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MODEL ESTIMATION AND PREDICTION FOR A WATER MANAGEMENT SYSTEM

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ABSTRACT

A water management model has been developed using identification techniques for predicting water table elevations. Data recorded near the Aurora site in the North Carolina coastal plains over a two- year period was used to develop and test the model.

Rainfall and water table elevations were recorded continuously at this site and the observed water table elevations were compared to predicted day end values. The identification of both linear and non-linear difference equation models is described to represent the relationship between the three inputs (rainfall, potential evaporation, and ditch water elevation) and the output (water table elevation).

INTRODUCTION

Skaggs (1982) developed a simulation model DRAINMOD which is based on a water balance for a thin soil section of unit surface area. DRAINMOD is a computer program developed from simulation of the mathematically derived non-linear models which characterize the response of the soil water regime to various combinations of surface and subsurface water management. DRAINMOD can be used to predict the response of the water table and soil water above the water table to rainfall and evaporation (ET), given degrees of surface and sub-surface drainage, and the use of water table control or subirrigation practices. Similar work has been carried out by others (SWATRE model developed by Feddes, 1978; Belman et al., 1983) but in all cases analytical models were derived based on the laws of physics.

In this paper a totally different technique of obtaining a model of the water management system is presented. It is based on linear and non-linear system identification methods. It will be shown how a model can be estimated directly by using input/output data only. In this way a concise mathematical description of the system can be built which can then be used as a basis for analysis, design, and prediction.

The major advantage of using the identification technique described in the present study is that it will reduce the model development and simulation time dramatically. It is also a useful alternative for analytical modeling of complex and difficult systems.

The aim of the present paper is to demonstrate the effectiveness of the identification technique using preliminary results and to compare the predictions with Skaggs' results (1982) .

BACKGROUND

Figure 1 illustrates a typical water management system. The soil is nearly flat and has an impermeable layer at a relatively shallow depth. Subsurface drainage is provided by drain tubes or parallel ditches spaced at a distance L, apart and at a distance d, above the impermeable layer. Water infiltrates at the surface and percolates through the profile raising the water table and increasing the subsurface drainage rate when the rainfall occurs. Water begins to collect on the surface if the rainfall rate is greater than the capacity of the soil to infiltrate. Most of the surface water will run off if good surface drainage (smooth and on a gradient) is provided. For poor surface drainage, a certain amount of water is stored in depressions before runoff can begin. Even after rainfall stops, the water stored in surface depressions infiltrates into the soil. Therefore poor surface drainage lengthens the infiltration event and permits more water to infiltrate; hence, resulting in a larger rise in the water table.

Water is drained from the profile at a rate depending on the hydraulic conductivity of the soil, the drain depth and spacing, the effective profile depth, and the depth of water in the drains. When the water level is raised in the drainage ditches, the drainage rate will be reduced and water may move from the drains into the soil profile (supplying water to the root zone of the crop, see Figure 1).

Skaggs(1974) shows how a high water table reduces the amount of storage available for infiltration. Water may also be removed from the profile by evaporation (ET), and by deep seepage. The water balance for soil profile which is the basis for the computer model is illustrated in **Figure 2**.

DRAINMOD was first developed in 1975 and has been continuously modified to improve its capabilities since that time. The result is a physical-based model that can reliably simulate the performance of the system described above. It has been tested for a variety of soil, crop and climatological conditions, (Skaggs, 1982; Skaggs et al., 1981; Gale et al., 1985; Fouss et al., 1987; Rogers, 1985; McMahan et al., 1988). The model simulates the performance of a water management system over a long period of climatological record and has been used successfully to design and evaluate the multicomponent water management systems on shallow water table soils.

The methods presented in this paper require much less time and expense in the development phase, but also result in a reliable simulation model.

MODEL ESTIMATION AND PREDICTION USING IDENTIFICATION TECHNIQUES

It can be shown that under some mild assumptions, any discrete time multivariable nonlinear stochastic system with r inputs and m outputs can be described by the model:

$$\underline{y}(t) = \underline{\mathbf{f}}\,[\underline{y}(t\text{-}1), \ldots, \underline{y}(t\text{-}n_y), \underline{u}(t\text{-}1), \ldots, \underline{u}(t\text{-}n_u), \underline{e}(t\text{-}1), \ldots, \underline{e}(t\text{-}n_e)] + \underline{e}(t) \tag{1}$$

where

$$\underline{y}(t) = \begin{bmatrix} y_1(t) \\ \cdot \\ \cdot \\ \cdot \\ y_m(t) \end{bmatrix}, \quad \underline{u}(t) = \begin{bmatrix} u_1(t) \\ \cdot \\ \cdot \\ \cdot \\ u_r(t) \end{bmatrix}, \quad \underline{e}(t) = \begin{bmatrix} e_1(t) \\ \cdot \\ \cdot \\ \cdot \\ e_m(t) \end{bmatrix}$$

represent the system output, input and noise, respectively; n_y , n_u and n_e are the maximum lags in the output, input and noise, e(t) is a zero mean white sequence and $\underline{\mathbf{f}}(\,.\,)$ is some vector-valued nonlinear function. A typical row in the model of eqn. 1 takes the form

$$\begin{aligned} y_i(t) = \mathbf{f}_i\,[&y_i(t\text{-}1), \ldots, y_1(t\text{-}n^i_{y1}), \ldots, y_m(t\text{-}1), \ldots, y_m(t\text{-}n^i_{ym}), u_1(t\text{-}1), \ldots, \\ &u_1(t\text{-}n^i_{u1}), \ldots, u_r(t\text{-}1), \ldots, u_r(t\text{-}n^i_{ur}), e_1(t\text{-}1), \ldots, e_1(t\text{-}n^i_{e1}), e_m(t\text{-}1), \ldots, \\ &e_1(t\text{-}n^i_{em})] + e_i(t) \qquad i = 1 \ldots\ldots m \end{aligned} \tag{2}$$

Where the maximum lags for each output, input, and noise have been assigned to different values to allow flexibility in the model structure.

The nonlinear form of $\mathbf{f}_i(\,.\,)$ in eqn. 2 can be very wide but in the present study only polynomial expressions will be considered. When m = r = 1 the model of eqn. 2 reduces to the single-input single-output (SISO) case

$$y(t) = \mathbf{f}\,[y(t-1), \ldots, y(t-n_y), u(t-1), \ldots, u(t-n_u), e(t-1), \ldots, e(t-n_e)] + e(t) \qquad (3)$$

which is called NARMAX model or Nonlinear AutoRegressive Moving Average model with eXogenous input (Leontaritis and Billings, 1986). Whilst the present application involves the estimation of a multivariable model it is notationally easier to introduce some of the identification methods in terms of the SISO NARMAX model of eqn. 3.

A NARMAX model with first order dynamics expanded as a second order polynomial would for example be represented as

$$x(t) = C_1x(t-1) + C_2u(t-1) + C_{11}x^2(t-1) + C_{12}x(t-1)u(t-1) + C_{22}u^2(t-1) \qquad (4)$$

If the output is measured with additive noise

$$y(t) = x(t) + e(t)$$

the model of eqn. 4 becomes

$$\begin{aligned} y(t) = {} & C_1y(t-1) + C_2u(t-2) + C_{11}y^2(t-1) + C_{12}y(t-1)u(t-1) + C_{22}u^2(t-1) + e(t) - C_1e(t-1) \\ & -2C_{11}y(t-1)e(t-1) + C_{11}e^2(t-1) - C_{12}e(t-1)u(t-1) \end{aligned} \qquad (5)$$

and cross product noise terms appear in the expansion. In general the noise may enter in a variety of ways and multiplicative and other nonlinear noise effects may be present in the data. Any estimation routine must therefore be capable of providing unbiased estimates in the presence of general and possibly multiplicative noise terms. Note that even in the case of simple additive noise the noise on the model eqn. 5 is dependent on the input and the output amplitudes.

Several parameter estimation algorithms have been derived for the NARMAX model. Most of these are based on representing eqn. 5, for example, in the form

$$\begin{aligned} x_t^{\,T} = {} & [y(t-1), u(t-1), y^2(t-1), y(t-1)u(t-1), u^2(t-1), \hat{\epsilon}(t-1), \hat{\epsilon}(t-1)y(t-1), \\ & u(t-1)\hat{\epsilon}(t-1), \hat{\epsilon}^2(t-1)] \end{aligned}$$

$$\begin{aligned} \hat{\beta}^T &= [\hat{C}_1, \hat{C}_2, \ldots\ldots, \hat{C}_s] \\ \hat{\epsilon}(t+1) &= y(t+1) - x_{t+1}^{\,T}\hat{\beta} \end{aligned} \qquad (6)$$

where $\hat{\epsilon}(.)$ are the prediction errors. Parameter estimation is relatively straightforward if the structure of the model is known exactly, however there are many processes where the form or structure of the describing equations are unknown. If the system is linear then determination of the model structure just consists of choosing the order or number of lags in the input, output, and noise. In the nonlinear case, structure determination becomes more complex because the number of possible terms increases very rapidly as n_u, n_y, n_ϵ and the degree of polynomial expansion of $\mathbf{f}(\,.\,)$ is increased. This problem is of course compounded if the system is multi-input multi-output (MIMO). For example if in eqn. 2 m = r = 2, all the maximum lags are set to 2 and $\mathbf{f}_1(\,.\,)$ and $\mathbf{f}_2(\,.\,)$ are expanded as quadratic polynomials, then there are 182 possible terms in the model. Often many of these terms will be redundant and including them in the model will create a large estimation problem, may well induce numerical problems and will lead to a very complex model which is not easy to use or analyze. It is for these reasons that all the estimation routines devised for the NARMAX model eqn. 3 and the MIMO variant eqn. 1 include methods for detecting the model structure or selecting the significant terms in the model prior to estimation. In practice this ensures that the simplest model that captures all the independent information in the data is produced. Even when analyzing real industrial data this usually gives a model which contains a very small number of terms in each loop.

Once the significant terms have been identified and estimates of the associated parameter values have been obtained, then the one step ahead prediction of the output

$$\hat{y}(t) = \hat{\mathbf{f}}(y(t\text{-}1), \ldots, y(t\text{-}n_y), u(t\text{-}1), \ldots, u(t\text{-}n_u), \epsilon(t\text{-}1, \hat{\Theta}), \ldots \epsilon(t\text{-}n_\epsilon, \hat{\Theta})) \tag{7}$$

the prediction error or residual sequence

$$\epsilon(t,\hat{\Theta}) = y(t) - \hat{y}(t) \tag{8}$$

and the model predicted output

$$\hat{y}_d(t) = \hat{\mathbf{f}}(y_d(t\text{-}1), \ldots, y_d(t\text{-}n_y), u(t\text{-}1), \ldots, u(t\text{-}n_u), 0 \ldots 0) \tag{9}$$

can be computed. Each of these sequences provide useful information on the properties and validity of the estimated model. For example the identified model will only be unbiased if the residuals are unpredictable from all linear and nonlinear combinations of past inputs and outputs and this will be true if the following correlation tests are satisfied

$$\phi_{\epsilon\epsilon}(\tau) = \delta(\tau)$$
$$\phi_{u\epsilon}(\tau) = 0 \;\forall\; \tau$$

$$\phi_{u^2\epsilon}(\tau) = 0 \ \forall \ \tau$$
$$\phi_{u^2\epsilon^2}(\tau) = 0 \ \forall \ \tau$$
$$\phi_{\epsilon\epsilon u}(\tau) = 0 \ \forall \ \tau \geq 0 \qquad (10)$$
where $\phi_{ab}(\tau) = E[a(t-\tau)b(t)]$.

In practice the estimation of both the structure and unknown parameters for nonlinear models is an iterative and interactive procedure. The model produced will often be concise and the individual terms in the model can often be related to the physical characteristics of the system under study.

Once a model has been estimated it is often used both to study the properties of the underlying system and to predict the system output for other input excitations. Notice that the one-step ahead predicted output eqn. 7 is not very useful because it uses past outputs one step back in time. It is for this reason that one-step ahead predicted outputs $\hat{y}(t)$ are often close to the measured system output y(t) even if the model which has been fitted is significantly in error. Many authors use the one-step ahead predicted output as a measure of goodness of fit. Unfortunately, this can often lead to wrong results. The model predicted output $\hat{y}_d(t)$ eqn. 9 is a far better indicator of model performance. Notice that in eqn. 9 only the predicted output $\hat{y}_d(\ .\ )$ is used not the measured output as in eqn. 7. Eqn. 9 can therefore be used to predict the output into the future, or over different data sets.

ESTIMATION RESULTS

The objective in the present study is to identify a model of the form of equation (1) when $\underline{\mathbf{f}}(\ .\ )$ is expanded as a polynomial relating the water table (WT) to the inputs rainfall, potential evapotranspiration (PET) and ditch water elevation. The model is therefore defined by one output m=1 and three inputs r=3 and these will be assigned as

$y_1(t)$ = water table elevation

$u_1(t)$ = rainfall

$u_2(t)$ = PET

$u_3(t)$ = ditch water elevation

Models were fitted using the 365 daily values of the above variables for 1975. The data set is illustrated in Figure 3. Initially a linear multivariable model with the specification $n_y = n_{u1} = n_{u2} = 14$ and $n_{u3} = 28$ was considered. A much larger lag was used for the ditch because from physical reasoning it might be expected that the relationship between the ditch water elevation and the water table would involve a large delay. This was confirmed by the data analysis. The algorithm selected the significant terms by searching over the complete model set defined by $n_y = n_{u1} = n_{u2} = 14$, $n_{u3} = 28$. Results for the one step ahead predicted output sequence are given in Figure 4. The model predicted output for the estimated linear model is shown in Figure 5. Both Figure 4 and Figure 5 represent data over the estimated set. The results of using the model estimated with the data for 1975 to predict the water table for 1976 are illustrated in Figures 6 and 7. Figure 6 represents the one step ahead predicted output and Figure 7 represents the model predicted output superimposed on the measured water table data of 1976.

A nonlinear multivariable model identified from the model set defined by $n_y = n_{u1} = n_{u2} = 14$, $n_{u3} = 28$ and using second degree polynomial expansions was estimated. The estimation algorithms searched through the many hundreds of possible terms in the model and produced the following result

$$\begin{aligned} y(t)=&0.919y(t-1)+0.24u_1(t-3)+0.26u_1(t-1)u_1(t-2)-0.99u_1(t-1)u_3(t-28)+0.99u_2(t-1)u_3(t-24) \\ &-0.268u_2(t-10)u_2(t-8)+2.26u_1(t-1)u_2(t-1)+0.108u_1(t-2)u_1(t-28)-1.132u_2(t-1)u_2(t-2) \\ &+0.157u_2(t-10)-0.323e_1(t-1)+e_1(t) \qquad (11) \end{aligned}$$

The one step ahead predicted output and the model predicted output for both the estimation set (i.e. the 1975 data used to estimate the model) and the test set (i.e. 1976 data used to test the prediction of the model) are illustrated in Figures 8,9, 10 and 11 respectively.

The model describes the estimation set (Figure 9) very well. A comparison of the nonlinear model predicted output (Figure 9) with the linear case (Figure 5) shows that the nonlinear model is much better at predicting the peaks especially in the latter half of 1975. Similar comments apply when comparing the model predicted output linear model Figure 7 with that of the nonlinear model Figure 11. The correlation model validity tests (developed by Billings, 1986) of eqn. 10 were all satisfied by the nonlinear model eqn. 11 suggesting that this model provides a good description of the 1975 and 1976 data. The linear model however did not satisfy the model validity test indicating that it was biased due to omission of nonlinear terms from the model.

CONCLUSIONS

All the above results must be considered as preliminary in the sense that the present study was more an investigation into the potential of NARMAX modelling for this type of problem rather than a definitive study to develop the optimal description for this specific system. However, the results are very encouraging. The final nonlinear model is concise and produces predicted results close to the measured values. The study is limited because only one year's data was used. The reliability of the model could be improved and further tested by considering a larger data set. However results presented herein are sufficient to demonstrate the potential of this approach for hydrologic systems.

The new methods presented in this paper are not critically needed to quantify the hydrology of shallow water table soil. Existing models are available to do this. However, this application demonstrates that the methods can be used to describe these processes. There are often processes such as the movement and rate of fertilizer nutrients, pesticides and other potential pollutants which are more complex and for which reliable models do not exist. Based on results in this study it appears possible that the methods presented herein might be used to describe these processes.

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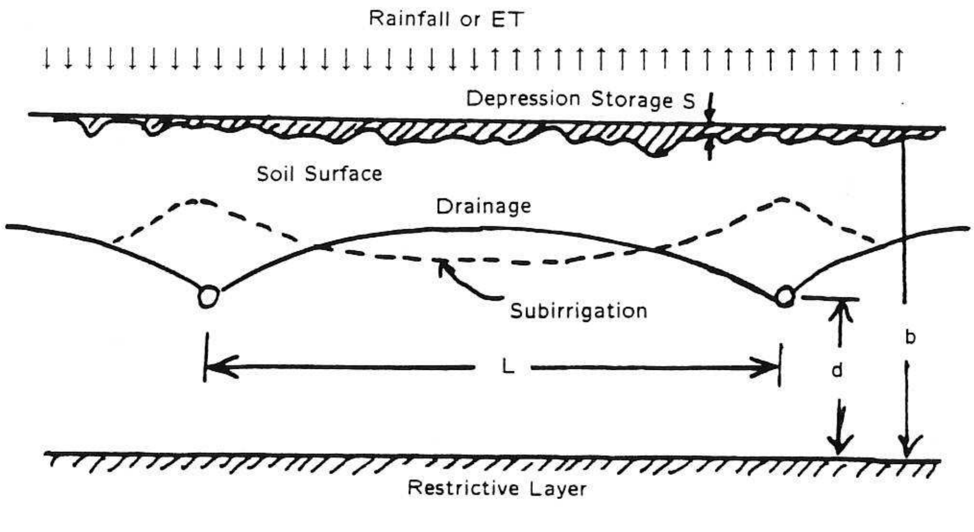

Figure 1. Schematic of water management system with subsurface drains.

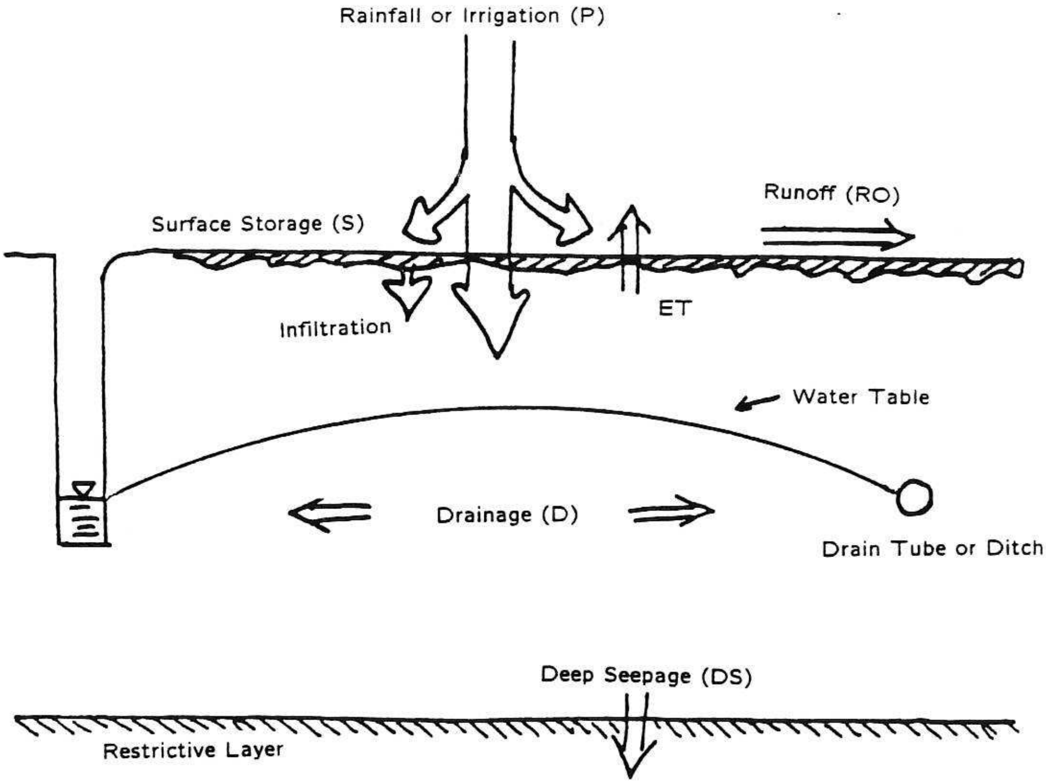

Figure 2. Schematic of water management system with drainage to ditches or drain tubes.

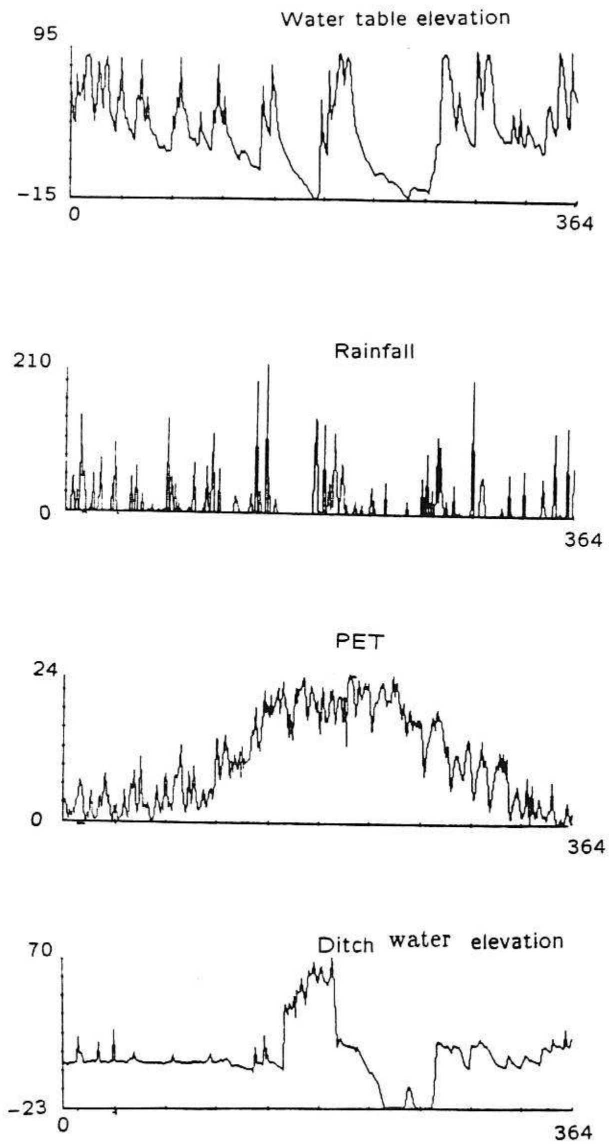

Figure 3. Input and output data for the water management system.

Year 1975

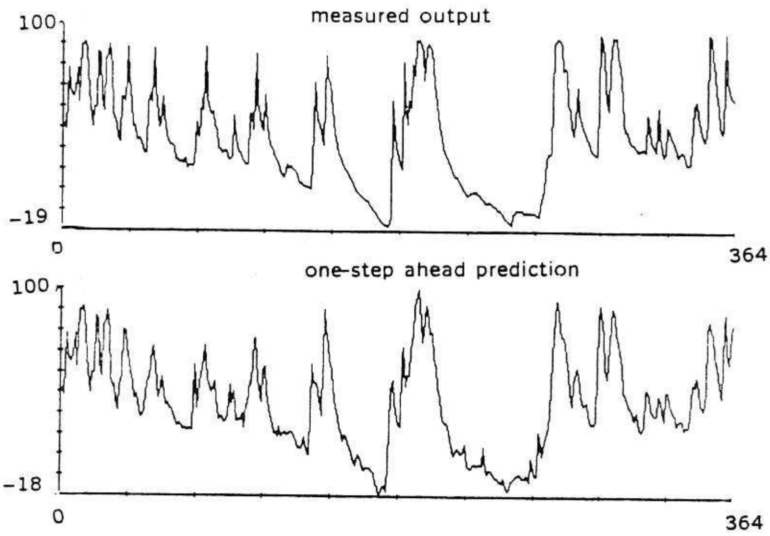

Figure 4. One step ahead prediction of the linear model on estimation set.

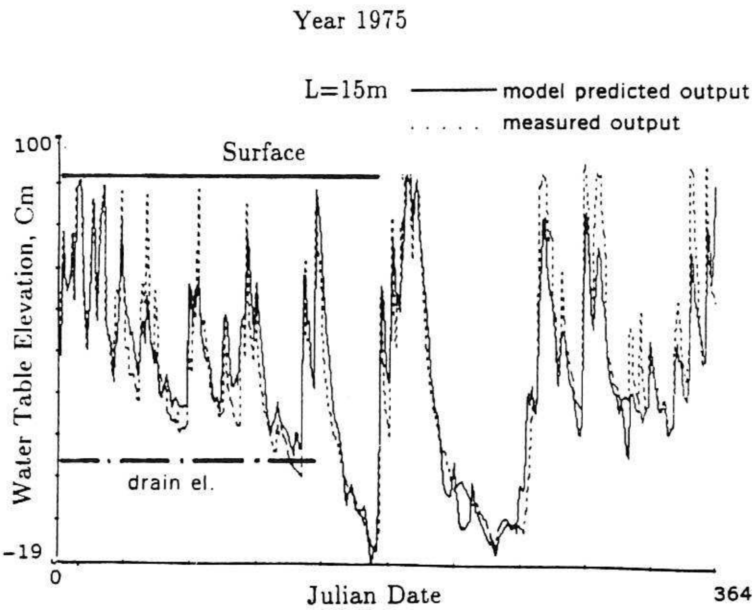

Figure 5. Model predicted output superimposed on observed output of the linear model on estimation set.

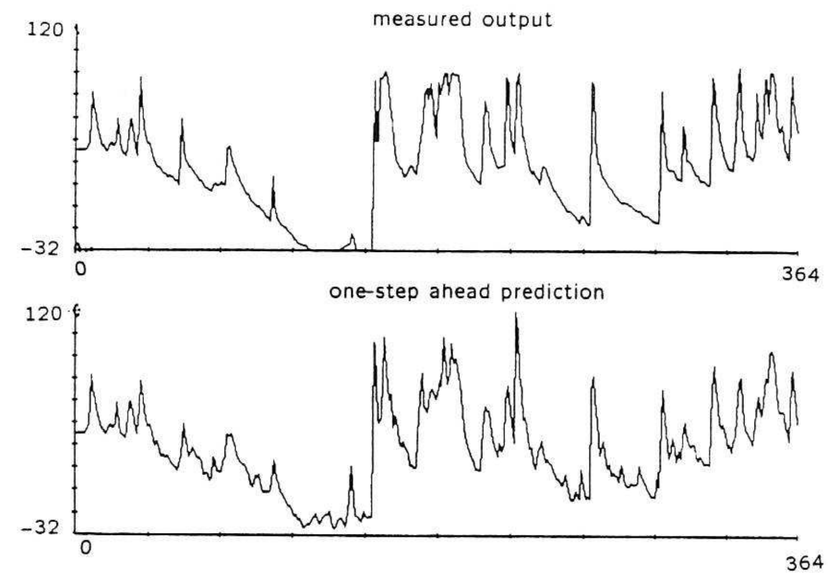

Figure 6. One step ahead prediction of the linear model on test set.

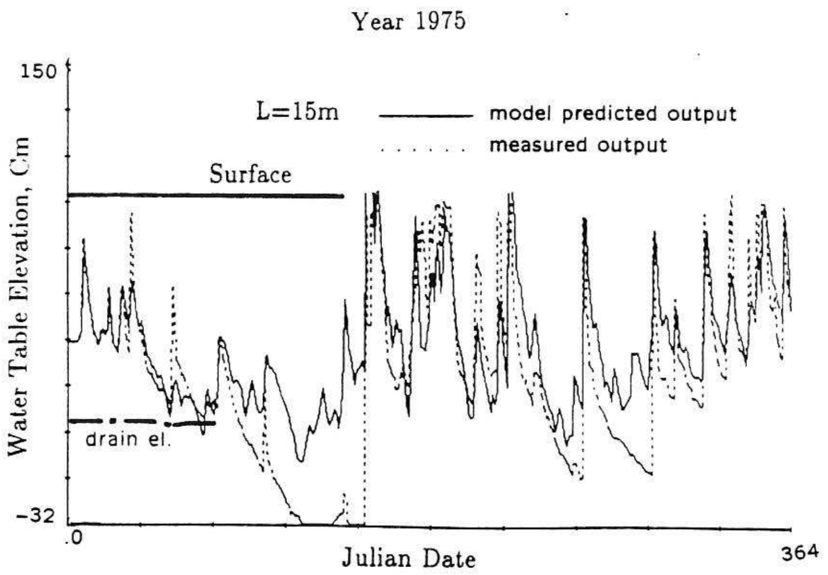

Figure 7. Model predicted output superimposed on observed output of the linear model on test set.

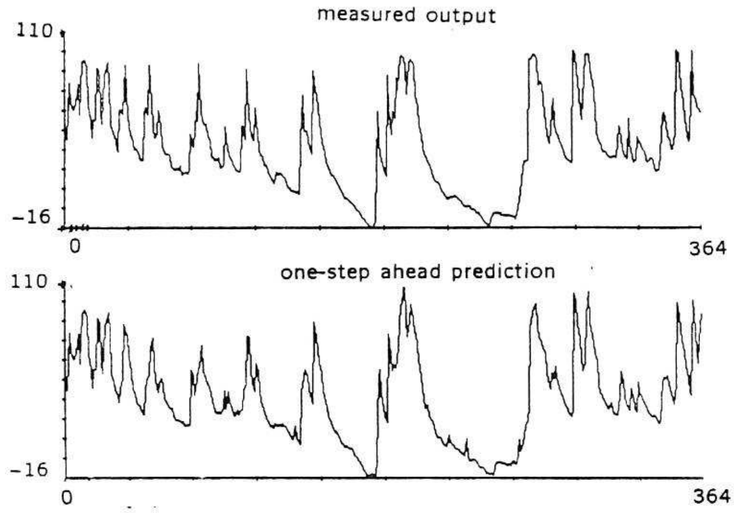

Figure 8. One step ahead prediction of the nonlinear model on estimation set.

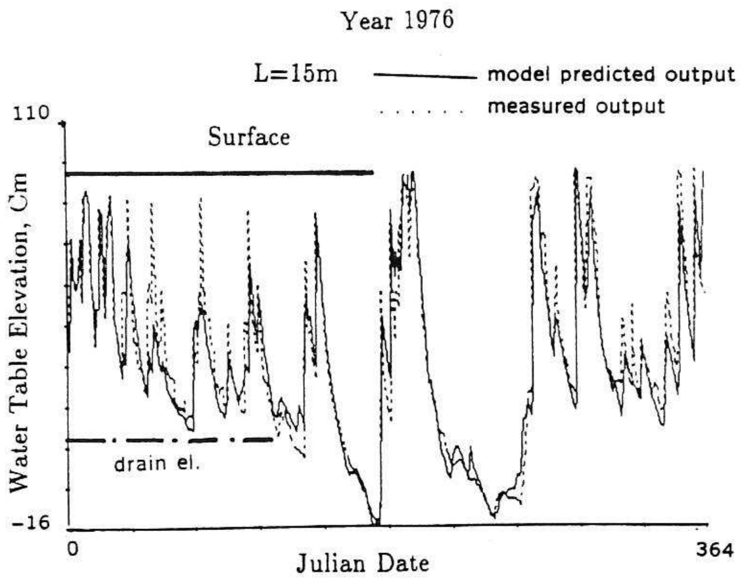

Figure 9. Model predicted output superimposed on observed output of the nonlinear model on estimation set.

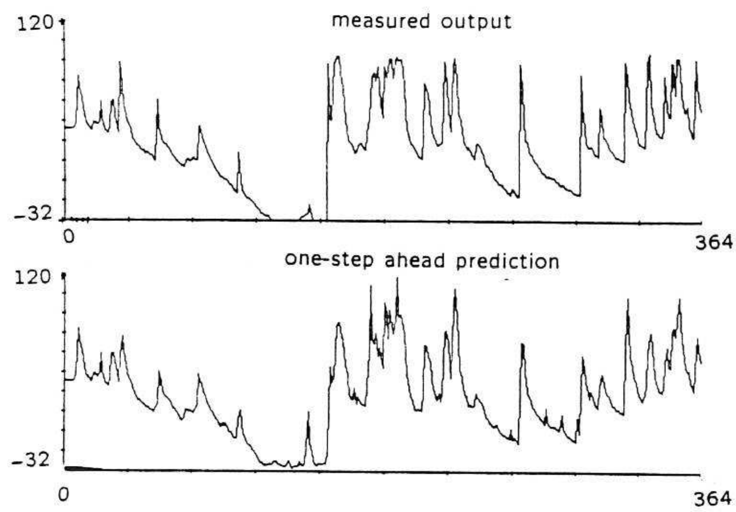

Figure 10. One step ahead prediction of the nonlinear model on test set.

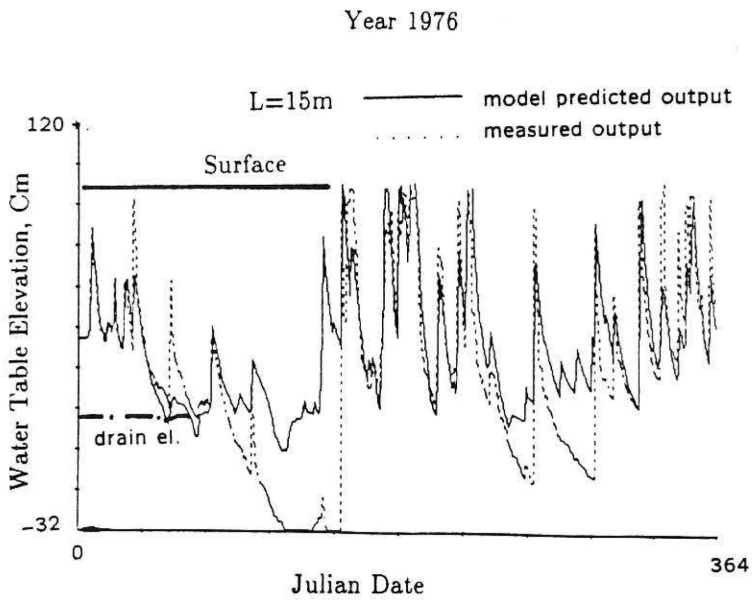

Figure 11. Model predicted output superimposed on observed output of the nonlinear model on test set.