BULLETIN *of the*
Malaysian Mathematical
Sciences Society
http://math.usm.my/bulletin

Bull. Malays. Math. Sci. Soc. (2) **32**(2) (2009), 137–150

Visualization of Rainfall Data Distribution Using Quintic Triangular Bézier Patches

[1]Azizan Saaban, [2]Ahmad Abd Majid and [3]Abd Rahni Mt Piah
[1]College of Arts and Sciences, Universiti Utara Malaysia,
06010 Sintok Kedah, Malaysia
[2,3] School of Mathematical Sciences, Universiti Sains Malaysia,
11800 USM Pulau Pinang, Malaysia
[1]azizan.s@uum.edu.my, [2]majid@cs.usm.my, [3]arahni@cs.usm.my

Abstract. This paper discusses the use of a C^2 interpolant which is positive everywhere and the need to preserve positivity in the case of visualization of rainfall data distribution of Peninsular Malaysia. The results from our previous work, where sufficient conditions on Bézier points have been derived, will be used in order to ensure that surfaces comprising quintic Bézier triangular patches are always positive. The first and second derivatives at the data sites are calculated and modified (if necessary) to ensure that these conditions are satisfied. A number of examples are presented based on the average monthly rainfall data in a particular year at various locations in Peninsular Malaysia.

2000 Mathematics Subject Classification: 41A05, 41A10, 65D05, 68U07

Key words and phrases: Visualization, triangular Bézier, positivity, interpolation.

1. Introduction

The need for researchers to obtain an accurate representation for an entity from incomplete data is common. Furthermore, these data are sometimes needed to be visualized by some interested parties. Visualization can be seen as a process of reconstruction. The problem we are addressing is the interpolation of scattered data that occurs in many practical situations where data are gathered experimentally. This paper will focus on the shape preserving interpolation where the constructed interpolants obey the shape of given positive data points. This problem could arise if one has data points on one side of a plane and wishes to have an interpolating surface which is also on the same side of this plane. Various methods concerning visualization of positive data of curves and surfaces can be found in the literature (see for example [14], [7], [10], [1], [2], [11], [4], [9], [3], [12]). Most of the visualization techniques used to visualize rainfall data is of 2D colour-coded contour map.

Received: June 24, 2008; *Revised:* October 25, 2008.

The significance of positivity lies in the fact that sometimes it does not make sense to talk of some quantity to be negative. In this paper we shall look at rainfall measurements gathered from a set of measuring stations in Peninsular Malaysia, using average rainfall data in the months of March and May of 2007 which we have obtained from the Malaysian Meteorological Department where preservation of positivity of data values must be taken into consideration. The given data represent only a sample and may not be sufficient to let one represent the entire entity accurately. As such, the interpolation of these scattered data will be used in order for us to construct an empirical model which matches the data samples. We will use a method which we have developed in [13] to display the data in a 2D or 3D interpolant which preserves positivity of the data. Another approach to this problem uses the idea of "meshless" surfaces such as radial basis functions and Shepard-type methods. For instance, the modified quadratic Shepard (MQS) method in 3D illustrated in [3] uses rainfall data for 2^{nd} May 2002 that were collected at some 133 stations throughout New Zealand. The data include the heights of the weather stations as well as the latitude and longitude. A 3D Shepard interpolant is subsequently created. To display the rainfall data, they used the unconstrained MQS to create a surface approximating the terrain, and then evaluated the 3D rainfall interpolant over this surface by first using the 3D unconstrained interpolant (where negative rainfall values can occur) and then using the 3D constrained interpolant (where positivity of the data are preserved).

In contrast to [3], the rainfall data points in this paper are triangulated, leading to a piecewise construction of the surface. A local positivity-preserving scheme, which we have developed in [13] using a convex combination of quintic Bézier triangular patches, is employed. An input of rainfall data by our proposed interpolant does not require the height of weather stations where the 3D rainfall interpolant is evaluated over the longitude-latitude plane in order to preserve the positivity of the data. The proposed interpolant can also be used to approximate unknown entities (amount of rainfall) at intermediate locations (refer to Table 2 in Section 4 of this paper) within the convex hull of triangulation domain.

2. Sufficient positivity conditions for a quintic Bézier triangular patch

A quintic Bézier triangular patch is given by

$$\begin{aligned} P(u,v,w) &= b_{500}u^5 + b_{050}v^5 + b_{005}w^5 \\ &\quad + 5(b_{410}u^4v + b_{401}u^4w + b_{140}uv^4 + b_{104}uw^4 + b_{041}v^4w + b_{014}vw^4) \\ &\quad + 10(b_{320}u^3v^2 + b_{302}u^3w^2 + b_{230}u^2v^3 + b_{203}u^2w^3 + b_{032}v^3w^2 + b_{023}v^2w^3) \\ &\quad + 20(b_{311}u^3vw + b_{131}uv^3w + b_{113}uvw^3) \\ &\quad + 30(b_{221}u^2v^2w + b_{212}u^2vw^2 + b_{122}uv^2w^2) \end{aligned} \tag{2.1}$$

where u, v, w are the barycentric coordinates (i.e. $u + v + w = 1$) and $u, v, w \geq 0$, and b_{ijk} are the Bézier ordinates with $i + j + k = 5$. We will refer to b_{500}, b_{050}, b_{005} as the vertices; b_{410}, b_{401}, b_{140}, b_{041}, b_{014}, b_{104}, b_{320}, b_{302}, b_{230}, b_{032}, b_{023}, b_{203} as boundary Bézier ordinates and b_{311}, b_{131}, b_{113}, b_{122}, b_{212}, b_{221} as inner Bézier ordinates, respectively (see Figure 1).

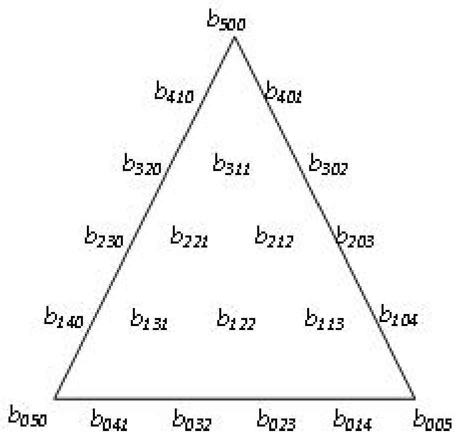

Figure 1. Control points of a quintic triangular patch

Let the Bézier ordinates at vertices be strictly positive, i.e. $b_{500}, b_{050}, b_{005} \quad > 0$. Sufficient conditions on the remaining Bézier ordinates have been derived in [13] to ensure that the entire Bézier patch is positive. For simplicity in writing, let the Bézier ordinates at vertices be $A = b_{500}$, $B = b_{050}$, and $C = b_{005}$. Our approach in [13] is to find lowest bound on the remaining Bézier ordinates, such that if all the Bézier ordinates apart from A, B, C are assigned this value, then $P(u, v, w) = 0$. We thus assume that, the remaining Bézier ordinates have the same value, $-r$(where $r > 0$). By treating r as a parameter, equation (2.1) can now be written as

$$P(u, v, w, r) = (A + r)u^5 + (B + r)v^5 + (C + r)w^5 - r. \tag{2.2}$$

Clearly, $P(u, v, w, r) > 0$ when $r = 0$. Using the corresponding values of u, v, and w at the minimum value of P, which for simplicity denoted as $P_{\min}(r)$, we want to find lowest bound of r, say r_0 where $P_{\min}(r_0) = 0$. The partial derivatives of P in equation (2.2) with respect to u, v and w are given by

$$\frac{\partial P}{\partial u} = 5(A + r)u^4, \ \frac{\partial P}{\partial v} = 5(B + r)v^4 \quad \text{and} \quad \frac{\partial P}{\partial w} = 5(C + r)w^4. \tag{2.3}$$

At the minimum value of P,

$$\frac{\partial P}{\partial u} - \frac{\partial P}{\partial v} = 0$$

and

$$\frac{\partial P}{\partial u} - \frac{\partial P}{\partial w} = 0.$$

Thus,

$$\frac{\partial P}{\partial u} = \frac{\partial P}{\partial v} = \frac{\partial P}{\partial w}. \tag{2.4}$$

Using equations (2.3) and (2.4), we have

$$\frac{u^4}{v^4} = \frac{B+r}{A+r} \quad \text{and} \quad \frac{u^4}{w^4} = \frac{C+r}{A+r}.$$

Hence,

$$u^4 : v^4 : w^4 = \frac{1}{A+r} : \frac{1}{B+r} : \frac{1}{C+r}.$$

Since $u+v+w=1$, we obtain

$$u = \frac{1/(A+r)^{1/4}}{1/(A+r)^{1/4} + 1/(B+r)^{1/4} + 1/(C+r)^{1/4}},$$

$$v = \frac{1/(B+r)^{1/4}}{1/(A+r)^{1/4} + 1/(B+r)^{1/4} + 1/(C+r)^{1/4}}$$

and

$$w = \frac{1/(C+r)^{1/4}}{1/(A+r)^{1/4} + 1/(B+r)^{1/4} + 1/(C+r)^{1/4}}.$$

Using the above values of u, v, w, we obtain the minimum value of $P(u,v,w,r)$in equation (2.2) as

$$P_{\min}(r) = \frac{r}{\left[1/(A/r+r)^{1/4} + 1/(B/r+r)^{1/4} + 1/(C/r+r)^{1/4}\right]^4} - r. \tag{2.5}$$

We need to find a value of $r = r_0$, where $P_{\min}(r) = 0$ or

$$\frac{1}{(A/r+1)^{1/4}} + \frac{1}{(B/r+1)^{1/4}} + \frac{1}{(C/r+1)^{1/4}} = 1. \tag{2.6}$$

By letting $s = 1/r$ and

$$G(s) = \frac{1}{(As+1)^{1/4}} + \frac{1}{(Bs+1)^{1/4}} + \frac{1}{(Cs+1)^{1/4}},$$

equation (2.6) can be written as

$$G(s) = 1, \; s \geq 0. \tag{2.7}$$

We will use a method in [13] to determine the value of $s_0 = 1/r_0$ for each triangular patch. For $s \geq 0$, $G'(s) < 0$ and $G''(s) > 0$. If $M = \max(A,B,C)$ and $N = \min(A,B,C)$, then

$$\frac{3}{(Ms+1)^{1/4}} \leq G(s) \leq \frac{3}{(Ns+1)^{1/4}},$$

with $G(80/M) \geq 1$ and $G(80/N) \leq 1$. Figure 2 shows the graph of $G(s)$, $s \geq 0$ with relative locations of $80/M, 80/N$ and s_0.

To obtain the value of s_0 for given values of A, B and C, we only need to solve equation (2.7) that will give us a lower bound on the remaining Bézier ordinates, i.e. $r_0 = 1/s_0$. We can use a simple iterative scheme which must ensure a one-sided convergence, i.e. s_0 is approached from above. The convexity of $G(s)$ means that this can be achieved using the method of false-position (see [6] for further details). An initial estimate for the root will be the value of s for which the line joining $80/N$

and $80/M$ has the value 1. The following is a proposition from [13] which establishes sufficient conditions for a positive interpolant to be used in this paper.

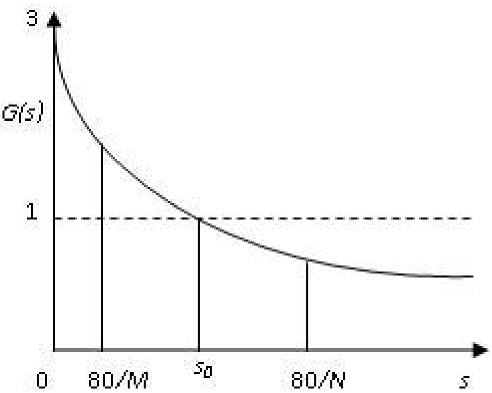

Figure 2. Function $G(s)$ for $s \geq 0$

Proposition 2.1. *Let the quintic Bézier triangular patch* $P(u, v, w)$ *in equation (2.1) with* $b_{500} = A,\ b_{050} = B,\ b_{005} = C$, *where* $A,\ B,\ C > 0$. *If* $b_{ijk} \geq -r_0 = -1/s_0, \quad (i, j, k) \neq (5, 0, 0), (0, 5, 0)$ *and* $(0, 0, 5)$, *where* s_0 *is the unique solution of*

$$\frac{1}{\sqrt[4]{As+1}} + \frac{1}{\sqrt[4]{Bs+1}} + \frac{1}{\sqrt[4]{Cs+1}} = 1,$$

then

$$P(u, v, w) \geq 0, \forall\ u, v, w \geq 0,\ u + v + w = 1.$$

If any of the values of A, B, or C is zero (i.e. the given data are not strictly positive), we will assign the value zero to r_0 for that triangle.

3. Construction of positivity-preserving interpolating surface of rainfall data

We are now able to construct the interpolating surface for Peninsular Malaysia rainfall data of a particular month. Given positive rainfall data $(x_i, y_i, z_i),\ z_i \geq 0,\ i = 1, 2, \cdots, N$, where $x_i,\ y_i$ are the longitude and the latitude of rainfall measuring station, respectively and z_i, the monthly average of rainfall data. We want to construct a C^2 positivity-preserving surface $z = F(x, y)$ that interpolates the given data. The surface comprises a piecewise convex combination of quintic Bézier triangular patches, each of which is guaranteed to remain positive.

We use a Delaunay type of triangulation to triangulate the convex hull of the data points (x_i, y_i). An estimation of the first order partial derivatives of F will be obtained using a proposed method in [8] while for the second order partial derivatives estimation, we will use the quadratic approximation of least squares method mentioned in [13]. Let $V_i,\ i = 1, 2, 3$ be the vertices of a triangular patch, such that

$F(V_i) = z_i$. Then, from the given data together with the estimated derivative values at all (x_i, y_i) we can now determine the 15 ordinates of b_{ijk} except for the three inner control points $b_{122}, b_{221}, b_{212}$ (see [5] for further details). However, the initial estimate of the above ordinates may not satisfy the positivity conditions for P. In view of Proposition 2.1, we need these Bézier ordinates to be greater or equal to $-r_0$. If they are not, then the magnitudes of derivatives at the vertices need to be reduced so that the non-negativity condition on the Bézier ordinates is satisfied. The modification of these partial derivatives at vertex V_i is achieved by multiplying each derivative at that vertex, by a scaling factor $0 < \alpha_i < 1,\ i = 1, 2, 3$. The smallest value of α_i is obtained by considering all triangles that meet at a common vertex, which satisfy the positivity condition of all these triangles. For example,

$$(b_{410})_j = F(V_1) + \alpha \frac{D_{(e_{12})_j}(V_1)}{5} \geq -(r_0)_j,$$

$$(b_{311})_j \ = \ F(V_1) \ + \ \frac{\alpha}{5} \left[D_{(e_{12})_j}(V_1) - D_{(e_{31})_j}(V_1) - \frac{D^2_{(e_{31})_j(e_{12})_j}(V_1)}{4} \right] \geq \ -(r_0)_j$$

where subscript j represents a quantity corresponding to triangle j, e_{jk} is the triangle edge joining (x_j, y_j) to (x_k, y_k), $D_{e_{ik}}(V_1)$ is first order directional derivative at vertex V_1 along edge e_{ik} and $D^2_{(e_{31})(e_{12})}(V_1)$ is the mixed derivative at vertex V_1.

For each triangle, the inner Bézier ordinates $b_{122},\ b_{221},\ b_{212}$ remain to be calculated, in such a way to guarantee preservation of positivity and C^2 continuity across patch boundaries. We shall use a similar method as in [5] to determine the initial estimates of these ordinates. A local scheme $P_i,\ i = 1, 2, 3$ is defined by replacing $b_{122},\ b_{212},\ b_{221}$ with $b^i_{122},\ b^i_{212},\ b^i_{221}$ respectively which will satisfy C^2 conditions across boundary e_i. Ordinates $b^1_{122},\ b^2_{212},\ b^3_{221}$ are obtained using cross boundary derivatives on edges $e_1,\ e_2,\ e_3$ respectively. These ordinates will then be used to estimate the remaining local ordinates i.e. $b^1_{212},\ b^1_{221},\ b^2_{122},\ b^2_{221},\ b^3_{122},\ b^3_{212}$. Initial estimates of these Bézier ordinates in each triangle may not satisfy the positivity condition of $P(u, v, w)$ as stated in Proposition 2.1 and we need to adjust $b^i_{122},\ b^i_{212},\ b^i_{221}$ for each local scheme in order to satisfy the proposition (see [13] for further details).

The final interpolating surface P which satisfies the positivity conditions and C^2 continuity on all sides of the triangles is then defined as a convex combination of all the local schemes given by

$$P(u, v, w) = c_1 P_1(u, v, w) + c_2 P_2(u, v, w) + c_3 P_3(u, v, w)$$

or

$$P(u,\, v,\, w) \ = \sum_{\substack{i+j+k=5,\\ i\neq 1, j\neq 2, k\neq 2,\\ i\neq 2, j\neq 1, k\neq 2,\\ i\neq 2, j\neq 2, k\neq 1}} b_{ijk} B^5_{ijk}(u, v, w) + 30uvw(c_1 Q_1 \ + \ c_2 Q_2 \ + \ c_3 Q_3)$$

where

$$c_1 = \frac{vw}{vw + vu + uw},$$
$$c_2 = \frac{uw}{vw + vu + uw},$$

Table 1. Average rainfall (in mm) in March and May 2007 of 25 major rainfall measuring stations in Peninsular Malaysia

Station	**Location**		**Amount (in mm)**	
	Longitude	**Latitude**	**March 2007**	**May 2007**
Chuping	100.2667	6.4833	61.0	88.0
Langkawi	99.7333	6.3333	40.6	166.0
Alor Setar	100.4000	6.2000	277.8	67.4
Butterworth	100.3833	5.4667	58.9	143.2
Prai	100.4000	5.3500	208.1	153.4
Bayan Lepas	100.2667	5.3000	125.2	144.4
Ipoh	101.1000	4.5833	364.2	42.6
Cameron Highlands	101.3667	4.4667	252.0	223.2
Lubok Merbau	100.9000	4.8000	156.4	98.4
Sitiawan	100.7000	4.2167	44.4	26.8
Subang	101.5500	3.1167	329.2	68.2
Petaling Jaya	101.6500	3.1000	321.0	196.2
KLIA	101.7000	2.7167	186.2	188.8
Malacca	102.2500	2.2667	113.8	183.4
Batu Pahat	102.9833	1.8667	182.0	195.0
Kluang	103.3100	2.0167	92.4	130.2
Senai	103.6667	1.6333	148.6	296.0
Kota Bharu	102.2833	6.1667	115.2	109.2
Kuala Krai	102.2000	5.5333	166.0	238.7
Kuala Terengganu Airport	103.1000	5.3833	121.0	64.8
Kuantan	103.2167	3.7833	79.2	270.4
Batu Embun	102.3500	3.9667	146.2	256.2
Temerloh	102.3833	3.4667	114.2	324.2
Muadzam Shah	103.0833	3.0500	131.6	204.8
Mersing	103.8333	2.4500	183.4	196.2

$$c_3 = \frac{vu}{vw + vu + uw},$$
$$Q_i = uvb^i_{122} + uwb^i_{212} + vwb^i_{221}$$

for $i = 1, 2, 3$ and u, v, w are the barycentric coordinates.

Our proposed method can also be used to estimate the amount of rainfall at any location which lies in the convex hull of triangulation domain. As an example, estimated amounts of rainfall for the 10 chosen locations in Peninsular Malaysia are displayed in Table 2.

4. 2D and 3D visualization

We now consider a visualization of rainfall data obtained from the Malaysian Meteorological Department. The data were collected at 25 major rainfall measuring

Table 2. Estimated average amount of rainfall (in mm) in March and May 2007 at 10 chosen locations

Name of Town	**Location**		**Estimated average amount of rainfall (in mm)**	
	Longitude	**Latitude**	**March 2007**	**May 2007**
Jitra	100.4167	6.2667	244.6	66.12
Kepala Batas	100.4333	5.5167	58.19	139.62
Kuala Kangsar	109.3333	4.7667	151.17	105.29
Cheras	101.7667	3.05	283.25	271.29
Genting Highlands	101.8000	3.4000	524.35	87.26
Jertih	102.5	5.75	122.25	88.80
Nilai	101.8	2.8167	216.32	229.29
Segamat	102.8167	2.5	132.57	174.85
Rawang	101.5833	3.3167	398.57	70.81
Kangar	100.2	6.4333	83.73	95.50

stations throughout Peninsular Malaysia, and represent the average monthly measurement in millimeters. For the purpose of this paper, we have chosen the months of March and May 2007 as illustrated in Table 1. Figure 3 shows the triangulation domain of these data sites. Figures 4 and 6 show the 2D visualization of average rainfall data in March and May 2007, respectively where the colour codes represent the amounts of rainfall. In 3D visualization, the positive interpolant of our proposed method is evaluated on a longitude-latitude plane and the results are shown in Figures 5 and 7, respectively.

5. Conclusion

We have shown how a convex combination of quintic Bézier triangular patches can be constrained in order to preserve the positivity of data points. This is achieved by imposing a lower bound on all the Bézier ordinates (except at the vertices) of each of the triangles. Our proposed method has been applied in both 2D and 3D visualization schemes, respectively using data of average monthly amount of rainfall obtained from the Malaysian Meteorological Department. We have also shown that the amount of rainfall at other places which are located within the triangulation domain can be estimated using our proposed method.

Acknowledgements. The authors are very grateful to the Malaysian Meteorological Department for providing us with the data in Table 1. We also like to extend our gratitude to the Ministry of Higher Education of Malaysia and Universiti Sains Malaysia for providing us with the fundamental research grant scheme (FRGS) account number 203/PMATHS/671040 which has enabled us to pursue this research.

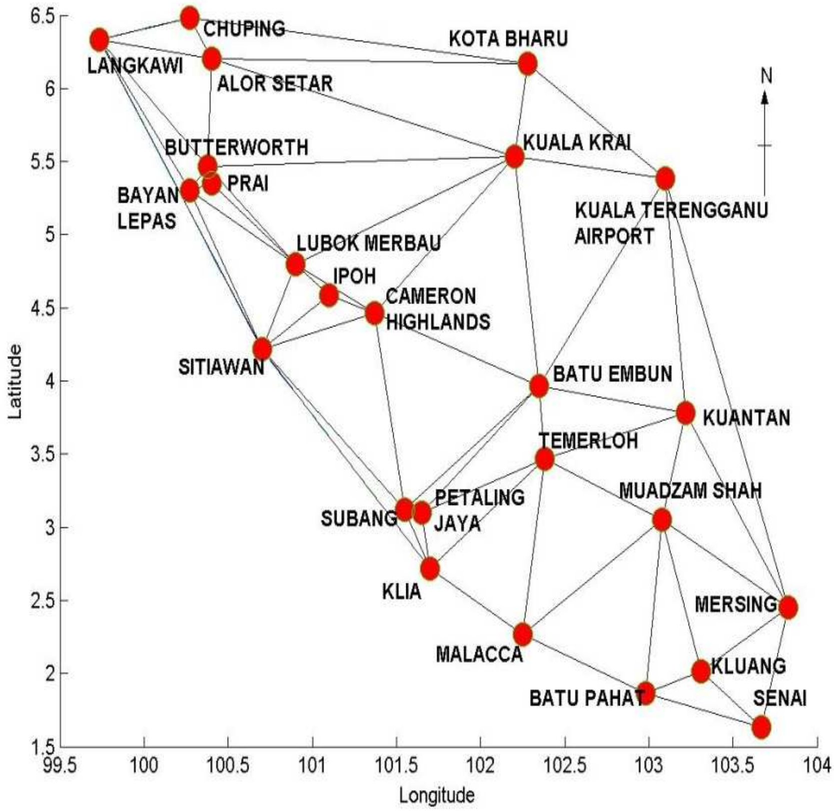

Figure 3. Triangular domain of rainfall measuring stations

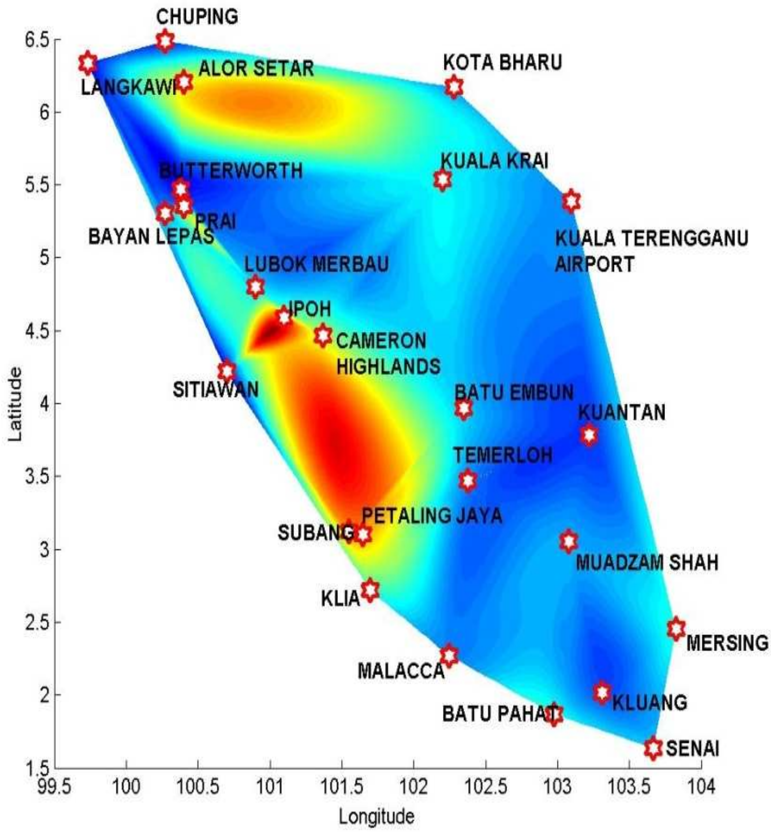

Figure 4. 2D colour-coded visualization of average amount of rainfall in March

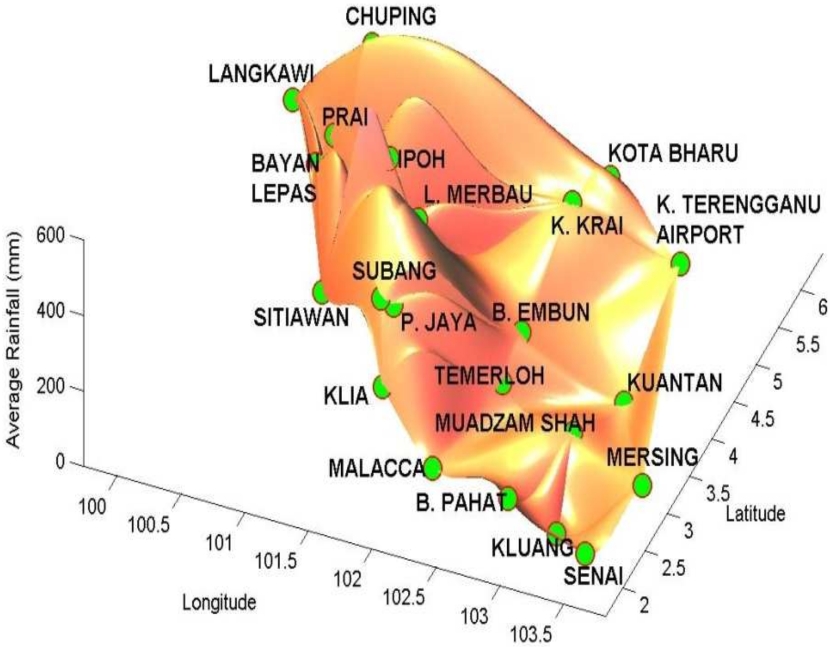

Figure 5. 3D interpolating surface of average amount of rainfall in March

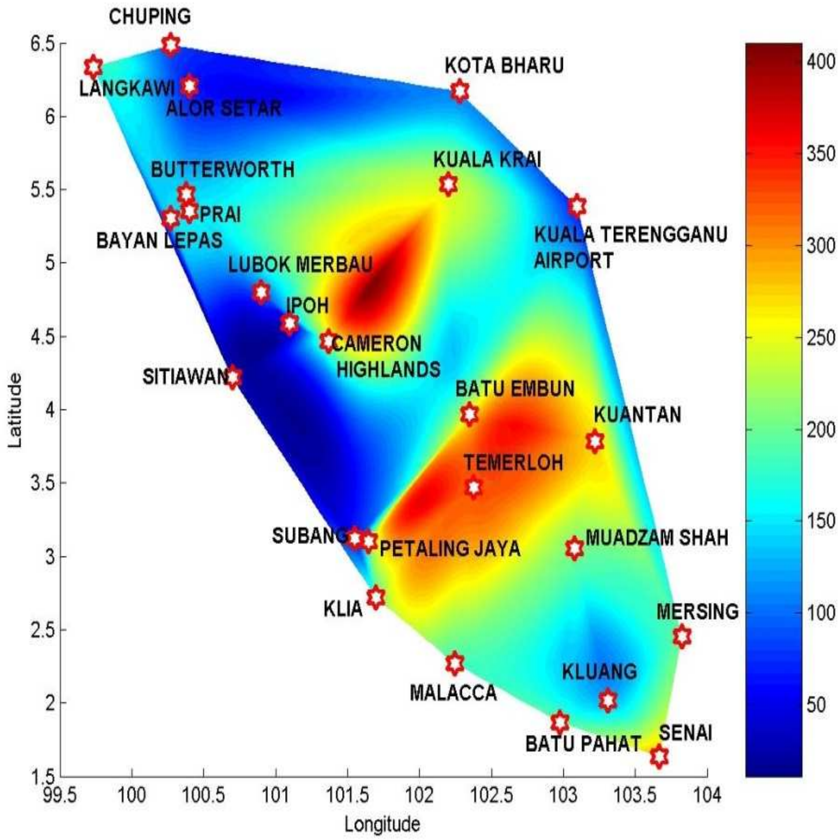

Figure 6. 2D colour-coded visualization of average amount of rainfall in May

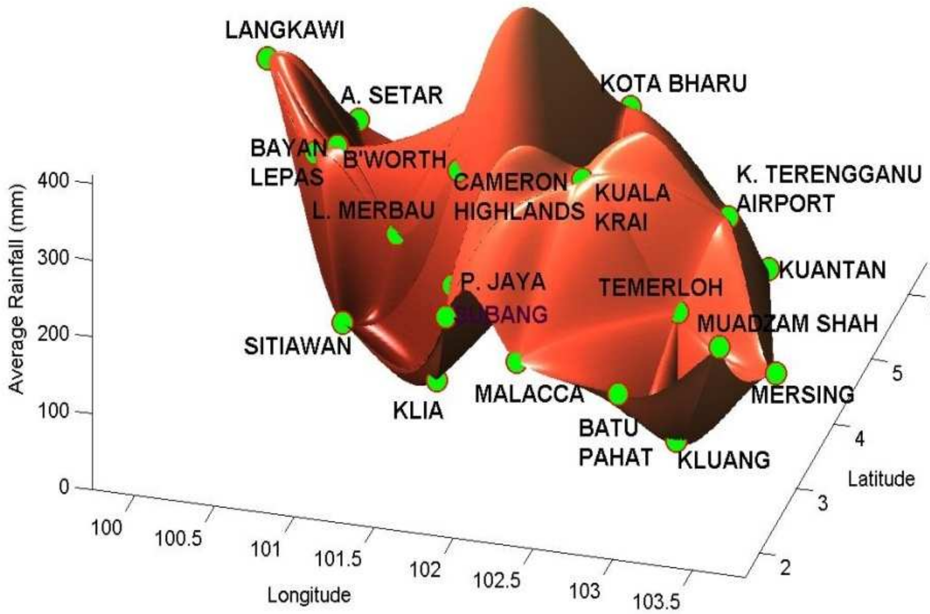

Figure 7. 3D interpolating surface of average amount of rainfall in May

References

[1] K. W. Brodlie and S. Butt, Preserving positivity using piecewise cubic interpolation, *Computers and Graphics* **17** (1993), 55–64.

[2] K. W. Brodlie, S. Butt and P. Mashwama, Visualisation of surface data to preserve positivity and other simple constraints, *Computers and Graphics* **19** (1995), 585–594.

[3] K. W. Brodlie, M. R. Asim and K. Unsworth, Constrained visualization using the Shepard interpolation family, *Computer Graphics Forum* **24** (2005), no. 4, 809–820.

[4] E. S. Chan and B. H. Ong, Range restricted scattered data interpolation using convex combination of cubic Bézier triangles, *J. Comput. Appl. Math.* **136** (2001), no. 1-2, 135–147.

[5] L. H. T. Chang and H. B. Said, A C^2 triangular patch for the interpolation of functional scattered data, *Computer Aided Design* **29** (1997), no. 6, 407–412.

[6] S. D. Conte and C. de Boor, *Elementary Numerical Analysis*, McGraw-Hill, Tokyo, 1992.

[7] T. N. T. Goodman, B. H. Ong and K. Unsworth, Constrained interpolation using rational cubic splines, in *NURBS for curve and surface design (Tempe, AZ, 1990)*, 59–74, SIAM, Philadelphia, PA.

[8] T. N. T. Goodman, H. B. Said and L. H. T. Chang, Local derivative estimation for scattered data interpolation, *Appl. Math. Comput.* **68** (1995), no. 1, 41–50.

[9] A. Kouibia and M. Pasadas, Variational bivariate interpolating splines with positivity constraints, *Appl. Numer. Math.* **44** (2003), no. 4, 507–526.

[10] B. H. Ong and K. Unsworth, On nonparametric constrained interpolation, in *Mathematical methods in computer aided geometric design, II (Biri, 1991)*, 419–430, Academic Press, Boston, MA.

[11] B. H. Ong and H. C. Wong, A C^1 positivity preserving scattered data interpolation scheme, in *Advanced topics in multivariate approximation (Montecatini Terme, 1995)*, 259–274, World Sci. Publ., River Edge, NJ.

[12] A. R. M. Piah, T. N. T. Goodman and K. Unsworth, Positivity-preserving scattered data interpolation, in R. Martin *et al.*(eds.), *Mathematics of Surfaces 2005*, Springer-Verlag Berlin Heidelberg (2005), 336–349.

[13] A. Saaban, A. R. M. Piah and A. A. Majid, Range restricted C^2 interpolant to scattered data, in E. Banissi, M. Sarfraz and N. Dejdumrong (eds.), *Computer Graphics, Imaging and Visualisation: New Advances*, IEEE, Los Alamitos, California (2007), 183–188.

[14] J. W. Schmidt and W. Hess, Positivity of cubic polynomials on intervals and positive spline interpolation, *BIT* **28** (1988), no. 2, 340–352.