Floquet Mechanism for Non-Abelian Fractional Quantum Hall States

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Three-body correlations, which arise between spin-polarized electrons in the first excited Landau level, are believed to play a key role in the emergence of enigmatic non-Abelian fractional quantum Hall (FQH) effects. Inspired by recent advances in Floquet engineering, we investigate periodic driving of anisotropic two-body interactions as a route for controllably creating and tuning effective three-body interactions in the FQH regime. We develop an analytic formalism to describe this Floquet-FQH protocol, which is distinct from previous approaches that instead focus on bandstructure engineering via modulation of single-particle hopping terms. By systematically analyzing the resulting interactions using generalized pseudopotentials, we show that our Floquet-FQH approach leads to repulsive as well as attractive three-body interactions that are highly tunable and support a variety of non-Abelian multicomponent FQH states. Finally, we propose an implementation of the protocol in optically dressed ultracold polar molecules with modulated Rabi frequencies.

Topological phases exhibit enticing prospects for fault-tolerant quantum computation [1, 2] owing to their exotic quasiparticle excitations [3–5]. These phases are believed to arise from an interplay between the Coulomb interaction, Landau level quantization and complete spin polarization in 2D electronic systems [6], as suggested by the observation of even-denominator FQH plateaus in semiconductors [7] and recently in bilayer graphene [8, 9]. The unexpected even-denominator plateaus are explained by adiabatic continuity [10–12] between the underlying gapped many-electron state and the ground state of a system with special 3-body electronic interactions [13, 14]. Such 3-body interactions condense the electrons into a strongly-correlated quantum state where they fractionalize into non-Abelian Ising anyons [3]. More generally, multi-body interactions are anticipated to give rise to other types of non-Abelian anyons [15–17].

Conventionally, effective 3-body interactions arise due to Coulomb interactions and virtual excitations between Landau levels (LLs) [18–22], a process suppressed by the LL splitting in a magnetic field, given by the cyclotron energy $\hbar\omega_c = \frac{\hbar eB}{mc}$. At the same time, the incompressibility gap, which determines the stability of a FQH state, scales as $e^2/\epsilon\ell_B$, where $\ell_B = \sqrt{\hbar/eB}$ is the magnetic length. Thus, the effect of 3-body interactions can typically only be enhanced at the expense of reducing the energy gap, which weakens the FQH state.

Inspired by recent progress in "Floquet engineering" [23–28], we propose an alternative method to realize effective 3-body interactions and hence stabilize various non-Abelian FQH states. Our approach consists of periodically modulating (2-body) interactions, specifically the repulsion between spatially separated electrons. Key to our idea is the non-commutativity of the Girvin-MacDonald-Platzman (GMP) algebra [29, 30] describing the electron density operators projected to a LL, which is the defining property of both continuum FQH states [29] and their lattice analogs, the fractional Chern insulators (FCIs) [31]. We show that, owing to this algebra, the effective, *static* Hamiltonian that arises when a generic anisotropic FQH system is driven at high frequencies contains a rich set of many-body interactions which scale with the inverse driving frequency, rather than the LL gap. In particular, desired 3-body multicomponent (spin) interactions can be engineered by time modulation of realistic 2-body interactions. More generally, we systematically analyze the interactions resulting from our "Floquet-FQH" protocol using the framework of generalized pseudopotentials [32], and show that the drive can also generate *attractive* 3-body interactions.

Finally, we discuss a realistic implementation of the Floquet-FQH protocol in ultracold molecules optically dressed with modulated Rabi frequencies, whose static version was previously established to host FCI states [33]. We note that our approach is conceptually different from previous Floquet proposals [34–37] and experiments [38] which focused on topological band engineering via modulation of (single-body) kinetic terms; it is also distinct from works [39–42], which modulated *on-site* 2-body interactions to probe tunnelling phenomena and Mott/superfluid phases.

Two key inspirations:–(i) A defining feature of FQH systems [29] is the GMP algebra

$$[\bar{\rho}^{\sigma}_{\mathbf{q}}, \bar{\rho}^{\sigma'}_{\mathbf{q}'}] = 2i\delta_{\sigma,\sigma'} \sin\frac{\hat{\mathbf{z}}\cdot(\mathbf{q}\times\mathbf{q}')\ell_B^2}{2}\bar{\rho}^{\sigma}_{\mathbf{q}+\mathbf{q}'}, \qquad (1)$$

obeyed by the density operators $\bar{\rho}^{\sigma}_{\mathbf{q}} \equiv \mathcal{P}\rho^{\sigma}_{\mathbf{q}}\mathcal{P} = \sum_j e^{i\mathbf{q}\cdot\mathbf{R}^{\sigma}_j}$, projected to a given LL via $\mathcal{P}$. Here, $R^{\sigma,a}_j \equiv r^{\sigma,a}_j + \ell_B^2\epsilon^{ab}\Pi^{\sigma}_{j,b}$ denote the guiding-center coordinates of j-th particle with spin σ [6], ϵ^{ab} is the antisym-

metric tensor ($a = x, y$). Note that $R_j^{\sigma,a}$ differ from the position coordinates r_j^σ by the canonical momentum $\Pi_j^{\sigma,a} = q_j^{\sigma,a} - eA^a$ in a magnetic field $B = \epsilon^{ab}\partial_a A_b$, and thus $R_j^{\sigma,a}$ do not commute. The same density algebra, Eq. (1), is also obeyed by the FCIs in the thermodynamic and long-wavelength limit [31, 43, 44], with magnetic field replaced by mean Berry curvature [44]. We will work in this limit, and will henceforth not distinguish between FQH and FCI. Our Floquet-FQH approach is based on the observation that the repeated application of the GMP algebra produces $(2N-1)$-body terms from the commutator of two N-body terms. In particular, the commutator of two 2-body terms yields a potentially desirable 3-body term.

(ii) At high frequencies $\Omega = 2\pi/T$, the stroboscopic dynamics of a periodically-driven system, $H(t) = \sum_l e^{il\Omega t} H_l$, can be captured by the static effective Hamiltonian

$$H^{\text{eff}} = H_0 + \frac{1}{\hbar\Omega}\sum_l \frac{1}{l}[H_l, H_{-l}] + \cdots , \qquad (2)$$

obtained, e.g., from the Magnus or other equivalent high-frequency expansions [24, 45, 46]. Most saliently, Eq. (2) involves commutators which represent the renormalizing effects of the drive on the interactions. Thus, we see that dynamically modulating a FQH system, combined with the structure of the GMP algebra in (i), is a natural way to realize higher-body interaction terms, Fig. 1(a).

Importance of anisotropy.–A necessary condition for our Floquet-FQH approach is that the commutators in Eq. (2) do not vanish (this does happen if the system is rotationally symmetric). Such commutators, however, can be shown to generically survive in *anisotropic* FQH systems. Our protocol is thus targeted at FQH systems with anisotropic interactions; note that this is not a major restriction because anisotropy is ubiquitous in many setups: it can be induced by tilting the magnetic field [47–49], and it is intrinsically large in FCIs [32].

We remark that the anisotropy of FQH/FCI systems can be quantified using standard Haldane pseudopotentials (PPs) and their generalizations to N-particle interactions with internal degrees of freedom [50–52], which we briefly review now. First, one defines a relative angular momentum eigenbasis, $|m\rangle$, in the LL-projected Hilbert space of N particles[53] with a given permutation symmetry type λ [50, 51]. Any isotropic interaction potential $V_{\mathbf{q}}$ can be expanded in terms of PPs, $U^{N,\lambda}_{m,\mathbf{q}}$, which form a complete orthonormal basis for N-body operators. Below we will use the coefficients in this expansion, c^λ_m for fixed $N = 3$, in order to characterize the 3-body interactions generated by the Floquet-FQH protocol. The same formalism can describe anisotropy by a redefinition of $U^{N,\lambda}_{m,\mathbf{q}} \to U^{N,\lambda}_{m,\Delta m,\pm,\mathbf{q}}$, where $\Delta m = 0, 2, 4, \ldots$ and $\pm$ denote the discrete symmetry ($U^{N,\lambda}_{m,\Delta m} \propto (q_x + iq_y)^{\Delta m}$) and directionality of the anisotropic PP [32, 44]. The coefficients of generalized PPs, $c^{N,\lambda}_{m,\Delta m,\pm}$, completely characterize any translation-invariant interaction and determine which FQH states are energetically favored [32]. Note that for $\Delta m = 0$, anisotropic PPs reduce to the standard Haldane PPs [6, 54].

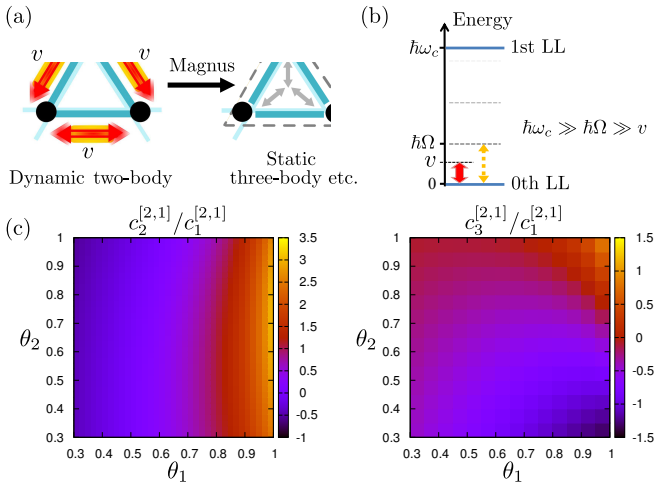

FIG. 1. (a) Time-modulated 2-body interactions give rise to an effective 3-body static interaction connecting different sublattices at leading order in Ω^{-1} (Eq. 6). (b) Energy hierarchy for the setup, with LL (or band) gap $\hbar\omega_c$ much larger than the driving frequency scale $\hbar\Omega$, which should also dominate the interaction v. (c) 3-body PP coefficient ratios, c_2/c_1 and c_3/c_1, for particles with opposite spins ($\lambda = [2, 1]$). The driven two-body interaction is given in Eq. (7).

Floquet-FQH system.–Building on the two key inspirations above, we consider periodically driving an anisotropic FQH/FCI system such that its two-body interaction term is time-modulated while the single-body term remains static, for instance by ultrafast rotation or by appropriate optical driving as detailed later:

$$H_{\text{FQH}}(t) = H_{\text{nonint}} + H_{\text{int}}(t). \qquad (3)$$

Here, $H_{\text{nonint}} = \frac{1}{2m}\sum_{i,\sigma} g^{ab}\Pi^\sigma_{a,i}\Pi^\sigma_{b,i}$, and the metric tensor g^{ab} encodes the anisotropy [47, 55]. We characterize the 2-body interaction $H_{\text{int}}(t)$ with its Fourier harmonics V_l and their momentum-space profiles $V^{\sigma\sigma',l}_{\mathbf{q}}$:

$$H_{\text{int}}(t) = \sum_l e^{i\Omega lt/\hbar} V_l = \sum_{\mathbf{q},l,\sigma\sigma'} e^{i\Omega lt/\hbar} V^{\sigma\sigma',l}_{\mathbf{q}} \bar{\rho}^\sigma_{\mathbf{q}} \bar{\rho}^{\sigma'}_{-\mathbf{q}}. \qquad (4)$$

Let us comment on the three relevant energy scales, shown in Fig. 1(b), that are behind Eq. (3): (i) the cyclotron frequency $\hbar\omega_c$, set by the single-body term, (ii) the driving frequency $\hbar\Omega$, and (iii) the typical interaction strength v, given by the averaged $|V^l_{\mathbf{q}}|$. The cyclotron frequency splits the Hilbert space into energetically separated LLs, while the dynamically modulated interaction connects LLs with amplitude v, whilst simultaneously allowing energy to be absorbed or emitted in multiples of $\hbar\Omega$. To achieve interesting physics, we consider smooth (strictly low-harmonic) driving obeying the

hierarchy $\hbar\omega_c \gg \hbar\Omega \gg v$, i.e., with driving being "high frequency" compared to v but not to $\hbar\omega_c$.

The above considerations allow us to derive an effective static description of the system at stroboscopic times, such that there is approximate energy conservation and an effective long-lived ground state [56]. To see this, note that LL mixing is suppressed due to large LL gaps, high frequency driving and the absence of high order harmonics. Hence we obtain, via a generalized Schrieffer-Wolff transformation, an effective dynamical description of the system within each LL [44]:

$$H^{\rm LL}_{\rm FQH}(t) = H_{\rm nonint} + \mathcal{P}H_{\rm int}(t)\mathcal{P} \to \mathcal{P}H_{\rm int}(t)\mathcal{P} \qquad (5)$$

and $H_{\rm nonint}$ drops out as an irrelevant constant.

We can further employ Eq. (2) on the effective dynamical Hamiltonian Eq. (5) to obtain the effective *static* description $H^{\rm eff}$ of the system within the lowest LL. This description persists up to the exponentially long heating timescale $t_h \sim \frac{\hbar}{v}\exp(\text{const.}\times\Omega/v)$ [57–59], which is estimated to be on the order of years for the example of a cold-atom setup in Fig. 3 below. Assuming a single driving frequency Ω, we have $H^{\rm eff} \approx V_0 + \frac{1}{\hbar\Omega}[V_1, V_{-1}]$. Using Eq. (1), after some commutator algrebra [44], we obtain $H^{\rm eff} \approx H_{2b} + H_{3b}$, where the 2-body term H_{2b} is the original static profile V_0 modified by an operator ordering correction [44], and the effective 3-body term is

$$\begin{aligned} H_{3b} = -\frac{4}{3\hbar\Omega}\sum_{\alpha,\beta,\gamma=\uparrow,\downarrow}\sum_{\mathbf{q},\mathbf{q}'}{\rm Im}^{-}\Big\{&2V^{\beta\alpha *}_{\mathbf{q}}V^{\beta\gamma}_{\mathbf{q}'} + V^{\beta\gamma *}_{\mathbf{q}'}V^{\gamma\alpha}_{\mathbf{q}-\mathbf{q}'} \\ &+ V^{\alpha\gamma *}_{\mathbf{q}'-\mathbf{q}}V^{\beta\alpha}_{\mathbf{q}} + V^{\gamma\beta *}_{\mathbf{q}'-\mathbf{q}}V^{\alpha\gamma}_{\mathbf{q}} + V^{\gamma\alpha *}_{\mathbf{q}'}V^{\alpha\beta}_{\mathbf{q}-\mathbf{q}'}\Big\} \\ &\times \sin\frac{\hat{\mathbf{z}}\cdot(\mathbf{q}\times\mathbf{q}')}{2}\bar{\rho}^{\alpha}_{\mathbf{q}}\bar{\rho}^{\beta}_{\mathbf{q}'-\mathbf{q}}\bar{\rho}^{\gamma}_{-\mathbf{q}'}, \end{aligned} \qquad (6)$$

where ${\rm Im}^{-}\{f_{\mathbf{q},\mathbf{q}'}\} \equiv (f_{\mathbf{q},\mathbf{q}'} - f^{*}_{-\mathbf{q},-\mathbf{q}'})/(2i)$ and $\ell_B = 1$.

The 3-body interaction in Eq. (6) is our central result. This interaction emerges from the products of Fourier components $V^{\sigma\sigma'}_{\mathbf{q}}$ of the original interaction, see Fig. 1(a). Due to ${\rm Im}^{-}$, Eq. (6) does not vanish only if $V^{\sigma\sigma'}_{\mathbf{q}}$ (and index permutations) are complex, i.e., only if the system breaks inversion symmetry, and phase differences exist between the modulations of different interaction components. Consequently, H_{3b} is non-zero only in multicomponent anisotropic FQH systems i.e. FCIs with multiatomic unit cells. This peculiar component dependence makes our Floquet approach particularly suited for engineering multicomponent FQH parent Hamiltonians. Finally, we observe that $H^{\rm eff}$ is not constrained to be repulsive, and could be used to cancel other repulsive interaction terms in the original interaction.

Illustrative examples.–We now illustrate the versatility of the Floquet-FQH approach by some examples of interactions and many-body states it could stabilize. First, consider driving a 2-body interaction $e^{i\Omega t}\sum_{\mathbf{q}}V_{\mathbf{q}}\bar{\rho}^{\uparrow}_{\mathbf{q}}\bar{\rho}^{\downarrow}_{-\mathbf{q}}$ + h.c., which consists of the simplest anisotropic PPs with $\Delta m = 2$ [32, 44]:

$$V_{\mathbf{q}} = \cos\theta_1 U_{0,2} + \sin\theta_1\cos\theta_2 U_{1,2} + \sin\theta_1\sin\theta_2 U_{2,2}, \quad (7)$$

where θ_1,θ_2 are free parameters that keep the overall interaction strength fixed, while the prefactors of $U_{m,\Delta m}$ can be negative. Eq. (7) produces a range of Floquet 3-body interactions between particles with opposite spins via Eq. (6). The resulting PP coefficient ratios, $c^{[2,1]}_2/c^{[2,1]}_1$ and $c^{[2,1]}_3/c^{[2,1]}_1$, are shown in Fig. 1(c). We see that PP ratios span a wide range, and can become attractive in certain parameter regimes or strongly suppressed, e.g., $U^{3,[2,1]}_1$ and $U^{3,[2,1]}_2$ might be of comparable strength to each other and twice larger than $U^{3,[2,1]}_3$.

Having demonstrated the tunability of 3-body Floquet PPs, we next consider two examples of exotic FQH states that they could naturally stabilize: the interlayer Pfaffian (iPf) state [60, 61] and the $\nu = 1$ permanent state ("111-perm") introduced in Ref. 3 (see also Ref. 62). The iPf state is a gapped state at filling factor $\nu = 2/3$ with non-Abelian Ising anyons, as well as spin-charge separation [63–66]. By contrast, the 111-perm state is an intriguing gapless state[14, 67–69] that is governed by a non-unitary conformal field theory [3, 70], and represents a critical point between the integer quantum Hall ferromagnet and a paramagnet [14].

The stability of these FQH states is determined not only by the generated 3-body PPs, which scale as v^2/Ω, but also by original 2-body PPs, which scale as v, and operator ordering corrections to them from the drive (also of the order v^2/Ω) [44]. Thus, if we target a specific state, the original 2-body interaction should be sufficiently "close" to its model interaction. Many non-Abelian FQH states can be realized in this way, e.g., the ground state of 2-body PPs, $U^{[1,1]}_1$ and $U^{[1,1]}_3$, is believed to be in the Moore-Read phase [71]. In the presence of weak anisotropy, the drive could then further enhance such states by amplifying the 3-body correlations, and thus the robustness of the FQH state. We now illustrate this using exact diagonalizations of continuum FQH systems on the sphere [44][72].

For the iPf we choose the initial "hollow core" interaction consisting of 2-body PPs, $U^{[2]}_1$ and $U^{[1,1]}_1$, whose strength is fixed to 1. The dominant Floquet corrections are 2-body $U^{[2]}_0$, and 3-body $U^{N=3,[2,1]}_1$ and $U^{N=3,[2,1]}_2$. In Fig. 2(a) we show the extrapolated neutral gap of the system in the presence of these perturbations. We assume, for simplicity, that 3-body PPs are of equal magnitude. The full line in Fig. 2(a) marks the value of the gap $\Delta E = 0.2$, while the dashed line denotes points where the overlap of the ground state and the iPf state is equal to 90% [44]. Thus, we see that a combination of 2-body and 3-body Floquet terms results in the large region of a robust iPf phase with non-Abelian correlations and a large gap (top right corner of Fig. 2(a)).

Similarly, our Floquet approach is also suited for sta-

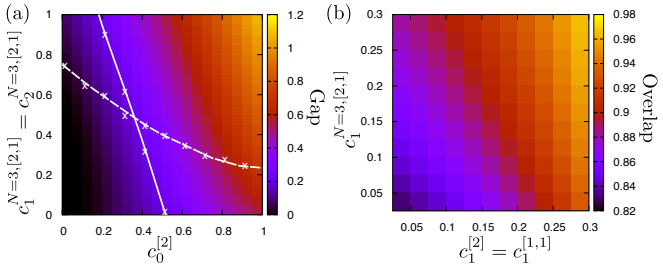

FIG. 2. (a) Extrapolated neutral gap (for system sizes $N \leq 10$) as a function of Floquet 2-body PP, $c_0^{[2]}$, and 3-body PPs, $c_1^{N=3,[2,1]} = c_2^{N=3,[2,1]}$. Full line traces gap $\Delta E = 0.2$, while dashed line denotes values of the PPs for which the overlap of the ground state and the iPf state (for $N = 10$ electrons) is 90%. The robust iPf phase is stabilized in the top right corner of the phase diagram. (b) Overlap with 111-perm state is increased by a combination of Floquet 2-body, $c_1^{[1,1]} = c_1^{[2]}$, and 3-body $c_1^{N=3,[2,1]}$ perturbations. Data is for 14 electrons on the sphere.

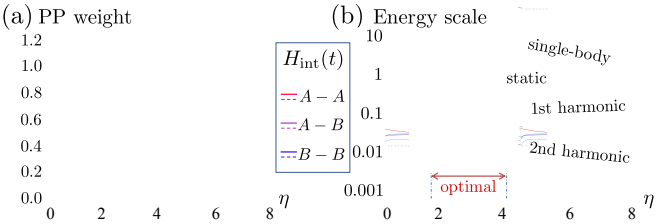

FIG. 3. (a) Coefficients of 2-body PPs c_1, c_2, c_3 and c_4 as a function of η for illustrative parameters yielding a band of flatness ≈ 3 [44]. Solid/Dashed curves represent dynamic/static contributions, which are colored according to whether they act between AA, AB or BB sites. (b) Comparison between the energy scales of the single-body, static 2-body (v) and lowest two harmonics of the dynamic 2-body parts of $H_{\rm FCI}(t)$ for different η. For all purposes, the 2nd harmonic can be neglected. In the optimal shaded regime we have $v \ll \hbar\Omega \ll \hbar\omega_c$.

bilizing the 111-perm state, which crucially relies on a strong $U_1^{[2,1]}$ [62, 73]. In Fig. 2(b), we fix the initial interaction to be $U_0^{\uparrow\downarrow}$ of magnitude 1. The driving is assumed to generate 2-body PPs $c_1^{[2]} = c_1^{[1,1]}$ and 3-body PPs $c_1^{N=3,[2,1]}$, predominantly. By evaluating the overlap with the 111-perm state, we see that the 111-perm phase is enhanced by these perturbations, with the overlap approaching 1. At the same time, the neutral gap of the system remains very small ($\ll 1$) throughout the phase diagram [44], which is consistent with the gapless phase in the thermodynamic limit [62]. At appropriate filling in bosonic systems, $H^{\rm eff}$ with its tail of higher PPs may also stabilize the related 221-permanent state [73, 74].

Experimental proposal.–In the continuum FQH case, the Floquet protocol can be implemented by modulating the component of the parallel magnetic field. For magnetic fields $B \sim 20$T, this however requires a very large frequency of $\Omega \sim 1$THz. Instead, a more flexible experimental platform to implement the protocol are FCIs [75–81], which naturally possess large anisotropy, non-trivial unit cell structure and tunable interactions [33, 82–85]. We now propose a FCI model of optically driven dipolar spins, realized by trapped dipolar molecules in a 2D optical lattice, which features directional interactions that lead to a direct analogue of [2, 1] 3-body PPs studied above in the continuum FQH case.

Each molecule in the setup possesses a rovibrational ground state, $|\downarrow\rangle = |0,0\rangle$, and three next-lowest $J = 1$ states ($|1,0\rangle$ and $|1,\pm1\rangle$), which are optically dressed to form a single 'dark' state $|\uparrow\rangle = s|1,-1\rangle + v|1,1\rangle + w|1,0\rangle$, where s, v and w are rational functions of the Rabi frequencies associated with optical driving [33, 44]. The $|\uparrow\rangle, |\downarrow\rangle$ states form the effective spin degrees of freedom, which are conserved when the molecules are sufficiently separated such that the physical dipole-dipole interaction between them is much weaker than the bare rotational energy (approximately the Zeeman splitting). In this case, the dipole interaction, together with a strong applied DC field that determines the quantization axis and orbital mixing, is effectively described by hardcore bosons on a lattice with the Hamiltonian [33]: $H_{\rm FCI} = -\sum_{ij} t_{ij} a_i^\dagger a_j + \frac{1}{2}\sum_{i\neq j} V_{ij}\rho_i\rho_j$ where $a_i^\dagger = |\uparrow\rangle_i\langle\downarrow|_i$ is the spin-flip operator and $\rho_i = a_i^\dagger a_i$. Both the effective hopping t_{ij} and Hubbard strength V_{ij} originate from the same physical dipole interaction, and can be independently tuned through $\mathbf{E}$ field and the Rabi parameters s, v, w to give rise to FCI states [33].

By modulating the Rabi parameters, it is possible to keep t_{ij} static while V_{ij} is made time-dependent. For an FCI with 2 components A, B, we can achieve this by dynamically modulating the Rabi parameters:

$$s_A(t) = s_A e^{i\Omega_1 t},\ \ s_B(t) = s_B e^{i\Omega_2 t},$$
$$v_A(t) = v_A e^{i\Omega_2 t},\ \ v_B(t) = v_B e^{i\Omega_1 t},$$
$$w_A(t) = W + W' v_A v_B^* e^{-i\Omega t},\ \ w_B(t) = W + W' s_A^* s_B e^{-i\Omega t},$$

where $\Omega = \Omega_2 - \Omega_1$ sets the driving frequency, and $W = \sqrt{\Lambda(1\mp\gamma)/2}$, $W' = \sqrt{\Lambda(1\pm\gamma)/(2v_A^* v_B s_A^* s_B)}$, $\gamma = \sqrt{1-(v_A^* v_B s_A^* s_B/\Lambda^2)(d_{01}/d_{00})^4}$, with Λ a real tuning parameter and $d_{01} = \langle 1,\pm1|d_z|0,0\rangle$, $d_{00} = \langle 1,0|d_z|0,0\rangle$ dipole transition matrix elements that depend on the applied $\mathbf{E}$ field. The Rabi parameter magnitudes are chosen to optimize the band flatness of the resultant tight-binding FCI Hamiltonian [44], leaving a dynamic 2-body interaction with a single tunable parameter $\eta = 2EId/\hbar^2$, the ratio of the molecular dipole energy Ed to its rotational energy scale $\hbar^2/2I$, I being the moment of inertia. Coefficients of various 2-body PPs are plotted as a function of η in Fig. 3(a), and we see that interactions between A and B sites (purple) dominate for most η. For very small η, the interaction is mostly dynamical, and its rapid sign fluctuations may destabilize the Floquet ground state. The relevant energy scales are shown

in Fig. 3(b). In the optimal regime, $1.5 \lesssim \eta \lesssim 4$, the single-body hoppings (and hence gap) are one to two orders larger than the interaction, thereby satisfying the requisite hierachy $v \ll \hbar\Omega \ll \hbar\omega_c$. At the same time, the static interaction between sublattices is still larger than the dynamic part. Thus, for $\eta \approx 3$ we achieve a direct analog of the above $U_m^{[2,1]}$ 3-body interaction, assuming we are in the thermodynamic limit where the GMP algebra is valid. Away from this limit, details of the Bloch wave functions, inter-band transitions and imperfections of the band flatness [86] could affect the stability of the Floquet FCI state.

Conclusions.–We have presented an approach for generating multicomponent 3-body FQH/FCI interactions, Eq. (6), via driving anisotropic 2-body interactions with inhomogeneous phase offsets. Our approach is valid in the regime $v \ll \hbar\Omega \ll \hbar\omega_c$, and yields an effective 3-body interaction whose magnitude scales like Ω^{-1}, rather than the conventional ω_c^{-1} due to LL mixing. The approach relies on the unique properties of the GMP algebra, and thus applies to both FQH and FCI systems in the thermodynamic limit. We have demonstrated that this approach provides a new route for the exploration of both gapped and gapless multicomponent non-Abelian FQH states, and proposed its implementation in a Floquet FCI of optically dressed dipolar molecules, where time reversal is broken by the asymmetry between the left and right-circularly polarized optical driving.

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