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Impact Analysis of Electricity Supply Unreliability to Interdependent Economic Sectors by an Economic-Technical Approach

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Abstract—This paper proposes a novel framework to quantify the economic impact of electricity supply interruptions to other economic sectors considering their interdependency and increasing penetration of wind power. It is achieved by a novel integrated model that combines economic interdependency and electricity supply reliability. Leontief Input-Output model is used to determine the dependency of other economic sectors on electricity supply and electricity reliability theory is utilised to quantify electricity supply interruptions. The two models are combined to quantify two key indexes: the inoperability of different economic sectors and their losses under electricity supply unreliability. Further, an optimal model is designed to allocate available electricity to minimise the economic losses of these sectors when electricity supply is interrupted. Two UK electricity generation scenarios are used to demonstrate the concept. It is found that economic sectors have various degrees of dependency on electricity supply and their losses also differ significantly. In addition, more wind power penetration could jeopardize electricity supply adequacy and consequences to other sectors. The findings can assist policy makers to understand the importance of electricity security to other sectors and quantify potential economic losses so that new policies and regulations can be designed to mitigate the adverse consequences, such as developing the capacity market.

Keywords **—**Inoperability, Interdependency, Leontief input-output, Reliability, Wind power, Electricity supply.

1. Introduction

The modern society is growingly dependent on electricity supply, which is accomplished by the effort to decarbonise the energy sector to reduce greenhouse gas emissions [1]. Different economic sectors and infrastructure are now becoming closely linked, for example, natural gas and electricity systems are linked gas-fired generation and new power-to-gas techniques. The increasing interdependency brings many benefits but also challenges, where one consequence is that the failure impact in a system can propagate to others [2]. For example, the 2003 North America blackout was estimated to cause a total cost of about $6 billion [3]. The consequence can be further worsened when electricity supply is interrupted with increasing non-dispatchable renewable, such as wind power [4].

Thus, it is essential to understand the dependency of economic sectors on electricity supply and quantify their losses in case of electricity shortage so that mitigation solutions can be adopted. Some pioneering work has studied the interdependency between electricity and other sectors from the technical

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aspect [5]. Paper [6] proposes an economic feasibility analysis for a standalone house operated with a hybrid power plant consisting of gas generation, photovoltaic and wind generation. In [7], the authors propose a new concept of energy hub which consists of various resources for energy conversion and optimization. In paper [8], the authors introduce a new configuration for natural gas pressure drop stations by employing a solar thermal system. The introduced concept is assessed in terms of fuel providence and exergy destruction rate. Paper [9] introduces a new optimization model to analyse the interdependency between different energy infrastructures, such as natural gas, coal, and electricity. It is noted that the previous studies are mainly studying the physical interdependency of a limited number of energy infrastructures. Due to the lack of data, they use simplified models of some sectors and networks when examining the impact of failures on others. In addition, they have not quantified the economic loss of one sector due to the supply failure of other sectors/infrastructure.

The reliability of electricity systems assesses the impact of electricity supply interruptions to other sectors. Electricity system reliability mainly focuses on quantifying some key reliability indexes in predefined contingency events, such as potential load loss and occurrence probabilities of the events. It can be roughly divided into two categories: security which is to measure system’s ability to withstand stability in response to disturbances, and adequacy which is to quantify the existence of sufficient supply to satisfy demand [10] [11]. In this paper, the reliability is referred to adequacy, which is also called operability in economics domain. They are assumed to be interchangeable here. In quantifying the economic loss of other sectors due to electricity shortage, electricity demand is normally classified into various categories, such as domestic, commercial, and industrial. Each type is assigned a specific Value of Lost Load (VOLL) [12] and the total economic loss is the summation of VOLL from all curtailed demand [13]. This concept has served the electricity system industry for decades, but it ignores the interdependency of other economic sectors, i.e. the propagating effect of electricity supply interruptions.

From the economic aspect, some work has investigated the impact of electricity supply interruptions on other economic sectors by using Inoperability Input-Output Model (IIM). The IIM developed from the original Input-Output (IO) model contributes to understanding infrastructure interdependency in abnormal conditions [14]. It has been applied to many areas for risk identification and mitigation, and is applicable to calculate the economic impacts of a given change to the economy. In paper [15], the authors use the IIM to measure the financial and inoperability impact of the 2003 Northeast Blackout. Paper [16] develops a static IO framework to analyse energy issues in the short run and discusses the potential barriers to its application. Papers [17, 18] use the IIM to assess and manage inherent risks in different interconnected economic systems. Paper [19] studies the marginal cost of GDP due to electricity deficits but the deficit probability is predefined but not quantified. Conclusively, the disadvantage of existing work in economics is that the inoperability of electricity supply is normally prefixed i.e. hypothetically assumed or obtained from historical data. It is unable to reflect electricity system’s stochastic features. On the other hand in the electricity system domain, the impact quantification of supply unreliability is fairly rudimentary, which cannot reflect the interdependency of different economic sectors. Thus, it is essential to integrate the economic and technical interdependency techniques together to quantify the potential impact of electricity interruptions on other economic sectors in a coherent way.

This paper proposes a novel integrated technical-economic framework to assess and manage the unreliability of electricity supply to other economic sectors. The IIM technique is adopted to analyse the economic dependency and energy system reliability is used to capture the technical dependency of other enonomic sectors on electricity supply considering the peneration of wind power. The IIM is built by using national economic statistic data and the reliability of wind power is analytically modelled by a Markov Model. Three key inoperability indexes are designed to measure the dependency degree of different economic sectors on electricity supply. An optimal model is also introduced to manage available electricity to minimise the economic losses of other sectors when electricity supply is partially interrupted. The UK electricity supply scenarios with various wind penetration levels are utilized to illustrate the concept. Results reveal that the proposed framework can effectively measure and manage the impact of electricity supply inoperability on other sectors.

The key innovations of this paper are that: i) it integrates economic IIM and technical reliability approaches to quantify the impact of electricity supply interruptions; ii) it designs new indexes to measure the inoperability of electricity systems on other economic sectors, and iii) it studies the impact of increasing wind power on the operability and economic losses of other sectors; iv) it introduces an optimal management strategy to minimise the economic losses of sectors with electricity supply interrupted. The study can benefit electricity system operators and policymakers to understand the importance of electricity supply security and take remedy actions to ensure supply reliability to other economic sectors.

The rest of this paper is organized as: Section 2 introduces the IO and IIM models. In section 3, electricity supply reliability with wind power is presented and Section 4 defines some interdependency indexes. Section 5 proposes an approach to quantify the impact of inoperable electricity supply and in Section 6, an optimal management model is proposed. Section 7 employs the UK case to demonstrate the proposed method. Section 8 concludes this paper.

2. Interdependency and The Input-output Model

This section briefly introduces the Leontief IO model and IIM for interdependency analysis, and the application to electricity supply.

2.1 Leontief Input-output Model for Interdependency Analysis

Figure 1 illustrates the interdependency of three sectors and their external demand. By taking the electricity supply as an example, one part of its output is consumed by itself, one part is exported to Sectors A and B for their production, and the remaining part is to meet external demand. In turn, electricity generation also needs the output from Sectors A and B to produce electricity. Thus, any deficits of electricity to either Sector A or B will adversely affect electricity production. This indicates the importance of considering the natural linkage/interdependency between different sectors in quantifying electricity supply security.

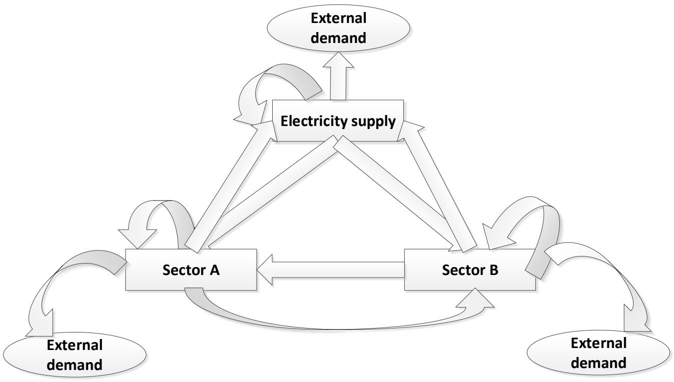

Fig.1. Input-output of a three-sector system

The Leontief IO model designed by Noble Laureate Leontief is an effective tool for examining the interactions and interdependency of different economic sectors. It provides a framework to analyse the economic impact for an equilibrium economic system with a set of interdependent subsystems or sectors. The IO model employs matrices to display the supply and demand between different sectors, called transactions table. An illustrative example is given in Table I with two sectors and one external demand column, where a row in the table displays the distribution of a producer's output and a column is the composition of inputs required by a sector in order to produce its output [20]. The total output of one section is the sum of the output to all sectors including itself and external demand.

Table I A two-sector Input-Output Table

From sector	**To sector**		**External demand**	**Total output**
	Sector 1	Sector 2		
Sector 1	A(1,1)	A(1,2)	C(1)	A(1,1)+A(1,2)+ C(1)
Sector 2	A(2,1)	A(2,2)	C(2)	A(2,1)+A(2,2)+C(2)

Mathematically, the traditional Leontief I-O model is

$$\boldsymbol{X} = \boldsymbol{A} \cdot \boldsymbol{X} + \boldsymbol{c} \tag{1}$$

where, $\boldsymbol{X}$ is the production vector, $\boldsymbol{A}$ is the Leontief technical coefficient matrix, and $\boldsymbol{c}$ is the final/external demand vector. All these parameters can be easily obtained by using the economic data in transactions tables [20].

Leontief inverse is defined as

$$\boldsymbol{B} = (1 - \boldsymbol{A})^{-1} \tag{2}$$

The most popular IO model is an economic formula and it can be easily transformed into an amount-based technical model to be applied for inoperability analysis [20].

2.2 Application of Input-output Model to Electricity Supply

Leontief IO model is a quantitative economic technique to represent the interdependency between different sectors of a national economy or regional economies. It has been widely used for investigating the interdependency of different economic sectors [19, 21]. It can also be applied to analysing electricity supply when it is considered to be as one sector so that its importance to other sectors can be quantified. Take Table I as an example, where sector 1 is assumed to be the electricity supply sector. It produces electricity to support its own production (with electricity amount *A(1,1)*) and that of sector 2 (with electricity amount *A(1,2)*). Meanwhile, electricity is also consumed by other external sectors/demands which do not produce products, such as domestic customers (with electricity amount *C(1)*), called external demand. Thus, if the total electricity production changes, it will affect not only its own production but also the production of sector 2 and final demand. However, the standard IO model does not provide the insight during abnormal cases and thus IIM is derived.

2.3 Inoperability Input-output Model

Leontief IIM proposed in [14] is widely used to evaluate the economic impact of supply perturbations in one sector on other interdependent sectors. In the model, inoperability is defined as the output difference between an as-planned scenario and the actual scenario. Electricity supply inoperability is quantified by assessing the reliability of the power system, where LOLP is defined as its inoperability index. The general form of IIM is [17]

$$\boldsymbol{q} = \boldsymbol{A}^* \cdot \boldsymbol{q} + \boldsymbol{c}^* \tag{3}$$

which can be reorganized as

$$\boldsymbol{q} = (\boldsymbol{I} - \boldsymbol{A}^*)^{-1} \cdot \boldsymbol{c}^* \tag{4}$$

where $\hat{\boldsymbol{c}}$ is defined as the reduced level of final demand and $\hat{\boldsymbol{x}}$ is the reduced level of production. Other variables in (3) and (4) are defined as follows:

i) $\boldsymbol{c}^{\boldsymbol{*}}$ is demand side perturbation vector expressed in terms of normalized degraded final demand

$$\mathbf{c}^* = [\mathrm{diag}(\boldsymbol{X})](\boldsymbol{c} - \hat{\boldsymbol{c}}) \tag{5}$$

ii) $\boldsymbol{A}^{\boldsymbol{*}}$ is the magnitude of interdependency of sectors in an inoperable case, derived from Leontief coefficient

$$\mathbf{A}^* = [\mathrm{diag}(\boldsymbol{X})]^{-\mathbf{1}}\mathbf{A}[[\mathrm{diag}(\mathbf{X})]^{-\mathbf{1}}] \tag{6}$$

iii) $\boldsymbol{q}$ is the ratio of unrealized production, defined as "as-planned" production minus degraded production divided by as-planned production

$$\boldsymbol{q} = [\mathrm{diag}(\boldsymbol{X})]^{-\mathbf{1}}(\boldsymbol{x} - \hat{\boldsymbol{x}}) \tag{7}$$

3. Reliability of Electricity Supply

This paper is mainly concerned with the balance between electricity supply and demand, and the impact of electricity supply unreliability on other sectors. Thus, only generation system adequacy is quantified but power system modelling is not included [22]. Normally, Monte Carlo simulation and analytical approaches are employed to assess system reliability when considering wind power. In this paper, an analytical approach is adopted here because of its simplicity.

3.1 Reliability of Wind Turbine

Figure 2 provides a typical wind power output curve. For simplicity, the impact of air density, swipe area of wind turbines, air pressure, etc. are not considered. A wind turbine is supposed to have the rated capacity of P_r and its actual output under each wind speed group in healthy conditions can be obtained by using (8) [23]. If the wind turbine is unhealthy, the power output is assumed to be zero.

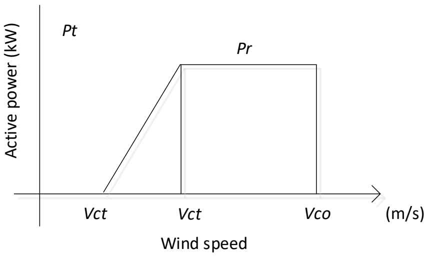

Fig.2 A typical wind turbine power curve.

$$P_t = \begin{cases} 0, & 0 < v < V_{ci} \\ (a + b \cdot v + c \cdot v^2) \cdot P_r, & V_{ci} < v < V_r \\ P_r, & V_r < v < V_{co} \\ 0, & v > V_{co} \end{cases} \tag{8}$$

where, P_t is wind turbine output, V_{ci} is wind cut-in speed, V_{co} is cut-out speed, V_r is rated output speed, P_r is rated output, and v is actual wind speed. a, b and c are coefficients.

Because wind turbine output is between zero and its rated capacity, the active power output can be split into a finite number of steps to reduce total output states by using Markov Chain based on wind speed distributions. The transition rates and durations of all states can be obtained by analysing historic wind speed and turbine output data. For simplicity, it is assumed that a wind turbine has three states, given in Figure 3 [24]. In state 1, the wind speed is within the cut-in speed and rated output speed and thus it can produce electricity (up state); in state 2, the wind speed is within rated output speed and cut out speed and it can produce electricity (up state); in state 3, the wind speed is below cut-in speed or above cut-out speed, and thus the wind turbine is shut down (down state). The arrows between the states represent the transition of state from one to the other as long as the wind speed (indicated by v in the figure) fails into the defined group. For example, assuming the turbine initially resides in State 3 (down state), when wind speed v is bigger than cut-in speed V_{ci} but lower than rated speed V_r, it will transit to state 1. If v is smaller than cut-out speed V_{co} but higher than rated speed V_r, it will transit to State 2.

The classification is given in equation (9)

$$\begin{cases} State\ 1\ (up\ state): & V_{ci} \le v < V_r \\ State\ 2\ (up\ state): & V_r \le v < V_{co} \\ State\ 3\ (down\ state): 0 \le V_{ci}\ or\ \ v \ge V_{co} \end{cases} \tag{9}$$

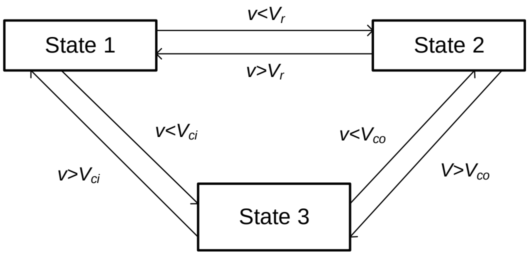

Fig.3. State transition for a three-stage wind turbine.

The probability for a wind turbine to reside in each state in Figure 3 is decided by the probability of wind turbine in health status multiplied by the probability of wind speed in corresponding ranges. The probability of each state thus is

$$\begin{cases} S1: \qquad p = (1 - FOR) \cdot p(V_{ci} \le v < V_r) \\ S2: \qquad p = (1 - FOR) \cdot \ p(V_r \le v < V_{co}) \\ S3: p = (1 - \mathrm{FOR}) \cdot \{p(0 \le v < V_{ct}) + p(v \ge V_{co})\} \end{cases} \tag{10}$$

where, FOR is the forced outage rate of a wind turbine and $p(\cdot)$ represents the probability of the corresponding wind speed.

The number of wind turbine states is normally arbitrary, decided by anticipated accuracy. More accurate results will need more turbine states in order to model its performance under various wind speed steps. Because wind turbine and wind farm capacity is normally smaller compared to other conventional generation units, thus only a few states are desirable for reducing computational burden. Once the reliability modelling for a wind turbine is obtained by (10), the reliability of wind farms with multiple turbines can be derived based on the single-turbine model via using Markov model. It is not covered here due to space limitation and more details can be found in [25].

3.2 Reliability Quantification of Electricity Supply

Once wind farm reliability is obtained, a complete capacity outage probability table for a generation system with wind power can be easily derived by a recursive algorithm through the following steps [10]:

- To develop capacity model, i.e. capacity outage probability table, for individual generation unit;
- To develop load model from given historic load profiles;
- To combine generation capacity outage probability table with the load model to obtain the probabilistic model of system capacity adequacy.

The final reliability indexes are the summation of those calculated in each wind speed step multiplied by the occurrence probability. Mathematically, a given reliability index *W* can be obtained by

$$W = \sum_{i=1}^{m} w_i \cdot Prob_i \tag{11}$$

where, w_i is the reliability index in a wind speed step *i*, $Prob_i$ is its probability, and *m* is the total number of wind speed steps.

4. Inoperability dependency Indexes

Apart from inoperability, some dependency indexes are also defined in this paper to examine the dependency degree of one sector on electricity supply under inoperable cases. Here, the diagonal and off-diagonal elements of matrix $(\boldsymbol{I} - \boldsymbol{A}^*)^{-1}$ are represented by $\{a_{ii}^*\}$ and $\{a_{ij}^*\}$. Correspondingly, the following three indexes are designed:

i) The diagonal entity is defined as the self-dependency inoperability index to illustrate the inoperability degree of sector *i* due to its own output shortage

$$\gamma_{i,i} = a_{ii}^* \tag{12}$$

ii) The off-diagonal entity is defined as the cross-dependency inoperability index to measure the inoperability of sector *i* due to the output shortage of sector *j*

$$\gamma_{i,j} = a_{ij}^*, \quad where\ j > i \tag{13}$$

iii) Index of Power Dispersion (IPD) is used to measure the impact of one sector on all other sectors, which is derived from Leontief inverse, defined as

$$IPD_i = \frac{\sum_{j=1}^{n} B_{ij}}{\frac{1}{n}\left(\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}\right)} \ (i, j = 1,2, \dots n) \tag{14}$$

where, B_{ij} is the element in Leontief inverse matrix given in equation (2).

The three indexes will be used in this paper to measure the importance of electricity supply to other economic sectors.

5. Inoperability of Electricity Supply System and Its Impact on Other Sectors

In this section, electricity supply reliability is integrated into the economic IIM to examine the dependency of different economic sectors on electricity supply and quantify the potential economic losses during electricity supply outage.

5.1 Integrating Reliability to Generation System

Suppose that an electricity supply system has *N* scenarios of generation loss and *M* wind speed stepss, and thus in total there are *M* ×*N* scenarios to be considered. In one scenario, there might be load curtailment, i.e. demand perturbation. The inoperability of electricity supply in the *i*th wind speed state under the *j*th generation loss case is defined as

$$\tilde{q}_{i,j} = \begin{cases} 0 & OP_{i,j} \geq Pd_{i,j} \\ \frac{OP_{i,j} - Pd_{i,j}}{Pd} & \mathrm{OP}_{i,j} < Pd_{i,j} \end{cases} \tag{15}$$

where, $\tilde{q}_{i,j}$ is electricity supply inoperability, $Pd_{i,j}$ is electricity system demand, and $OP_{i,j}$ is available electricity supply in the period.

Thus, the total electricity system inoperability is

$$\tilde{q} = \sum_{i=1}^{M}\left\{\left(\sum_{j=1}^{N} \tilde{q}_{i,j} \cdot Pro_j\right) \cdot Pw_i\right\} \tag{16}$$

where, Pro_j is generation loss probability, Pw_i is the probability of the *i*th wind speed step, *M* is the total number of wind speed steps, and *N* is the total number of generation loss scenarios.

The impact of electricity supply inoperability will propagate to other sectors because of their interdependency, thus causing more perturbations in other sectors where consequence could affect in turn affect electricity production. This impact will be studied by integrating electricity system inoperability with the IIM.

5.2 Integration of Supply Reliability to IIM

Suppose that electricity supply is inoperable/unreliable and its inoperability level is specified as $\tilde{\boldsymbol{q}}$. The final external demand perturbation is to be determined, defined as $\boldsymbol{c}^*$. For other economic sectors, their final external demand perturbations are specified beforehand and their inoperability indexes are to be determined, defined as $\boldsymbol{q}$. In a matrix format, the IIM matrix $(\boldsymbol{I} - \boldsymbol{A}^*)$ can be broken down into the following submatrices

$$(\boldsymbol{I} - \boldsymbol{A}^*) = \begin{pmatrix} \boldsymbol{A_{11}}^* & \boldsymbol{A_{12}}^* \\ \boldsymbol{A_{21}}^* & \boldsymbol{A_{22}}^* \end{pmatrix} \tag{17}$$

Reorganizing (4) and submitting (17) into it produces

$$\begin{pmatrix} \boldsymbol{A_{11}}^{*} & \boldsymbol{A_{12}}^{*} \\ \boldsymbol{A_{21}}^{*} & \boldsymbol{A_{22}}^{*} \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{q} \\ \widetilde{\boldsymbol{q}} \end{pmatrix} = \begin{pmatrix} \widetilde{\boldsymbol{c}^{*}} \\ \boldsymbol{c}^{*} \end{pmatrix} \tag{18}$$

By resolving (18), the following solutions are obtained

$$\begin{cases} \boldsymbol{q} = \boldsymbol{A_{11}}^{*-1} \cdot (\widetilde{\boldsymbol{c}^{*}} - \boldsymbol{A_{12}}^{*} \cdot \widetilde{\boldsymbol{q}}) \\ \boldsymbol{c}^{*} = \boldsymbol{A_{21}} \cdot \left(\boldsymbol{A_{11}}^{*-1} \cdot (\widetilde{\boldsymbol{c}^{*}} - \boldsymbol{A_{12}}^{*} \cdot \widetilde{\boldsymbol{q}})\right) + \boldsymbol{A_{22}}^{*} \cdot \widetilde{\boldsymbol{q}} \end{cases} \tag{19}$$

To summarise, the known variables are: i) inoperability level of electricity supply, i.e. unreliability, which is obtained by implementing electricity supply reliability in Section III; and ii) the final demand perturbation of other economic sectors, which are assumed to be zero if not specified. The variables to be quantified are: i) the final demand perturbation of electricity supply; and ii) the inoperability indexes of other economic sectors.

5.3 Economic Loss due to Inoperability

The inoperability indexes provide the impact of electricity supply interruptions to other dependent sectors. Here, an economic loss index is defined as well to capture the economic losses of other sectors due to electricity supply shortage. For a specific sector, the index under each inoperable case of electricity supply is quantified by multiplying the inoperability with as-planned output

$$Loss_k = \tilde{q}_k \cdot x_k \tag{20}$$

where, x_k is the planned production of sector *k* and $\tilde{q}_k$ is its inoperability.

The total expected economic loss for sector *k* due to electricity supply shortage is

$$Loss_k = \sum_{i=1}^{M} \left\{ \left(\sum_{j=1}^{N} \tilde{q}_{i,j} \cdot Pro_j \right) \cdot Pw_i \cdot x_i \right\} \tag{21}$$

6. Minimising economic loss

This section proposes an optimization model to minimize the total economic loss of all other sectors due to electricity supply interruptions by optimally allocating the available electricity after interruptions. The control variables are allocated available electricity supply to various sectors, q_k. The objective is to reduce the total economic loss for other sectors, defined in (22). At the same time, the optimisation should meet three constraints: equality constraints derived from (18), upper and lower boundaries of demand perturbations and sector inoperability indexes given in (23).

$$obj. \min cost = \sum_{i=k}^{n} q_k \cdot x_k \tag{22}$$

$$s.t. \begin{cases} \boldsymbol{A_{11}}^{*} \boldsymbol{q} = \widetilde{\boldsymbol{c}^{*}} - \boldsymbol{A_{12}}^{*} \cdot \widetilde{\boldsymbol{q}} \\ 0 \leq \boldsymbol{c}^{*} \leq 1 \\ 0 \leq q_k \leq 1 \end{cases} \tag{23}$$

where, the external demand perturbation is

$$\boldsymbol{c}^{*} = \boldsymbol{A_{21}}^{*} \cdot \boldsymbol{q} + \boldsymbol{A_{22}}^{*} \cdot \widetilde{\boldsymbol{q}} \tag{24}$$

By rearranging (24) and submitting it to the first inequity in (23), the following two equations can be obtained

$$\begin{cases} \mathbf{0} \le \boldsymbol{A}_{\mathbf{21}}{}^{*} \cdot \boldsymbol{q} + \boldsymbol{A}_{\mathbf{22}}{}^{*} \cdot \tilde{\boldsymbol{q}} \\ \boldsymbol{A}_{\mathbf{21}}{}^{*} \cdot \boldsymbol{q} + \boldsymbol{A}_{\mathbf{22}}{}^{*} \cdot \tilde{\boldsymbol{q}} \le \mathbf{1} \end{cases} \tag{25}$$

By reorganising (25), it produces

$$\begin{cases} -\boldsymbol{A}_{\mathbf{21}}{}^{*} \cdot \boldsymbol{q} \le \boldsymbol{A}_{\mathbf{22}}{}^{*} \cdot \tilde{\boldsymbol{q}} \\ \boldsymbol{A}_{\mathbf{21}}{}^{*} \cdot \boldsymbol{q} \le \mathbf{1} - \boldsymbol{A}_{\mathbf{22}}{}^{*} \cdot \tilde{\boldsymbol{q}} \end{cases} \tag{26}$$

The two inequality constraints in equation (26) can be integrated as one in (27), where the new coefficient matrixes are formed by the coefficient matrixes of the original two inequality constrains.

$$[-\boldsymbol{A}_{\mathbf{21}}{}^{*},\ \boldsymbol{A}_{\mathbf{21}}{}^{*}] \cdot \boldsymbol{q} \le [\boldsymbol{A}_{\mathbf{22}}{}^{*},\ (\mathbf{1} - \boldsymbol{A}_{\mathbf{22}}{}^{*})] \cdot \tilde{\boldsymbol{q}} \tag{27}$$

This linear optimisation is resolved by many mathematical solvers, such as GAMS and CPLEX.

7. Case Study

This section uses UK electricity generation to demonstrate the proposed framework and quantify the impact of electricity interruptions on other economic sectors. It also examines the optimal allocation of electricity to other sectors to minimise economic loss when electricity supply shortage appears.

7.1 Input Data

Two electricity development scenarios by UK's National Grid for 2018/19, Slow Progression (SP) and Low Carbon Life (LCL) under various wind penetrations are used for demonstration [26]. The LCL scenario is a world of high affordability and low sustainability. High penetration of low carbon generation and demand is desirable in this scenario. Government policies are mainly focused on the long-term with consensus around decarbonisation, which is delivered through purchasing power and macro policy. The SP scenario is a world of high affordability and low sustainability. By contrast, the penetration rate of low carbon generation and demand is relatively slow compared to low carbon life scenario. Although there are political wills and market interventions, slower economic recovery in this scenario delays delivery against environmental targets. The UK planned electricity generation capacity in 2018-2019 under the two scenarios is provided in Table II. The installed capacity is 75.3GW and 79.6GW respectively, and the LCL scenario has more renewable energy particularly wind power. The peak demands in SP and LCL are fairly close, 54.1GW and 54.2GW respectively. The typical data from UK's National Grid is used to represent electricity demand profiles [27] and the peaks are enlarged to match the peaks in the two SP and LCL scenarios. The economic data from UK Office for National Statistics is utilized for deriving the Input-Output model [28].

The availability levels of all generation technologies in the first column of Table II are from UK's electricity regulator the Office of Gas and Electricity Markets (Ofgem) [29]. As seen, traditional generation ith relatively simple engineering complexity has much higher availability, such as pump storage of 0.97, but complex generation such as nuclear has a low reliability of 0.81. Wind power plant has a high reliability of 0.95.

Table II Installed Generation Capacity in the UK (MW)

Generation	Availability	SL	LCL
Biomass	0.88	2,353	2,353
Coal	0.88	12,342	13,316
Gas-CCGT	0.94	31,408	30,346
Gas-CHP	0.94	1,699	1,544
Hydro	0.84	1,122	1,122
Nuclear	0.81	8,981	8,981
OCGT	0.94	735	735
Oil	0.82	0	0
Pump storage	0.97	2,744	2,744
Tidal	0.84	0	20
Wind	0.95	15,566	18,486
Total	N/A	75276	79647

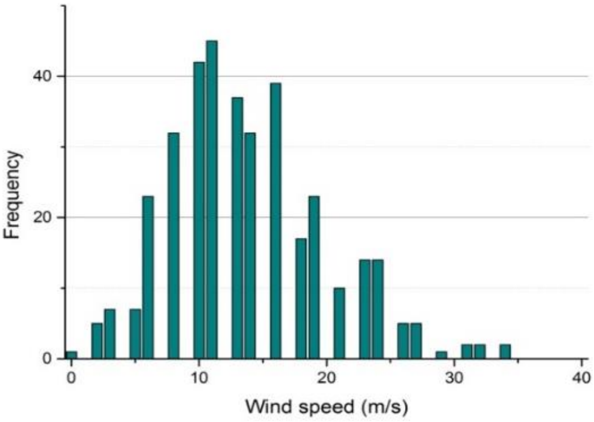

Fig. 4. Distribution of typical UK wind speed.

Table III Wind Power Output and the Probabilities

	W1	W2	W3	W4	W5	W6
Output level of wind power	100%	80%	60%	40%	20%	0%
Probability	0.32	0.19	0.24	0.09	0.08	0.08

The distribution of wind speed from a typical UK wind farm in Figure 4 is used to quantify wind power output. For simplicity, the wind power output is modelled as a generator with six output states and the following parameters are assumed: cut-in speed of 3 m/s, rated output speed of 14m/s, and cut-out speed of 25m/s. Table III gives the probability of the wind power output in relation to the total installed capacity and wind speed in Figure 4. The percentage means the output level of wind power corresponding to different wind speed. For example, the 100% output means that the wind power can produce its rated capacity output as the wind speed is above rated speed but below cut-out speed. When wind speed drops below rated speed, the output will decline as well. As seen, the wind power can provide rated power with a probability of 0.32, followed by 60% output with a probability of 0.24. For output lower than 40% of rated capacity, the probability is relatively small with a sum of 0.25.

7.2 Interdependency Analysis

By utilisting the proposed method on the UK generation case, all obtained indexes are given in Table IV, including the self-interdependency and cross-dependency inoperability indexes of other sectors on electricity, and the power dispersion degree of electricity supply on other sectors.

As seen, electricity sector has the largest self-interdependency index of 2.21, indicating that it is the most highly dependent on its own output to produce electricity. By contrast, S7-public administration sector has the least index of 1.01, which means it is less dependent on its own output. In terms of dependency on electricity, S2-mining has the highest ratio of 0.23 followed by S4-construction sector of 0.12, indicating that their production can be seriously interrupted by electricity supply shortage. The IPD reflects the dispersion level of one sector to others. When bigger than 1, it indicates the sector has a higher impact on other sectors: the higher the value is, the bigger the impact. As seen, electricity supply's IPD is 1.28, ranking the fourth after sectors S2-S4. It means that electricity supply is fairly important to other sectors. By contrast, S3-manufacturing sector has the largest IPD index of 1.68.

Table IV Indexes for all sectors

Sector	Self-dependency	On electricity	IPD
S1:Agriculture	1.39	0.02	0.92
S2:Mining	1.10	0.23	1.31
S3:Manufacturing	1.61	0.04	1.68
S4:Construction	1.97	0.12	1.61
S5:Distribution and catering	1.04	0.01	0.65
S6:Transport and communication	1.38	0.04	0.80
S7:Finance and business	1.51	0.07	0.87
S8:Public administration	1.01	0.00	0.69
S9:Education health	1.08	0.00	0.57
S10:Other	1.14	0.02	0.61
S11:Electricity	2.21	-----	1.28

7.3 Inoperability of Different Economic Sectors under Electricity Interruptions

This subsection provides the inoperability indexes of all sectors in response to electricity supply interruptions under both SP and LCL scenarios.

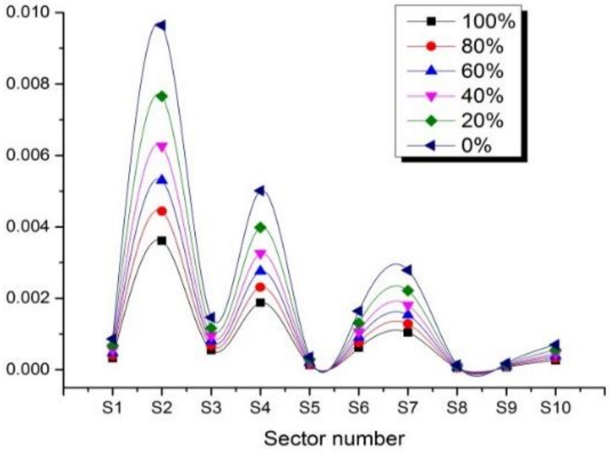

Fig.5. Inoperability under various wind speed (SP scenario).

Figures 5 and 6 are results of inoperability of all sectors under various wind speed levels from 0% to 100% in Table IV. Apparently, in both scenarios, S2-mining has the highest inoperability level around 0.01with zero wind output, followed by S4-construction around 0.005. With increasing wind output, inoperability indexes drop gradually. At all wind output levels, the inoperability indexes in LCL scenario are lower than those in SP scenario. It is because that although LCL scenario has slight more demand, the wind installed capacity is much higher by around 4.5GW compared to SP scenario. Thus, the shortage of electricity supply can be partially picked up by the increasing generation capacity.

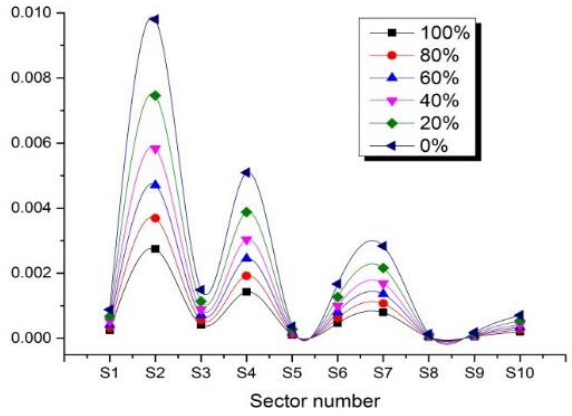

Fig.6. Inoperability under various wind speed (LCL scenario).

The expectations of inoperability indexes of all sectors are given in Figure 7, calculated by using the results in Figures 5 and 6. Generally, they are all lower in LCL scenario than those in SP scenario. For example, S2 has the highest inoperability indexes of 0.0052 in SP scenario and 0.0046 in LCL scenario, and S8 has the least of 0.003 and 0.001 respectively. It is mainly due to that although peak demand in LCL scenario is approximately 0.1GW higher than that in SP scenario, the installed generation capacity is also higher by 4.6GW. From another aspect, it can be seen that the inoperability indexes in the two scenarios match well with the proposed interdependency indexes on electricity supply in Table IV (the third column). For example, S2 has the biggest inoperability and its interdependency inoperability index on electricity supply is also the biggest of 0.232. By contrast, sectors S4, S7, and S6 have relatively high inoperability indexes in a descending order, which match the interdependency inoperability indexes on electricity supply as well.

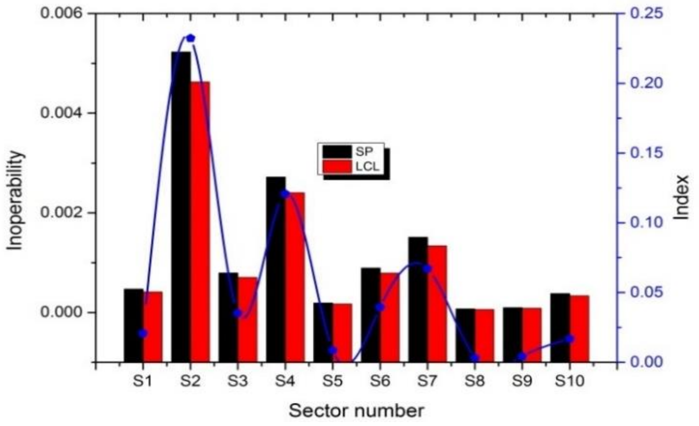

Fig.7. Comparison of inoperability and interdependency indexes.

7.4 Economic Loss

The expected economic losses of other sectors due to electricity supply interruptions are quantified in Table V. Clearly, all sectors have higher losses in the SP scenario than in the LCL scenario. Particularly, Sector S7-finance and business suffers the largest loss of £38.77 million in SP and £34.31 million in LCL. It is because that although S7 has small inoperability indexes of 0.0015 and 0.0013 in both cases, it has the largest output of £25,642 million. Thus, small perturbations of electricity supply could cause severe disruptions to its production. It is followed by sector S4- construction with a loss of £12.16 million and £10.76 million respectively in both scenarios. Sector S8-public administration has the least economic loss across all sectors.

Table V Expected Economic Loss of Sectors (£million)

Sector	SP	LCL	Sector	SP	LCL
S1	1.08	0.96	S6	8.12	7.18
S2	5.81	5.15	S7	38.77	34.31
S3	5.60	4.96	S8	0.91	0.81
S4	12.16	10.76	S9	1.55	1.37
S5	3.21	2.84	S10	2.44	2.16

7.5 Sensitivity Analysis of Wind Power Penetration

The impact of increasing wind power on other economic sectors is investigated by studying two extra scenarios in this subsection, scenario 3 and scenario 4. In both two new scenarios, system peak demands are assumed to be equal to that in the LCL scenario, i.e. 54.234GW. However, in scenario 3, it is assumed that 50% of coal generation is replaced with wind power, and in scenario 4 all coal generation (100%) is replaced with wind power.

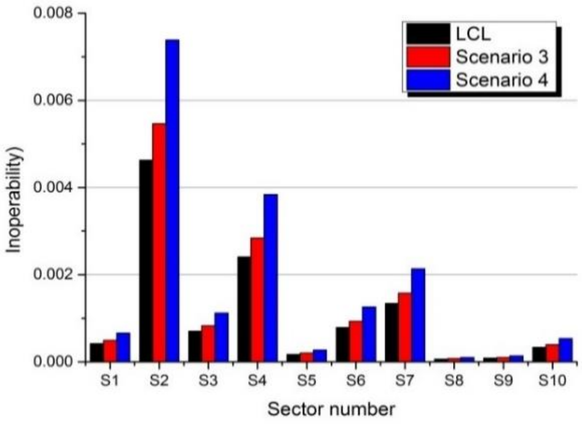

Fig.8. Comparison of inoperability across scenarios.

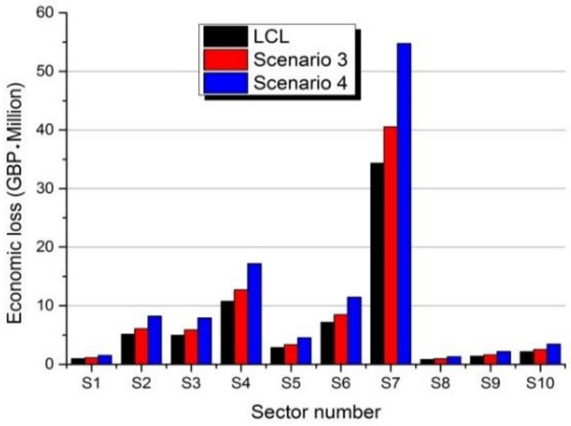

Fig.9. Comparison of economic loss across scenarios.

Figure 8 compares the inoperability indexes of all 10 sectors in LCL, 3 and 4 scenarios. Apparently, the indexes dramatically increase with more wind power penetration, particularly for those sectors severely dependent on electricity supply. For example, the inoperability of S2-mining increases from 0.0007 in LCL scenario to 0.0008 in scenario 3, and to 0.0011 in scenario 4. The values for S4-construciton and S7-finace also increase by 0.0004 from LCL scenario to scenario 3. The inoperability indexes for other sectors follow the same growing patterns.

The economic losses of all 10 sectors under the three scenarios are depicted in Figure 9, which also climb dramatically with increasing wind power generation. Complying with the results in Table V, sector S7 has the highest economic losses in all three scenarios, where the value is as high as £54.76 million in scenario 4. S4-constrution also suffers high losses, £10.76 million, £12.71 million and £17.18 million in LCL, scenarios 3 and 4 respectively. For all sectors, S8 and S1 have the least economic losses because of their lower dependency levels on electricity supply, which are £1.29 million and £1.53 million in scenario 4.

The results in the two figures illustrate that the inoperability and economic losses of all economic sectors increase exponentially with growing wind power penetration. It is mainly because that wind power is relatively intermittent compared to traditional generation, and thus reliability is low. The sectors with higher dependency indexes on electricity supply have bigger inoperability indexes if electricity supply is interrupted. By contrast, economic losses are decided by both inoperability indexes and their total production, which vary over sectors.

7.6 Optimal Available Electricity Allocation

This section provides the results of optimal allocation of electricity supply to its own external demand and other sectors when the supply is interrupted. Both electricity supply inoperability and the perturbations of external demand of other sectors are assumed to be 0.1. The objective is to minimise the total economic loss of other sectors. The obtained economic losses are presented in Table VI, which are also compared with the case without optimal allocation. As seen, the total reduced cost for all sectors is £114.5 million, with sector 2 enjoying the maximum reduction of £48.2 million.

Table VI Optimal Economic Loss of Sectors (£million)

Sector	**Without optimisation**	**With optimisation**	Sector	**Without optimisation**	**With optimisation**
S1	640.3	635.9	**S6**	612.9	604.6
S2	833.3	784.9	**S7**	562.5	548.5
S3	609.6	602.2	**S8**	252.3	251.6
S4	855.4	830.2	**S9**	272.5	271.6
S5	316.3	314.5	**S10**	362.2	358.7
			Sum	5317.3	5202.8

In order to obtain more insights, a sensitivity analysis of other sector's inoperability and economic losses with respect to electricity supply inoperability is in conducted here in two scenarios: the old case without optimal management and with the optimal management.

Figure 10 provides the inoperability of three selected sectors, S5, S6, and S7. As seen, with increasing electricity supply inoperability from 0.05 to 0.5, the operability indexes of all three sectors increase, represented by the dashed lines, particularly for S6 and S7, clime very quickly. The inoperability of S5 increases slightly always around 0.14 as it is less dependent on electricity supply. For the inoperability indexes in the optimal management case represented by the solid lines, they are always below those in the old case, particularly for S6 and S7 whose indexes drop with increasing electricity supply shortage.

The economic losses of the three sectors are depicted in figure 11. Apparently, S7 has the largest economic loss in all electricity supply shortage cases with the largest, which reaches £6500 million with electricity inoperability of 0.5. This value drops to around £5700 million with the optimal management. For both S5 and S6, their economic losses are always below £2500 million in all electricity shortage case. Apparently, their losses are much lower with the optimal management compared to the case without any optimal management.

The results here justify that without more unreliable electricity supply, different economic sectors will suffer losses that are decided by their dependency degree on electricity and their planned output. The new optimal management can effectively reduce the losses by allocating avoidable electricity to various sectors during electricity shortage.

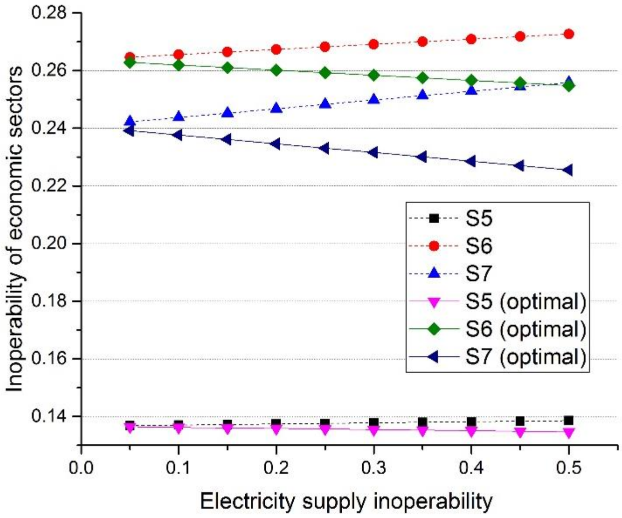

Fig.10. Comparison of inoperability under various electricity supply inoperability

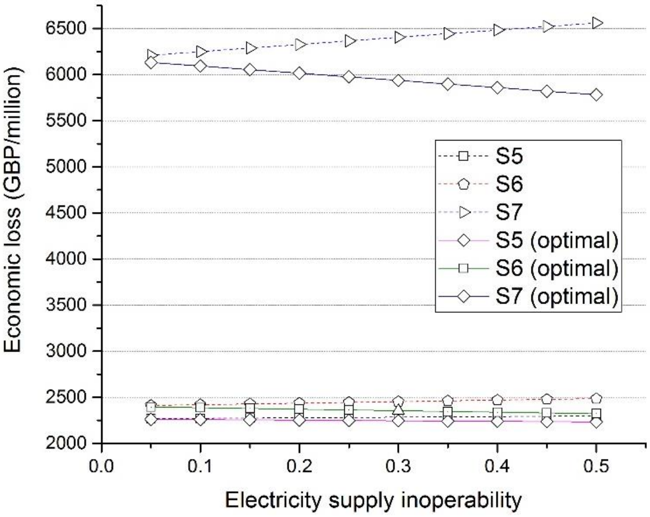

Fig.11. Comparison of economic losses under various electricity supply inoperability

8. Conclusions

This paper proposes a new framework to quantify the impact of electricity supply interruptions on other sectors by introducing a novel technical-economic model. It is the first effort to integrate economic interdependency with technical electricity supply reliability for interdependency analysis. The designed indexes can effectively capture the dependency degree of economic sectors on electricity supply and their economic losses in electricity shortage. By extensive demonstration, the following key findings are observed:

- The value of economic losses of different sectors due to electricity supply interruptions is decided by the scales of both inoperability indexes and their total normal production.
- The growing penetration of wind power jeopardizes electricity system reliability and consequently affects other sectors in electricity shortage. The inoperability indexes and economic losses increase exponentially with increasing wind power penertration.
- The proposed optimal management model can effectively reduce economic loss for other sectors by efficient allocating available electricity during interruptions.

The research can enable policymakers to understand the impact of electricity interruptions on different economic sectors and quantify their losses so that more informative policies can be designed to ensure secure electricity supply, such as the capacity market. In the future, more accurate demand profiles and detailed network models of each typical sector will be included to conduct more accurate modelling and analysis.

Acknowledgement

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