsymbol{f}}\|^2+\lambda\|\boldsymbol{u_f}\|^2\\ s.t.\quad & Eq.\ (15),\ Eq.\ (21)\ \&\ Eq.\ (26)\end{aligned} \tag{28}$$

where $\boldsymbol{r_f}$ is the reference trajectory and λ is the control weight coefficient.

4 Simulation results

In the simulation, all the transfer functions in S domain are transfered into the form of Eq.(9) according to the backward difference method, that is

$$\begin{aligned}&z^{-1}=e^{-T_s s}\approx 1-T_s s\\ &\Rightarrow s=\frac{z-1}{T_s z}\end{aligned} \tag{29}$$

4.1 Study of the control weight

In this section, the P controller synthesis for a VSCs system is first introduced to verify the proposed on-line tuning method and the effect of the control weight on the system performance in the cost function is illustrated. The model of a two-level VSCs for high voltage direct current (HVDC) is given as

$$G_p(s)=\frac{1}{Ts}e^{-\tau s} \tag{30}$$

where $T=0.02$ is the time constant and equals to the capacitor value in a VSCs system, and time delay $\tau=7.4\times10^{-4}$s. For the detailed physical meaning of the parameters, see Wu et al. (2016). The sampling time T_s is chosen to be 0.74×10^{-4}s, so the delay t_d in the discrete domain is 10 sampling intervals. The change of the P controller parameter K_p, the dynamic response and the system input under different control weighting λ compared with the off-line simulation results in the continous domain are shown in Figures 3, 4 and 5. The numerical results are given in Table 1, where E_y and E_u are the 2-norm of the tracking error and control signal multiplying the square root of the sampling time T_s, that is the square root of the integral square error (ISE), which are equivalent to the definitions in continuous system in Wu et al. (2016).

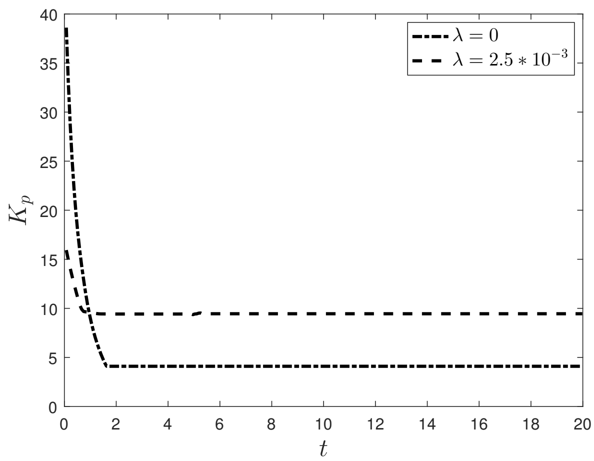

Figure 3 Change of K_p with different control weight.

Table 1 Numerical results of P control.

Type	E_y	E_u	$t_s(ms)$
$\lambda=0$	0.0305	0.6739	2.00
$\lambda=2.5\times10^{-3}$	0.0361	0.4088	5.33
$K_p=22.4$	0.0328	0.7355	6.60
$K_p=11.2$	0.0369	0.4130	3.73

Figures 3-5 and Table 1 reveal the following observations:

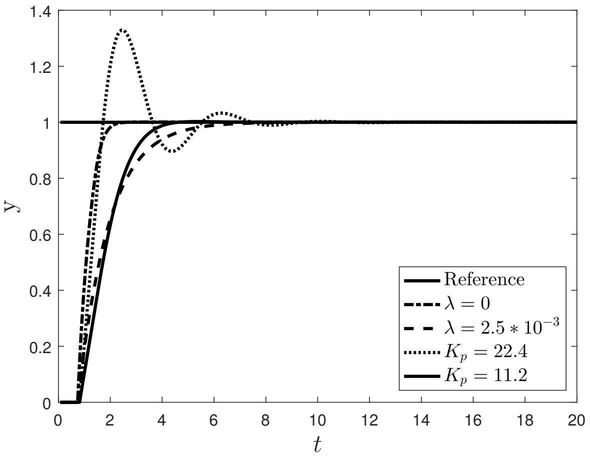

Figure 4 System response with different control weight.

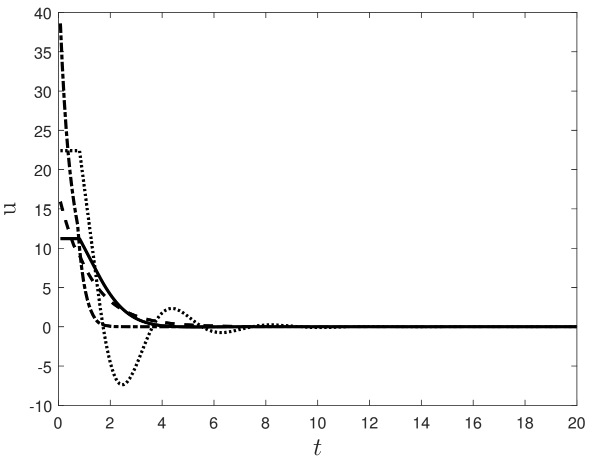

Figure 5 System input with different control weight.

Our previous work provides a range of K_p from 11.2 to 22.4 which makes the control energy and tracking error minimum respectively. The final value of K_p in this range is user-decided according to the system requirements and in this paper the change of K_p depends on the choice of λ. As the weighting for the control energy increases, the tracking performance gets worse. However, the energy consumption is reduced. To make the energy consumption minimum, the value of K_p is the lower boundary of its range and accordingly λ should increase infinitely which makes K_p close to zero and inadvisable.

The system performance of $K_p = 22.4$ is compared to the results of $\lambda = 0$, both are to minimize the value of ISE. Obviously, the on-line parameter optimization not only produces a better tracking performance, but also reduces the control energy and the settling time, compared to the fixed off-line numerical result. However, the control signal at the beginning is very big. In the next section, we are discussing the effect of the constraints in the proposed method.

4.2 Study of the constraints

A heating, ventilation, and air conditioning (HVAC) system can be modelled as a FOPDT system (Xu and Li (2007)), which is also widely used to describe many industrial processes. The three parameters, namely amplification gain K, time constant T and time delay τ can be estimated from the step response curve of the system.

$$G_p(s) = \frac{K}{Ts+1}e^{-\tau s} \tag{31}$$

The parameters are chosen as $K = 72$, $T = 60$ and $\tau = 5m$ from Bai and Zhang (2007). Figures 6-7 show the on-line tuning of the controller parameters K_p and K_i. The system response and control signal are given in Figures 8-9 under the constraints on the system output and input, which are compared to the results obtained in our previous work. The numerical results are summarised in Table 2.

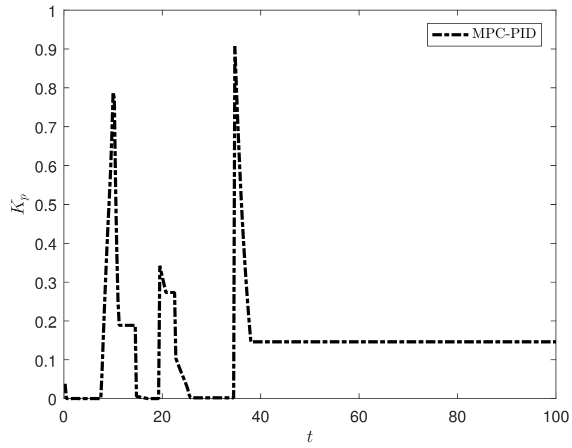

Figure 6 On-line tuning of K_p.

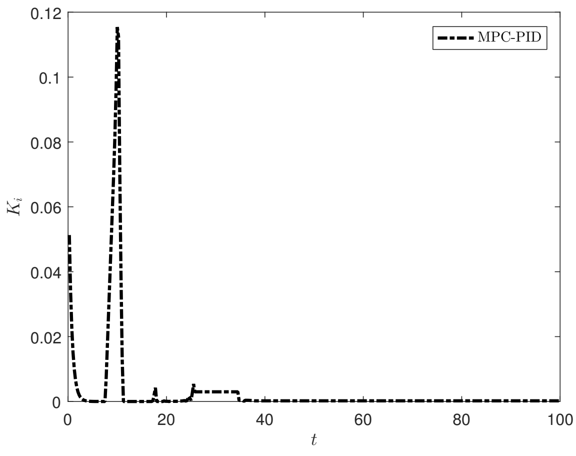

Figure 7 On-line tuning of K_i.

Figures 8-9 and Table 2 show that the controller parameters obtained with the method we proposed in this paper can produce a good tracking performance and give a low energy consumption with the consideration of the constraints on the system input and output, which is very intuitional to control the overshoot and control signal to the desired requirements. It provides a convenient and effective method to adjust

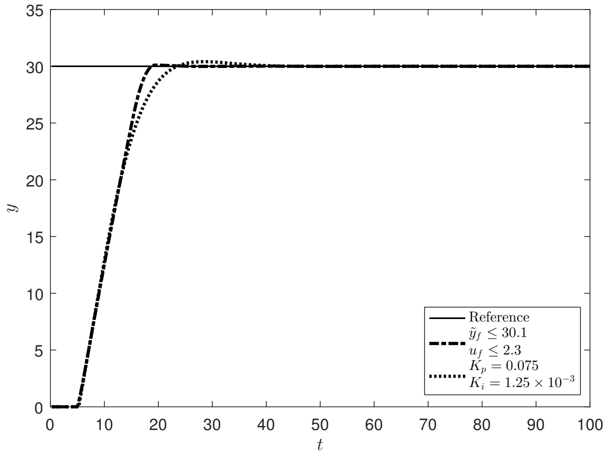

Figure 8 System response under the constraints.

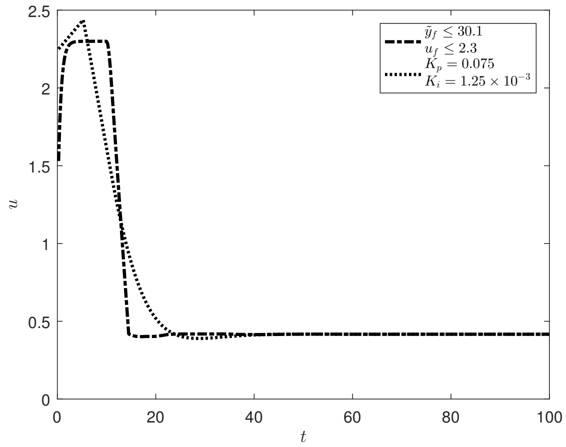

Figure 9 Control signal under the constraints.

Table 2 Numerical results of PI control.

Constraints	E_y	E_u	y_{max}	u_{max}	$t_s(m)$
$\tilde{y}_f \leq 30.1$ $u_f \leq 2.3$	89.59	8.68	30.1	2.3	18.75
$K_p = 0.075$ $K_i = 1.25 \times 10^{-3}$	89.85	8.47	30.41	2.44	21.30

PI controller parameters to meet the requirements on tracking performance, control energy, overshoot and settling time by adding the constraints to the optimal function.

4.3 Study of the change of the setpoint

In this section, the PID controller parameters on-line optimization with the change of the setpoint is verified and compared to the control effect with the numerical optimization approach NOA-PID algorithm proposed in C. B. Kadu and S. B. Lukare (2015). Likewise, Figures 10-12 show the change of PID controller parameters. The control effect and control sequence are given in Figures 13-14. The numerical results are summarized in Table 3. The model below is one of the simulation example in C. B. Kadu and S. B. Lukare (2015).

$$G_p(s) = \frac{e^{-2.5s}}{s^2 + 2s + 1} \tag{32}$$

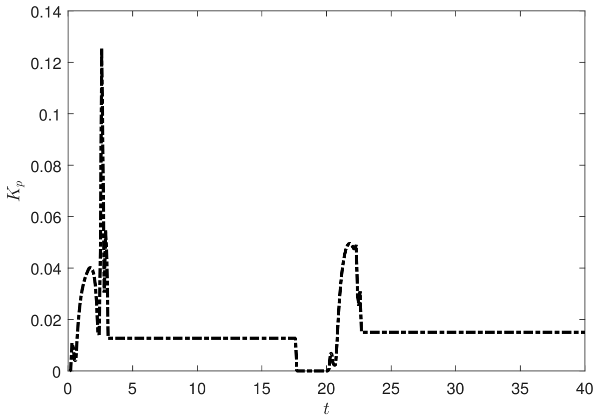

Figure 10 On-line tuning of controller parameter K_p.

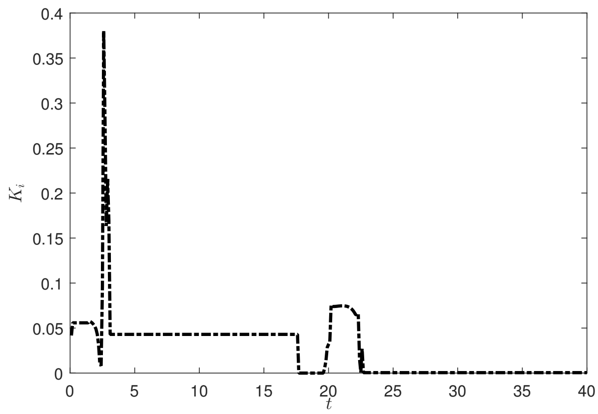

Figure 11 On-line tuning of controller parameter K_i.

Figures 10-14 and numerical results show the effectiveness of the proposed MPC-PID method. It greatly improves the control effect than the fixed controller parameters.

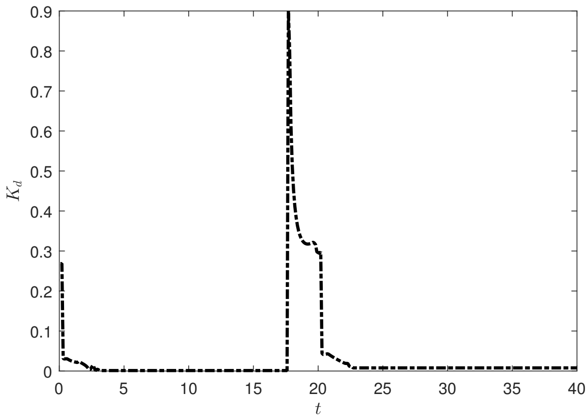

Figure 12 On-line tuning of controller parameter K_d.

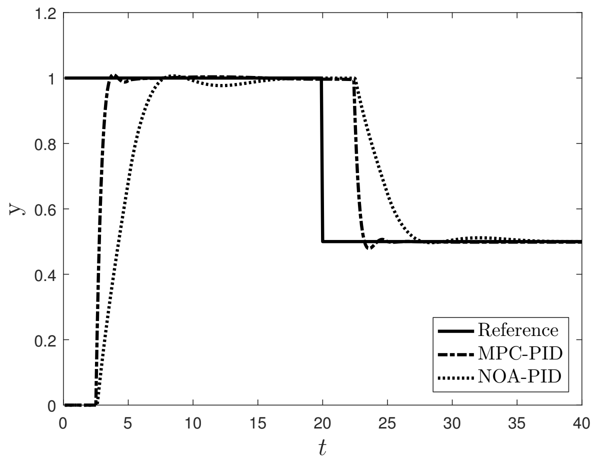

Figure 13 System response with the change of setpoint.

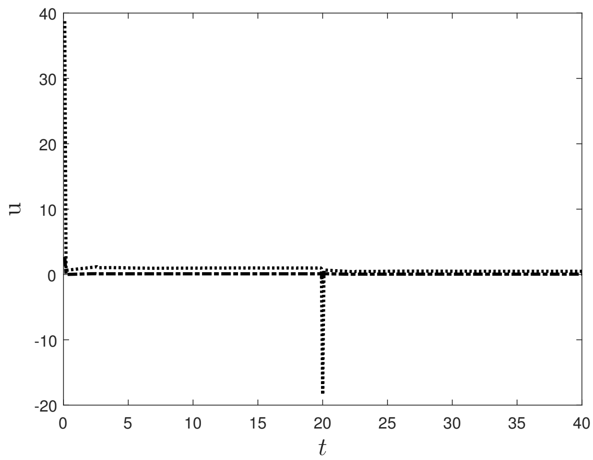

Figure 14 System input with the change of setpoint.

Table 3 Numerical results of PID control.

Type	E_y	E_u
MPC-PID	1.8136	1.0759
NOA-PID	2.1969	14.4545
Improvement	17.45%	92.56%

5 Conclusions and future work

Inspired by the framework of MPC, this paper extends the off-line P and PI controller parameters optimization in continuous systems to the on-line PID parameter adjustment in discrete systems. The two main contributions of this paper are: first, we proposed a convenient method to realize on-line optimization for PID controller parameters, which not only produces good tracking performance but also gives a low control energy; second, it is easy to deal with constraints in the PID controller design to meet the desired requirements on the overshoot and operation limit. The simulation results verify the effectiveness of the proposed MPC-PID method.

As a future work, the idea of this paper will be extended to the MIMO sytem. And the model mismatch, disturbance or some other situations will be considered in the on-line PID controller design. Though comparing with tuning three parameters of the PID controller, selecting a control weight λ is much easier, the effect of λ on the control performance will be further studied.

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