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# OPTIMIZATION PRINCIPLE AND ITS' APPLICATION IN OPTIMIZING LANDMARK UNIVERSITY BAKERY PRODUCTION USING LINEAR PROGRAMMING

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## ABSTRACT

*This paper deals with the applications of optimization principle in optimizing profits of a production industry using linear programming to examine the production cost and determine its optimal profit. Linear programming is an operation research technique which is widely used in finding solutions to managerial decision problems. However, many enterprises make more use of the trial-and-error method. As such, firms have been finding it difficult in allocating scarce resources in a manner that will ensure profit maximization and/or cost minimization.*

*This paper makse use of secondary data collected from the records of the Landmark University Bakery on five types of bread produced in the firm which include Family loaf, sliced family bread, Chocolate loaf, medium size bread, small size bread. A problem of this nature was identified as a linear programming problem, formulated in Mathematical terms and solved using AMPL software. The solution obtained revealed that Landmark bakery unit should concentrate much more in production of 14,000 loaves of Family loaf and 10,571 loaves of Chocolate bread while others type should be less produced since their value is turning to zero in order to achieve a maximum monthly profit of N1,860,000. From the analysis, it was observed that Family loaf and the Chocolate bread contributed objectively to the profit. Hence, more of Family loaf and Chocolate bread are needed to be produced and sold in order to maximize the profit.*

**Keywords**: Linear, programming, production. Optimization, maximization, enterprises

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## 1. INTRODUCTION

The aim of every organization, company or firm is to make profit as that will guarantees its continuous existence and productivity. In this modern day, manufacturing industries at all levels are faced with the challenges of producing goods of right quality, quantity and at right time and more especially at minimum cost and maximum profit for their survival and growth. Thus, this demands an increase in productive efficiency of the industry.

Linear programming (LP) can be defined as a mathematical technique for determining the best allocation of a firm's limited resources to achieve optimum goal. It is also a mathematical technique used in operation research or Management Sciences to solve specific problems such as allocation, transportation and assignment problems that permits a choice or choices between alternative courses of action. It is one of the most widely used optimization techniques and perhaps the most effective method. The term 'linear programming was coined by [1] which refers to problems in which both the objective function and constraints are provided as the Simplex method as published in [2].

[3] Studied the optimal production cost of raw materials to its production output using Linear programming solver to solve and to optimize its monthly production output. Based on their result, the monthly optimal production output was 1.2252E-08. The company has to budget at least the optimal result to achieve their monthly cost of production. The result helps the company to eliminate excess waste that incurs in their cost of production. Likewise, [4] examined optimization principle and its application in solving the problem of over-allocation and under-allocation of the classroom space using Linear Programming in Landmark University where linear programming model was formulated based on the data obtained from the examination and lecture timetable committee on the classroom facilities, capacities and the number of students per programme in all the three (3) Colleges to maximize the available classroom space and minimizes the congestion and overcrowding in a particular lecture room using AMPL software which revealed that 16 out of 32 classrooms available with a seating capacity of 2066 has always been used by the current student population of 2522 which always causes overflow and congestion in those concentrated classroom while the remaining 16 classrooms with the seating capacity of 805 were underutilized. Meanwhile it was revealed by the AMPL software if all these 32 classroom with seating capacity of 3544were fully utilized, this indicated that an additional 1022 i.e.(3544-2522) students can be fully absorbed comfortably with the existing 32 classrooms in both of the three (3) colleges if the seating capacities are fully managed and maximized while the school management will generate additional income using the same classroom facility with the existing seating capacity.

[5] Focused on linear optimization for achieving product-mix optimization in terms of the product identification and the right quantity in paint production for better profit and optimum firm performance in Nigeria. Their result showed that only two out of the five products they

considered in their computational experiment are profitable. [6] Empirically examined the impact of linear programming in entrepreneur decision making process as an optimization technique for maximizing profit with the available resources. Their work drew examples from a fast food firm that encountered some challenges in the production of meat pie, chicken pie and doughnut due to an increment in the price of raw materials. Their results showed that there should be discontinuity in the production of chicken pie and doughnut and that they should concentrate with production of meat pie.

Using farm activities, [7] developed a linear program that reflects choices of selection that is feasible given a set of fixed farm constraints and maximizing income while achieving other goals such as food security. Their result obtained using linear programming is compared with the traditional methods. Their results obtained using the linear programming model shows that they are more superior. [8] showed that the product-mix problem can be used efficiently not only to determine the optimal operational points but also to provide information on how those optimal points could be further increased through changing the constraints of the optimization problem. Their results showed that this information could be used to enhance production by informing expansion plans in which management identifies and take advantage of the capacity of under-utilized constraints and use them to expand the capacity of over-utilized or limiting constraints. Therefore, this paper deal with the application of linear programming (LP) as an optimization principle to optimize profit of manufacturing industries such as Landmark University Bakery and determine the optimal solution for production and verify the output under normal operational environment using AMPL software.

## 2. MATHEMATICAL FORMULATION

This paper investigates the overall quantity and quality combination of the five products produced by Landmark University Bakery and the allocation of resources to the various products through the records kept by the manager of Landmark University Development Ventures (LMDV) and the Bakery Unit manager relating to the different brands of bread products produced by the firm, the technical coefficients, the raw materials available and their relative prices is shown in Table 1 below.

We present Linear programm as a general standard form to display all properties required of a linear programming problem. This consists of a linear objective function $f(x)$ such that real numbers $c_1, c_2 .........c_n$, then the function $f$ of real variables $x_1, x_2 .........x_n$ can be defined as:

$$f(x) = C_1X_1 + \cdots + C_nX_n = \sum_{j=1}^{n} C_j x_j \tag{1}$$

Other properties include a linear constraint (which is one that is either a linear equation or linear inequality) and a non-negativity constraint. These can be written in mathematical Notations as:

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \;\; \forall\, i \in \{1, \dots, m\} \quad \text{(Linear constraint)} \tag{2}$$

$$x_j \geq 0 \;\; \forall\, j \in \{1, \dots, n\} \quad \text{(Non-negative constraint)} \tag{3}$$

Hence every linear program in the standard form can be generally presented as:

$$Z = \max. \sum_{j=1}^{n} C_j x_j \tag{4}$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \ \forall\, i \in \{1, \dots, m\}$$
$$x_j \ge 0 \ \forall\, j \in \{1, \dots, n\} \quad (5)$$

$(X_1, X_2, \dots, X_n)$ satisfy all the constraints of linear program, then the assignment of values to these variables are called a feasible solution of the linear program.

## 2.1. Linear programming

We consider a linear programming of the form:

$$Maximize\, F = \sum_{j=1}^{n} C_i X_j$$

*Subject to*

$$\sum_{j=1}^{n} a(i\, j) X_j = b_i . i = 12 \dots n$$
$$l_j \le X \le u_j . j = 12 \dots n \quad (6)$$

*Where* $C_j$ *arethen objective functioncoefficients* $a(i\ j)$ *and b are parametersin them linearinequalityconstrantsand* $l_j$ *and* $u_j$ *arelower and upperboundswith* $l_j \le u_j$.
*Both* $l_j$ *and* $u_j$ *maybe positiveornegative*

## 2.2. Formulation of LP model

Mathematical models were constructed for the production of various type of bread produced by the LMU bakery unit. The objective of the model was to minimize cost of producing a particular product after satisfying a set of constraints. These constraints were mainly those from nutrients requirements of the bread and the ingredients. The variables in the models were the ingredients while the cost of each ingredients and the nutrient valued of each ingredient was the parameter.

The specified L.P model for the attainment of the objective function is as follows:

$$maximixe\ z = a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 + a_5 x_5 \quad (7)$$

$$\textbf{subject to}:$$
$$xj \ge 0 \qquad j = (1, 2, \dots \dots n)$$

$$\begin{aligned}
c_{11}x_1 + c_{12}x_2 + c_{13}x_3 + c_{14}x_4 + c_{15}x_5 &\le b_1 \\
c_{21}x_1 + c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 &\le b_2 \\
c_{31}x_1 + c_{32}x_2 + c_{33}x_3 + c_{34}x_4 + c_{35}x_5 &\le b_3 \\
c_{41}x_1 + c_{42}x_2 + c_{43}x_3 + c_{44}x_4 + c_{45}x_5 &\le b_4 \\
c_{51}x_1 + c_{52}x_2 + c_{53}x_3 + c_{54}x_4 + c_{55}x_5 &\le b_5 \\
c_{61}x_1 + c_{62}x_2 + c_{63}x_3 + c_{64}x_4 + c_{65}x_5 &\le b_6 \\
c_{71}x_1 + c_{72}x_2 + c_{73}x_3 + c_{74}x_4 + c_{75}x_5 &\le b_7 \\
c_{81}x_1 + c_{82}x_2 + c_{83}x_3 + c_{84}x_4 + c_{85}x_5 &\le b_8
\end{aligned} \quad (8)$$

## 3. DATA COLLECTION AND ANALYSIS OF RESULTS

Table 1 below presents five different types of breads produced by LMU bakery, their production cost, selling price and profit. Table 2: shows basic eight (8) raw materials used for the production of bread at Landmark University Bakery, The combinations of the quantities of these eight basic raw materials (raw material mix) for bread production per loaf (in grams), and the maximum quantity of each raw material held in stock for monthly production is also captured in the table. This information is used to determine the production cost (in terms of raw materials) per loaf of bread produced by the bakery.

**Table 1** Shows types of Bread, Cost and selling price with the profits

| | Name of Product | Production cost per loaf(₦) | Selling price per loaf (₦) | Profit (₦) |
|---|---|---|---|---|
| 1 | Family loaf $(x_1)$ | 220 | 300 | 80 |
| 2 | Family loaf slice $(x_2)$ | 240 | 300 | 60 |
| 3 | Chocolate bread $(x_3)$ | 280 | 350 | 70 |
| 4 | Medium size loaf $(x_4)$ | 150 | 200 | 50 |
| 5 | Small size loaf $(x_5)$ | 70 | 100 | 30 |

Source: Landmark bakery 2018

**Table2** shows the raw material Mix used for Bread Production per Baking

| Raw Materials | Type of Bread and their Raw Material Mix | | | | | Total Quantity Per month in (grams) (approx.) |
|---|---|---|---|---|---|---|
| | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | |
| Flour | 400 | 450 | 350 | 320 | 200 | 9300000 |
| Yeast | 30 | 25 | 20 | 15 | 10 | 2800000 |
| Milk | 30 | 35 | 45 | 25 | 15 | 6500000 |
| Egg | 100 | 100 | 80 | 75 | 50 | 6000000 |
| Water | 280 | 280 | 220 | 180 | 150 | 8800000 |
| Flavour | 30 | 30 | 50 | 20 | 10 | 1200000 |
| Butter | 45 | 45 | 35 | 25 | 15 | 1000000 |
| Sugar | 30 | 30 | 35 | 20 | 10 | 1500000 |

Source: Landmark Bakery records 2018

### 3.1. FORMULATION OF LINEAR PROGRAMMING

Both the objective function and the constraints values were inserted into the linear programming model as shown below

$$\begin{aligned}
&Maximize\,P: 80x_1 + 60x_2 + 70x_3 + 50x_4 + 30x_5 \\
&Subject\ \ to: \\
&Flour: 400x_1 + 450x_2 + 350x_3 + 320x_4 + 200x_5 \le 9300000 \\
&Yeast: 30x_1 + 25x_2 + 20x_3 + 15x_4 + 10x_5 \le 2800000 \\
&Milk: 30x_1 + 35x_2 + 45x_3 + 25x_4 + 15x_5 \le 6500000 \\
&Egg: 100x_1 + 100x_2 + 80x_3 + 75x_4 + 50x_5 \le 6000000 \\
&Water: 280x_1 + 280x_2 + 180x_3 + 220x_4 + 150x_5 \le 8800000 \\
&Flavour: 30x_1 + 30x_2 + 50x_3 + 20x_4 + 10x_5 \le 1200000 \\
&Butter: 45x_1 + 45x_2 + 35x_3 + 25x_4 + 15x_5 \le 1000000 \\
&Sugar: 30x_1 + 30x_2 + 45x_3 + 20x_4 + 10x_5 \le 15000000 \\
&x_j \ge 0 \\
&j = (1,2...n) \rightarrow non-negativity
\end{aligned}$$

### 3.2. Formation of Slack Variables

In order to represent the above LP model in canonical form, six slack variables $w_i\,(i = 1,2...........6)$ were introduced into the model. This changed the inequalities signs in the constraint aspect of the model to equality signs. A slack variable will account for the unused quantity of raw material (if any) at end of the production.

As a result, the above LP model yields:

$$\begin{aligned}
&Maximize\,P: 80x_1 + 60x_2 + 70x_3 + 50x_4 + 30x_5 \\
&Subject\ \ to: \\
&Flour: 400x_1 + 450x_2 + 350x_3 + 320x_4 + 200x_5 + w_1 = 9300000 \\
&Yeast: 30x_1 + 25x_2 + 20x_3 + 15x_4 + 10x_5 + w_2 = 2800000 \\
&Milk: 30x_1 + 35x_2 + 45x_3 + 25x_4 + 15x_5 + w_3 = 6500000 \\
&Egg: 100x_1 + 100x_2 + 80x_3 + 75x_4 + 50x_5 + w_4 = 6000000 \\
&Water: 280x_1 + 280x_2 + 180x_3 + 220x_4 + 150x_5 + w_5 = 8800000 \\
&Flavour: 30x_1 + 30x_2 + 50x_3 + 20x_4 + 10x_5 + w_6 = 1200000 \\
&Butter: 45x_1 + 45x_2 + 35x_3 + 25x_4 + 15x_5 + w_7 = 1000000 \\
&Sugar: 30x_1 + 30x_2 + 45x_3 + 20x_4 + 10x_5 + w_8 = 15000000 \\
&x_j \ge 0 \\
&j = (1,2...n) \rightarrow non-negativity
\end{aligned}$$

We analyse this program by Simplex method proposed by George Danzig (1947 and published in Danzig(1963) which have been found to be more efficient and convenient for computer software implementation (AMPL program) which is a present day application used for solving Mathematical equations.

### 3.3. Program written in AMPL to generate the Results

```
var x1>= 0; #family size bread
var x2>= 0; #sliced family size bread
var x3>= 0; #chocolate bread
var x4>= 0; #medium
var x5>= 0; #100 naira
Maximize z: 80*x1 + 60*x2 + 70*x3 + 50*x4 + 30*x5;
s.t. M1: 400*x1 + 450*x2 + 350*x3 + 320*x4 + 200*x5 <= 9300000; #flour
s.t. M2: 30*x1 + 25*x2 + 20*x3 + 15*x4 + 10*x5 <= 2800000; #yeast
s.t. M3: 30*x1 + 35*x2 + 45*x3 + 25*x4 + 15*x5 <= 6500000; #milk
s.t. M4: 100*x1 + 100*x2 + 80*x3 + 75*x4 + 50*x5 <= 6000000; #egg
s.t. M5: 30*x1 + 30*x2 + 45*x3 + 20*x4 + 10*x5 <= 1500000; #sugar
s.t. M6: 280*x1 + 280*x2 + 180*x3 + 150*x4 + 220*x5 <= 8800000; #water
s.t. M7: 30*x1 + 30*x2 + 50*x3 + 20*x4 + 10*x5 <= 1200000; #flavour
s.t. M8: 45*x1 + 45*x2 + 35*x3 + 25*x4 + 15*x5 <= 1000000; #butter
reset;
model bsc.mod;
solve;
display x1, x2, x3, x4, x5, z;
ampl: include bsc.run;
MINOS 5.51: optimal solution found.
3 iterations, objective 186000
x1 = 14000
x2 = 0
x3 = 10571.4
x4 = 0
x5 = 0
z = 1860000
```

## 4. ANALYSIS OF RESULTS GENERATED BY THE AMPL

Results from the analysis carried out on the Linear Programming model using Simplex method through AMPL software estimated the value of the objective function to be ₦1860000. The contributions of the five decision variables $x_1, x_2, x_3, x_4, x_5$. into the objective function are 14000, 0, 10571, 0 and 0 respectively. This simply shows that only $x_1$ and $x_3$ variables contributed meaningfully to improve the value of the objective function of the Linear Programming model with 14000 and 10571 respectively.

From the results of the Linear Programming model, it is therefore desirable and profitable for Landmark University Bakery unit to concentrate much more on the production of $x_1$ (family size bread) and $x_3$ (chocolate bread) production. By this, total sales of about 14000 loaves of $x_1$ and 10571 loaves of $x_3$ would be sold by the LMU Bakery per month. This would fetch the Bakery an optimal profit of about ₦1, 860,000 per month based on the costs of raw materials and the capacity of the oven only

## 5. CONCLUSION

In this paper, we have successfully examines various type, quantities and the cost of Landmark University Bakery production. We determine its' optimal solution using the secondary data collected from the records of the Landmark University Bakery on five types of bread produced in the firm through a linear programming problem formulated as a Mathematical terms using AMPL software. The solution revealed that the bakery manager should concentrate much more in production of loaves of Family loaf and loaves of Chocolate bread while others type should be less produced since their value is gradually turning to zero in order to achieve a maximum monthly profit of ₦1,860,000. From the analysis, it was also revealed that Family loaf and the Chocolate bread contributed objectively to the highest and optimal profit. Hence, more of Family loaf and Chocolate bread are needed to be produced and sold in order to maximize the profit.

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