Problemas de módulos para una clase de foliaciones holomorfas.

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Given an holomorphic foliation $\mathcal{F}_0$ on a complex surface S, we want to find out how many different analytic classes there are in the class of all the holomorphic foliations topologically conjugated to $\mathcal{F}_0$. In other words, we want to identify the moduli space of $\mathcal{F}_0$.

We have studied two different contexts. The first one is the local context, where $\mathcal{F}_0$ is an holomorphic germ of foliation in $\mathbb{C}^2$ with a isolated singularity at the origin. The second one is the global context, where $\mathcal{F}_0$ is an holomorphic singular foliation on a compact complex surface which is the total space of a fibre bundle over a non-singular complex curve.

The main results belong to the local context. There are several precedents to this work. Between the most important ones we can cite to D. Cerveau, X. Gómez-Mont, A. Lins Neto, F. Loray, J-F. Mattei, R. Moussu, M. Nicolau and P. Sad. Although most of them treat on a weak moduli space because they use unfoldings or topologically trivial deformations of $\mathcal{F}_0$.

In this memoir we have obtained analogous counterparts and generalizations to the results formulated by the authors cited above, in relation to the weak problem. For instante, we can stand out the generic rigidity for foliations defined by 1-forms of order two. There is another remarkable result about the moduli space for foliations whose annulation order in the singularity is three. This moduli space is a covering over $\mathbb{C} \setminus \{0, 1\}$ whose fundamental group can be interpreted as *symmetries* of their projective holonomy representation. The method for the proof has been adapted to a more general situation: the quasi-homogeneous foliations, for which we have obtained similar results.

Concerning the global context we have obtained a rigidity theorem on fibred surfaces, similar to Ilyashenko's Theorem on $\mathbb{CP}^2$. We particularize this result to the case of ruled surfaces (in which the fibre is the projective line $\mathbb{CP}^1$), where we have studied too the moduli space for some non-rigid foliations (Riccati foliations).

We emphasize that the techniques used in both contexts have a common point: the existence of an holonomy representation which represents quite well the dynamics of the foliation. According to this viewpoint, in the last part of the memoir we propose a new notion of generalized holonomy which could play a similar role for foliations in $\mathbb{CP}^2$ or $\mathbb{CP}^1 \times \mathbb{CP}^1$ other than Riccati foliations. This work finishes with a detailed study of this new notion in a explicit example.