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**Monetary Aggregates and Monetary
Policy in Peru**

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MONETARY AGGREGATES AND MONETARY POLICY IN PERU.

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Abstract

This paper investigates empirically the usefulness of monetary aggregates as information variables in the conduct of monetary policy. For this purpose, some recent advances on the topic were used, which include the analysis of both real-time and revised final data, and the application of Bayesian model averaging to allow for model uncertainty regarding the lag length and number of cointegrating relationships. In this paper, money is considered as an information variable for W_t (e.g. output or prices) if the following two criteria are satisfied: (i) M_t is strongly exogenous, and (ii) M_t Granger-causes W_t . Strong exogeneity is relevant because it validates conditional forecasting of W_t using monetary aggregates as conditioning variables. The results show no strong evidence supporting the usefulness of monetary aggregates as information variables for prices or output. However, this does not preclude their potential use as information variables for other macroeconomic targets such as financial stability.

JEL Classification : C32, E52, E58

Key Words : Bayesian Model Averaging, cointegration, Granger causality, monetary aggregates, monetary policy, real-time data, strong exogeneity.

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1 Introduction

Monetary aggregates constitute a main part of the statistics collected by most central banks. However, the theoretical and practical relevance of money in the conduct of monetary policy has changed over time. During the 1960's and the 1970's monetary aggregates were considered key instruments for monetary policy, but their usefulness was questioned in the early 1980's due to instabilities displayed by well-established empirical relations connecting monetary aggregates to prices, output, and interest rates (Goldfeld and Sichel, 1990; Benati and Goodhart, 2010; Lucas Jr. and Nicolini, 2015).¹ With the emergence of the new Keynesian approach in the 1990's,² monetary policy switched focus to interest rate rules, leaving no key role to money (Clarida et al., 1999; Woodford, 2003).³ Recently, the international financial crisis has shown the limits of interest rate rules (the “zero lower bound problem”) and led to the implementation of unconventional monetary policies in major developed countries, which have motivated new research that suggests that monetary aggregates can be useful in the conduct of monetary policy (e.g. Benchimol and Fourcans, 2012; Dreger and Wolters, 2015).

Currently, only 24 countries out of 192 use monetary aggregates as monetary policy instruments under a monetary targeting regime (IMF, 2016).⁴ However, the praxis of some major central banks such as the European Central Bank, the Bank of England, and the Bank of Japan, among others, suggests that monetary aggregates can be useful as information variables because they provide relevant information about future inflation and output.⁵ This possibility is also supported by several recent papers (e.g. Dotsey and Hornstein, 2003; Nelson, 2003; Coenen et al., 2005; Beck and Wieland, 2007; Aksoy and Piskorski, 2006; Hafer et al., 2007; Benchimol and Fourcans, 2012; Dreger and Wolters, 2015, among others).⁶

This paper investigates empirically the usefulness of monetary aggregates as information variables in the conduct of monetary policy in Peru. To this end, a particular monetary aggregate M_t will be considered as an information variable for W_t (e.g. output or

¹By the end of the 1960's the consensus was that money had a prominent role in monetary policy and could be used as an intermediate target (Friedman and Schwartz, 1963; Andersen and Jordan, 1968; Sims, 1972, among others).

²This new framework, which Clarida et al. (1999) denominated “New Keynesian Perspective”, was motivated by monetary policy rules proposals such as Taylor (1993)'s recommendation of a simple interest rate rule and the growing adoption of so-called “inflation targeting” regimes during the 1990's.

³Furthermore, it has been argued that monetary policy can be theoretically studied without making any specific reference to money, and that money has no role as an additional tool for stabilisation policies (Woodford, 2008; Cúrdia and Woodford, 2011)

⁴These countries include low-income and emerging markets. Among the remaining countries, 38 middle-income and advanced countries follow an inflation targeting regime, 82 countries use an exchange rate anchor, and 48 countries - which include many of the largest economies - follow a monetary regime that is not committed to any particular target.

⁵The evolution of monetary aggregates has always had a prominent role for the European Central Bank, which is not the case with the FED. In particular, the FED decided to cease the publication of M3 in 2006 arguing that it did not contain any additional information about economic activity beyond that contained in M2. See <http://www.federalreserve.gov/Releases/h6/discm3.htm>.

⁶As pointed out by Friedman and Kuttner (1992), Kareken et al. (1973) appears to be the first paper which formally introduced the “information-variable” concept into the analysis of monetary policy.

prices) if the following two criteria are satisfied: (i) M_t is strongly exogenous, and (ii) M_t Granger-causes W_t . If a monetary aggregate M_t is strongly exogenous (for some relevant parameters),⁷ then it can be safely used as a conditional variable to perform conditional forecasts of W_t n periods ahead (Engle et al., 1983; Banerjee et al., 1993; Hendry, 1995; Pedregal and Young, 2002); if M_t is only weakly exogenous, then valid conditional forecasts are restricted to one period ahead. Testing for strong exogeneity of M_t requires testing for weak exogeneity of M_t and Granger noncausality from W_t to M_t (Hendry, 1995). On the other hand, if M_t Granger-causes W_t , then it is said that M_t has predictive content for W_t .

To illustrate the usefulness of the concept of strong exogeneity, consider a simple linear dynamic relationship between money (M_t) and GDP (y_t). If money has predictive content for GDP, then it is possible to perform unconditional forecasts of future values of GDP i periods ahead, y_{t+i} , for $i = 2, 3, \dots$; however, these unconditional forecasts require the simultaneous calculation of future values of y_t and M_t . In contrast, if M_t is strongly exogenous, then a conditional forecast of y_{t+i} can be performed using future values of M_t that are calculated independently of future values of y_t .⁸ Thus, strong exogeneity of monetary aggregates is useful because it allows to perform valid forecasts of the variable of interest under different scenarios (i.e. different paths of future values) of M_t , which can be obtained from a marginal model. Furthermore, strong exogeneity is particularly useful because monthly *GDP* data are usually available after two or three periods; thus, forecasting GDP_{t+2} using M_{t+1} will not be possible because it requires information about GDP_t (to forecast M_{t+1}) which will be available at $t + 2$.

Strong exogeneity and Granger causality of monetary aggregates are tested empirically using two approaches. Under the first approach, which will be referred to as “model certainty”, a single “best” vector error correction model (VECM) is estimated for each monetary aggregate M_{i_t} ($M0$, $M1$, $M2$, and $M3$); then Granger non-causality from M_t to the variable of interest and strong exogeneity are tested following the procedure suggested in Hendry (1995). Under “model certainty”, the selection of the best VECM for each monetary aggregate is done using standard econometric procedures that include testing for well-behaved errors, parameter stability, optimal lag length, and correct number of cointegrating relationships. Each VECM includes as endogenous variables a domestic price index, real output, a domestic interest rate, the exchange rate, and a specific monetary aggregate ($M0$, $M1$, $M2$, and $M3$). The second approach, which will be referred to as “model uncertainty”, does not rely on a single best VECM but on a combination of VECMs with different lag lengths and different number of cointegrating relations, which are combined using Bayesian model averaging (BMA). Under “model uncertainty”, the relevance of monetary aggregates is based on the probability that M_{i_t} is strongly exogenous, and the probability that M_{i_t} has predictive content for W_{i_t} .

Under both approaches, each VECM is estimated using revised final data and a novel real-time data set for monetary aggregates. The real-time data set was specially con-

⁷For simplicity, we will omit the clarification that “exogeneity” is with respect to “some relevant parameters”, unless it is necessary.

⁸Thus, strong exogeneity permits valid forecasts of y_t from a conditional model, given forecasts of M_t from a marginal model.

structed for this paper and covers the period January 1994 - December 2011.⁹ Following the existing literature, real-time data is defined as the data that were available to policy-makers and forecasters at each point in time.

The empirical analysis is performed for the period January 1994 - December 2011, which covers two monetary policy regimes in Peru: (i) monetary targeting (1993-2001), based on the control of monetary aggregates, and (ii) inflation targeting (since January 2002), based on the announcement of an inflation target and the use of an official interest rate (the so called “reference interest rate”) as the operational target or policy instrument. Taking into account this change in the monetary policy regime, the empirical analysis is also performed for the sub-sample January 2002 - December 2011.

The results do not support the relevance of monetary aggregates as information variables in the case of Peru. Under model certainty, the recursive parameter stability tests do not provide strong evidence of a fully stable VECM for any monetary aggregate, a result that is obtained using both real-time and revised final data. Therefore, monetary aggregates may provide a poor signal of current and future output growth or inflation. However, if one is able to tolerate some subjective degree of instability, the results show that M0 and M2 have been strongly exogenous only for the recursive samples covering the inflation targeting period January 2002 - December 2011, whereas the predictive content of M0 and M2 using revised final data has been significant only until the beginning of 2008. Under model uncertainty, the probability that M0 is strongly exogenous for prices displayed a stable evolution along the recursive samples that cover the periods January 1994 - December 2011 and January 2002 - December 2011; however, these probabilities have remained below 55%. The probability that M2 is strongly exogenous for real output has displayed a similar evolution, both with real-time and revised final data.

Although these results do not make a strong case for the use of monetary aggregates as information variables in Peru, it could be cautious not to fully discard them (see Thornton, 2014). In fact, Drake and Fleissig (2006) and Lucas Jr. and Nicolini (2015) show that an alternative definition of monetary aggregates could display a stable relation with prices and output. Furthermore, monetary aggregates can still be useful if they contain information about any other variable relevant for monetary policy; for instance, if monetary authorities are also concerned with securing financial stability through the implementation of macroprudential policies, monetary aggregates may be useful in the construction of financial stability indicators (e.g. Kim et al., 2013).

This paper follows closely the empirical approach developed by Garratt et al. (2009), which includes two ingredients from the recent literature about the predictive content of money: the use of real-time data and the application of Bayesian model averaging. The results obtained with real-time data might be very different compared to those obtained with revised final data because the latter usually incorporate important revisions; thus, the use of real-time data is more realistic for policy analysis (Orphanides, 2001; Bernanke and Boivin, 2003; Croushore and Stark, 2003). Amato and Swanson (2001), and Garratt

⁹The original database for this paper was completed in April 2012 and covered the period until December 2011. The update of the real-time database is left for a future version of this paper.

et al. (2009) use real-time data to analyse the predictive content of monetary aggregates for output and prices. On the other hand, Bayesian model averaging (BMA) allows for model uncertainty regarding the lag length and the number of cointegrating relationships; thus, instead of relying on a single “best” model specification, BMA constructs a weighted average of all available model specifications, where the weights are given by the posterior model probabilities. Garratt et al. (2009) is the first to use Bayesian methods to measure the model uncertainty in this literature; furthermore, they compare BMA results with those from the standard approach of model selection that in this paper is referred to as “model certainty”.

One contribution of this paper is that it extends the empirical framework proposed by Garratt et al. (2009) by testing weak and strong exogeneity of monetary aggregates. Garratt et al. (2009) and Amato and Swanson (2001) focus on the marginal contribution of monetary aggregates to forecast a variable of interest; however, they do not assess whether monetary aggregates can be used to perform conditional forecasts.

This paper is also related to some previous attempts that study the role of monetary aggregates for monetary policy in Peru (León, 1999; Berg and Borensztein, 2000; Lahura and Rodriguez, 2005, among the most relevant). However, this is the first paper that performs a recursive analysis using both real-time and revised final data together with BMA. Furthermore, this paper provides the first real-time data set for Peruvian monetary aggregates.

The results obtained in this paper are in line with previous studies that provide evidence against the use of monetary aggregates in the conduct of monetary policy, which include Friedman and Kuttner (1992), Estrella and Mishkin (1997), Woodford (2003), Kandil (2005), Lippi and Neri (2007), Woodford (2008), Binner et al. (2009), Garratt et al. (2009), among others.

The remainder of this paper is organised as follows. Section 2 presents the statistical properties of revised final data and describes the construction of the real-time data for monetary aggregates. Section 3 presents the empirical methodology. Section 4 presents and discusses the results. Finally, the main conclusions are presented in Section 5.

2 Data

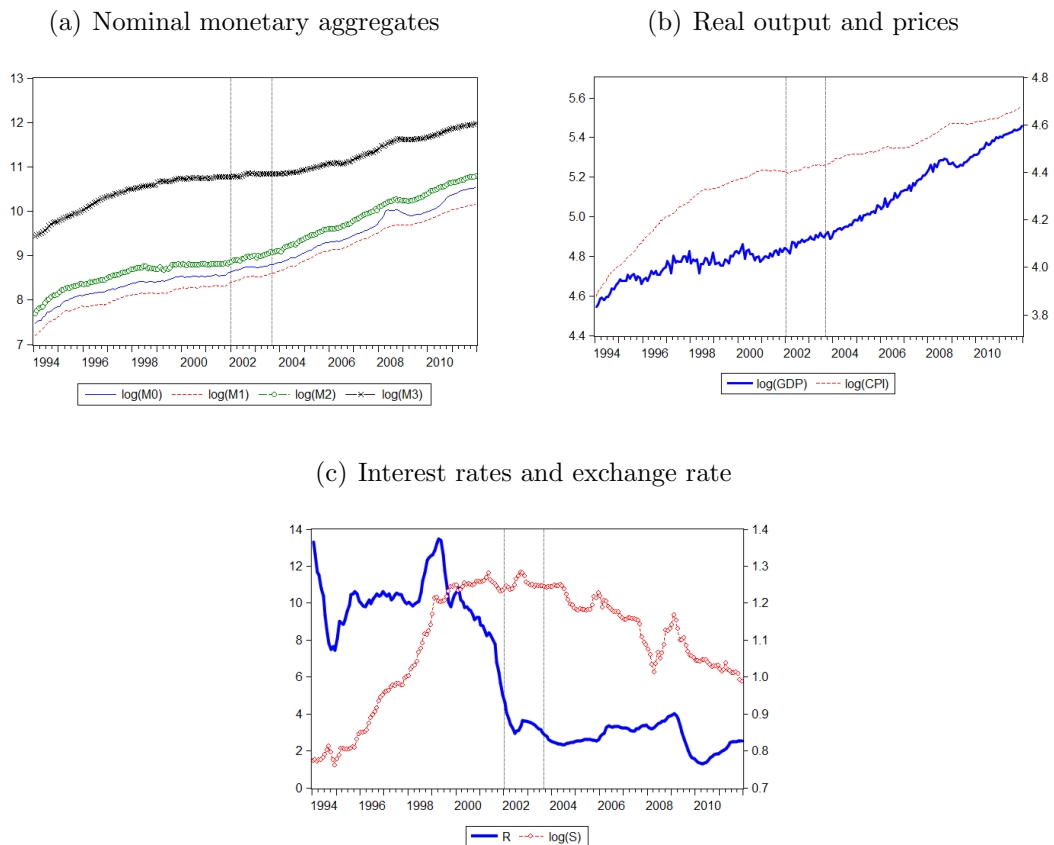
The paper uses two data sets. The first one consists of final revised final data for all the variables considered. The second data set is the same as the first one but replaces final revised data for monetary aggregates by a novel real-time data set that was specially constructed for this paper with information up to December 2011. The empirical analysis is performed for two sub-samples: January 1994 - December 2011 (full sample) and January 2002 - December 2011 (inflation targeting sample).

Revised final data were obtained from the Central Reserve Bank of Peru. Four nominal monetary aggregates were chosen: monthly average monetary base ($M0$), monthly average currency ($M1$), currency plus demand deposits ($M2$), and total liquidity in domestic cur-

rency ($M3$).¹⁰ Economic activity, prices and the interest rate are measured by real Gross Domestic Product (GDP), the Consumer Price Index (CPI), and the average interest rate in domestic currency (R), respectively. The nominal exchange rate (S) is measured by the monthly average exchange rate of the banking system. All variables, except interest rates, were seasonally-adjusted and transformed into log levels.

Figure 1 shows the evolution of revised final data up to December 2011. All variables are expressed in logs except for interest rates. The dotted vertical lines indicate the beginning of the inflation targeting regime (January 2002) and the announcement of the reference rate as the operational target (September 2003). Inspection of Figure 1 suggests that: (i) money, prices, output, and interest rates are all non stationary, and (ii) there might be some possible breaks in the log of CPI, the interest rate in domestic currency, and the exchange rate.

Figure 1. *Money, Output, Prices and Interest Rates in Peru: 1994-2011*



NOTE: In panels 1(a), and 1(b), the vertical axes are measured in logs, whereas in panel 1(c) the vertical axis is measured in percentages. R and S represent the average interest rate in domestic currency and the nominal exchange rate, respectively.

¹⁰Although the labels $M0$, $M1$, $M2$, and $M3$ do not coincide with the standard definition of monetary aggregates, they represent successively broader monetary aggregates.

The presence of unit roots in the series was tested using the Augmented Dickey-Fuller (ADF) test. Because the series display a trending behaviour, the DF-GLS test (Elliott et al., 1996) was also applied because of its efficiency in detecting unit roots when the series contain deterministic components. Given the possible presence of breaks in the series, we also apply Perron’s unit root test with endogenous breaks (Perron, 1997). The results are shown in Table 1. According to the ADF and DF-GLS tests, the unit root hypothesis cannot be rejected in any case. Furthermore, the Perron (1997) test does not show evidence of trend-stationarity with breaks. Therefore, all the time series considered can be treated as unit root processes.

Table 1. *Unit root tests*

Test	M0	M1	M2	M3	Real GDP	CPI	R	R*	S
ADF	-1.48	-1.26	-1.08	-2.49	-1.04	-3.18*	-2.21	-2.19	-1.76
DF-GLS	-1.60	-1.65	-1.64	-1.03	-1.24	-1.07	-1.85	-1.72	-0.26
PERRON	-5.47*	-4.82	-4.24	-3.18	-5.50*	-4.18	-4.42	-4.94	-4.83

Notes: ADF and DF-GLS test the null hypothesis of unit root against the alternative of trend stationarity, whereas Perron tests the null of unit root with a break against the alternative of stationarity with a break. ADF critical values: -4.01 (1%), -3.43 (5%), -3.14 (10%). DF-GLS critical values: -3.46 (1%), -2.93 (5%), and -2.64(10%). Perron critical values: -6.22 (1%), -5.55 (5%), and -5.25 (10%). Rejection of no significance at 1%, 5%, and 10% are represented by “***”, “**”, and “*”, respectively. At 10% level of significance, Perron test suggests that nominal M0 has a break in 2002m7, whereas real GDP has a break in 2000m2. All series are stationary in first differences.

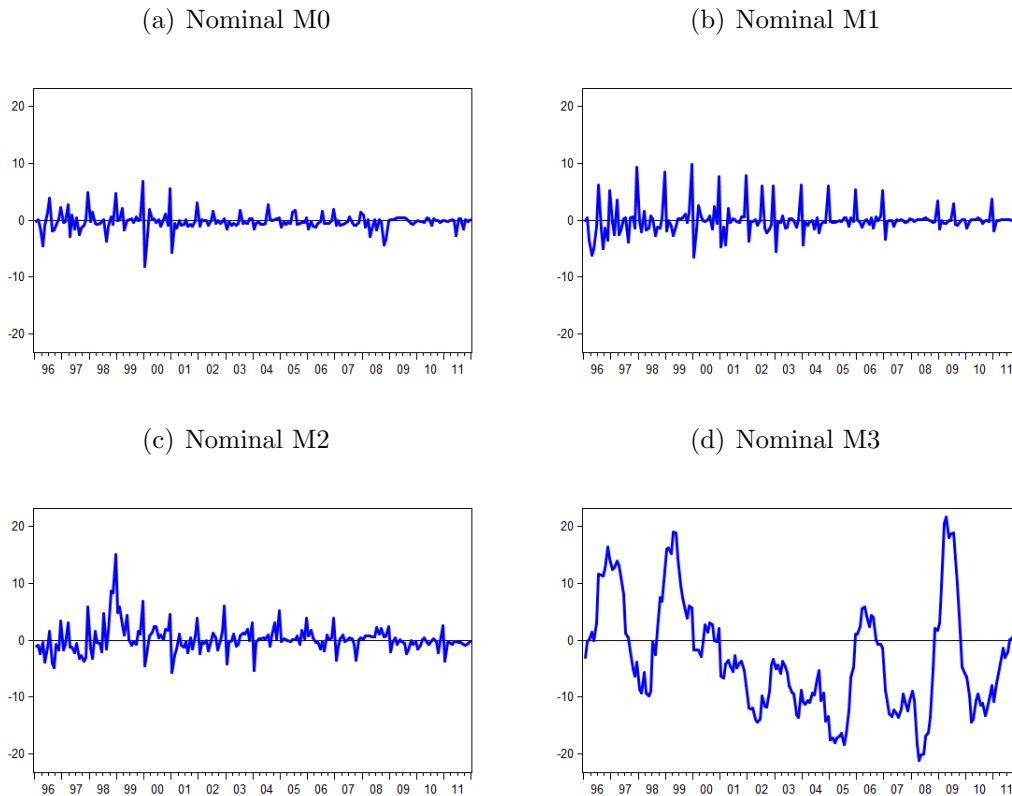
Real time data are defined as the data that were available to policymakers and forecasters at every period of time. For instance, a real-time series for M1 available in December 2011, denominated “December 2011 vintage”, contains the latest available time series for M1 up to December 2011, which typically includes information up to November 2011 or earlier. Real-time data is usually revised in the following vintages (January 2012, February 2012, and so on), so that a comparison between real-time data and revised final vintage data might yield important differences. The use of real-time data in empirical research is more realistic from a policy perspective as stressed by Orphanides (2001), Bernanke and Boivin (2003), and Croushore and Stark (2003).

In Peru, there is no official real-time data set for monetary aggregates. Therefore, we constructed a real-time data set for monetary aggregates using printed versions of the Weekly Report published by the BCRP and other internal sources,¹¹ which covers the period January 1994 - December 2011. The construction and use of this real-time data set for monetary aggregates is one of the key contributions of this paper.

The real-time data set contains 168 vintages for each monetary aggregate. Each vintage available at time t contains information up to $t - 1$ (i.e. the previous month). Thus, the first vintage is given by the data available in January 1998, which includes information from January 1992 to December 1997; ... ; the last vintage is given by the data available in January 2012, which includes information from January 1992 to December 2011. The size of the difference between real-time and revised final data can be observed in Figure

¹¹Most recent real-time data is based on internal sources, which was available for this paper only until December 2011. We are grateful to Guillermo Guevara, Head of Monetary Statistics, for providing the available internal real-time information.

Figure 2. *Real-time versus revised final vintage data: 1996-2011*



NOTE: *The figures show the difference in the annual growth rate between real-time and revised final monetary aggregates. The vertical axes are measured in percentage points.*

2, which shows the differences in annual growth between real-time and revised final data for each monetary aggregate. This comparison suggests that monetary aggregates in Peru have been subject to important revisions, especially in the case of M3. Therefore, results from revised final data should be taken with caution as they might be obscuring some key information constraints faced by policymakers in real time, such as “model uncertainty”.

3 Econometric Methodology

The empirical methodology is based on the use of recursively estimated vector error correction models (VECM), and the assessment of weak and strong exogeneity of monetary aggregates. The specification and estimation of each VECM, and the assessment of exogeneity are performed under two alternative approaches: “model certainty” and “model uncertainty”.

3.1 Vector error correction model (VECM)

Let X_t be a vector of order $(n \times 1)$ that contains n variables $X_{t1}, X_{t2}, \dots, X_{tn}$. An unrestricted VAR model of order q for X_t is defined as:

$$X_t = A_1 X_{t-1} + A_2 X_{t-2} + \dots + A_q X_{t-q} + \Phi D_t + \varepsilon_t \quad (1)$$

where A_j is the $(n \times n)$ matrix of autoregressive coefficients for $j = 1, 2, \dots, q$, D_t is a vector of deterministic components (which can include an intercept, a linear trend, seasonal dummies, intervention variables, or weakly exogenous stationary random variables), and ε_t is a vector $(n \times 1)$ of white noise errors characterised by:

$$E(\varepsilon_t) = 0$$

$$E(\varepsilon_t \varepsilon_\tau') = \begin{cases} \Omega, & \text{for } t = \tau \\ 0, & \text{otherwise} \end{cases}$$

If the series contained in X_t are non-stationary, then the unrestricted VAR (1) can be reformulated in terms of differences, lagged differences, and levels of the process X_t , as:

$$\Delta X_t = \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_q \Delta X_{t-q} + \Phi D_t + \varepsilon_t \quad (2)$$

where $\Pi = -(I - \sum_{i=1}^q A_i)$ and $\Gamma_i = -\sum_{j=i+1}^q A_j$ are $(n \times n)$ matrices.

If all variables contained in X_t are unit root processes and there exist r linear combinations of elements of X_t that are stationary, then it is said that X_t is cointegrated and contains r cointegrating vectors. In this case, the non-stationary VAR given by (2) has an alternative representation known as a vector error correction model:

$$\Delta X_t = \alpha \beta' X_{t-1} + \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_q \Delta X_{t-q} + \Phi D_t + \varepsilon_t \quad (3)$$

where α and β are two $(n \times r)$ matrices with full-column rank. The expression $\beta' X_t$ represents the stationary linear combinations of X_t , in which each column of β is called a cointegrating vector. Mathematically, the existence of r cointegrating vectors imply that the rank of Π is equal to r , with $r < n$, and that $\Pi = \alpha \beta'$, where the matrix α contains coefficients α_{ij} that measure the response of ΔX_{it} to a shock in the j -th cointegrating relation.¹²

Following Garratt et al. (2009), X_t contains monetary aggregates, prices, real output, interest rate, and exchange rates, so that $n = 5$. Thus, if X_t has r cointegrating relations then the VECM can be written as:

¹²Notice that if there is no cointegrating relationship, then $\Pi = \alpha \beta' = 0$ and thus the non-stationary VAR can be written as a VAR in first differences.

$$\begin{aligned}
\Delta y_t &= \sum_{i=1}^r \alpha_i^y \epsilon_{i,t-1} + \sum_{i=1}^q [\gamma_i^y \Delta y_{t-i} + \theta_i^y \Delta p_{t-i} + \varphi_i^y \Delta M_{t-i} + \vartheta_i^y \Delta r_{t-i} + \pi_i^y \Delta s_{t-i}] + u_t^y \\
\Delta p_t &= \sum_{i=1}^r \alpha_i^p \epsilon_{i,t-1} + \sum_{i=1}^q [\gamma_i^p \Delta y_{t-i} + \theta_i^p \Delta p_{t-i} + \varphi_i^p \Delta M_{t-i} + \vartheta_i^p \Delta r_{t-i} + \pi_i^p \Delta s_{t-i}] + u_t^p \\
\Delta M_t &= \sum_{i=1}^r \alpha_i^M \epsilon_{i,t-1} + \sum_{i=1}^q [\gamma_i^M \Delta y_{t-i} + \theta_i^M \Delta p_{t-i} + \varphi_i^M \Delta M_{t-i} + \vartheta_i^M \Delta r_{t-i} + \pi_i^M \Delta s_{t-i}] + u_t^M \\
\Delta R_t &= \sum_{i=1}^r \alpha_i^R \epsilon_{i,t-1} + \sum_{i=1}^q [\gamma_i^R \Delta y_{t-i} + \theta_i^R \Delta p_{t-i} + \varphi_i^R \Delta M_{t-i} + \vartheta_i^R \Delta r_{t-i} + \pi_i^R \Delta s_{t-i}] + u_t^R \\
\Delta s_t &= \sum_{i=1}^r \alpha_i^s \epsilon_{i,t-1} + \sum_{i=1}^q [\gamma_i^s \Delta y_{t-i} + \theta_i^s \Delta p_{t-i} + \varphi_i^s \Delta M_{t-i} + \vartheta_i^s \Delta r_{t-i} + \pi_i^s \Delta s_{t-i}] + u_t^s
\end{aligned} \tag{4}$$

where M is the log of nominal money balances (seasonally adjusted), p is the log of prices, y is the log of real output (seasonally adjusted), R is the interest rate for deposits denominated in domestic currency, s the log of the exchange rate, and $\epsilon_{i,t-1}$ represents the i -th cointegrating relation normalised with respect to either y or p . For example, when $r = 1$, $\epsilon_t = y_t - \beta_0 - \beta_1 M_t - \beta_2 p_t + \beta_4 R_t + \beta_5 s_t$, or $\epsilon_t = p_t - \beta'_0 - \beta'_1 M_t - \beta'_3 y_t + \beta'_4 R_t + \beta'_5 s_t$.

Based on the general VECM 4, it is possible to analyse the usefulness of monetary aggregates as information variables in terms of the marginal contribution of money to forecast output and/or prices. For the case of output, the null hypothesis to be tested is:

$$H_0 : \alpha_1^y = 0, \dots, \alpha_r^y = 0 ; \varphi_1^y = \dots = \varphi_q^y = 0 \tag{5}$$

as in Garratt et al. (2009). The non-rejection of this joint hypothesis will mean that money has no predictive content for output. The analysis of the marginal contribution of money to forecast prices is analogous.

3.2 Weak and strong exogeneity

The usefulness of monetary aggregates as information variables can also be analysed using the concepts of weak and strong exogeneity. The concept of weak exogeneity was proposed in Richard (1980) and analysed by Engle et al. (1983) together with the concept of strong exogeneity. Hendry (1995) provides an excellent analysis of these concepts when the relevant empirical model contains unit root processes. As shown in Hendry (1995), weak exogeneity does not necessarily imply strong exogeneity; however, one necessary condition required to achieve strong exogeneity is weak exogeneity.

Weak and strong exogeneity are useful to determine whether it is appropriate to use one variable as a conditioning variable in order to perform one-period ahead or multi-step conditional forecasting (Banerjee et al., 1993; Hendry, 1995; Pedregal and Young, 2002). As an illustration, consider a dynamic relationship between money (m_t), GDP (y_t), and other variables. If money has predictive content for GDP, then an unconditional forecast

of future values of GDP i periods ahead, y_{t+i} , for $i = 2, 3, \dots$, requires the simultaneous calculation of future values of y_t and m_t . In contrast, a conditional forecast of y_{t+i} can be performed using future values of m_t that are calculated independently of future values of y_t . Following Banerjee et al. (1993), Hendry (1995), Pedregal and Young (2002), if m_t is weakly/strongly exogenous, then it is valid to forecast y_{t+i} conditional on m_{t+i} , and statements about future y_{t+i} conditional on m_{t+i} , for $i = 1/i = 1, \dots, H$ are not distorted by intermediate values y_{t+j} altering m_{t+i} .¹³

In this paper, weak and strong exogeneity tests are performed following Hendry (1995). To assess whether M_t can be used to make inferences about β_1 and perform conditional forecasts of y_t one-period ahead, the null hypothesis to be tested is:

$$H_0 : \alpha_1^M = \dots = \alpha_r^M = 0 \quad (6)$$

The non-rejection of this joint hypothesis means that M_t is *weakly exogenous* for β_1 , which we will simply refer to as “weak exogeneity of money for output”. To assess whether M_t can be used to make inferences about β_1 and perform conditional forecasts of y_t h -periods ahead for $h = 1, 2, \dots$, the null hypothesis to be tested is:

$$H_0 : \alpha_1^M = \dots = \alpha_r^M = 0 ; \gamma_1^M = \dots = \gamma_q^M = 0 \quad (7)$$

The non-rejection of this joint hypothesis means that M_t is *strongly exogenous* for β_1 , which we will simply refer to as “strong exogeneity of money for output”. Notice that the first part of the null hypothesis is equal to the null hypothesis of weak exogeneity, whereas the second part is testing whether y does not Granger cause M . An analogous procedure can be implemented to test weak and strong exogeneity of money with respect to prices.

If weak (strong) exogeneity is satisfied, then money can be safely used as a conditional variable to perform conditional forecasts of output or prices one-period (or more-than-one periods) ahead. Furthermore, given weak exogeneity of money, it is possible to “reduce” the VECM to a smaller system and thus obtain a parsimonious forecasting model for output and prices (see details in Engle et al., 1983; Johansen, 1992; Hendry, 1995; Juselius, 2006). Although it is very important for policy purposes to find a parsimonious forecasting model for output and prices, we do not attempt to do it in this paper and leave it for a future paper. Instead, we simply focus on determining whether money has the potential to be used as a conditioning variable in such a forecasting model (i.e. whether money is weakly and/or strongly exogenous for the relevant parameters) and on testing the marginal contribution of money given the initial specification of the VECM.

Amato and Swanson (2001) and Garratt et al. (2009) focus their analysis on the marginal contribution of money to predict both output growth and inflation. However, under their approach it is possible that, simultaneously, money has predictive content for output growth (i.e. $\alpha_1^y \neq 0$) and that money is not weakly exogenous (i.e. $\alpha_1^M \neq 0$). Therefore, in this paper a monetary aggregate will be considered as an information variable only if it is strongly exogenous and it has predictive content for prices and/or output, which requires the analysis of hypotheses 5, 6, and 7.

¹³Thus, strong exogeneity permits valid forecasts of p_t from a conditional model, given forecasts of m_t from a marginal model.

3.3 Empirical analysis under model certainty

For every monetary aggregate considered, the empirical strategy is to choose a single “best” VECM specification in terms of well-behaved errors, optimal lag length, correct number of cointegrating relationships, and parameter stability. Then, the three proposed hypotheses will be tested using a unique VECM specification.

Cointegration and diagnostic tests

Under the assumption that the vector of disturbances ε_t is normally distributed, the existence of cointegration can be tested using the rank test proposed by Johansen (1991), which evaluates the null hypothesis of r cointegrating vectors against the alternative of n cointegrating vectors, for $r = 0, 1, 2, \dots, n - 1$. Following Hamilton (1994), the construction of the corresponding test statistic, which is a likelihood ratio statistic denominated “trace statistic”, requires some auxiliary calculations. First, we need to regress ΔX_t on $\Delta X_{t-1}, \dots, \Delta X_{t-q+1}$, and X_{t-1} on $\Delta X_{t-1}, \dots, \Delta X_{t-q+1}$ by ordinary least squares (OLS), and collect the corresponding $(n \times 1)$ vectors of OLS residuals \hat{u}_t and \hat{v}_t , respectively. Then we calculate the sample variance-covariance matrices of the OLS residuals $\hat{\Sigma}_{\hat{u}\hat{u}}$, $\hat{\Sigma}_{\hat{v}\hat{v}}$, and $\hat{\Sigma}_{\hat{v}\hat{u}} = \hat{\Sigma}_{\hat{u}\hat{v}}$, where for instance $\hat{\Sigma}_{\hat{u}\hat{u}} = \frac{1}{T} \sum_{t=1}^T \hat{u}_t \hat{u}_t'$. Then the trace statistic is given by:

$$2(\mathcal{L}_A^* - \mathcal{L}_0^*) = -T \sum_{i=h+1}^n \log(1 - \hat{\lambda}_i)$$

where $\hat{\lambda}_1 > \hat{\lambda}_2 > \dots > \hat{\lambda}_n$ are the eigenvalues of the matrix $\hat{\Sigma}_{\hat{v}\hat{v}}^{-1} \hat{\Sigma}_{\hat{v}\hat{u}} \hat{\Sigma}_{\hat{u}\hat{u}}^{-1} \hat{\Sigma}_{\hat{u}\hat{v}}$, and \mathcal{L}_A^* and \mathcal{L}_0^* represents the maximum value achieved by the log likelihood function subject to the constraint that there exist n and r cointegrating vectors, respectively. The distribution of the trace statistic is non standard and requires simulated critical values as discussed in Johansen (1996).

Ideally, cointegration tests must be applied to a correctly specified VAR. Thus, before testing for cointegration, it is important to verify that the VAR in levels is described by well-behaved error terms (i.e. not serially correlated, homoskedastic and normally distributed) and that the number of lags is correctly specified. The conventional practice suggests the application of standard diagnostic tests for autocorrelation (e.g. Breusch-Godfrey LM test), conditional heteroskedasticity (e.g. ARCH-LM test), normality (e.g. Doornik and Hansen test), and lag-length specification (e.g. the sequential Likelihood Ratio (LR) test).

Recursive tests of parameter constancy

The relevance of monetary aggregates depends on the stability of their co-movements with relevant variables for monetary policy. Therefore, we need to establish whether the full sample period $1, \dots, T$ defines a constant parameter regime in each estimated VECM. For this purpose, we perform a set of recursive stability tests proposed by Hansen and Johansen (1999) in order to test for the stability of each estimated VECM and each cointegrating vector (in case they can be identified). These tests are called “recursive” because they are applied to a baseline model estimated using an initial subsample period $1, \dots, t_1$, and then applied successively to increasing samples obtained by extending the final point t_1 ,

for $t_1 = T_1, T_1 + 1, \dots, T$, until the full sample is covered.¹⁴ In particular, we employ the recursive test of the likelihood function (RLF), the recursively calculated trace test (RTT), the fluctuations test (FT), the max test of constant β ($C\beta$), and the test of “ β_t equal to a known β ” ($C\beta k$).

The recursive test of the likelihood function (RLF) is a recursive test of the whole VECM which indicates whether the model is approximately acceptable or not, and is similar to the recursive Chow tests used in single equation models. The log likelihood test statistic is defined by:

$$Q_T(t_1) = \frac{t_1}{T} \sqrt{\frac{T}{2n}} \{ \log|\hat{\Omega}_{t_1}| - \log|\hat{\Omega}_T| + B \}$$

where n is the number of variables in the VECM, t_1 is the last observation in the recursion sample, T is the last observation in the full sample period, $\hat{\Omega}_j$ is the estimated variance covariance matrix of the VECM residuals for the sample $1, \dots, j$, and $B = T^{-1}\{(n(n+1)/2) + r + n(k-1) + 1\}\{1 - t_1/T\}$ is a bias correction term that slightly improves the size properties of the test. Under the null hypothesis of constant parameters, the limiting distribution of $\max_{t_1} |Q_T(t_1)|$ is that of a maximum of a Brownian bridge, from which the 95% critical value of the test is 1.36.

The recursively calculated trace test (RTT), is obtained calculating the trace statistic for each recursive subsample $1, \dots, t_1$, for $t_1 = T_1, T_1 + 1, \dots, T$ s, and is given by:

$$\mathcal{T}_j(t_1) = \frac{1}{C_{0.95}^*(j)} [-t_1 \sum_{i=1}^j \log(1 - \hat{\lambda}_i)]$$

where $j = 1, \dots, r$, and $C_{0.95}^*(j)$ is the corresponding 95% critical value of the rank test. The RTT provides a first visual impression of whether the cointegration relations are reasonably constant or not. Thus, if α_i and β_i are reasonably constant, then λ_i will also be constant, which implies that the graph of the test will be upward sloping for $j \leq r$ and constant for $j > r$.

The fluctuations test (FT) is a recursively calculated constancy test for each λ_i , $i = 1, 2, \dots, r$, and can be considered as a recursive way to check the joint constancy of β_i and α_i , for $i = 1, 2, \dots, r$. The fluctuations test is defined as:

$$F(t_1) = \frac{t_1}{T} \sqrt{T} \Sigma_{ii}^{-1/2} (\hat{\lambda}_{i,t_1} - \hat{\lambda}_{i,T})$$

where Σ_{ii} is the variance of λ_i defined in Hansen and Johansen (1999). The test is a supremum test and is likely to be very conservative with respect to the null hypothesis. This means that a non-rejection by this test does not rule out non-constancies, whereas a

¹⁴These tests are denominated “forward recursive tests” because the recursion goes forward in time. When the recursions are reversed, the tests are denominated “backward recursive tests”. The results reported in this paper are based on forward recursive tests; however, the backward recursive tests provide similar results.

rejection is often a strong signal of parameter non-constancy.

The max test of constant β (which we denote as “C β ”) and the test of “ β_t equal to a known β ” (which we denote as “C β k”) focuses on testing changes in β ; however, these tests do not discriminate between individual β_i . On the one hand, the null hypothesis of C β is that each recursively estimated β is equal to the estimated β using the full sample $1, \dots, T$, i.e. $\hat{\beta}_{t_1} = \beta_T$ for $t_1 = T_1, \dots, T$. As detailed in Hansen and Johansen (1999), the test is a *sup* test and its asymptotic distribution is a function of Brownian motions and its critical values are obtained by simulation. This test is also very conservative, and therefore the rejection of the null should be considered as strong evidence of non-constancy. On the other hand, C β k tests whether the estimated β for a reference period (e.g. the full sample period, or any period where it is believed that there is no regime change) is in the space spanned by each recursively estimated β_{t_1} , for $t_1 = T_1, \dots, T$. The test statistic is asymptotically distributed as a chi-square with $(n - r)r$ degrees of freedom (Hansen and Johansen, 1999, for details).

Finally, in those cases where it is possible to identify each cointegrating vector, we analyse their stability using the recursively calculated coefficients (RCC) of each cointegrating vector (RCC). These are obtained by estimating each VECM using recursive samples of the form $1, \dots, t_1$, for $t_1 = T_1, \dots, T$. The RCC can be calculated only when the cointegrating vectors are (over) identified and provide a visual impression of the stability and significance of every estimated coefficient of the cointegrating vectors.

Weak and strong exogeneity tests

As detailed in Johansen (1996), the null hypothesis of weak exogeneity can be tested using a likelihood ratio test which is asymptotically distributed as a chi-square with $r \cdot m$ degrees of freedom, where r is the number of cointegrating vectors and m the number of weakly exogenous variables. To test strong exogeneity of money we impose weak exogeneity and perform a Granger non-causality test. For each (real-time and revised final) data set, these two tests are applied to a unique VECM specification and for each monetary aggregate.

3.4 Model uncertainty and Bayesian model averaging

In some cases, the diagnostic tests applied to the residuals of an estimated VECM might fail in choosing a unique model, i.e. a VECM with a specific number of lags and well-behaved errors. As a result, the number of cointegrating vectors might also be uncertain. If there exists model uncertainty with respect to the lag length and the number of cointegrating vectors, the choice of a single specification might result in unstable estimates of either the cointegrating vectors or the short-run parameters of the VECM or both, which will bias any inference on the parameters. In order to deal with this model uncertainty, we take into account a weighted average of all possible specifications (i.e. models with different lag length and number of cointegrating vectors) using Bayesian model averaging (BMA) following the procedure proposed by Garratt et al. (2009). Specifically, we use BMA to analyse whether money is weakly or strongly exogenous and its predictive content.

Bayesian techniques use the rules of conditional probability to make inferences about unknown features (e.g. the probability that money is weakly or strongly exogenous, the probability that money has predictive content for inflation, and so on) given known entities (data available). If we assume that there are s competing models, V_1, V_2, \dots, V_s , then the rules of conditional probability imply that the probability that money has predictive content for, say, inflation (f) given the available data (D) can be calculated as follows:

$$Pr(f|Data) = \sum_{i=1}^s Pr(f|Data, V_i)Pr(V_i|Data) \quad (8)$$

where $Pr(f|Data, V_i)$ is a summary of what is known about f in a particular model V_i and $Pr(V_i|Data)$ provides the probability that model V_i generated the data. Expression (8) states that inference about f involves taking a weighted average of $Pr(f|Data, V_i)$ across all models, where the weights are given by the posterior model probabilities $Pr(V_i|Data)$. This procedure is known as Bayesian model averaging or *BMA*.

In order to quantify the components of expression (8), we use the novel procedure proposed by Garratt et al. (2009) which is based on approximate Bayesian methods.¹⁵ First, the posterior model probability $Pr(V_i|Data)$ can be written using Bayes' rule as:

$$Pr(V_i|Data) \propto Pr(Data|V_i)Pr(M_i) \quad (9)$$

where $Pr(Data|V_i)$ is known as the marginal likelihood and $Pr(M_i)$ is the prior model probability (i.e. the prior weight attached to this model). Following Garratt et al. (2009), we give the same prior weight to each model and use the Schwarz or Bayesian Information Criterion (BIC) proposed by Schwarz (1978) to approximate the marginal likelihood:

$$\ln Pr(Data|V_i) \approx l - \frac{K \ln(T)}{2} \quad (10)$$

where l represents the log of the maximised value of the likelihood function, K is the number of parameters in the model and T is the sample size. Specifically, the weights will be the exponential of $\ln Pr(Data|V_i)$.¹⁶

The probability $Pr(f|Data, V_i)$ is quantified using the standard noninformative prior as follows. First, we consider VECMs with $q = 1, \dots, 8$ lags¹⁷ and $r = 1, \dots, 4$ cointegrating vectors given by:

¹⁵As mentioned in Garratt et al. (2009), this Bayesian approach is an approximate one because it is based on likelihood functions estimated using conventional econometric techniques.

¹⁶As mentioned in Garratt et al. (2009), these weights provide results closely related to those obtained using many of the benchmark priors used by Bayesians (see Fernandez, Ley and Steel 2001).

¹⁷The number of lags is constrained by the size of the available sample.

$$\begin{aligned}
\Delta y_t &= \sum_{i=1}^r \alpha_i^y \epsilon_{i,t-1} + \sum_{i=1}^q [\gamma_i^y \Delta y_{t-i} + \theta_i^y \Delta p_{t-i} + \varphi_i^y \Delta M_{t-i} + \vartheta_i^y \Delta R_{t-i} + \pi_i^y \Delta s_{t-i} + u_t^y] \\
\Delta p_t &= \sum_{i=1}^r \alpha_i^p \epsilon_{i,t-1} + \sum_{i=1}^q [\gamma_i^p \Delta y_{t-i} + \theta_i^p \Delta p_{t-i} + \varphi_i^p \Delta M_{t-i} + \vartheta_i^p \Delta R_{t-i} + \pi_i^p \Delta s_{t-i} + u_t^p] \\
\Delta M_t &= \sum_{i=1}^r \alpha_i^M \epsilon_{i,t-1} + \sum_{i=1}^q [\gamma_i^M \Delta y_{t-i} + \theta_i^M \Delta p_{t-i} + \varphi_i^M \Delta M_{t-i} + \vartheta_i^M \Delta R_{t-i} + \pi_i^M \Delta s_{t-i} + u_t^M] \\
\Delta R_t &= \sum_{i=1}^r \alpha_i^R \epsilon_{i,t-1} + \sum_{i=1}^q [\gamma_i^R \Delta y_{t-i} + \theta_i^R \Delta p_{t-i} + \varphi_i^R \Delta M_{t-i} + \vartheta_i^R \Delta R_{t-i} + \pi_i^R \Delta s_{t-i} + u_t^R] \\
\Delta s_t &= \sum_{i=1}^r \alpha_i^s \epsilon_{i,t-1} + \sum_{i=1}^q [\gamma_i^s \Delta y_{t-i} + \theta_i^s \Delta p_{t-i} + \varphi_i^s \Delta M_{t-i} + \vartheta_i^s \Delta R_{t-i} + \pi_i^s \Delta s_{t-i} + u_t^s]
\end{aligned} \tag{11}$$

i.e a total of 32 competing models. If we are interested in $f = \text{“money is weakly exogenous for output”}$, then the probability $Pr(f|Data, V_i)$ is calculated as follows:

$$Pr(f|Data, V_i) \equiv Pr(\alpha_1^M = \dots = \alpha_r^M = 0 | Data, V_i) = \frac{\exp(BIC_R)}{\exp(BIC_R) + \exp(BIC_U)} \tag{12}$$

where BIC_R and BIC_U denote the BIC for the restricted and unrestricted models, respectively. The restricted model is the VAR with where $\alpha_1^M = \dots = \alpha_r^M = 0$. Furthermore, if we are interested in $f = \text{“money is strongly exogenous for output”}$, the probability $Pr(f|Data, V_i)$ is calculated as follows:

$$\begin{aligned}
Pr(f|Data, V_i) &\equiv Pr(\alpha_1^M = \dots = \alpha_r^M = 0, \gamma_1^M = \dots = \gamma_q^M = 0 | Data, V_i) \\
&= \frac{\exp(BIC_R)}{\exp(BIC_R) + \exp(BIC_U)}
\end{aligned} \tag{13}$$

where the restricted model is the VECM with $\alpha_1^M = \dots = \alpha_r^M = 0$ and $\gamma_1^M = \dots = \gamma_q^M = 0$. The analysis for $f = \text{“money is strongly exogenous for prices”}$ is analogous. Thus, by averaging the information contained in all possible models, BMA provides an overall assessment of whether money is useful to predict output and prices, allowing the possibility of model uncertainty. Finally, if we are interested in $f = \text{“the predictive content of money for output conditional on money being strongly exogeneous”}$, the probability $Pr(f|Data, V_i)$, is calculated as follows:

$$\begin{aligned}
Pr(f|Data, V_i) &\equiv 1 - Pr(\alpha_1^y = \dots = \alpha_r^y = 0, \varphi_1^y = \dots = \varphi_q^y = 0 | Data, V_i) \\
&= 1 - \frac{\exp(BIC_R)}{\exp(BIC_R) + \exp(BIC_U)}
\end{aligned} \tag{14}$$

where V_i includes all models with $\alpha_1^M = \dots = \alpha_r^M = 0, \gamma_1^M = \dots = \gamma_q^M = 0$. Again, the analysis for prices is analogous.

As discussed in Garratt et al. (2009), there has recently been concerns about the properties of hypothesis testing procedures in a recursive testing exercise, especially regarding the choice of the correct critical values for such tests. However, that is not an issue under BMA because the Bayesian approach does not involve critical values. In particular, the analysis of this paper focuses on the calculation of probability statements such as “ the probability that money is strongly exogenous for output” in each period of time.

4 Results

This section contains the results under model certainty and model uncertainty. The basic econometric model is a VECM with five variables: $X_t = [p_t, y_t, Mi_t, R_t, s_t]$, where $i = 0, 1, 2, 3$. The models were estimated using both real time and final revised data for each monetary aggregate $M0, M1, M2$, and $M3$.

4.1 Results under model certainty

The main results for model adequacy analysis are reported in Table 2 and Figures 3 and 4. The top panel of Table 2 displays the results of lag length tests and specification tests (autocorrelation, normality and ARCH tests) for each model using revised final data.

For all the models considered, the conditions for the rank test to be valid do not hold. Conditional on each VAR’s lag length, the multivariate LM test for autocorrelation shows no evidence of autocorrelation for the full sample period and all monetary aggregates; however, there is evidence of autocorrelation for the sub-sample January 2002 - December 2011 when money is measured by M0 or M2. The multivariate LM tests for conditional heteroskedasticity reject the null of homoskedasticity in all cases; however, the presence of conditional heteroskedasticity might not be a problem in detecting the number of cointegrating vectors given that the rank test is robust to moderate ARCH effects (Rahbek et al., 2002; Cavaliere et al., 2010). Doornik and Hansen’s normality test indicates that the error terms are not normally distributed; in contrast to the presence of ARCH errors, the non-normality of the error terms might bias the results of the rank test and thus generate uncertainty regarding the lag length and the number of cointegrating vectors.¹⁸ These results suggest that there is no unique model that fits the data under model certainty and thus all the inferences that follow in the rest of this subsection might be inaccurate; however, they are presented for comparison purposes.

The bottom panel of Table 2 shows the results of the rank test. In all cases, there is evidence of only one cointegrating vector for the sample 1994m1-2011m12. However, as shown in Figure 3, this result is not robust when the rank test is calculated recursively. For

¹⁸In order to alleviate the non-normality problem, dummy variables that account for extreme values were added to each specification; however, the problem persisted.

Table 2. *Model adequacy tests: lag length determination, specification tests, and trace test for cointegration.*

	1994m1-2011m12				2002m1-2011m12			
	M0	M1	M2	M3	M0	M1	M2	M3
Lag reduction LM tests								
Number of lags	8	8	8	4	5	7	7	3
Residual Analysis								
Test for autocorrelation								
LM test								
Ho: No autocorrelation at lag 1	0.33	0.08	0.99	0.31	0.00	0.27	0.04	0.06
Ho: No autocorrelation at lag 2	0.17	0.14	0.22	0.95	0.02	0.15	0.10	0.64
Test for normality								
Doornik and Hansen test								
Ho: Residuals are normally distributed	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tests for ARCH								
LM Test (Lutkepohl and Kratzig)								
Ho: No ARCH(1) in the residuals	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Ho: No ARCH(2) in the residuals	0.00	0.00	0.00	0.00	0.00	0.03	0.10	0.00
Trace test for cointegration rank								
Ho: No cointegrating vector	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.03
Ho: One cointegrating vector	0.06	0.14	0.05	0.31	0.15	0.17	0.17	0.50
Ho: Two cointegrating vectors	0.56	0.75	0.42	0.39	0.60	0.34	0.49	0.43
Ho: Three cointegrating vectors	0.53	0.63	0.61	0.25	0.79	0.38	0.65	0.71
Ho: Four cointegrating vectors	0.60	0.64	0.87	0.69	0.76	0.37	0.66	0.77

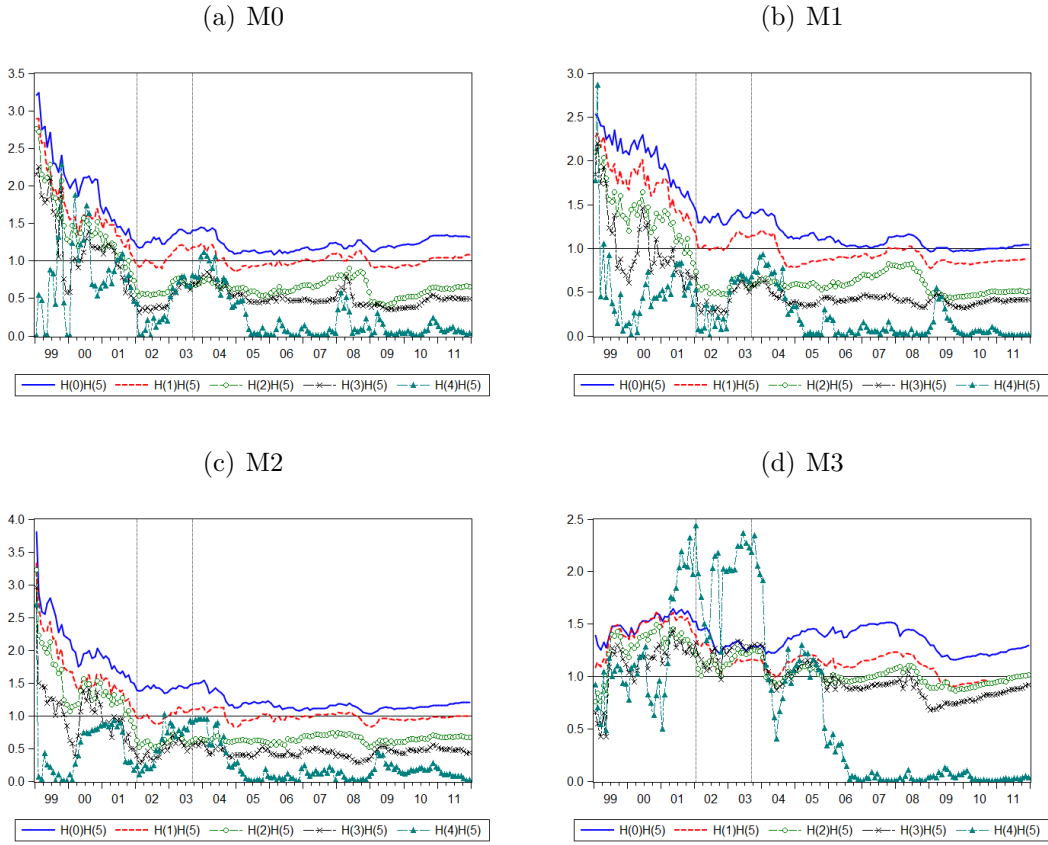
Note: All entries represent p-values from the corresponding tests. Each model includes Nominal Money (M0, M1, M2, or M3), Real Output, Prices, Interest Rate, and Nominal Exchange Rate

instance, Panel (d) shows that in the case of M3 the null hypothesis of one cointegrating vector, $H(1)H(5)$, is rejected in favour of two cointegrating vectors for all the recursive samples ending between late 2005 and early 2009.

Figure 4 shows the recursive likelihood function (RLF) test for each model considered. The RLF test is based on two types of estimated VECMS: (i) Model X, in which both the long-run and the short-run parameters are re-estimated at every point of the recursion, and (ii) Model R1, in which only the long-run parameters are re-estimated and the short-run parameters are fixed to their full-sample estimated values.¹⁹ The null hypothesis of parameter constancy is rejected when the RLF test is above 1 (the normalised critical value). As shown in Figure 4, the null of parameter constancy is rejected with Model X and Model R1, except for the recursive samples ending between mid-2010 and December 2011. In addition, the fluctuations test, the $C\beta$ and $C\beta k$ tests displayed in Figure 9 in the appendix, also suggest instabilities for M0, M1, and M2, especially for the first recursive samples. Again, in order to proceed with the analysis, the symptoms of parameter insta-

¹⁹See Juselius (2006), Chapter 9, for more details.

Figure 3. *Recursive Trace test*



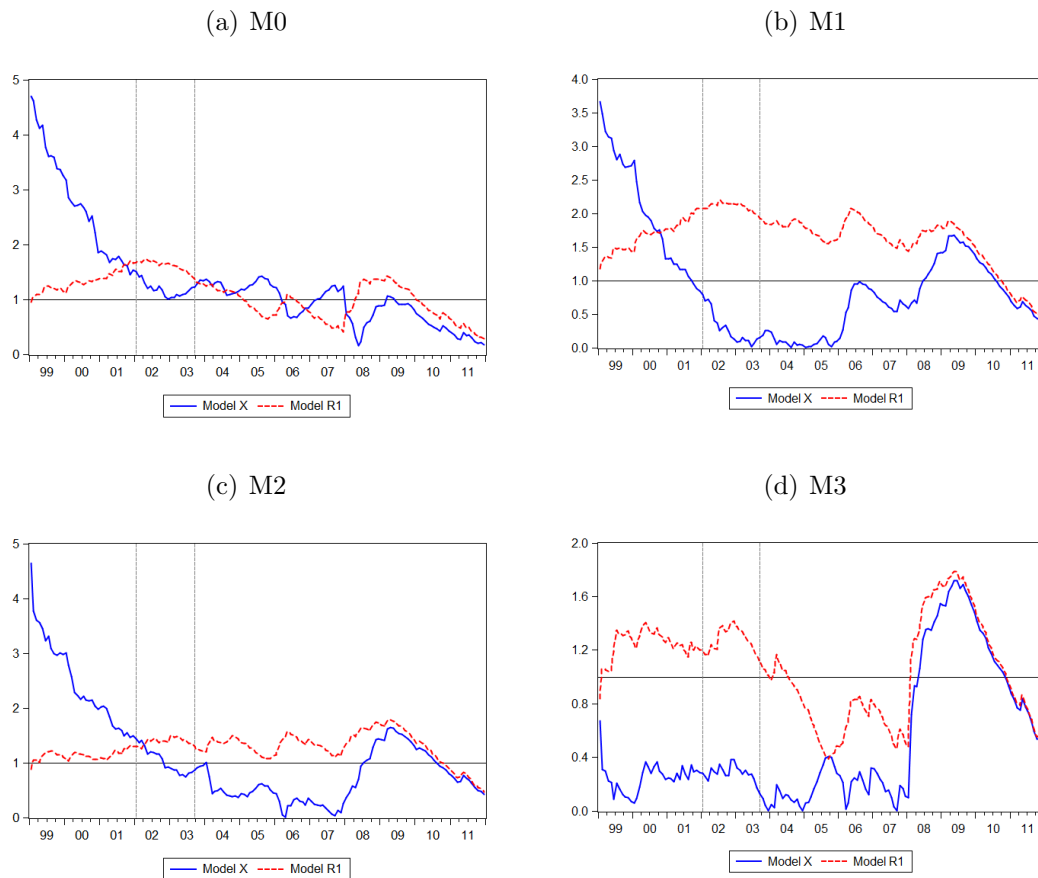
NOTE: $H(j)H(5)$ represents the evolution of the recursively calculated trace test under the null that there exist j cointegrating vectors, for $j = 1, \dots, 4$; the null hypothesis is rejected if the test statistic is above 1 (which is the normalised critical value). The results are based on estimated VECMs in which both the long-run and the short-run parameters are re-estimated at every point of the recursion (Model X).

bility are ignored.

The results of the recursive analysis of the three hypotheses tested in this paper (marginal predictive content, weak and strong exogeneity), using both real-time and revised final data for monetary aggregates, are shown in Figures 10 to 13 in the Appendix. Overall, the results show no strong evidence of money as an information variable according to the three criteria tested in this paper. Given this, the rest of the section will describe two particular cases in which there is some evidence that support the hypotheses under assessment: $M0$ as information variable for prices, and $M2$ as information variable for output.

Figure 5 shows the results for the case of $M0$ as information variable for prices. The recursive analysis is performed for the samples 1994m11-2011m12 (Panels (a) and (b)) and 2002m1 2011m12 (Panel (c) and (d)), using both real-time and revised final data. The interpretation of Figure 5 is based on the existence of one cointegrating vector normalised with respect to prices, i.e. $\epsilon_t = p_t - \beta'_0 - \beta'_1 M_t - \beta'_3 y_t + \beta'_4 R_t + \beta'_5 s_t$. The null hypothesis that $M0$ is weakly exogenous for β'_1 is rejected when the p-value, represented by the shaded

Figure 4. *The recursive likelihood function test*

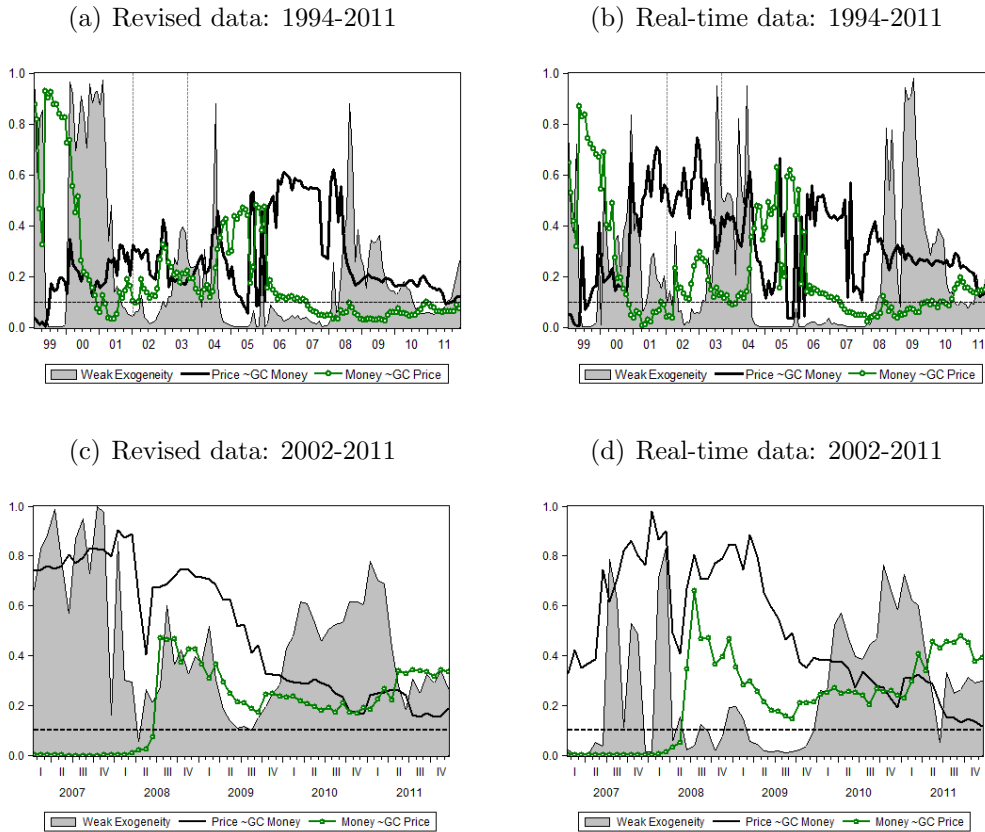


NOTE: The horizontal lines represent critical values which have been normalised to 1. Model R1 indicates that the recursive estimation is performed fixing the short-run parameters to their full-sample estimated values. Model X indicates that both the long-run and the short-run parameters are re-estimated at every point of the recursion. The graphs show the results of the recursive test of the likelihood function; when the value of the statistic is above 1 the null of parameter constancy is rejected.

area “Weak exogeneity”, is below 0.10. The null hypothesis that “Prices” do not Granger cause M0 is rejected when the p-value, represented by the line “Prices \sim GC Money”, is below 0.10. M0 is strongly exogenous for β'_1 if it is both weakly exogenous and “Prices” do not Granger cause M0. The null hypothesis that M0 does not Granger cause “Prices” is rejected when the p-value, represented by “Money \sim GC Prices”, is below 0.10, which suggests that M0 has predictive content for “Prices”.

Based on the sample 1994m11-2011m12, the results do not provide systematic evidence that M0 is either (weakly or strongly) exogenous or has predictive content for prices. As shown in Panel (a), the null hypothesis that M_t is weakly exogenous for β'_1 is rejected in several of the recursive samples that cover the period 1994m1-2011m12, such as those in which the final date lies between 2004m9 and 2007m12. Panel (b) shows a similar result when real-time data is used. In both cases, the null hypothesis that prices do not Granger cause M0 is not rejected in most periods where M0 is weakly exogenous for β'_1 ,

Figure 5. Recursive Weak and strong exogeneity tests of $M0$ and its predictive content for Prices



NOTE: Each vertical axis measures the probability of rejecting the corresponding null hypothesis. The null hypothesis that $M0$ is weakly exogenous is rejected when the p-value, represented by the shaded area “Weak exogeneity”, is below 0.10. The null hypothesis that “Prices” do not Granger cause $M0$ is rejected when the p-value, represented by the line “Prices \sim GC Money”, is below 0.10. $M0$ is strongly exogenous for the long-run parameter of “Prices” if it is both weakly exogenous and not Granger caused by “Prices”. The null hypothesis that $M0$ does not Granger cause “Prices” is rejected when the p-value, represented by “Money \sim GC Prices”, is below 0.10, which suggests that $M0$ has predictive content for “Prices”.

and thus $M0$ is also strongly exogenous for β'_1 . Therefore, $M0$ could have been used as a conditioning variable to perform conditional forecasts of prices only during some specific periods. However, both real-time and revised final show that $M0$ is strongly exogenous and has predictive content for prices only for some recursive samples, such as the ones ending between 2008m1 and mid 2010.

The results based on the sample 2002m1-2011m12 show evidence of weak and strong exogeneity of $M0$ when revised-final data are used. As shown in Panel (c), weak and strong exogeneity of M_t for β'_1 is not rejected for any recursive samples defined over the period 2002m1-2011m12 (except for the recursive sample that ends in March 2008); however, $M0$ has predictive content for prices only up to June 2008. Panel (d) indicates that weak and strong exogeneity of $M0$ measured in real time is rejected in several recursive samples

between 2007 and 2009, whereas M0 has predictive content for prices only up to June 2008.

Overall, these results do not provide systematic and strong evidence that M0 can be useful as an information variable for prices in terms of the three criteria considered in this paper (predictive content, weak and strong exogeneity). However, if conditional forecasts are not a desirable feature, then it is possible that some monetary aggregates can be useful to perform unconditional forecasts. For instance, Panel (g) in Figure 10 (see Appendix) shows that M3 has predictive content for prices for any recursive sample between 2002m1 and 2011m12, however, M3 is not even weakly exogenous for those samples.

To illustrate the usefulness of money as an information variable for output, Figure 6 displays the results for the case of M2, based on the existence of one cointegrating vector normalised with respect to real output, y_t , i.e. $\epsilon_t = y_t - \beta_0 - \beta_1 M_t - \beta_2 p_t + \beta_4 R_t + \beta_5 s_t$. Using the same reasoning as for the case of M0, it can be concluded that M2 is weakly and strongly exogenous for most of the recursive samples defined for the period 2002m1-2011m12, and thus can be used as a conditioning variable to perform conditional forecasts of output; however, the predictive content of M2 for output is significant only until April 2008.

Overall, under model certainty (i.e. based on a single best specification for each monetary aggregate), there is no strong evidence that supports the relevance of money as information variable with either real-time or revised final data. The results for the remaining monetary aggregates (shown in Figures 10 to 13) are less supportive.

4.2 Results under model uncertainty

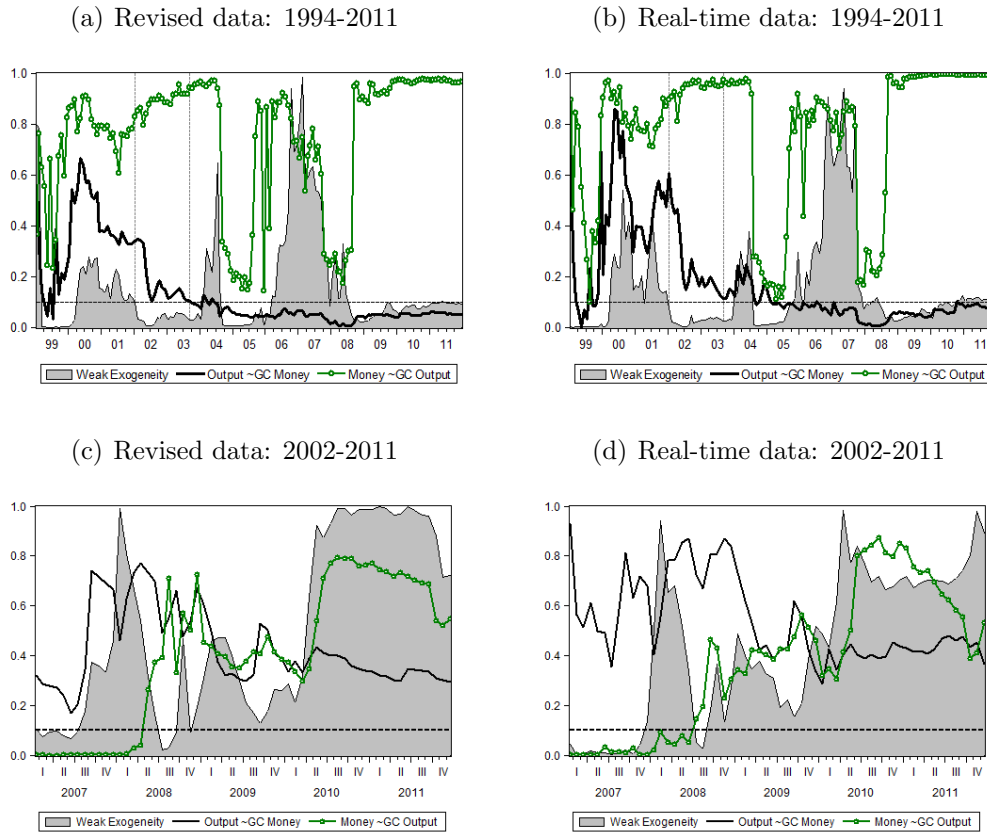
The results under model uncertainty are based on the estimation of VECMs with $r = 1, \dots, 4$ cointegrating relationships and $q = 1, \dots, 8$ lags²⁰. For each monetary aggregate, all the VECMs are combined using BMA. Then, three probabilities are calculated: “the probability that money is weakly exogenous” (PWE), “the probability that money is strongly exogenous” (PSE), and “the probability that money has predictive content for x ” (PPC).²¹ Figures 14 to 18 display the results for all monetary aggregates.

Figures 7 and 8 show the recursively calculated probabilities PSE and PPC for the case of M0 and M2, using both real-time and revised final data. Panel (a) of Figure 7 shows that the probability that M0 is strongly exogenous for prices has been roughly stable between 51% and 54% for all the recursive samples that covered the period 2001m1-2011m12, reaching 52% in the last recursive sample (1994m1-2011m12). Compared to this, Panel (b) shows that the probability is smaller and converges to 50% if the estimation sample is restricted to the period 2002m1-2011m12. In both cases, the results with real-time data and revised final data are very similar. Under a Bayesian approach, and following Jeffreys (1961), these results suggest there is no strong evidence that M0 can be used to perform conditional forecasts of prices more than one-period ahead because the probability that

²⁰The lag length is restricted by the sample size of the real-time data set.

²¹This probability is calculated imposing strong exogeneity of money.

Figure 6. Recursive Weak and strong exogeneity of M2 and its predictive content for output



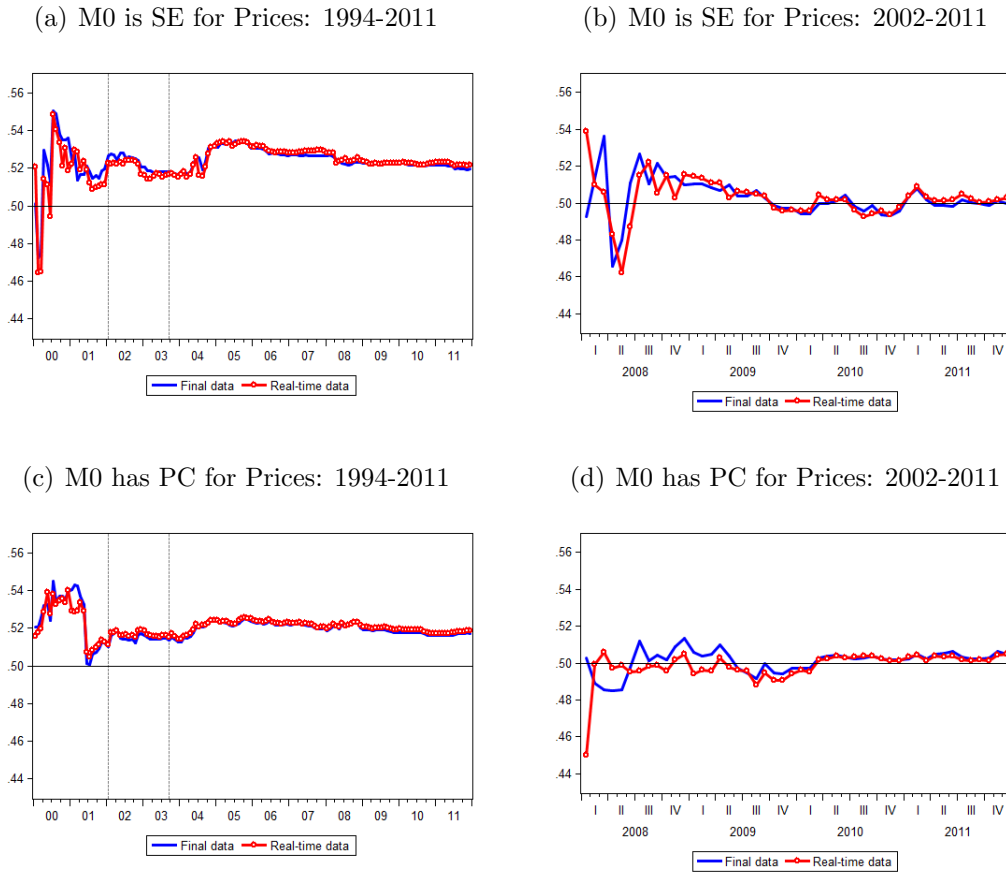
NOTE: Each vertical axis measures the probability of rejecting the corresponding null hypothesis. The null hypothesis that “Money” is weakly exogenous is rejected when the p-value, represented by the shaded area “Weak exogeneity”, is below 0.10. The null hypothesis that “Output” does not Granger cause “Money” is rejected when the p-value, represented by the line “Prices ~ GC Money”, is below 0.10. “Money” is strongly exogenous for the long-run parameter of “Output” if it is both weakly exogenous and not Granger caused by “Output”. The null hypothesis that “Money” does not Granger cause “Output” is rejected when the p-value, represented by “Money ~ GC Prices”, is below 0.10, which suggests that “Money” has predictive content for “Output”.

M0 is strongly exogenous for prices is very close to 50%.²²

In terms of the predictive content of money, Panel (c) indicates that the probability that M0 has predictive content for prices has fluctuated between 50% and 55% for all the recursive samples that cover the period 1994m1-2011m12, converging to 52%. However, when the estimation sample is restricted to 2002m1-2011m12, the probability falls as shown in Panel (d), and fluctuates around 50%. In both cases, the results with real-time data are similar to those based on revised final data. Based on these results, we conclude that there is no strong evidence in favour of the predictive content of M0 for prices.

²²Based on Jeffreys (1961), a probability greater than 91% could be interpreted as strong evidence in favour of the exogeneity hypothesis.

Figure 7. Probability that $M0$ is strongly exogenous (PSE) and its predictive content (PPC) for Prices

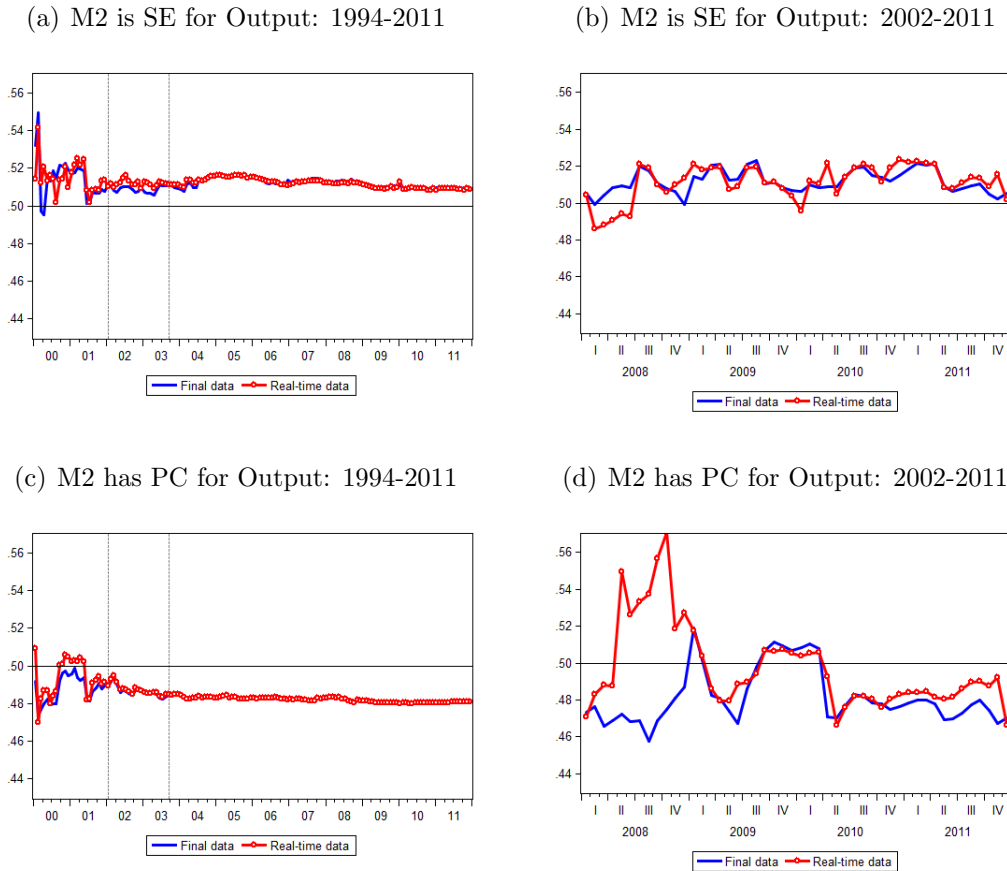


NOTE: Each vertical axis represents probabilities between 0 and 1. The horizontal line indicates the date of the last observation of each recursive sample. For instance, January 2008 represents the recursive sample 1994m1-2008m1 in Panels (a) or (c), and the recursive sample 2002m1-2008m1 in Panels (b) and (d).

Figure 8 shows the results of strong exogeneity of $M2$ and its predictive content for output. Panel (a) shows that the probability that $M2$ is strongly exogenous for output is systematically above 50% and fluctuates around 51% for most of the recursive samples that cover the period 1994m1-2011m12, using both real-time and revised final data. For the period 2002m1-2011m12, Panel (c) indicates that the probability still fluctuates around 51% but is less smooth than in Panel (c). Therefore, we can conclude that there is no strong evidence that $M2$ is strongly exogenous for output.

The results of the predictive content of $M2$ for output are shown in Panels (c) and (d) of Figure 8. Based on the recursive samples that cover the period 1994m1-2011m12, Panel (c) indicates that the probability that $M2$ has predictive content for output has been stable around 48%. For the recursive samples that cover the period 2002m1-2011m12, Panel (d) indicates that the probability becomes more volatile but still around 48%, displaying some discrepancies between real-time and revised final data. Therefore, there is no strong

Figure 8. *Probability that M2 is strongly exogenous (PSE) and its predictive content (PPC) for Output*



NOTE: Each vertical axis represents probabilities between 0 and 1. The horizontal line indicates the date of the last observation of each recursive sample. For instance, January 2008 represents the recursive sample 1994m1-2008m1 in Panels (a) or (c), and the recursive sample 2002m1-2008m1 in Panels (b) and (d).

evidence that M2 has any predictive content for output.

In a nutshell, we conclude that there is no strong evidence supporting that money can be used as an information variable for prices or output in the conduct of monetary policy in Peru.

A further analysis based on out-of-sample tools (as in Garratt et al., 2009) is an important extension of the work presented here, but is left for a further study. However, it is reasonable to expect that such analysis will yield similar results to the ones obtained in this paper or even less supportive.

5 Conclusions

This paper provides an empirical analysis of the usefulness of monetary aggregates as information variables in the conduct of monetary policy in Peru. For this purpose, some recent advances on the topic were used, which include the analysis of both real-time and revised final data, and the application of Bayesian model averaging to allow for model uncertainty regarding the lag length and number of cointegrating relationships. In this paper, money is considered as an information variable for W_t (e.g. output or prices) if the following two criteria are satisfied: (i) M_t is strongly exogenous, and (ii) M_t Granger-causes W_t . Strong exogeneity is relevant because it validates conditional forecasting of prices and output using monetary aggregates as conditioning variables. The results show no strong evidence supporting that money can be used as information variable for prices or output in the conduct of monetary policy in Peru.

Although these results do not make a strong case for the use of monetary aggregates as information variables in Peru, it could be cautious not to fully discard them (see Thornton, 2014). In fact, Drake and Fleissig (2006) and Lucas Jr. and Nicolini (2015) show that an alternative definition of monetary aggregate could display a stable relation with prices and output. Furthermore, monetary aggregates can still be useful if they contain information about any other variable related to monetary policy; for instance, if monetary authorities are also concerned with securing financial stability through the implementation of macroprudential policies, monetary aggregates may be useful in the construction of financial stability indicators (e.g. Kim et al., 2013).

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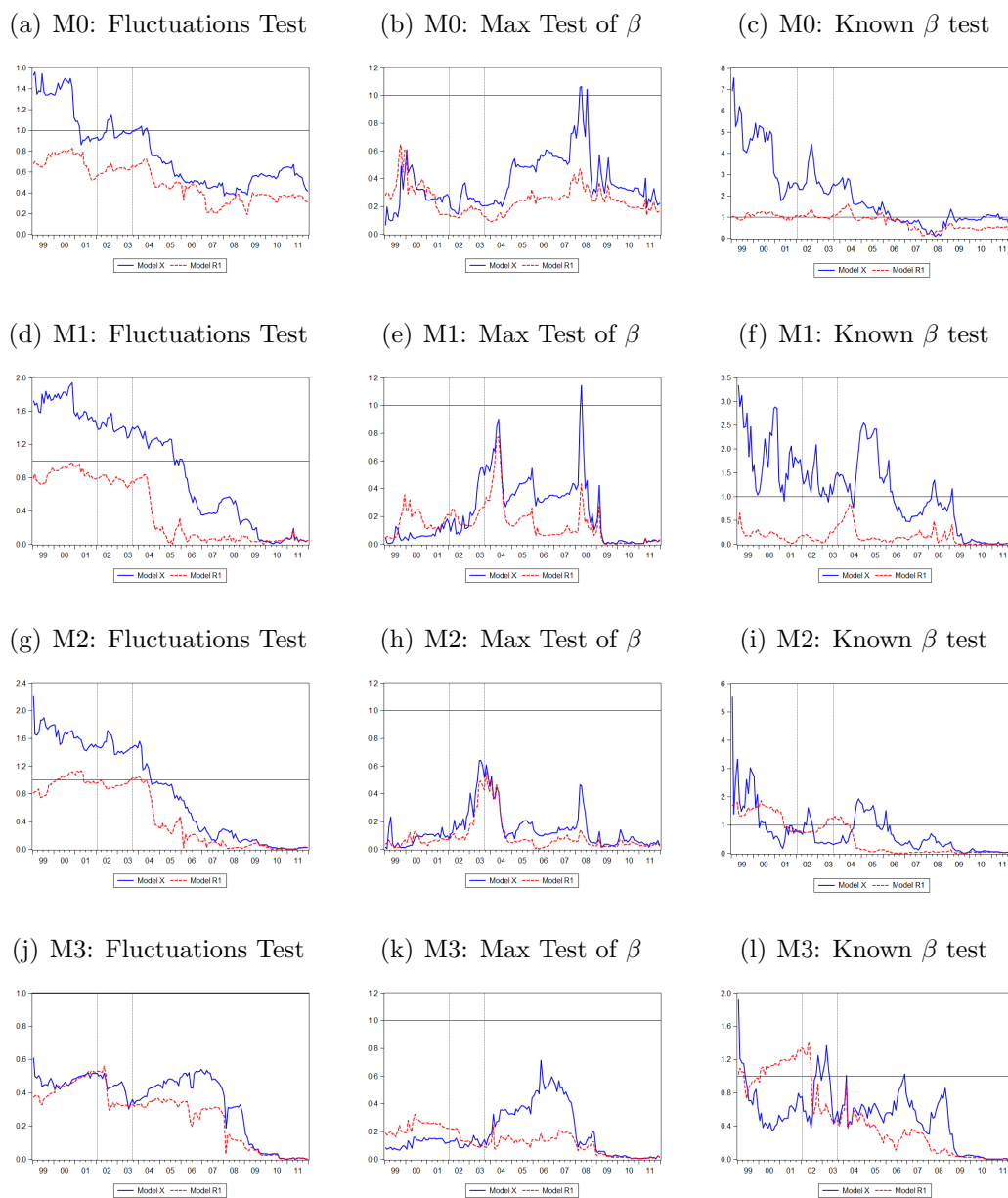
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Appendix

A Stability tests for VECMs

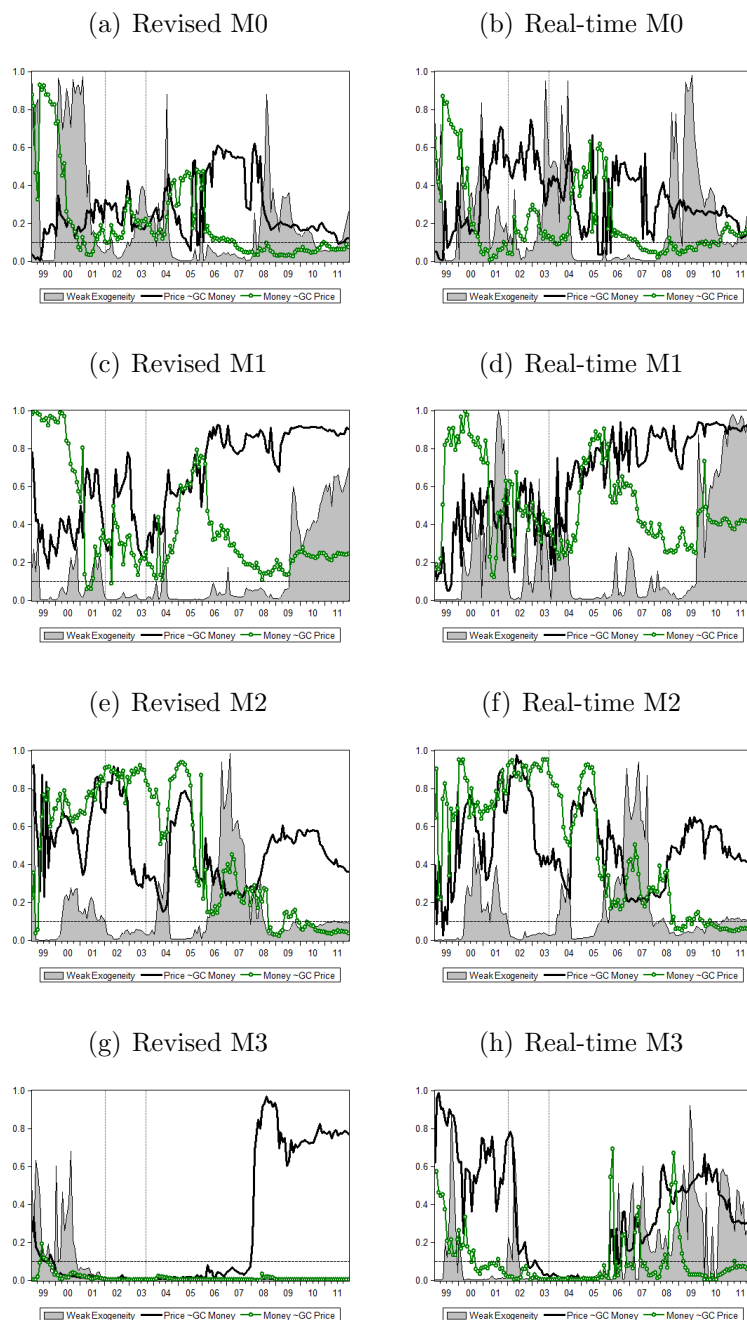
Figure 9. Other recursive tests for parameter constancy



NOTE: Recursive estimates are obtained re-estimating both the long-run and the short-run parameters of the corresponding VECM. In all cases, the null hypothesis of parameter constancy is rejected when the value of the statistic is above 1, which is the normalised critical value.

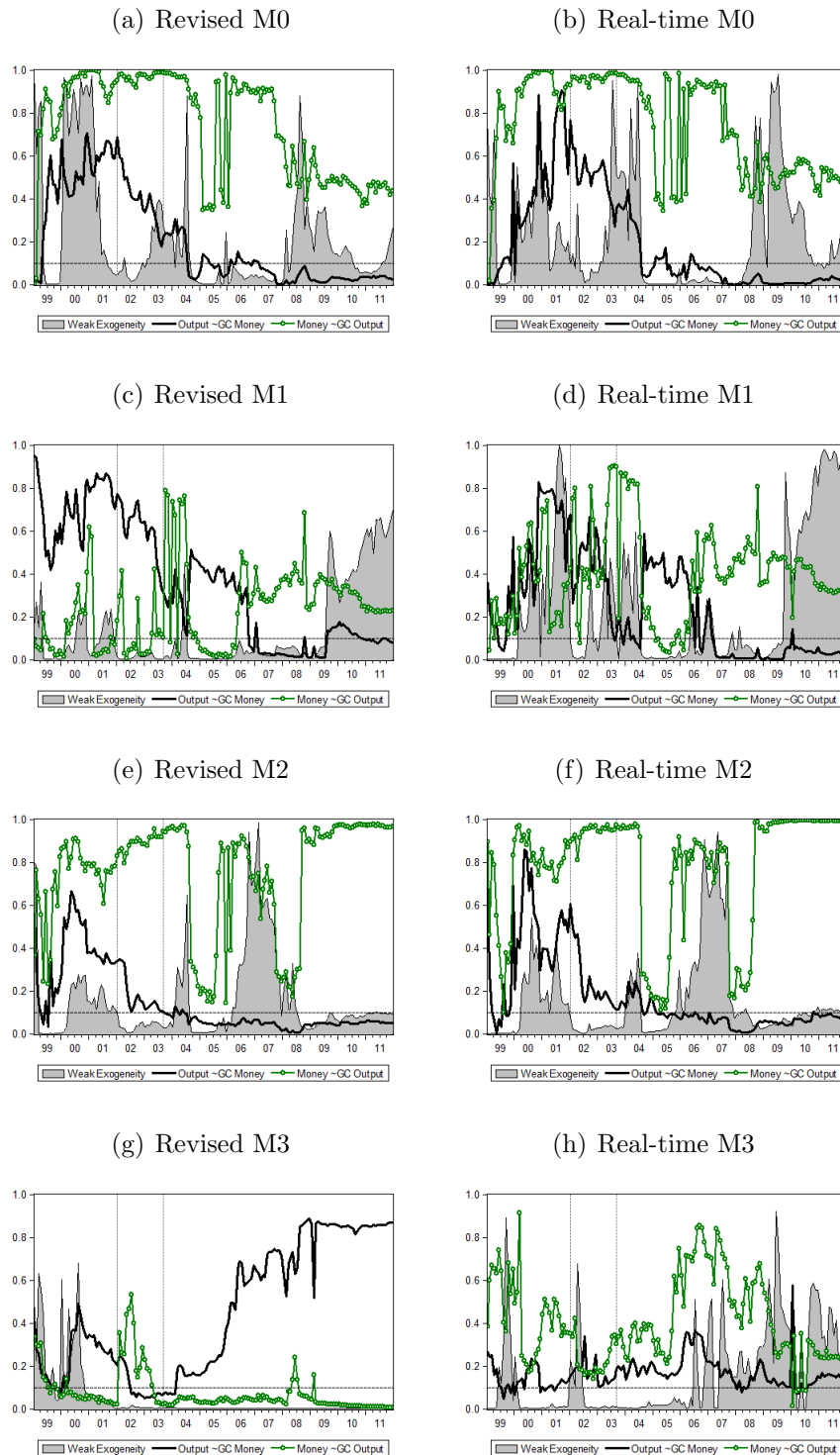
B Weak and strong exogeneity tests

Figure 10. Recursive Weak and strong exogeneity tests: Money and Prices



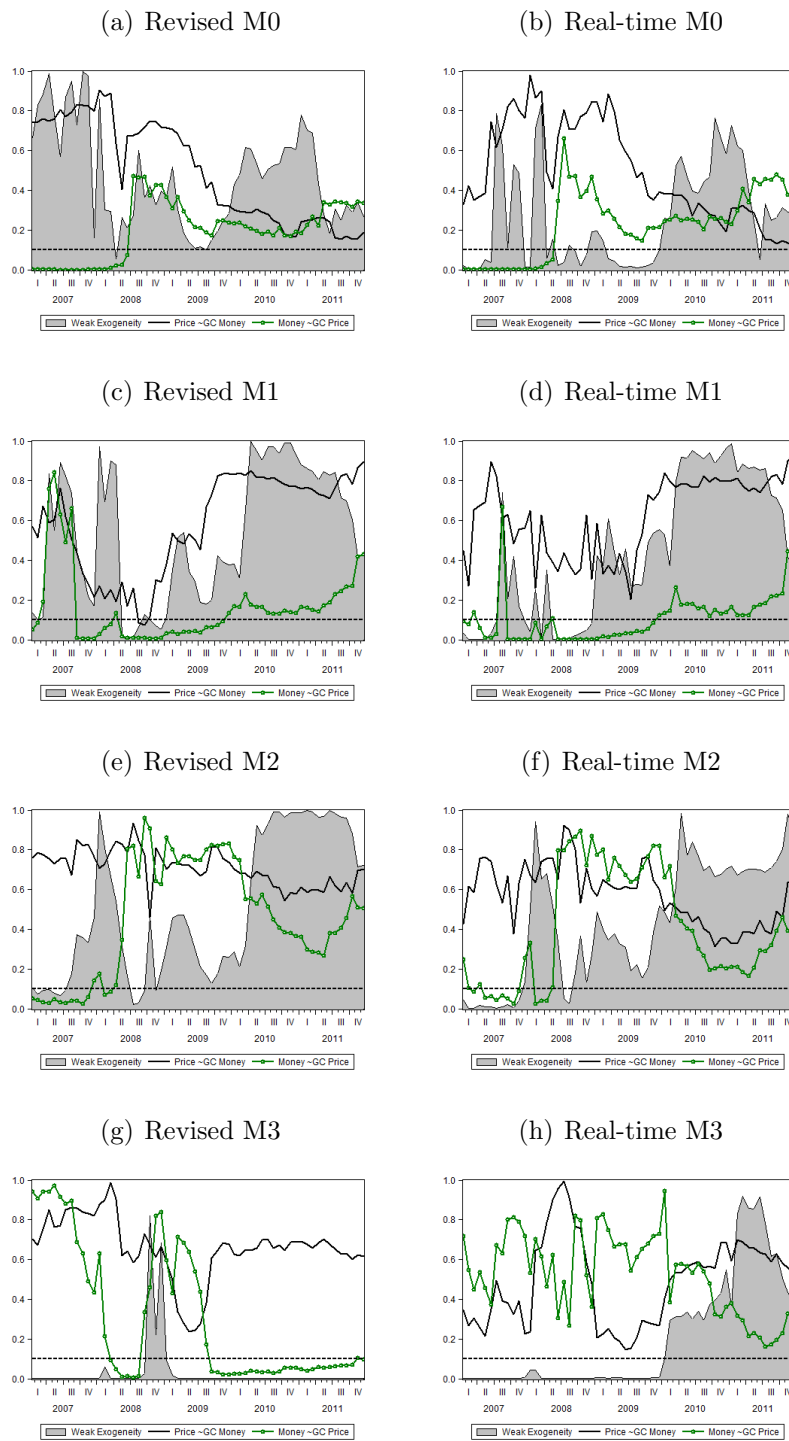
NOTE: The null hypothesis that “Money” is weakly exogenous is rejected when the p-value, represented by the shaded area “Weak exogeneity”, is below 0.10. The null hypothesis that “Prices” do not Granger cause “Money” is rejected when the p-value, represented by the line “Prices ~ GC Money”, is below 0.10. “Money” is strongly exogenous for the long-run parameter of “Prices” if it is both weakly exogenous and not Granger caused by “Prices”. The null hypothesis that “Money” does not Granger cause “Prices” is rejected when the p-value, represented by “Money ~ GC Prices”, is below 0.10, which suggests that “Money” has predictive content for “Prices”.

Figure 11. Recursive weak and strong exogeneity tests: Money and Output



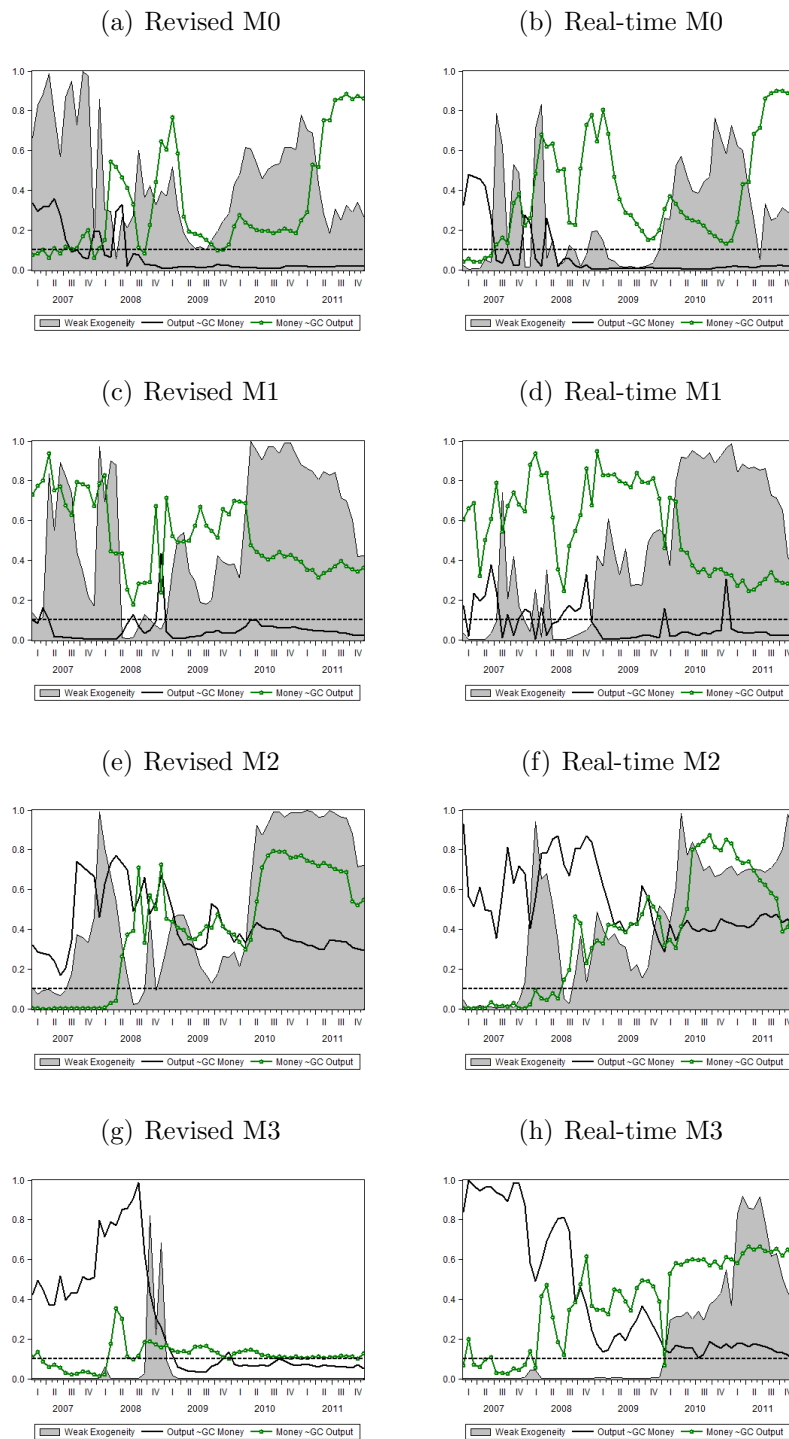
NOTE: The null hypothesis that “Money” is weakly exogenous is rejected when the p-value, represented by the shaded area “Weak exogeneity”, is below 0.10. The null hypothesis that “Output” does not Granger cause “Money” is rejected when the p-value, represented by the line “Prices ~ GC Money”, is below 0.10. “Money” is strongly exogenous for the long-run parameter of “Output” if it is both weakly exogenous and not Granger caused by “Output”. The null hypothesis that “Money” does not Granger cause “Output” is rejected when the p-value, represented by “Money ~ GC Prices”, is below 0.10, which suggests that “Money” has predictive content for “Output”.

Figure 12. *Recursive weak and strong exogeneity tests: Money and Prices, 2002-2011*



NOTE: The null hypothesis that “Money” is weakly exogenous is rejected when the p-value, represented by the shaded area “Weak exogeneity”, is below 0.10. The null hypothesis that “Prices” do not Granger cause “Money” is rejected when the p-value, represented by the line “Prices ~ GC Money”, is below 0.10. “Money” is strongly exogenous for the long-run parameter of “Prices” if it is both weakly exogenous and not Granger caused by “Prices”. The null hypothesis that “Money” does not Granger cause “Prices” is rejected when the p-value, represented by “Money ~ GC Prices”, is below 0.10, which suggests that “Money” has predictive content for “Prices”.

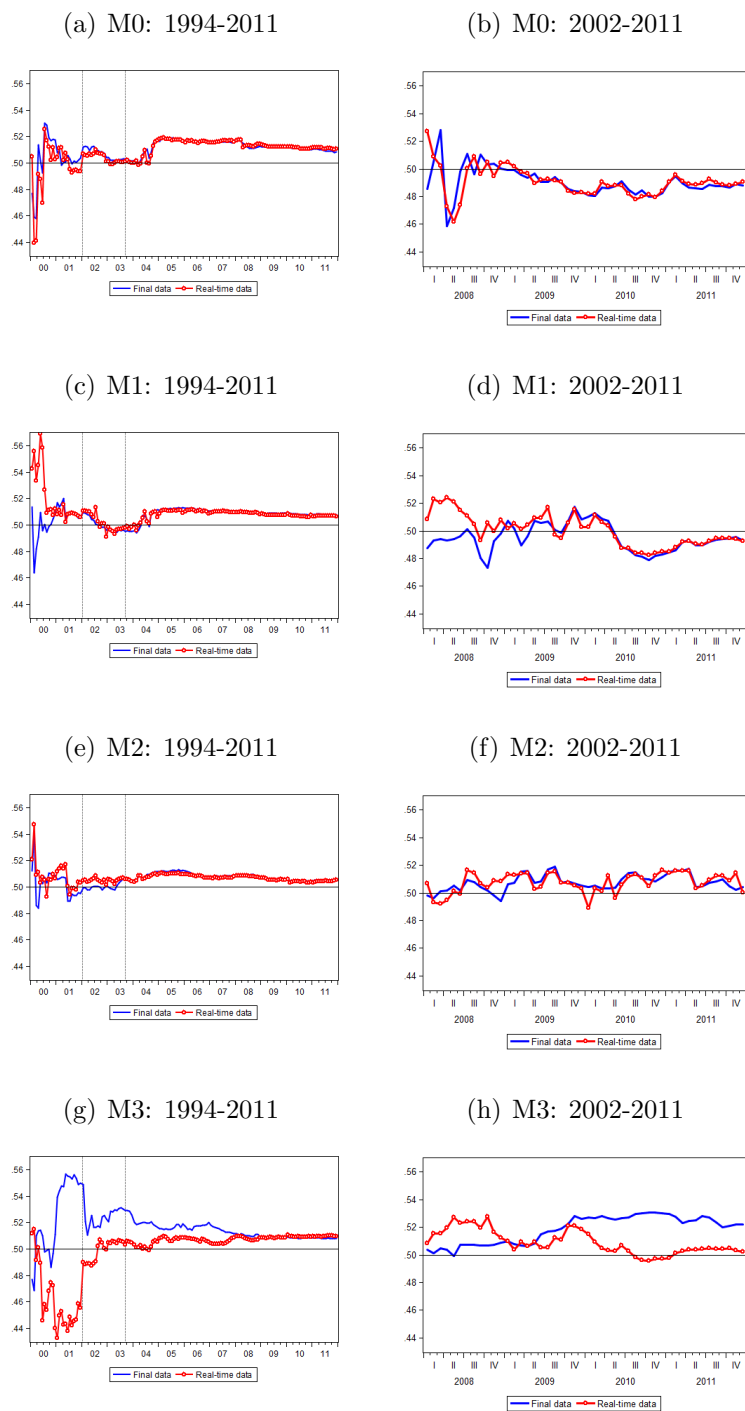
Figure 13. Recursive Weak and strong exogeneity tests: Money and Output, 2002-2011



NOTE: The null hypothesis that “Money” is weakly exogenous is rejected when the p-value, represented by the shaded area “Weak exogeneity”, is below 0.10. The null hypothesis that “Output” does not Granger cause “Money” is rejected when the p-value, represented by the line “Prices ~ GC Money”, is below 0.10. “Money” is strongly exogenous for the long-run parameter of “Output” if it is both weakly exogenous and not Granger caused by “Output”. The null hypothesis that “Money” does not Granger cause “Output” is rejected when the p-value, represented by “Money ~ GC Prices”, is below 0.10, which suggests that “Money” has predictive content for “Output”.

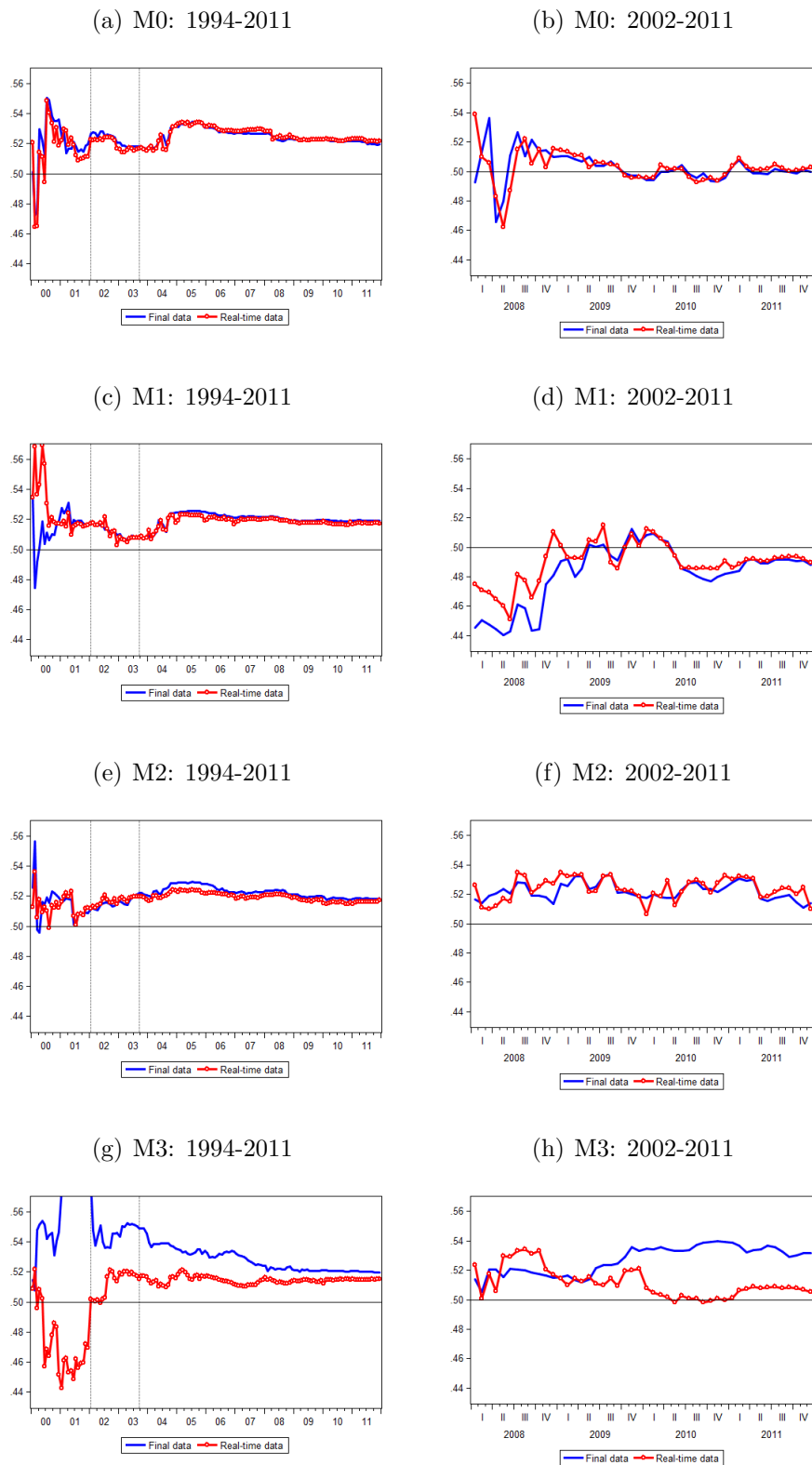
C Model Uncertainty

Figure 14. Probability that monetary aggregates are weakly exogenous.



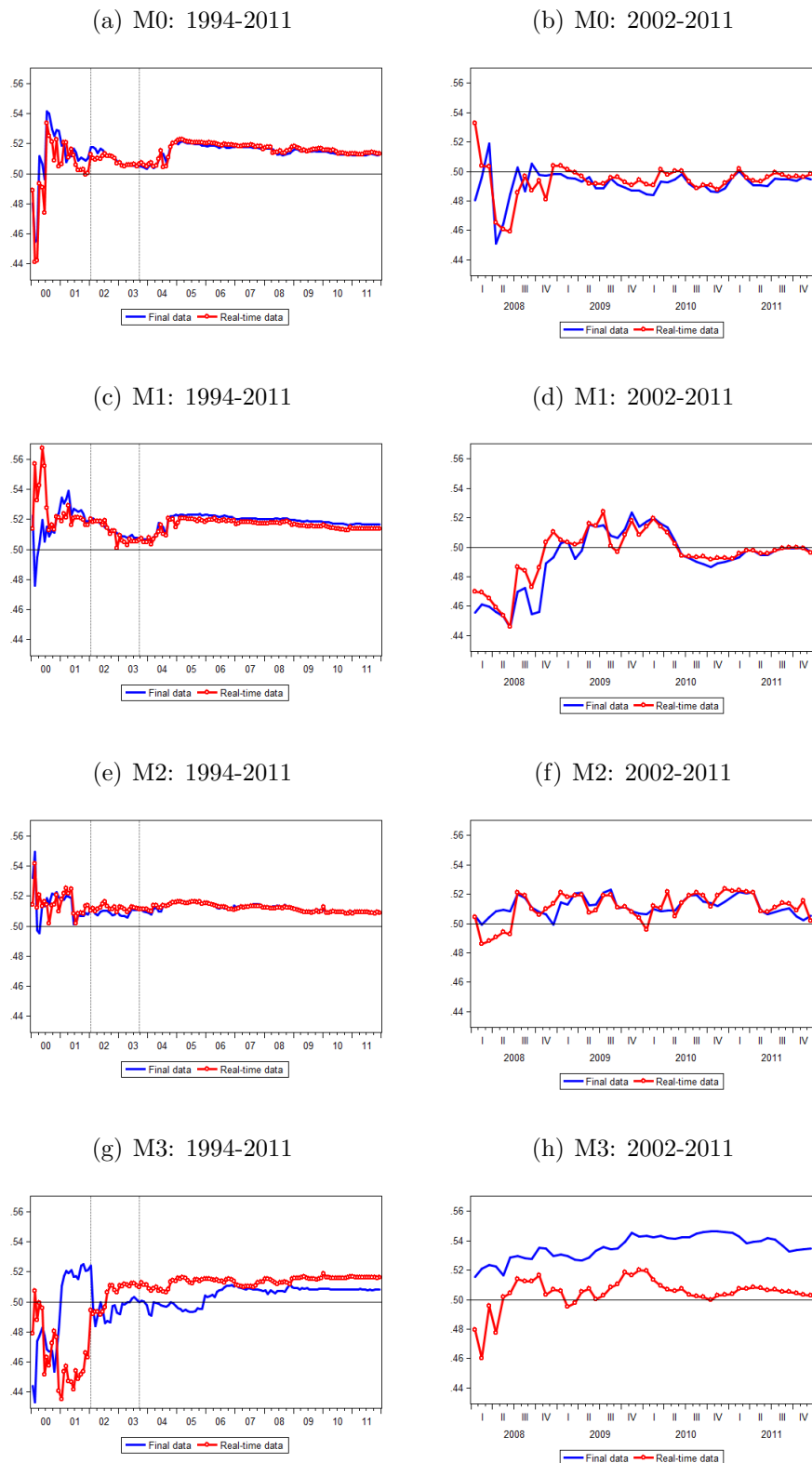
NOTE: Each vertical axis represents probabilities between 0 and 1. The horizontal line indicates the date of the last observation of each recursive sample. For instance, January 2008 represents the recursive sample 1994m1-2008m1 in Panels (a) or (c), and the recursive sample 2002m1-2008m1 in Panels (b) and (d).

Figure 15. *Probability that monetary aggregates are strongly exogenous: prices*



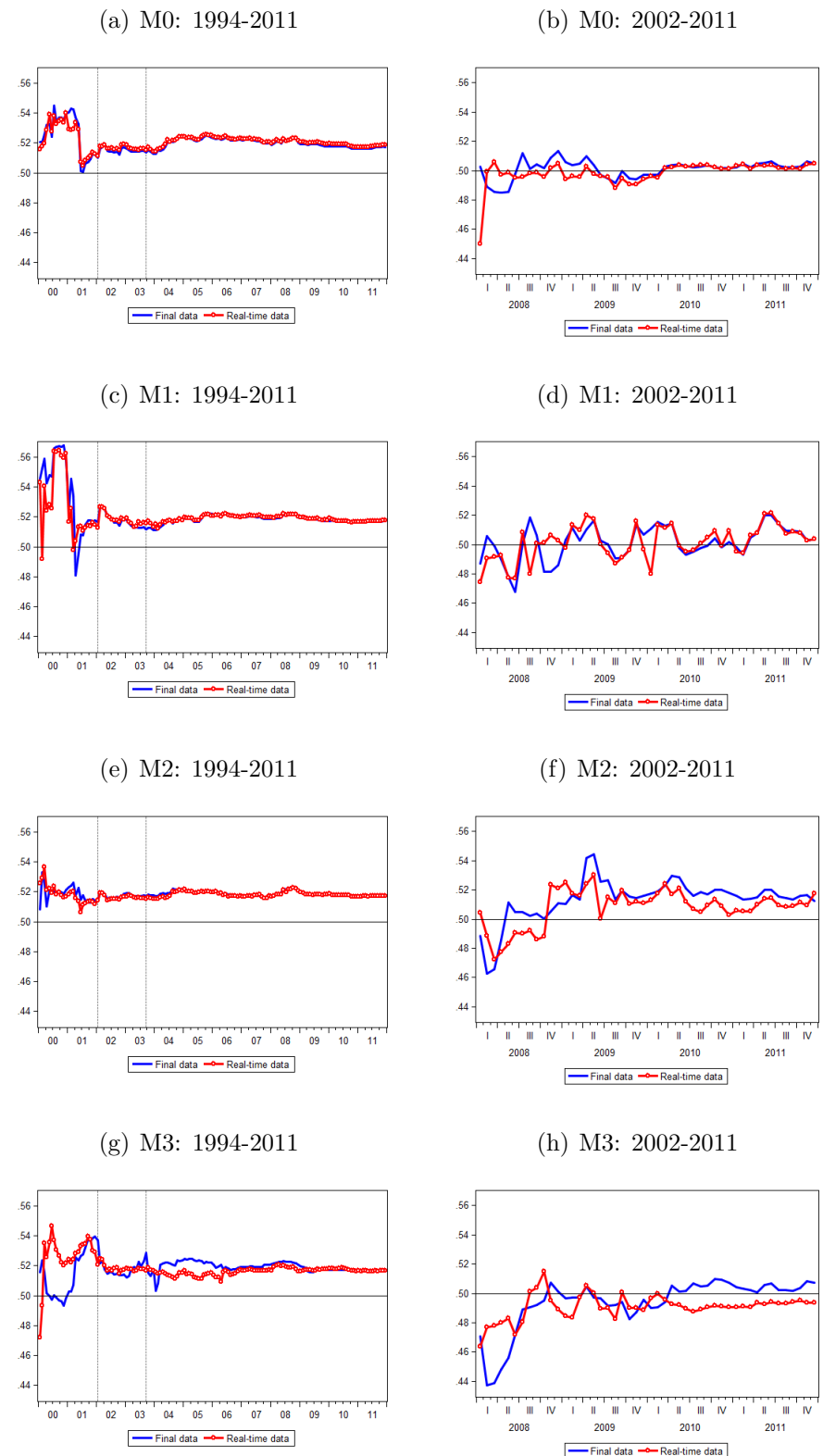
NOTE: Each vertical axis represents probabilities between 0 and 1. The horizontal line indicates the date of the last observation of each recursive sample. For instance, January 2008 represents the recursive sample 1994m1-2008m1 in Panels (a) or (c), and the recursive sample 2002m1-2008m1 in Panels (b) and (d).

Figure 16. *Probability that monetary aggregates are strongly exogenous: real output*



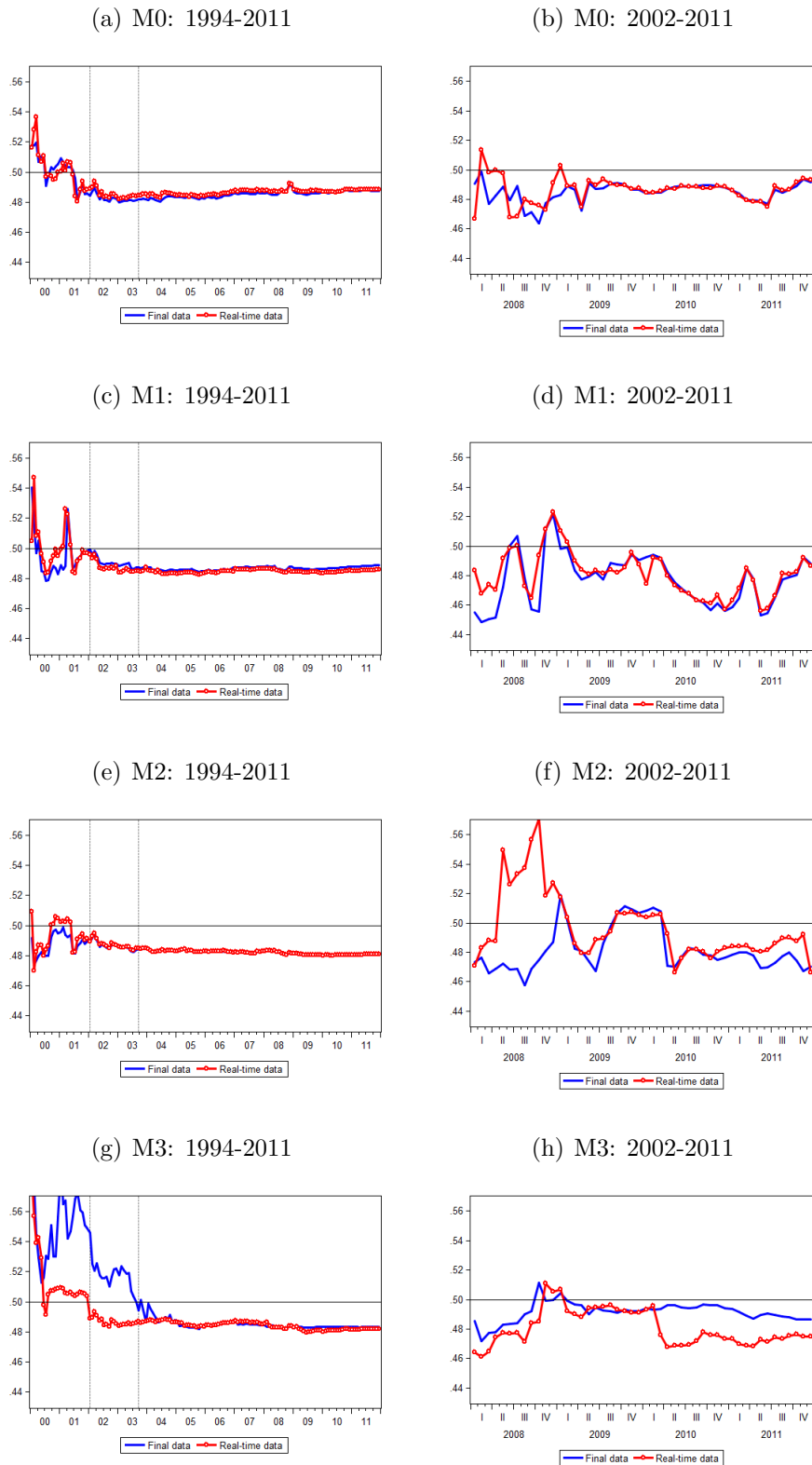
NOTE: Each vertical axis represents probabilities between 0 and 1. The horizontal line indicates the date of the last observation of each recursive sample. For instance, January 2008 represents the recursive sample 1994m1-2008m1 in Panels (a) or (c), and the recursive sample 2002m1-2008m1 in Panels (b) and (d).

Figure 17. *Probability of predictive content of money for prices*



NOTE: Each vertical axis represents probabilities between 0 and 1. The horizontal line indicates the date of the last observation of each recursive sample. For instance, January 2008 represents the recursive sample 1994m1-2008m1 in Panels (a) or (c), and the recursive sample 2002m1-2008m1 in Panels (b) and (d).

Figure 18. *Probability of predictive content of money for real output*



NOTE: Each vertical axis represents probabilities between 0 and 1. The horizontal line indicates the date of the last observation of each recursive sample. For instance, January 2008 represents the recursive sample 1994m1-2008m1 in Panels (a) or (c), and the recursive sample 2002m1-2008m1 in Panels (b) and (d).