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## Analysis of Stator Slots and Rotor Pole Pairs Combinations of Rotor-Permanent Magnet Flux-Switching Machines

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**Abstract**—*Abstract*—This paper investigates the influence of stator slots and rotor pole pairs combinations on torque performances in rotor permanent magnet flux switching machines. Based on a magnetomotive force permeance model, the candidates of stator slots and rotor pole pairs combinations with higher torque capability can be determined by analysing the permanent magnet magnetomotive force and winding factor. Meanwhile, the candidates with a lower torque ripple can be obtained by referring to the cogging torque, which is related to the greatest common divisor of stator slots and rotor pole pairs. In addition, from the field modulation principle, the rotor-permanent magnet flux-switching machines with the same fundamental magnetic loadings and

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winding factors exhibit identical fundamental harmonic torque, but different modulation harmonic components. Finally, four candidates with attractive torque performance are chosen, and the characteristics are verified by finite element analysis and experiments.

**Index Terms**—Analytical models, finite element analysis, flux switching, permanent magnet machines, rotors, slots and pole pairs combination, torque.

### Nomenclature

$B_{gv}$	Magnetic loading of $v$ -order harmonic			
$B_{Rw}$	Modulated armature reaction flux density			
$D_{so}, D_{si}$	Outer and inner diameters			
$F_{Rw}$	Armature reaction-MMF function			
$F_{RPMb}$	PM-MMF amplitude			
$H_c$	PM coercivity			
$I_{Rmax}$	Maximum value of the phase current			
$J_{sa\_rms}$	Armature current density			
$K_{sv}$	Electrical loading of $v$ -order harmonic			
$L_d, L_q$	$dq$ -axes inductances			
$N_{sp}$	Spoke number of conductors back EMF vectors			
$N_{Rc}$	Coil turns number			
$P_s, P_r, P_{PM}$	Numbers of stator slots, rotor pole pairs			
a	n	d	P	M

pole pairs.

$T_{cog}, T_e$	Cogging torque and electromagnetic torque
$T_{ef}, T_{em}$	Fundamental and modulation harmonic components
$g$	Air gap length
$h_{PM}$	PM height
$k_{fw}$	Flux weakening coefficient
$k_{wv}$	Winding factor of $v$ -order harmonic
$l_a$	Stack length
$w_{PM}, w_{rt}$	PM width and rotor tooth width
$\Lambda_{gap}, \Lambda_{PM}$	Air-gap and PM magnetic permeances
$\Phi_{PMg}$	Air-gap PM flux linkages
$\theta_{st}, \theta_{rt}, \theta_{rs}$	Stator tooth, rotor tooth and rotor slot arcs
$\omega_{rv}$	Rotation speed of $v$ -order harmonic
$\mu_0, \mu_{PM}$	Air and PM relative permeabilities
$\vartheta_0, \vartheta_r$	Initial rotor position and rotation angle

## 1. INTRODUCTION

For electric vehicles (EVs), special characteristics are desired for the traction machines, such as large torque capability, low torque ripple, strong overload ability, and high efficiency [1], [2]. Recently, a new rotor-permanent magnet (PM) flux-switching (FS) (RPM-FS) machine was proposed, which evolves from the topology of stator-PM flux-switching (SPM-FS) machines. The RPM-FS machines not only inherit the advantages of SPM-FS machines, including high torque (power) density and efficiency [3], [4], but also improve the overload capability due to relieving the saturation level of stator teeth [5]. Hence, the RPM-FS machines can be

identified as a promising candidate of traction machines in EVs.

For the flux-switching machines, the combination of stator slots number  $P_s$  and rotor pole-pairs number  $P_r$  significantly affects electromagnetic performances, which is a key parameter in the design process [6]. Generally, the feasible combinations of  $P_s$  and  $P_r$  for SPM-FS machines can be determined based on the back electro-motive-force (EMF) vectors distribution of slot conductors, and then can be chosen according to the winding factor  $k_w$  [7]-[8]. However, since the electromagnetic torque is not determined only by  $k_w$ , it is difficult to select the proper  $P_s/P_r$  combinations with higher torque capability. Hence, the torque capability of machines with different  $P_s/P_r$  are investigated further by the air-gap surface occupation factor (ASOF) [9] and pole ratio (PR) [10], which are helpful in reducing the selection range. Moreover, it is found that the combinations that satisfy  $P_s=P_r \pm k$  ( $k=1, 2$ ) can exhibit a higher electromagnetic torque, and they are chosen as the attractive candidates. Nevertheless, for the combinations, which are simply the integer multiples of the smallest equivalent machines with the lowest pole numbers, e.g., 12s/10p versus 6s/5p machines, the ASOF and PR between two machines are equal, i.e., ASOF=0.208 [9] and PR=2.5 [10]. Thus, it is still difficult to directly evaluate the torque capability of such combinations and choose the optimal candidates for SPM-FS machines. For RPM-FS machines, the torque performance is dramatically influenced by the  $P_s/P_r$  combinations, and the attractive combinations of  $P_s/P_r$  should also be provided at the preliminary design stage.

The purpose of this paper is to investigate the influence of  $P_s/P_r$  combinations on electromagnetic torque performances, ultimately contributing to the choice of candidates with attractive torque performances. The  $P_s/P_r$  combinations performance is investigated from three perspectives, i.e. magnetic loading, electrical loading, and cogging torque production. Based on the field modulation principle, the PM field and armature reaction field at different  $P_s/P_r$  combinations are analyzed. Meanwhile, the analytical expressions of cogging torque with different  $P_s/P_r$  combinations of RPM-FS machine are deduced in section II. Then, four RPM-FS machines 24-stator-slots/10-rotor-pole-pairs (24s/10p), 24s/11p, 24s/13p

and 24s/14p are selected as the candidates exhibiting higher torque output torque and lower torque ripple. In section III, a comprehensive electromagnetic characteristics comparison between four RPM-FS machines candidates and a typical 12s/10p SPM-FS machine with the same structure dimensions is conducted. Then the prototyped machines are manufactured and tested to verify the effectiveness of the proposed technique. It can be concluded that the RPM-FS machines candidates exhibit an attractive torque performance. Finally, conclusions are drawn in section IV.

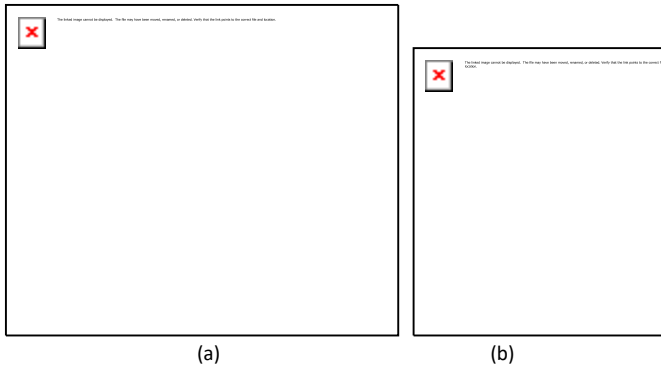


Fig. 1 A three-phase 24s/10p RPM-FS machine. (a) The cross section topology. (b) The key geometric parameters.

2. Slots and Pole-Pairs Combination Principles

The topology of a 24s/10p RPM-FS machine, shown in Fig. 1, is evolved from a typical 12s/10p SPM-FS machine and inherits the flux-switching principle [5]. Based on the slot-conductor back-EMF star vectors theory, feasible combinations of the stator slots number  $P_s$  and the rotor pole pairs number  $P_r$  can be determined by the spoke number  $N_{sp}$  in Fig. 2, where  $N_{sp}$  is expressed as [7], [10], [11]:

$$N_{sp} = \frac{P_s}{\text{GCD}(P_s, P_r)} \tag{1}$$

where  $\text{GCD}(P_s, P_r)$  is the greatest common divisor of  $P_s$  and  $P_r$ .

Obviously,  $N_{sp}$  must be an integer divisible by phase number (e.g.,  $m=3$ ). From equation (1), the possible  $N_{sp}$  of the RPM-FS machines with  $P_s=24$  and single-layer concentrated winding can be deduced as four cases, i.e.,  $N_{sp}=3, 6, 12$  and  $24$ , and the corresponding  $\text{GCD}(P_s, P_r)$  values are 8, 4, 2 and 1. Therefore, the feasible

combinations of  $P_s/P_r$  are listed in Table I. Similarly, the feasible  $P_s/P_r$  combinations in the cases of  $P_s=18, 12$  and  $6$  are respectively listed in Tables II, III and IV.

Based on the results shown in Tables I-IV, the combinations of  $P_s/P_r$  can be selected preliminarily according to the winding factor  $k_w$ ; e.g., the 12s/5p, 18s/10p, 24s/10p, 24s/11p, etc., can be adopted with higher  $k_w$ . However, for  $P_s/P_r$  combinations with the same  $k_w$ , the electromagnetic torque should be evaluated based on equation (2) [12].

$$T = \frac{3}{2} p \frac{D_{sj} l_a}{\pi} \sum_{\nu} B_{g\nu} K_{s\nu} \sin \varphi_{\nu} \tag{2}$$

where  $D_{sj}$  is stator inner diameter,  $l_a$  is stack length, and  $\varphi_{\nu}$  is the  $\nu$ -order harmonic angle between magnetic loading  $B_{g\nu}$  and electrical loading  $K_{s\nu}$ . The torque of RPM-FS machines is contributed not only by the fundamental component ( $\nu=P_{PM}$ , where  $P_{PM}$  is the PM pole-pair number, equal to  $P_r$  in RPM-FS machines) but also by the modulation harmonics ( $\nu=|P_{PM} \pm P_s|$ ) [12]. Hence, the harmonics of the winding factor  $k_{w\nu}$ , magnetic loading  $B_{g\nu}$  and electrical loading  $K_{s\nu}$  should be analysed for the selected combinations of  $P_s/P_r$ .

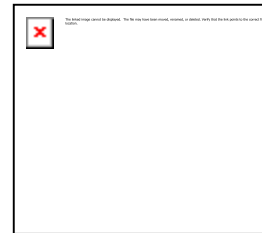


Fig. 2 The slot-conductors back-EMF vectors distribution.

Table I  
Combinations of  $P_s=24$  and  $P_r$  for RPM-FS machines

$\text{GCD}(P_s, P_r)$	$P_r$	$P_s=24$				
		$k_w$	1	5	7	11
$\text{GCD}=1, N_{sp}=24$	$k_w$	0.126	0.588	0.766	<b>0.958</b>	<b>0.958</b>
	$P_r$	2	<b>10</b>	<b>14</b>	22	26
$\text{GCD}=2, N_{sp}=12$	$k_w$	0.259	<b>0.966</b>	<b>0.966</b>	0.259	0.259

GCD=4, $N_{sp}=6$	$P_r$	4	20	28	...
	$k_w$	0.5	0.5	0.5	
GCD=8, $N_{sp}=3$	$P_r$	8	<b>16</b>	32	...
	$k_w$	0.866	<b>0.866</b>	0.866	

Table II

Combinations of  $P_s=18$  and  $P_r$  for RPM-FS machines

GCD( $P_s, P_r$ )	$P_s=18$					
GCD=1, $N_{sp}=18$	$P_r$	1	5	7	<b>11</b>	13
	$k_w$	0.167	0.735	0.9	<b>0.9</b>	0.735
GCD=2, $N_{sp}=9$	$P_r$	2	4	8	<b>10</b>	14
	$k_w$	0.328	0.617	0.945	<b>0.945</b>	0.617
GCD=3, $N_{sp}=6$	$P_r$	3	15	21	33	...
	$k_w$	0.5	0.5	0.5	0.5	
GCD=6, $N_{sp}=3$	$P_r$	6	<b>12</b>	24	30	...
	$k_w$	0.866	<b>0.866</b>	0.866	0.866	

Table III

Combinations of  $P_s=12$  and  $P_r$  for RPM-FS machines

GCD( $P_s, P_r$ )	$P_s=12$					
GCD=1, $N_{sp}=12$	$P_r$	1	5	7	11	13
	$k_w$	0.259	0.966	0.966	0.259	0.259
GCD=2, $N_{sp}=6$	$P_r$	2	10	14	22	26
	$k_w$	0.5	0.5	0.5	0.5	0.5
GCD=4, $N_{sp}=3$	$P_r$	4	8	16	20	28
	$k_w$	0.866	0.866	0.866	0.866	0.866

Table IV

Combinations of  $P_s=6$  and  $P_r$  for RPM-FS machines

GCD( $P_s, P_r$ )	$P_s=6$					
GCD=1, $N_{sp}=6$	$P_r$	1	5	7	11	13
	$k_w$	0.5	0.5	0.5	0.5	0.5
GCD=2, $N_{sp}=3$	$P_r$	2	4	8	10	14
	$k_w$	0.866	0.866	0.866	0.866	0.866

1. Magnetic loading

For the PM-excited field, the PM-MMF is modulated by the salient stator teeth, and then the magnetic loading  $B_g(\vartheta, t)$  of a RPM-FS machine with the primitive dimensions listed in Table V can be deduced as [12]:



(3)

where  $F_{RPMb}$  is the PM-MMF amplitude;  $F_{RPMn}$  are the Fourier series coefficients of the PM-MMF distribution;  $n$  is 1, 2, 3...;  $\Lambda_{S0}$ ,  $\Lambda_{Sb}$  and  $\Lambda_{Sk}$  are the corresponding Fourier series coefficients of stator air-gap permeance, respectively;  $k$  is 1, 2, 3...;  $\omega_r$  is rotor angular speed; and  $\vartheta_0$  is the initial rotor position. It can be established from equation (3) that  $B_g(\vartheta, t)$  is composed of fundamental components with  $nP_{PM}$  orders and modulation harmonic components with  $|nP_{PM} \pm kP_s|$  orders. The amplitudes of the harmonic components are mainly determined by PM-MMF amplitude  $F_{RPMb}$  and stator air-gap permeance  $\Lambda_s$ , where the PM-MMF amplitude  $F_{RPMb}$  can be deduced by the magnetic circuit equation (4), where the permeability of the iron core is assumed to be infinite.



(4)



(5)

where  $H_c$  is PM coercivity and  $w_{PM}$  and  $w_{cg}$  are PM width and cell gap width, equal to the rotor tooth width  $w_{rt} = \pi D_{si} / (4P_r)$  in the primitive design.  $\Lambda_{gap}$  and  $\Lambda_{PM}$  are the magnetic permeance of the air-gap and PM, respectively.  $\Phi_{PMg}$  is the air-gap PM flux linkage.  $l_g$  is the stack length,  $h_{PM}$  is the PM height,  $h_{rt}$  is rotor teeth height, and  $\mu_0$  and  $\mu_{PM}$  are the relative permeabilities of air and PMs, respectively.  $g$  is the air gap length. Then, the rotor PM-MMF  $F_{RPMb}$  in an air-gap field can be determined as follows:



(6)

Table V

### The Primitive Design Parameters of RPM-FS Machines

Items	Values
Stator outer diameter $D_{so}$ (mm)	128
Stator inner diameter $D_{si}$ (mm)	79.36
Stack length $l_a$ (mm)	75
Airgap length $g$ (mm)	0.35
Stator tooth arc $\theta_{st}$ (deg.)	$\pi/P_s$
Rotor tooth arc $\theta_{rt}$ (deg.)	$\pi/(2P_r)$
Rotor slot arc $\theta_{rs}$ (deg.)	$\pi/(2P_r)$

It can be found from equation (6) that the  $F_{RPMb}$  is dominantly determined by the  $D_{si}$  and  $P_r$ . The analytically predicted characteristics of  $F_{RPMb}$  vs.  $P_r$  are compared to those from FEA, as shown in Fig. 3. The analytical  $F_{RPMb}$  increases first and then decreases when  $P_r=16$  at  $D_{si}=79.36$  mm, which is similar to the trend of FEA-predicted result. For  $D_{si}=40$  mm, the highest  $F_{RPMb}$  are obtained at  $P_r=10$  by FEA and  $P_r=12$  by the analytical solution. The difference is attributed to the simplified MMF permeance model that ignores the iron core permeability. Overall, the variation trends from the analytical solution and FEA prediction agree well. In addition, since the PM volumes of RPM-FS machines with different  $P_s/P_r$  combinations are identical in the same  $D_{si}$ , e.g.  $D_{si}=79.36$  mm,  $V_{PM}=34733$  mm<sup>3</sup>, the waveforms of  $k_{MMF}/V_{PM}$  (the ratio of PM MMF to PM volume) with different  $P_r$  coincide exactly with the waveforms of PM-MMF vs.  $P_r$ , as shown in Fig. 3.

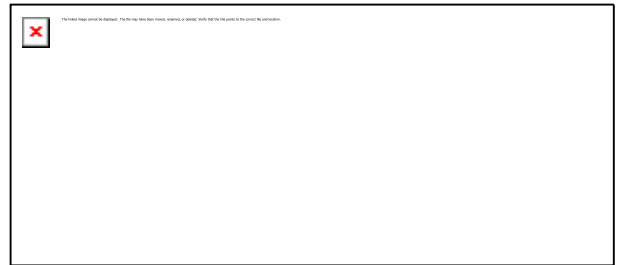


Fig. 3 The per-unit of PM-MMF amplitudes  $F_{RPMb}$  with different  $P_r$  of RPM-FS machines.

From the feasible combinations of  $P_s/P_r$  listed in Table I-IV, the influence of  $P_r$  on magnetic loading  $B_{gv}$  can be analyzed and compared in two cases, namely, the machines with the same  $P_s$  and the machines with different  $P_s$  but the same  $k_w$ .

For the RPM-FS machines with the same  $P_s=24$ , the cases of  $P_r=8, 10, 11, 13, 14$  and  $16$  are analyzed due to larger  $k_w$  values in Table I. The open-circuit air-gap flux density harmonic distributions are shown in Fig. 4. It can be found that the fundamental harmonic amplitude of PM flux density  $B_{gv}$  ( $v=P_{PM}$ ) is the lowest when  $P_r=8$  ( $B_{g8}=0.65T$ ), due to the minimum  $F_{RPMb}$  among 6 cases in Fig. 3. Hence, to obtain a larger  $B_{gv}$  ( $v=P_{PM}$ ) under the same  $P_s$  condition, a higher  $P_r$  should be chosen based on the  $F_{RPMb}$  distribution.

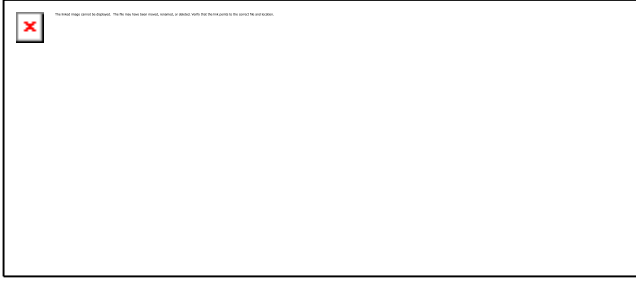
On the other hand, as shown in Fig. 4(b), the fundamental harmonics  $B_{gv}$  and  $k_{wv}$  ( $v=P_{PM}$ ) of the RPM-FS machines with  $P_r=10$  and  $14$  are equal, i.e.,  $B_{gv}=0.7T$  and  $k_{wv}=0.966$ , since the PM-MMF at  $P_r=10$  and  $14$  are approximately identical. However, the corresponding modulation harmonic ( $|P_{PM}-P_s|$ ) amplitudes are different, resulting in an identical fundamental torque  $T_{ef}$  and different modulation harmonic torque  $T_{em}$ . The influence on electromagnetic torque  $T_e$  is analyzed further in Part D. Similarly, the RPM-FS machines with  $24s/11p$  and  $24s/13p$  have the same fundamental harmonic  $B_{gv}$  and  $k_{wv}$  ( $v=P_{PM}$ ), but different modulated harmonics  $B_{gv}$  ( $v=|P_{PM}-P_s|$ ) (Fig. 4(c)).



(a)



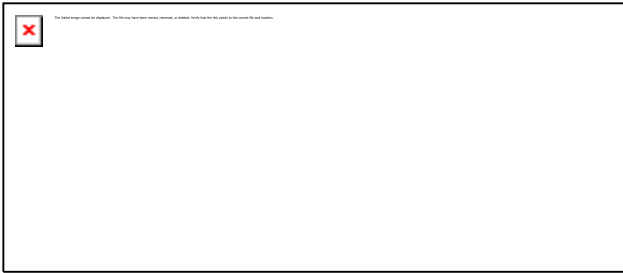
(b)



(c)

Fig. 4 Open-circuit PM flux density distributions with the same 24s. (a) 24s/8p and 24s/16p. (b) 24s/10p and 24s/14p. (c) 24s/11p and 24s/13p.

For the RPM-FS machines with different  $P_s$  but the same  $k_w$ , e.g., 12s/4p versus 24s/8p, the open-circuit air-gap flux density harmonic distributions are shown in Fig. 5. From Fig. 5(a), the amplitudes of fundamental harmonics with  $P_{PM}$  order and modulation harmonics  $|P_{PM} \pm P_s|$  of the 24s/8p machine are higher than those of the 12s/4p machine due to the larger  $F_{RPMb}$ , as shown in Fig. 3. Similarly, the fundamental harmonic  $P_{PM}$  and modulation harmonics  $|P_{PM} \pm P_s|$  of the 24s/10p and 24s/14p machines are higher than those of the 12s/5p and 12s/7p machines, respectively. Thus, it can be concluded that compared to the  $P_s=12$  machines, the  $P_s=24$  RPM-FS machines may exhibit larger  $B_{gv}$  ( $v=P_{PM}$ ), which is helpful to select the attractive  $P_s/P_r$  combinations of the RPM-FS machines with different  $P_s$  but the same  $k_w$ .



(a)



(b)



(c)

Fig. 5 Open-circuit air-gap flux density distributions with the same  $k_w$ . (a) 12s/4p and 24s/8p. (c) 12s/5p and 24s/10p. (d) 12s/7p and 24s/14p.

From equation (3), the PM air gap flux density harmonics rotation speed relative to the stator and rotor are listed in Table VI. The iron losses in RPM-FS machines with different  $P_s/P_r$  combinations are produced by the harmonics with the rotation speed  $\omega_{rv} \neq 0$ . For the stator parts, all harmonics rotation speeds relative to the stator are not zero in the PM air gap field. Hence, these harmonics generate iron losses in the stator cores. However, the rotation speed relative to the rotor of harmonics with  $nP_{PM}$  orders are zero, whereas the harmonics with  $|nP_{PM} \pm kP_s|$  orders are not equal to zero. Hence, the iron losses in the rotor cores are generated by the modulation harmonic components, i.e.,  $|nP_{PM} \pm kP_s|$  orders.

In addition, the PMs are mounted in the rotor, then the PM eddy current loss  $P_{PMloss}$  is dominantly produced by the air gap filed harmonics, of which the rotation speeds relative to the rotor are not zero. Thus, the  $P_{PMloss}$  is produced by the harmonics with  $|nP_{PM} \pm kP_s|$  orders.

Table VI

The Harmonic Components of PM Field Flux Density

Pole Pairs	Relative to stator	Relative to rotor
	Rotation speed	Rotation speed
$nP_{PM}$	$\omega_r$	0
$nP_{PM} + kP_s$	$nP_{PM}\omega_r / (nP_{PM} + kP_s)$	$kP_s\omega_r / (nP_{PM} + kP_s)$
$ nP_{PM} - kP_s $	$nP_{PM}\omega_r / (nP_{PM} - kP_s)$	$kP_s\omega_r / (nP_{PM} - kP_s)$

2. Electrical loading

The  $v$ -order harmonic amplitude of electrical loading  $K_{sv}$  can be expressed as [11], [12]:

$$\text{[Redacted Equation (7)]} \quad (7)$$

where  $J_{sa\_rms}$  is the armature current density; and  $A_{slot}$  is armature slot area per phase. It is found that the fundamental harmonic of  $K_{sv}$  ( $v=P_{PM}$ ) is only determined by the  $k_{wv}$  ( $v=P_{PM}$ ) in the primitive design, since  $J_{sa\_rms}=5A/mm^2$  and  $A_{slot}=74mm^2$  are constant values with different  $P_s$ .

Considering the salient stator and rotor cores, the armature reaction field is modulated by the salient teeth. Then, the electrical loading  $K_s(\vartheta, t)$  yields [13] [14],

$$\text{[Redacted Equation (8)]} \quad (8)$$

where  $M[\cdot]$  is the modulation operator of salient teeth,  $F_{Rw}(\vartheta, t)$  is armature reaction-MMF function, and  $B_{Rw}(\vartheta, t)$  is modulated armature reaction air-gap flux density function. Then, the  $v$ -order harmonic amplitude  $K_{sv}$  is proportional to the harmonic amplitude  $vB_{Rwv}$ , i.e.,  $K_{sv} \propto vB_{Rwv}$ , which is analysed below.

For the 24s/10p machine, the modulated armature reaction flux density in the primitive design can be expressed as [12]:

$$\text{[Redacted Equation (9)]} \quad (9)$$

where  $N_{Rc}$  is coil turns number;  $I_{Rmax}$  is the maximum value of the phase current;  $D_{Ri}$  is the Fourier coefficients of the armature reaction MMF;  $\Lambda_{Rr0}$ ,  $\Lambda_{Rrb}$  and  $\Lambda_{Rrp}$  are the Fourier coefficients of rotor air-gap permeance; and  $p$  is 1, 2, 3...

$$\text{[Redacted Equation (10)]} \quad (10)$$

$$\text{[Redacted Equation (11)]} \quad (11)$$

The  $\xi$ ,  $\theta_1$  and  $\theta_2$  can be expressed as follows:

$$\text{[Redacted Equation (12)]} \quad (12)$$

$$\text{[Redacted Equation (13)]} \quad (13)$$

According to equation (9), it can be found that certain harmonic components are generated in two ways, namely, fundamental components with  $(4i-2)$  orders are produced by the armature reaction MMF only and the resultant components with  $(pP_r \pm (4i-2))$  orders are due to the salient rotor modulation effect, as shown in Table VII. For the harmonic with 14-orders, the fundamental component  $(4i-2)$ -orders ( $i=4$ ) and resultant part  $|pP_r - (4i-2)|$  order ( $i=9, p=2$ ) are the positive values, and the harmonic amplitude with 14-pole-pairs is superimposed by the two components. Similarly, the harmonics with 34-orders are heightened else. However, the amplitude of the 38-order harmonic is cancelled because the fundamental component and resultant part are opposite. Hence, the 38th harmonic order is significantly lower than 14th and 34th harmonics, which is verified in Fig. 7.

Table VII

The Harmonic Components of 24s/10p RPM-FS Machines

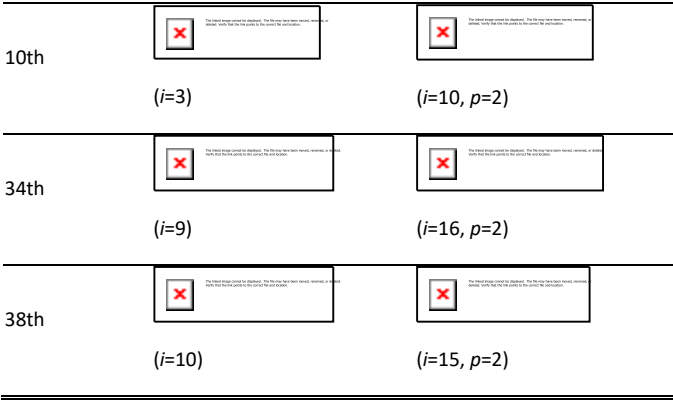
Harmonics	Fundamental components	Resultant components
$v=14th$	$(i=4)$	$(i=9, p=2)$
$v=34th$	$(i=9)$	$(i=4, p=2)$
$v=38th$	$(i=10)$	$(i=15, p=2)$

Table VIII

The Harmonic Components of 24s/14p RPM-FS Machines

Harmonics	Fundamental components	Resultant components
-----------	------------------------	----------------------





For the 24s/14p RPM-FS machine, the winding function is similar to the 24s/10p RPM-FS machine, resulting in the same armature reaction MMF distribution between two machines, as shown in Fig. 6. Considering the salient iron core, the armature reaction air-gap flux-density harmonic distribution is shown in Fig. 7. It can be found from Table VIII that the harmonic amplitudes with 10- and 38-orders are enlarged by the fundamental and resultant parts, whereas 34-order is cancelled. In addition, the modulation harmonics of electrical loading  $K_{sv}$  ( $v=P_s \pm P_r$ ) are mainly determined by the parameter  $vB_{Rwv}$ . For  $P_r=10$  RPM-FS machine,  $vB_{Rwv}=2.24T$  ( $v=|P_s-P_r|$ ) is higher, whereas  $vB_{Rwv}=1.88T$  ( $v=P_s+P_r$ ) is lower than the case of  $P_r=14$  ( $vB_{Rwv}=1.93T$ ,  $v=|P_s-P_r|$  and  $vB_{Rwv}=2.77T$ ,  $v=P_s+P_r$ ). The modulation torque components are influenced further in Part D.

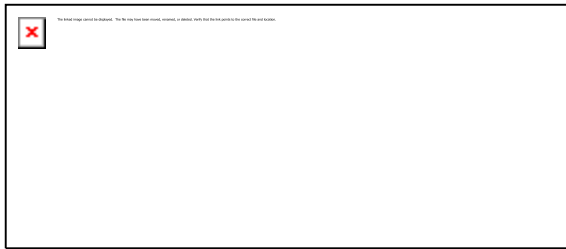


Fig. 6 Armature reaction MMFs of the 24s/10p and 24s/14p machines.

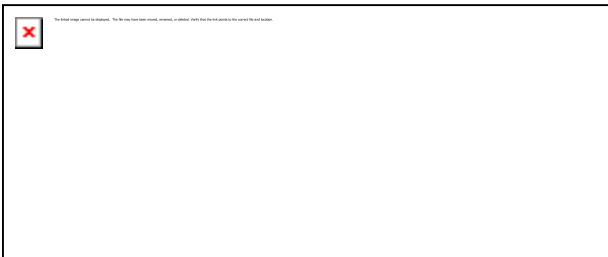


Fig. 7 Modulated armature reaction flux density in air-gap of the 24s/10p and 24s/14p machines.



Fig. 8 Armature reaction MMFs of the 24s/11p and 24s/13p machines.

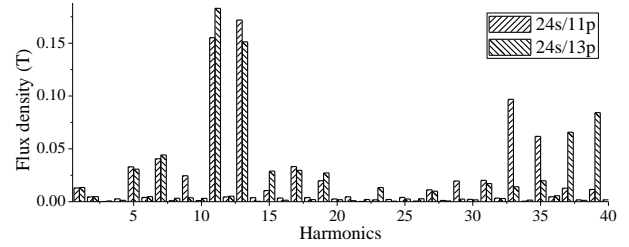


Fig. 9 Modulated armature reaction flux density in air-gap of the 24s/11p and 24s/13p machines.

The 24s/11p and 24s/13p RPM-FS machines also have the same winding function, and the armature reaction field harmonic distributions are the same as shown in Fig. 8. Taking the modulation effect due to salient teeth into consideration, the armature reaction air-gap flux density distributions are shown in Fig. 9. The 37-order harmonic in the  $P_r=11$  machine and 35-order harmonic in the  $P_r=13$  machine are weakened. In addition, the parameter  $vB_{Rwv}=2.43T$  ( $v=P_s+P_r=37$ th) in the  $P_r=13$  machine is higher, whereas the  $vB_{Rwv}=2.01T$  ( $v=|P_s-P_r|=11$ ) is lower than that of the  $P_r=11$  machine ( $vB_{Rwv}=2.16T$ ,  $v=35$ th and  $vB_{Rwv}=2.23T$ ,  $v=13$ th), which results in a larger modulation torque in Part D.

From equation (9), the harmonic components of the armature reaction air gap flux density are listed in Table IX. For the stator parts, all harmonic components of the armature reaction flux density contribute to stator core loss because the effective harmonics rotation speed relative to the stator is not zero. However, for the rotor of the RPM-FS machine, if the harmonics satisfy equation (14), the rotation speed relative to the rotor is not equal to zero. Hence, the rotor core loss and  $P_{PMloss}$  are mainly contributed by these harmonic components in armature reaction flux density.

$$\begin{cases} HC \neq 4i - 2, i = 3 \\ HC \neq 4i - 2 \pm pP_r, i = 3 \end{cases} \quad (14)$$

Table IX

## Harmonic Components of Armature Reaction Field Flux Density.

$i$	Pole Pairs	Relative to stator	Relative to rotor
		Rotation speed	Rotation speed
	$4i-2$	$-P_r\omega_r/(4i-2)$	$(4i-2-P_r)\omega_r/(4i-2)$
$i=3r-2$	$4i-2+pP_r$	$(p-1)P_r\omega_r/(pP_r+4i-2)$	$(4i-2+P_r)\omega_r/(pP_r+4i-2)$
	$ 4i-2-pP_r $	$(p+1)P_r\omega_r/[pP_r-(4i-2)]$	$-(4i-2+P_r)\omega_r/[pP_r-(4i-2)]$
	$4i-2$	$P_r\omega_r/(4i-2)$	$(4i-2+P_r)\omega_r/(4i-2)$
$i=3r$	$4i-2+pP_r$	$(p+1)P_r\omega_r/(pP_r+4i-2)$	$(4i-2-P_r)\omega_r/(pP_r+4i-2)$
	$ 4i-2-pP_r $	$(p-1)P_r\omega_r/[pP_r-(4i-2)]$	$-(4i-2-P_r)\omega_r/[pP_r-(4i-2)]$

### 3. Cogging torque production

The torque ripple of the RPM-FS machine is dominantly influenced by the cogging torque [12], which is considered in the preliminary design process. The cogging torque can be expressed as [15], [16]:

$$T_{cog}(\theta_r) = -\frac{\partial W(\theta_r)}{\partial \theta_r} \quad (15)$$

where  $\vartheta_r$  is the rotor rotation angle,  $\vartheta_r = \omega_r t$ , and  $\omega_r$  is rotor mechanical angular velocity.  $W(\vartheta_r)$  is magnetic energy, which can be expressed as:

$$W(\theta_r) = \frac{1}{16\mu_0} (D_{so}^2 - D_{si}^2) l_a \int_0^{2\pi} F_{RPM}^2(\theta, \theta_r) \Lambda_s^2(\theta) d\theta \quad (16)$$

where  $D_{so}$  and  $D_{si}$  are the outside and inside stator diameters,  $l_a$  is the stack length,  $\mu_0$  is air magnetic permeability, and  $F_{RPM}^2(\vartheta, \vartheta_r)$  and  $\Lambda_s^2(\vartheta)$  are the PM-MMF function and stator air-gap permeance function, respectively.

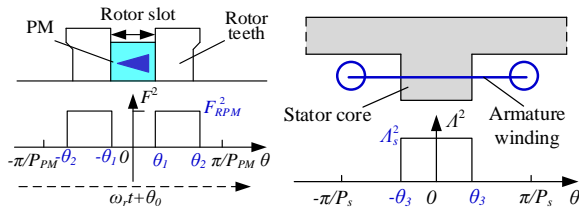


Fig. 10 PM-MMF-permeance model of RPM-FS machine.

Based on the PM-MMF permeance model shown in Fig. 10, the  $F_{RPM}^2(\vartheta, \vartheta_r)$  can be expressed as:

$$F_{RPM}^2(\theta, \theta_r) = F_{RPM0} + \sum_{n=1}^{\infty} F_{RPMn} \cos[nP_{PM}(\theta + \theta_r)] \quad (17)$$

where  $P_{PM}$  is PM pole pairs, which is equal to rotor pole pairs  $P_r$ ;  $n$  is 1, 2, 3...; and  $F_{RPM0}$  and  $F_{RPMn}$  are the Fourier coefficients.

The stator air-gap permeance  $\Lambda_s^2(\vartheta)$  can be expressed as:

$$\Lambda_s^2(\theta) = \Lambda_{s0} + \sum_{k=1}^{\infty} \Lambda_{sk} \cos(kP_s\theta) \quad (18)$$

where  $P_s$  is stator teeth number,  $\Lambda_{s0}$  and  $\Lambda_{sk}$  are the Fourier coefficients, and  $k$  is 1, 2, 3...

Substituting equations (17) and (18) into equation (16), the magnetic energy  $W(\vartheta_r)$  can be determined as:

$$W(\theta_r) = \frac{1}{16\mu_0} (D_{so}^2 - D_{si}^2) l_a [2\pi F_{RPM0} \Lambda_{s0} + \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} F_{RPMn} \Lambda_{sk} \pi \cos(nP_{PM}\theta_r)] \quad (19)$$

where the  $nP_{PM}$  and  $kP_s$  are identical to avoid  $W(\vartheta_r)=0$ , i.e.,

$$nP_{PM} = kP_s = j \text{LCM}(P_s, P_{PM}), j=1, 2, 3... \quad (20)$$

where the  $\text{LCM}(P_s, P_{PM})$  is the lowest common multiple of  $P_s$  and  $P_{PM}$ .

Hence, the cogging torque can be obtained as follows:

$$T_{cog}(\theta_r) = \frac{1}{16\mu_0} (D_{so}^2 - D_{si}^2) l_a \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} F_{RPMn} \Lambda_{sk} nP_{PM} \pi \sin(nP_{PM}\theta_r) \quad (21)$$

It is found that the period of cogging  $C_{cog}$  is obtained as:

$$C_{cog} = \frac{2\pi}{nP_{PM}} = \frac{2\pi}{\text{LCM}(P_s, P_{PM})} \quad (22)$$

From equation (21), the amplitude of cogging torque can be calculated as

$$T_{cog} = \frac{(D_{so}^2 - D_{si}^2) l_a F_{RPM}^2 \Lambda_s^2 \frac{P_s}{n}}{4\pi\mu_0} \quad (23)$$

Since the parameters  $D_{so}$ ,  $D_{si}$  and  $l_a$  are the constant values, as listed in Table V. Then, the  $T_{cog}$  is dominantly determined by

$$\frac{P_s}{n} = \frac{P_s P_{PM}}{\text{LCM}(P_s, P_{PM})} = \text{GCD}(P_s, P_{PM}) \quad (24)$$

It can be concluded that the appropriate  $P_s/P_{PM}$  combinations with lower cogging torque can be chosen by a lower  $GCD(P_s, P_{PM})$  during the preliminary design.

For the RPM-FS machines ( $D_{si}=79.36\text{mm}$ ) with the same stator teeth number, e.g.,  $P_s=24$ , the cogging torques are listed in Table X. Then, according to equation (24), the  $P_r=8$  and 16 machines exhibit higher cogging torques, whereas the  $P_r=11$  and 13 machines obtain lower values. In addition, although the 8 and 16 pole pairs machines have the same  $GCD(P_s, P_{PM})=4$ , the former exhibits a lower cogging torque due to the smaller  $F_{RPMb}$  as shown in Fig. 3.

Table X

The cogging torque of different  $P_s/P_r$  combinations at  $P_s=24$ .

$P_r$	8	10	11	13	14	16
$GCD(P_s, P_{PM})$	8	2	1	1	2	8
$T_{cog}$ (Nm)	1.4	0.9	0.12	0.12	0.8	4.6

#### 4. Electromagnetic torque production

The electromagnetic torque  $T_e$  with different  $P_s/P_r$  are shown in Fig. 11. It can be found that  $T_e$  of RPM-FS machines are dominantly contributed by fundamental harmonic components with the  $P_{PM}$ -order, which is produced from the interaction between magnetic loading  $B_{gv}$  and electrical loading  $K_{sv}$  ( $v=P_{PM}$ ) in equation (2). In addition, the fundamental harmonic of electrical loading  $K_{sv}$  is only influenced by  $k_{wv}$  ( $v=P_{PM}$ ). Hence, the torque capability of machines with different  $P_s/P_r$  combinations are investigated using parameters  $B_{gv}$  and  $k_{wv}$ .

For the RPM-FS machine with the same  $P_s$  and  $k_{wv}$  ( $v=P_{PM}$ ), e.g., 24s/8p versus 24s/16p in Fig. 11(a),  $T_e$  of the  $P_r=16$  machine is 12.86 Nm, which is larger than the  $P_r=8$  machine (10.81 Nm), due to the larger fundamental harmonic magnetic loading in the  $P_r=16$  machine shown in Fig. 4(a). Similarly, from Fig. 11, higher  $T_e$  of RPM-FS machines with the same  $P_s$  and  $k_{wv}$  ( $v=P_{PM}$ ) can be obtained by choosing a larger  $P_r$ .

However, compared to the 24s/10p and 24s/14p RPM-FS machines with the same  $P_s$  and  $k_{wv}$  ( $v=P_{PM}$ ), the fundamental torque component  $T_{ef}$  contributed by the fundamental harmonic ( $v=P_{PM}$ ) of two machines are

approximately equal to 11.5 Nm in Fig. 12(a) since the corresponding fundamental  $B_{gv}$  and  $K_{sv}$  of the two machines are identical to those analysed above. However, for the modulation harmonic torque  $T_{em}$  produced by the modulation harmonic ( $v=|P_{PM}-P_s|$ ), the 14-order in  $P_r=10$  machine contributes -1.35 Nm, which is higher than that of the 10th harmonic in the  $P_r=14$  machine (-1.1 Nm). This is attributed to the higher modulation of the harmonic amplitude of PM flux density  $B_{gv}$  and electrical loading  $K_{sv}$  ( $v=|P_{PM}-P_s|$ ) in the  $P_r=10$  machine. On the other hand, for the modulation harmonic torque  $T_{em}$  produced by the modulation harmonic ( $v=P_{PM}+P_s$ ), the torque produced by the 34-order harmonic in the  $P_r=10$  machine (2.55 Nm) is lower than that of the 38-order in  $P_r=14$  machine (3.04 Nm). This is because the electrical loading  $K_{sv}$  ( $v=|P_{PM}+P_s|$ ) in the  $P_r=10$  machine is lower, and the corresponding magnetic loading harmonics with the  $|P_{PM}+P_s|$  order of both machines are approximately identical. Hence, the total electromagnetic torque  $T_e$  of the  $P_r=14$  machine (13.5 Nm) is larger than that of the  $P_r=10$  machine (12.7 Nm).

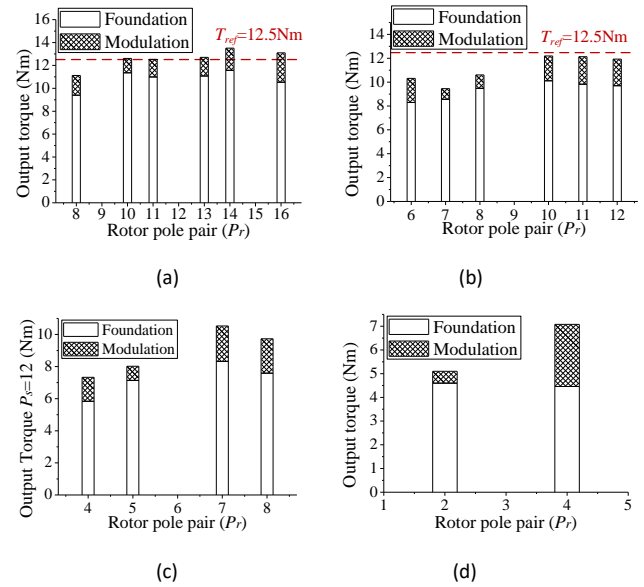


Fig. 11 The output torque with different  $P_s/P_r$  combinations of RPM-FS machines. (a)  $P_s=24$ . (b)  $P_s=18$ . (c)  $P_s=12$ . (d)  $P_s=6$ .

Similarly, the 24s/11p and 24s/13p RPM-FS machines also exhibit the same fundamental torque  $T_{ef}=11$  Nm ( $v=P_{PM}$ ), while the superimposed modulation torque  $T_{em}$  by  $v=|P_{PM}\pm P_s|$  order harmonics in the  $P_r=13$  machine (1.62N m) is higher than that of the  $P_r=11$  machine (1.44 Nm), as shown in Fig. 12(b) ( $v=|P_{PM}\pm P_s|$ ). Finally, a

larger  $T_e=12.6$  Nm is obtained in the  $P_r=13$  machine. Therefore, it can be concluded that for the RPM-FS machines with the same fundamental magnetic loading  $B_{gv}$  and winding factor  $k_{wv}$  ( $v=P_{PM}$ ), the fundamental harmonic contribution of electromagnetic torque are approximately equal, then the torque difference between RPM-FS machines is dominantly determined by the modulation harmonic torque components.

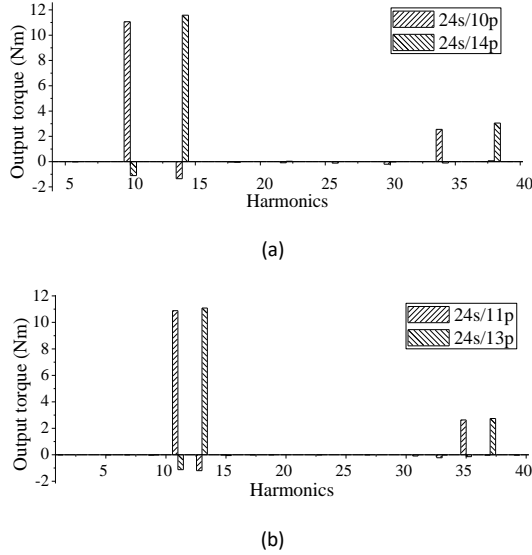


Fig. 12 Torque decompositions of the RPM-FS machine. (a) 24s/10p and 24s/14p. (b) 24s/11p and 24s/13p.

For RPM-FS machines with the same  $P_s$  but different  $k_{wv}$  ( $v=P_{PM}$ ), e.g., the 24s/14p versus the 24s/16p in Fig. 11(a), a higher fundamental harmonic component of  $B_{gv}$  and  $k_{wv}$  ( $v=P_{PM}$ ) in 14-pole-pairs machine results in a larger torque capability.  $T_e$  of the  $P_r=8$  machine is the lowest among the candidates with  $P_s=24$ , due to the lowest  $B_{gv}$  and  $k_{wv}$  ( $v=P_{PM}$ ). Similarly, for the machines with  $P_s=12$  and 18 in Figs. 11(b) and (c), the highest  $T_e$  are obtained in the cases of 12s/7p and 18s/10p.

For the RPM-FS machines with different  $P_s$  but the same  $k_{wv}$  ( $v=P_{PM}$ ), e.g., 12s/4p and 24s/8p with the same  $k_{wv}=0.866$ , it is found from Figs. 11(a) and (c) that  $T_e$  of the 12s/4p machine is 7.34 Nm, which is 68% of the 24s/8p machine, due to the larger fundamental harmonic of PM flux density in Fig. 5(a). Similarly, for the 12s/5p machine ( $k_{wv}=0.966$ ),  $T_e$  is 8.02 Nm, which is 65% of the 24s/10p machine. In addition,  $T_e$  of the 12s/7p machine is 10.52 Nm, which is 78% of the 24s/14p machine.

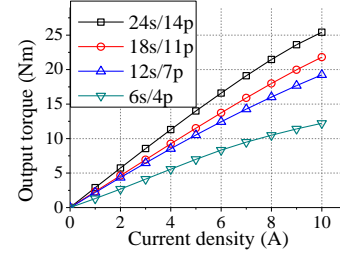


Fig. 13 The output torque of four RPM-FS machines versus different current densities.

To investigate the influence of overload capability by the  $P_s/P_r$  combinations, the 24s/14p RPM-FS machine is chosen, since this machine has a smaller rotor teeth width ( $\beta_{rt}=\pi/(2P_r)$ ), and exhibits a higher output torque at the  $P_s=24$  condition. Similarly, the 6s/4p, 12s/7p and 18s/11p RPM-FS machines are also selected. It can be found from Fig. 13 that the output torque of RPM-FS machines increases linearly as the current density rises, and then all four machines exhibit satisfactorily overload capability. Therefore, the overload capability is hardly influenced by the  $P_s/P_r$  combinations of RPM-FS machines.

Based on the above analysis, the fundamental harmonics of magnetic loading  $B_{gv}$  and the winding factor  $k_{wv}$  ( $v=P_{PM}$ ) are the key parameters to determine torque capability, where  $B_{gv}$  is mainly determined from the PM-MMF amplitude  $F_{RPMb}$  in equation (3). Therefore, based on the analysis of  $F_{RPMb}$  and  $k_{wv}$  in Fig. 3 and Tables I-IV, the attractive combinations of  $P_s/P_r$  of RPM-FS machines are chosen as follows.

(1) Feasible  $P_s/P_r$  combinations can be deduced based on the slot-conductor back-EMF star vector theory; meanwhile, the fundamental winding factor  $k_{wv}$  ( $v=P_{PM}$ ) is calculated.

(2) The optimal range of  $P_r$  can be determined through the analysis of PM-MMF. Since the base speed in this paper is only 1500 r/min, a higher  $P_r$  can be selected to obtain larger magnetic loading  $B_{gv}$  ( $v=P_{PM}$ ), i.e.,  $10 < P_r < 20$ .

(3) The attractive  $P_s=18$  and 24 are selected since the counterparts with low  $P_s$  and high  $P_r$  (e.g., 12s/16p, 6s/10p and 6s/14p machines) cannot exhibit the flux-switching principle, which results in a lower torque capability.

(4) Considering  $k_{wv}$  ( $v=P_{PM}$ ), the candidates of RPM-FS machines with 18s/10p, 18s/11p, 18s/12p, 24s/10p, 24s/11p, 24s/13p, 24s/14p and 24s/16p are chosen, which is highlighted in Tables I-IV.

(5) The  $P_s/P_r$  combinations with lower cogging torque can be chosen based on a smaller  $GCD(P_s, P_{PM})$ , which may result in small torque ripple for the RPM-FS machines. Thus, the 18s/12p and 24s/16p RPM-FS machines are removed.

The candidates chosen by  $F_{RPMb}$  and  $k_{wv}$  at preliminary design exhibits higher torque capability, which is verified in Fig. 11. Then, by considering the  $GCD(P_s, P_{PM})$ , the  $P_s/P_r$  combinations with lower cogging torque are obtained. To reduce the candidates further, the reference torque is set to be 12.5 Nm, which is equal to the output torque of the 12s/10p SPM-FS machine in the preliminary design. Consequently,  $T_e$  of the 24s/10p, 24s/11p, 24s/13p and 24s/14p RPM-FS machines in the primitive design are larger than the reference torque 12.5 Nm, these are selected and analysed further.

### 3. Performance Comparison and Verification

To verify the feasibility of above  $P_s/P_r$  combination principle, a torque performance comparison between four RPM-FS machines candidates with 24s/10p, 24s/11p, 24s/13p and 24s/14p, together with a 12s/10p SPM-FS machine is conducted. The key parameters of five machines are listed in Table XI, where each machine is pre-optimized under the identical stator outside diameter, stack length and air-gap length for a fair comparison. Furthermore, the armature winding current densities and DC-bus voltages are the same, i.e.,  $J_{sa\_rms}=5$  A/mm<sup>2</sup> and  $U_{dc}=600$  V. The PM operation temperature is 90°C.

#### 1. Electromagnetic Torque

The cogging torque waveforms of the five machines are shown in Fig. 14(a). The 12s/10p SPM-FS machine exhibits the highest peak-to-peak value of cogging torque (3.4 Nm), which results in the largest torque ripple in Fig. 14(b). For the RPM-FS machines, the 24s/11p and 24s/13p machines have smaller cogging torques (0.2 Nm), which is approximately 15% and 40% of those with 10- and 14-rotor-pole-pairs, respectively.

Fig. 15(a) shows the output torque of five machines with different current densities. Since the  $dq$ -axes inductances are nearly identical, the  $i_d=0$  control is utilized in the five machines. It is found that the output torques of the four RPM-FS machines are always larger than the SPM-FS machine, which verifies that the RPM-FS machines have better torque capability. The output torque of the 24s/10p RPM-FS machine is 16.05 Nm at  $J_{sa\_rms}=5$  A/mm<sup>2</sup>, which is 1.08 times that of the 12s/10p SPM-FS machine. Obviously, the average torque of the 24s/14p machine reaches a maximum value of 17.6 Nm at rated current density of 5 A/mm<sup>2</sup>, which is 1.09 times, 1.11 times and 1.03 times that of the  $P_r=10, 11$  and 13 RPM-FS counterparts. In addition, the masses of magnetic material and the PM utilization ratios  $k_{T/MPM}$  (the ratio of the average torque to the PM mass) are listed in Table XI. It can be found that  $k_{T/MPM}$  of the SPM-FS machine is only 16.4 Nm/kg, which is significantly lower than the four RPM-FS machines. The core losses and PM losses are listed in Table XI, where the core losses are analyzed based on the improved Yamazaki's model [17, 18], and the PM eddy current losses are calculated by the 3D-FEA. The predicted efficiencies of two machines at rated operation point are almost same.

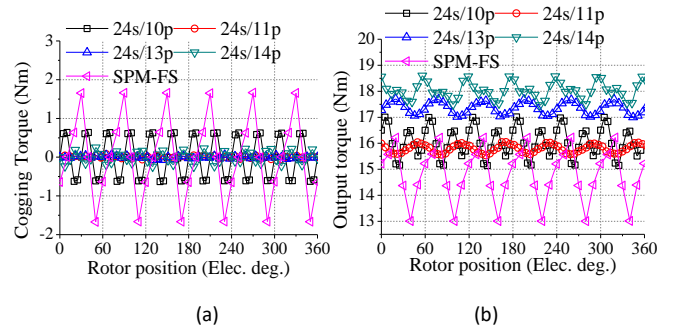


Fig. 14 The torque waveforms of RPM-FS machines and SPM-FS machine. (a) Cogging torque waveforms. (b) Output torque waveforms.

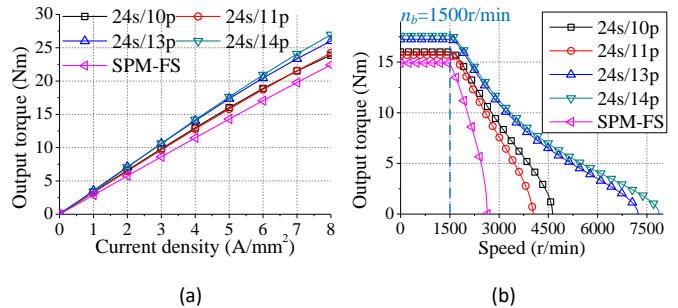


Fig. 15 The torque performances of four RPM-FS machines and the SPM-FS machine. (a) The output torque versus different current densities. (b) The output torque versus different speeds.

**Table XI**  
**The Optimal Dimensions and Performances of Five Machines**

Key Parameters	RPM-FS ( $P_s=24$ )				SPM-FS
	$P_r=10$	$P_r=11$	$P_r=13$	$P_r=14$	12s/10p
$D_{st}$ (mm)	76.80	79.36	76.80	79.36	70.4
$D_{ro}$ (mm)	76.10	78.66	76.10	78.66	69.7
$\theta_{st}$ (deg.)	7.20	6.55	6.92	6.43	7.5
$\theta_{rt}$ (deg.)	9.90	8.18	7.62	7.07	10.5
$\theta_{rs}$ (deg.)	7.20	7.36	5.54	5.14	—
PM width $w_{PM}$ (mm)	4.54	4.26	3.49	3.35	4.6
PM height $h_{PM}$ (mm)	9.79	9.41	9.80	9.4	28.8
Phase turns $N_p$	264	268	240	228	280
$R_{phase}$ ( $\Omega$ ) @25°C	1.2	1.24	0.94	0.89	1.9
$\Psi_{PM}$ (Wb)	0.124	0.116	0.094	0.084	0.185
$L_d$ (mH)	8.8	7.8	7.3	6.4	11.64
$L_q$ (mH)	9.2	7.8	7.3	6.5	14.97
$k_{fw}$	2.47	2.23	3.8	4.03	1.51
Cogging (Nm)	1.4	0.2	0.2	0.5	3.4
$T_e$ (Nm)	16.05	15.8	17.35	17.6	14.8
Torque ripple (%)	11%	3%	3%	6%	22%
Mass of PM (kg)	0.27	0.27	0.27	0.27	0.9
$k_{T/MPM}$ (Nm/kg)	59.4	58.5	64.3	65.1	16.4
Output power (W)	2521	2482	2725	2761	2324.6
Core loss (W)	60	53	65	69	51
Copper loss (W)	158.8	156.3	158	159.1	103.8
PM loss (W)	3.9	5.2	6.2	7.1	30.95
Efficiency (%)	90.2%	90.4%	90.6%	90.5%	90.9%
Power factor	0.89	0.88	0.85	0.83	0.91

**2. Flux-Weakening Capacity**

The flux-weakening capacity coefficient  $k_{fw}$  is another key parameter for evaluating the speed regulation range of traction machines for EV and HEV applications [19], [20]. expressed as:


(25)

where  $\Psi_{PM}$  is the peak value of PM flux linkage, and  $L_d$  and  $i_d$  are the  $d$ -axis inductance and armature current, respectively. Based on the PM flux-linkages, the  $d$ -axis inductances and the armature currents in Table XI, the flux-weakening coefficients  $k_{fw}$  of five machines can be calculated as listed in Table XI. Compared to the 12s/10p SPM-FS machine with the same structural dimensions, the four RPM-FS machines have a larger  $k_{fw}$  and exhibit a wider range of speed regulations under the same DC-bus voltage (600 V), as shown in Fig. 15(b). For the four RPM-FS machines, the maximum speed of the 24s/14p machine is 8000 r/min, which is higher than those with 24s/10p (4600 r/min), 24s/11p (3700 r/min) and 24s/13p (7200 r/min). In addition, the  $P_r=14$  machine exhibit the lowest  $L_d$  and  $L_q$  among four RPM-FS machines, since the per phase turns  $N_p$  of  $P_r=14$  machine is the smallest ( $N_p=228$ ).

**3. Experimental Verification**

To validate the torque performances of the candidate RPM-FS machines, the prototypes of a 24s/10p RPM-FS machine and a 12s/10p SPM-FS machine were manufactured as shown in Fig. 16 [12]. Two machines are chosen with the same rotor-pole-pairs to maintain identical operation frequencies, and the main design parameters are in accordance with those listed in Table XI. The experiment platform is shown in Fig. 17(a). A DSP-TMS320F28335 based digital controller is designed to control the prototype machines. The field current is supplied by a DC power source. The variable load is a dynamometer, which is composed of an induction motor and a variable-frequency driver ABB-ACS800. The torque transducer HBM-T40 is mounted on the platform as shown in Fig. 17(a), which can be used to measure the transient torque values, and then the values can be displayed in a monitor of ABB-ACS800.

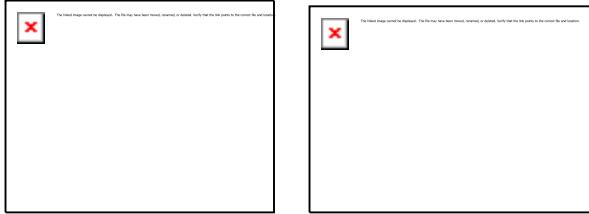


(a)



(b)

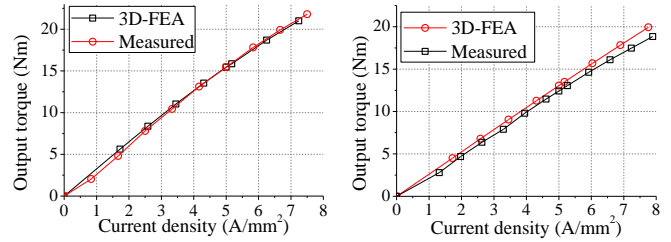
Fig. 16 The Prototype flux switching machines. (a) 24s/10p RPM-FS machine. (b) 12s/10p SPM-FS machine. [12]



(a)

(b)

Fig. 17 The experiment platforms of FS machines. (a) The output torque and power test platform. (b) The cogging torque test platform. The 3D-FEA simulated and measured output torque of the 24s/10p RPM-FS machine is shown in Fig. 18, for which the rated speed is 1500 r/min [12]. It can be found that the measured output torque increases linearly as current density rises, which agrees well with the 3D-FEA results. The measured torque of RPM-FS machine at the rated current density of 5 A/mm<sup>2</sup> is 15 Nm, which is slightly lower than that of the 3D-FEA result. For the 12s/10p SPM-FS machine, the measured torque at  $J_{sa\_rms}=5$  A/mm<sup>2</sup> is 11.8 Nm, which is 95% of that of the 3D-FEA result. The error between measured and 3D-FEA results in the RPM-FS and SPM-FS machines can be attributed to different manufacturer tolerances. The power factors of two machines at rated torque and speed 1500 r/min are measured to be 0.88, which are slightly lower than the FEA results in Table XI. Furthermore, the armature windings temperature of RPM-FS machine and SPM-FS machine are measured at rated  $J_{sa\_rms}=5$  A/mm<sup>2</sup>, and the corresponding results are 110°C and 97°C, which verifies the rated torque and speed for continuous operation.

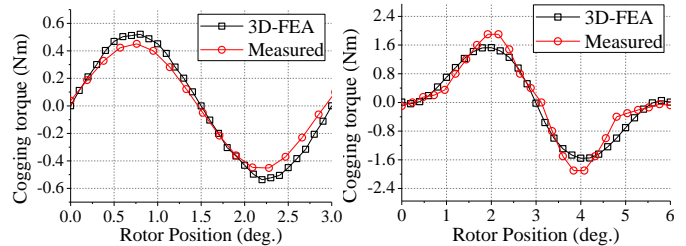


(a)

(b)

Fig. 18 Measured and predicted output torque vs. current densities. (a) 24s/10p RPM-FS machine. (b) 12s/10p SPM-FS machine [12].

The cogging torque experiment platform is shown in Fig. 17(b). The cogging torque meter is mounted on the platform. The principal computer is utilized to collect data from cogging torque meter, and display the cogging torque waveforms. Then, the cogging torque of 24s/10p RPM-FS machine and 12s/10p SPM-FS machine are measured as shown in Fig. 19. It can be found that the measured cogging torque waveforms of two machines are approximately coincide well with the 3D-FEA results. The difference between experiments and 3D-FEA can be attributed to the prototypes manufacture tolerance and the inaccurate assembling process. The measured peak value of RPM-FS machine is 0.9Nm, which is significantly lower than that of SPM-FS machine (4Nm).



(a)

(b)

Fig. 19 Measured and predicted cogging torque. (a) 24s/10p RPM-FS machine. (b) 12s/10p SPM-FS machine.

The output torque of 24s/10p RPM-FS machines and 12s/10p SPM-FS machine with different speed regulation ratios (the ratio of speed to base speed) are measured as shown in Fig. 20. It can be seen that the measured and 3D-FEA torque-speed waveforms are in good agreement. In the constant-torque region, the  $i_d=0$  control is utilized and the  $q$ -axis RMS current density of two machines are  $J_{sa\_rms}=5$  A/mm<sup>2</sup>. It can be found that the output torque of RPM-FS machine is 15Nm, which is 1.25 times of that SPM-FS machine. In the flux weakening region, two machines are operated under the same DC bus voltage, i.e. 200V. It can be seen that the maximum speed regulation ratio of RPM-FS

machine is 3, which means 3 times of base speed can be obtained in RPM-FS machine with flux weakening control. For the SPM-FS machine, the maximum speed regulation ratio is only 1.8, and the flux weakening region is significantly narrower than the RPM-FS machine.

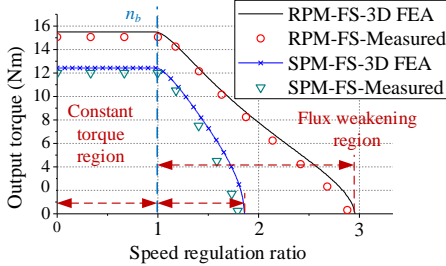


Fig. 20 The output torque versus different speed values of 24s/10p RPM-FS machines and 12s/10p SPM-FS machine.

The efficiency maps of 24s/10p RPM-FS machine and 12s/10p SPM-FS machine are shown in Fig. 21. It can be found that the measured operation points of two machines are slightly lower than the FEA results. For the rated operation point of RPM-FS machine ( $T_e=15\text{Nm}$ ,  $n_b=1500\text{r/min}$ ), the measured efficiency is 90.3%, which is approximately identical to that of the 3D-FEA result. For the SPM-FS machine, the measured rated operation point is  $T_e=11.8\text{Nm}$  and  $n_b=1500\text{r/min}$ . It can be found that a reduction of 0.3% of the measured efficiency (90.1%) is obtained compared with the 3D-FEA result.

(b)

Fig. 21 The efficiency maps of two machines. (a) The RPM-FS machine. (b) The SPM-FS machine.

4. Conclusion

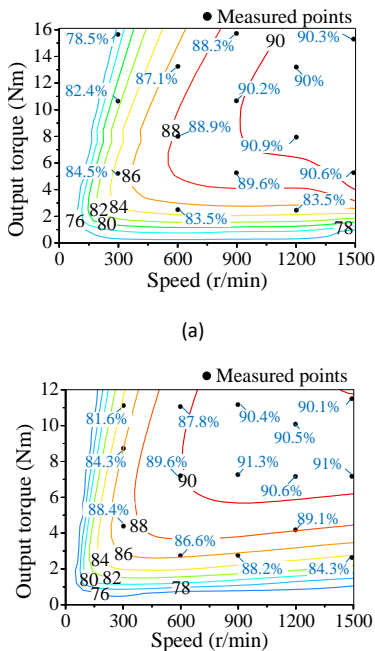
In this paper, the influence of  $P_s/P_r$  combinations on electromagnetic torque performances in RPM-FS machines is investigated, which is helpful to choose the candidates with higher torque capability. The conclusions are as follows:

- (1) The electromagnetic torque  $T_e$  of RPM-FS machine is mainly (>80%) contributed by the fundamental harmonic torque, which is determined by the fundamental PM MMF and winding factor  $k_{sv}$  in the preliminary design.
- (2) The attractive  $P_s/P_r$  combination candidates are changed at the different stator outer diameter  $D_{so}$ . Since the variation of PM MMF with  $P_r$  is influenced by the key parameter  $D_{so}$  based on the magnetic circuit analysis based on the magnetic circuit analysis.
- (3) For the machines with the same fundamental PM MMF and  $k_{wv}$ , the fundamental harmonic contributions of electromagnetic torque are approximately identical, then the difference of  $T_e$  between RPM-FS machines is dominantly determined by the modulation harmonic torque components.

- (4) Taking the cogging torque influence into consideration, the candidates exhibiting lower torque ripple can be obtained by lower  $GCD(P_s, P_r)$ .

Based on the analysis of PM MMF,  $k_{wv}$  and  $GCD(P_s, P_r)$ , the 24s/10p, 24s/11p, 24s/13p and 24s/14p RPM-FS machines are chosen to be the candidates at  $D_{so}=128\text{mm}$ . Compared with a typical 12s/10p SPM-FS machine with the same structure dimensions, these four RPM-FS machines exhibit higher torque capability and wider speed regulation range. The characteristics of  $P_s/P_r$  combinations are verified by the FEA and experiments. Therefore, employing the proposed techniques is helpful for choosing attractive  $P_s/P_r$  combinations promptly and effectively for certain space dimensions.

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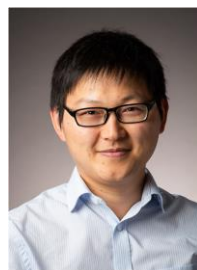
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