Summary of the work

"THE DEVELOPMENT AND APPRAISAL OF A LINEAR PROGRAMME FOR THE TEACHING OF COMPLEX NUMBERS"

After a brief review of the development of Programmed Learning the work reviews some techniques for producing linear programmes. The development of the linear programme for teaching the manipulation of complex numbers, and its preliminary trial in 1965 are described. The examination of the errors and subsequent modifications to the programmes are also shown.

In 1966 the programme was administered to 33 mature students (Mean age 38) in a College of Education, who were compared with another group of 12 in the same college. This latter group had been taught the same material in a conventional way. No significant differences were found in post test scores or times needed to complete the work by either method. Some other correlations are investigated.

The programme teaches successfully (Mean Gain + 72%), and methods of improving its performance for poorer students are discussed. A loop branching method is also suggested as an alternative way of helping less able students.

THE DEVELOPMENT AND APPRAISAL OF A LINEAR PROGRAMME FOR THE TEACHING OF COMPLEX NUMBERS

Thesis submitted for the degree of M.Ed.
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by

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CONTENTS

CHAPTER ONE

THE DEVELOPMENT OF PROGRAMMED LEARNING - A BRIEF REVIEW

Programmed Instruction has grown rapidly in little more than a decade under the main initial impulses of Professors B. F. Skinner and S. L. Pressey in the U.S.A. They have approached the problem from completely opposing poles, Skinner formulating a specific approach built on learning theories, and Pressey presenting an essentially pragmatic approach. It was only in 1960 that E. B. Fry referred to a dichotomy in the field of Programmed Learning, and called for much clarifying research. These are the approaches which led to the dichotomy.

Skinner has appreciated the great advances made in the techniques of making animals learn by a process of stimulus and reinforcement. He himself has made notable demonstrations of the power and rapidity of the technique by making pigeons learn a routine. Other experimenters have had outstanding success with many types of vertebrates. Provided that the stimulus and reinforcement are appropriate to the animals pattern of behaviour then it seems possible to shape the behaviour in a wide variety of patterns. The technique is essentially a process of leading the animal through a series of slightly advancing steps until the behaviour is that which is required. What is required is reinforced, usually by food in hungry animals: what is not required must be extinguished at the outset by lack of reinforcement. This success with 'lower' animals prompted the idea that a similar 'shaping' process in a classroom could be more efficient than the traditional methods. A gradual build up of a behaviour pattern as a result of following a series of graded steps (i.e. Programmed Learning), is according to Skinner, a linear process. (Skinner 1954).

What is known as the 'branching' method, has evolved from the works of Pressey (1926-34). During his experiments in automatic scoring of multichoice tests, he discovered that if, during a test, a student was informed of the correct answer to the question, he tended to retain this, and his performance on a subsequent test was markedly better. A result of the use of multichoice answers was that students were given the reason why their answer was wrong and told to try again. That is, they were given remedial work if it was needed. The remedial work, which might be of some length, came to be known as a 'branch' from the main sequence of work, hence the 'branching system'. The idea is expressed in Pressey's own words (Pressey, 1950) "When the most important function served by a test is instruction, each wrong answer should be made a real contribution to instruction. Each wrong answer should be one against which a warning is needed, or which elucidates the question in some way".

A great deal of recent work on branching methods of programming has also been carried out by N. Crowder whose approach is given thus:- "The essential problem is that of controlling a communication process by the use of feedback ...... the material is not constrained by any learning model" (Crowder 1959).

Thus we have two fundamental differences between the systems. Skinner avoids errors as far as possible, as they interfere with his conception of the learning process, whereas Pressey and Crowder allow for, and welcome them, as starting points for remedial work. In both systems of programming the programmes are written in such a way that a student should have more correct answers than errors to maintain his motivation. The second difference is that Skinner bases his programmes on the principle learned from the results of the 'animal learning' experiments, a gradual shaping of behaviour. Crowder and Pressey are content to follow a course of continual multichoice questioning, seizing their opportunities to confirm or correct a students progress whenever possible.

Whichever method of programming is followed, there are several features which are common to them both, and must be recognised.

In the learning process, a student is presented with some form of stimulus (S) which may take a number of forms. It is part of the desired behaviour pattern that he should indicate that he recognises the presence of the stimulus by emitting some response (R). If it is the correct response, there is some form of reinforcement given, usually an acknowledgement that it is the correct response. If the incorrect response is given, this is negatively reinforced (extinguished) either by showing the correct R, or showing why it was the wrong one. This leads to the next S in the series, and so on. If the student is not sufficiently motivated to emit a response, then probably no learning will take place.

A Skinnerian teaching programme calls for a constructed response to the stimulus material in the frame, the insertion of words, a sentence to write, a step in a mathematical process, or whatever it may be. This involves the recall function of memory. Many branching programmes, presenting multichoice answers, must show all the alternatives, and the correct one could be recognised by the student, though perhaps he could not recall it without the visual prompt. Most tests and examinations are set in the recall mode, and it would appear questionable whether branching programmes would be an adequate preparation for this type of test. Available evidence shows that there is not a great deal of difference between the results of using the two methods of response, though constructed response programmes function slightly better for 'recall' type tests (Coulson and Silberman 1961). There is, however, a saving of time in using a multiple choice programme, because there is less writing to do. With the present educational testing systems, work generally needs to be recalled, and the constructed response of a Skinnerian programme may be a better preparation for a student's future work.

A logical conclusion from the underlying ideas of linear programming is that intelligence is of secondary importance. A poor student can be led to appreciate almost any concepts provided that, in planning the programme, the order and size of step is fitted to his needs. He will take longer than a brighter student to achieve the goal but he will attain it (Skinner 1958). This has not been found to be so in the results of an experiment by Shay (1961). He found that intelligence was positively related to post test scores at .001 level of significance. A brighter student still does better than a weaker student in learning from a programme.

It was also thought that branching programmes could attempt to widen their scope by a process of multiple branching (Crowder 1960).

This would involve so called "wash back", i.e. returning to an earlier part of the programme, as well as remedial loops which would be parallel to short sections of the programme, but covering the same ground more thoroughly. The complexities of the patterns which become possible with these ideas led to experiments with a computor based programme (Coulson and Silberman 1961).

Some experiments, designed specifically to test one method against the other, as well as the computor based experiment mentioned above, tend to show that while both techniques will teach effectively, time can often be saved by using the branching method (Larkin 1964, Herringshaw and Hunter 1964).

As results, perhaps, of these and other tests, consideration is now being given to the processes required to make specific programmes produce optimum performances. Several factors are involved.

1. Student interest. A student will only pay attention to a stimulus if the result is (a) self satisfying and (b) meaningful. It was thought that knowledge of results was sufficient stimulation here to provide self satisfaction, (Skinner 1958), but some recent studies show that meaning is as important to the student. (Stones 1966, Conoley 1966). The student likes to know how the frames he is doing fit into an overall pattern.
2. Size of step. One of Skinner's original bases for Programmed Learning was that the size of step from one frame to the next must be small, and he worked using the criterion of an error rate of less than 5 per cent. Evidence is available, however, that

larger steps do not necessarily reduce the learning efficiency (measured by post testing), although it may increase the error rate. (Goldbeck 1960, Smith & Moore 1962). A succession of too large steps will affect motivation, leading to lack of attention. It would seem then that a programme must be written with a target population in view (Austwick 1962).

3. Repetition. It has been suggested that frequent repetition is needed in a programme to provide continual reinforcement for newly learned concepts (Klaus 1961). Too much repetition will not only cause a student to become bored and lose interest, but will take time to perform the unnecessary frames. Pressey's early experiments with his 'drum tutor' dropped an item out of his programme after it had been performed successfully twice, but he later modified this to three or four times, and found that this caused a drop in the error scores. It has also been suggested that the repetition should be frequent after the introduction of new material, and that, thereafter, it should be faded according to a negative exponential curve. (Gilbert 1958). The amount of practice must depend upon the individual student.

4. Method of response. Pressey and Skinner hold that an overt response is necessary; pressing a button, writing a word, or whatever it may be. But the covert response has the advantage of taking less time to emit than an overt response, and a programme needing only these could well be more efficient in time needed for training. Experiments by Stolurow and Walker (1962) and

Lambert, Miller and Wiley (1962) allowed one group of subjects under training to think their responses, and no significant difference in the amount learned was found. Similar results have also been produced in England by Widlake (1962).

The inter-action of these factors seems complex, and it would seem that a linear programme cannot be suitable for a large number of students, because, once written, it can only provide a single path for all students. A branching programme is slightly less limited, because multiple branching is possible, but even so, it has only a few pre-selected paths. These intrinsic limitations were discussed by Pask (1960) and he suggested the more sophisticated method of extrinsic programming. The description of 'extrinsic' is supplied to indicate that the variations in path through a programme should be due to external factors rather than an inbuilt pattern.

The essential feature of extrinsic programming is that it is capable of adapting the programme continuously to the needs of the students. This requires a complex machine which is able to do three things (a) monitor the students' responses (b) vary the content of the programme and (c) vary the pace of the programme. Pask sees the machine and student in 'conversation', the machine leading the student on through the amount of material appropriate to his capabilities, by continuous monitoring of concept error counters, and keeping his interest by increasing the pace on easy sections where there are low error scores. This adaptive machine would have to contain a large number of carefully graded frames which are accessible at random. The number and order of the frames seen by students will depend upon the outputs of the various error counters within the machine. Thus the machine will be presenting the student with a programme within his capabilities, but at the same time making him perform near the upper limits of his ability. Variations in level and pace will maintain his motivation.

An adaptive machine, not quite of this type as it was to teach a motor skill rather than increase intellectual content, has been built to Pask's specification. This is called SAKI (Solartron Automatic Keyboard Instructor) and is capable of prompting, fading and pacing in the process of teaching a student to use a punched-card machine.

Further advances in the direction of extrinsic programming have been made in the designing and building of an even more complex machine with the appropriate name of SOCRATES (System for Organising Content to Review And Teach Educational Subjects) (Stolurow and Davis 1965). This machine was built for the purpose of testing the technique of 'idiomorphic' programming (literally 'own-form') devised by Stolurow (1965). A student's attainment level, aptitude level and personality profile are fed into the machine, together with the requirement for training i.e. topic to be studied, level of attainment required at post test, and the time available. A suitable programme for the student is chosen by the machine from the available repertoire in a large capacity computor. If no suitable programme is available, it can suggest remedial or preparatory work for the student. Once a programme has been chosen, it is administered in much the same way as

Pask suggested, pacing the student carefully to maintain his motivation, and by monitoring his responses, finding an optimum path through the programme. This, Stolurow called the Teacher Function of the machine.

A second function (the Professor Function) has the task of comparing the student's responses with those of others who have taken the same programme, or some other previously calculated criterion. If the student did not seem to be performing sufficiently well, or performing too well, this could lead to a complete change of programme. The machine would always be trying to find a programme, or even a section of a programme, most appropriate to the student's own requirements. This function can also vary the schedules of knowledge of results, decide whether correctional material or non correctional material is more appropriate and give positive or negative evaluative feedback (e.g. you are doing well!). These variables, in conjunction with the knowledge of the student's personality, can arrange that the output to the student is such that he is working with maximum motivation. Thus the machine can adapt the programme and the feedback to suit the needs of any particular student; it is his personal programme.

Such complex programming can only be carried out using costly machines, which at present are very limited in number. Cheaper electronic circuiting, micro-modules etc. may eventually make their use more common. This should make it possible for large numbers of students to follow an optimum path through many programmes and reduce the time needed to learn the many requirements of modern education.

CHAPTER TWO

PROGRAMMING TECHNIQUES

A. The Skinner-Holland technique

When Skinner first wrote about programme construction in 1958, he pointed out that whereas a text book explanation can always be clarified by a teacher, a linear programme has to be adequate and stand by itself. A thorough examination of the field to be taught is required, the material must be systematically distributed throughout the programme, and the final arbiter of the whole programme is the student himself. He was not prepared to say if programming was to be an art or a scientific technology. His collaborator Holland (1960) suggested eight basic rules. These were:-

1. Immediate reinforcement of a response. In this type of programme, knowledge of the result is assumed to be a sufficient 'reward' to provide adequate reinforcement for continued learning. Thus in a machine, or other form of presentation, immediately the response is made, there must be some means of showing the correct answer.

2. Only overt responses, which have been suitably reinforced, are learned. Skinner wished to eliminate all non observable factors from the learning situation though he does not deny the existence of the other factors. Only by insisting on an overt response, can he be sure that the right response is being reinforced.

3. Errors have an adverse effect on learning. A subject who is

repeatedly 'punished', i.e. learns that his answers are wrong, will lose motivation. Thus the error rate in a programme must be kept low by careful grading of steps and 'prompting' where it is helpful.

4. Progress must be in small, successive steps. This follows from the effects of errors on learning. The effects of the small steps is that behaviour can be 'shaped' towards a final pattern. Unfortunately this is difficult to apply to rote learning, such as spelling, where words cannot be shaped.

5. Assistance to the subject must be withdrawn gradually. Early in a programme, a student will be told precisely what to do, and by gradual withdrawal (fading) of prompts or cues, he will be able in the end, to produce the correct response without help. Skinner particularly recommends this for vocabulary building, spelling and foreign languages.

6. The students' observing behaviour should be controlled. The point of this assertion is that efficient learning can only take place if the student 'pays attention' to the correct stimulus. It is not worth putting a frame into a programme if the answer can be written in without reading the information carefully. The degree of prompting or cueing at any stage in the programme must be very carefully considered in this connection. Other distracting stimuli, such as pictures on a wall, movement of other people, talking, or even other frames in a booklet programme will allow the behaviour to get out of control, and at best, learning will

be inefficient. This accounts for Skinner's use of machines in preference to texts, as they can only present one item at a time.

7. Extensive discrimination training is required to establish a concept. The learning of a rule does not mean that a student can apply it. Many frames working from abstraction to example, and from example to abstraction must be included to ensure discrimination between one concept and another. It follows, therefore that a linear programme cannot be short.

8. A programme must be continually tested and revised. The writing of responses means that these can be examined in detail, and the programme modified in the light of these findings. The criterion of a successful programme is that it can teach under the conditions set out by the programmer.

A difficulty encountered by Skinner and Holland in their experimental work was that of presenting all the information required in the small space of a programme frame. This was overcome by using 'panels' (Skinner & Holland 1958). These were printed material useful for reference. The programmer forced the student to read the panel by suitable questioning in the programme. The use of the panel is gradually faded as the programme is followed.

B. <u>The Ruleg System</u> (Evans, Homme & Glaser 1962)

This system of programming divides all verbal matter into (1) Rules to be learned (abbreviated RU) and (2) examples or illustrations (EG), hence 'RULEG' system. The RUs may be any form of generalisation, such as a definition, a mathematical formula, a principle or axiom. An EG is also a wide classification in that it can be anything which illustrates the RU. Thus the definition 'a noun is a naming word' and the equation of a circle $x^2 + y^2 = r^2$ are RUs, and the statements 'the word dog is a noun' and '$x^2 + y^2 = 3^2$ is a circle of radius 3 units' are EGs.

The authors of this system specified 12 steps in the preparation of a programme. Some of these are similar to Skinner and Holland's methods. These are the steps:-

1. The specification of the criterion behaviour. This must be done very precisely, stating exactly how the student is going to behave, e.g. learn a formula which gives the amount of heat flowing through a block of material when its temperature is steady, and be able to use it in calculations.
2. The specification of the Subject Matter rules. Every rule that the programmer can think of in connection with the subject matter should be written down. As the order does not matter at present, it is probably wiser to write each rule on a separate card to allow for easy rearrangement.
3. The collection of stimulus support material, in the form of texts, notes, advice etc. This will probably yield a mixture of further rules and examples which are indexed as in step 2.
4. The preliminary ordering of RUs. This should be done in accordance with the principle of gradual progression rather than following any text book order. If the RUs are on separate cards,

this allows for a number of trial arrangements to obtain the best. This gives a rough outline of the proposed programme.

5. Preparing RU matrices . The RUs are listed, in order, horizontally and vertically to allow a systematic search to be made for relationships between various RUs, e.g. how they are similar, how they are different etc.

<u>An example of an RU matrix</u>

Similarity	RU1	RU2	RU3
RU1	1	2	3
RU2	4	5	6
RU3	7	8	9

The word in the top left hand corner of the matrix is referred to as an operator and may be changed, to obtain the different relationships between the RUs. The cells in the matrix may be numbered in the order in which the RUs appear in the programme. Then cell number 7 would ask the question "In what way is RU1 similar to RU3?" There may not be any answer to this question, but at least the possibility has been examined. The major diagonal starting in the top left hand corner contain all the cells which relate the RU to itself, and this is suggested as a good place for definitions.

6. The example operator is placed into the RU matrix and the

construction of examples for the programme is begun. The programmer must note that sufficient examples are generated for every RU, i.e. enough for practice and review. All types of examples for any given RU must be generated, e.g. trivial cases, special cases, examples containing redundant information and so on. The EGs should be sufficiently diverse to ensure good generalisation of a RU.

7. The numbering of the RU cells in the matrices which are actually going to be used. The decision must be made at this stage whether, say comparison between a particular pair of RUs is needed or not.

8. The assembly of the RUs and EGs into frames. The frames are constructed from the material already prepared by judicious combination of RU and EG. To allow the student to make a response, key words may be omitted from a statement or rule and these are designated by $\widetilde{EG}$ (EG Tilde) and $\widetilde{RU}$ (RU Tilde). Very incomplete RUs or EGs (e.g. "what is a complex number?" and "add the complex numbers (7 + i3) and (2 + i9)") form good test frames and are denoted by a double tilde sign $\approx$. Nine frame types are listed by the authors which have been found to be useful in practice.

I. RU + EG + $\widetilde{EG}$. This combination is found to be a good starting frame as the rule is quoted, followed by an example of this, and then the student can complete the incomplete example. The strong prompt of RU + EG makes it very unlikely that the student will make an error.

II. $RU + \widetilde{RU}$. This type of frame is good for pointing out new terms in a frame. If the student is asked to place the new term in the $\widetilde{RU}$, his attention is drawn to its use in the adjacent RU.

III. $RU + \widetilde{EG}$. This frame represents a weaker prompt than that given in type I, as there is no direct comparison of examples. This can be the start of a fading process.

IV. $EG + \widetilde{RU}$. The incomplete RU here is asking the student to induce something from the given EG. This can only be used safely if the RU is already known.

V. $\widetilde{RU}_1 + \widetilde{RU}_2$. After RU_1 and RU_2 have been presented in the programme separately, a discrimination between them can be made by using this type of frame.

VI. $\widetilde{EG}_1 + \widetilde{EG}_2$. Again this can be used to compare and contrast examples of RU_1 and RU_2.

VII. $\overset{\approx}{EG}$. As previously mentioned, this very incomplete example would have no cues or prompts and would represent practically the terminal behaviour of the student.

VIII. $\overset{\approx}{RU}$. This frame type would be asking the student for a definition or rule, again without prompts.

IX. $\overline{EG}$. A special type used by the authors to forwarn students of traps or misunderstandings which could arise, by giving a negative example.

9. Using the cell numbers in the RU matrix as a guide to the assembly of the frames into a programme. Decisions must be made at this point about the actual numbers of frames illustrating the RUs and how they are to fit into the programme. This can only be an

estimate based upon experience.

10. The trial of the programme on students. This is followed by a systematic analysis of their responses. If the desired behaviour is being achieved and students can deal with EGs successfully then the frames are acceptable.

11. Revision of the programme on the basis of the results from step 10.

12. Repetition of steps 10 and 11. This revision process is continued until the programme is reliable and efficient.

C. Task Analysis

A similar approach to this has been made by the R.A.F. Education Branch. Basing the ideas on those of Systems Analysis, a definite procedure is laid down to ensure that the process is as efficient as possible (Davies 1965)

1. It begins with a task analysis. In this must be specified accurately and completely the topic or job to be studied, the duties or processes to be performed in this, the tasks, or steps in a process, and the task elements, which are the smallest possible steps. These are arranged in a definite heirarchy and must be broken down. Thus, in the job of solving quadratic equations we could identify the tasks as follows:-

i. Using the discriminant to decide if a solution is possible.

ii. Solving it by factoring.

iii. Solving it by completing the square.

iv. Solving it by using the formula.

v. Solving it by plotting a graph.

The second task could be split into the task elements of

i. Ensuring that the coefficients were integers.

ii. Factorizing the trinomial expression.

iii. Equating the two factors to zero.

iv. Solving the two equations.

When this inventory is complete it should be possible to identify all the cues, steps performed, and how they are performed. The task elements, or basic rules in the whole sequence of events can then be written.

2. Synthesis. The analysis is now examined from the point of view of deciding what learning structures are implicit in its content. Four types of sequences are generally discovered from the material.

(a) The simple chain sequence. This is a string of task elements which always occur in the same order in the tasks, for example, the steps in a long division problem. The chain may be reported many times to reinforce it, though using different material each time. In the long division, a different set of numbers would be used each time.

(b) Complex chain and branching sequences. In more complicated problems involving discriminative behaviour there can be alternative routes or branches - thus in the solving of quadratic equations, there is a choice of four ways of solving it.

(c) Discrimination sequences. These are needed wherever it is necessary to discriminate between cues and must be anticipated.

again referring to quadratic equations, the use of the discriminant ($b^2 > 4ac$, $b^2 = 4ac$, $b^2 < 4ac$) decides which of the subsequent branches must be used.

(d) Generalisation sequences. At the end of these sequences a student should have acquired some concept, and be able not only to generalise within a class of stimuli, but also to discriminate between items inside or outside of the class. Thus, if the concept was 'electronic apparatus', the student should recognise that amplifiers, radios, decade timers, valve voltmeters etc. are within the class, but that electric motors, jet engines, old fashioned gramophones, and ammeters are not. He is able to recognise the implication of the word 'electronic'.

3. Having assembled these sequences, the remaining problem is to decide upon the teaching strategy of the programmes. The students' backgrounds, motivation, numbers, and proposals to use the information are all relevant to this.

4. The programmer is now in a position to be able to write the objectives of the programme, i.e. a behavioural analysis. This is written in terms of what the student must <u>do</u>, not <u>that</u> which a student has learned. This collection of intents may be given to the student, and under certain circumstances this could be a guide to his own study, but generally the other steps of programme writing are required.

D. <u>Other methods</u> (Fry, 1963)

These, in general are all attempting to do similar things to the processes mentioned in B and C, but are not so detailed.

D. E. P. Smith suggested that there were four kinds of frames, and that systematic use of these would help programming. These were:-

1. Definition of a concept. This would define a concept and follow it up with responses.

2. A contrast frame to show what the concept is not. This could be compared with discrimination frames or sequences mentioned earlier.

3. Example frames. These would show actual uses of a concept first followed by a student response.

4. An anticipation frame would introduce a concept before it was needed, e.g. a technical word could be used in a self explanatory context to carry out practice of some previous concept.

A 10 category classification has been suggested by Gilbert (1958).

A different technique suggested by Barlow (1960) is that of so called 'Conversational Chaining'. The student reads the frame in the usual way and makes the necessary response. However, the correct answer is not given before the main content of the next frame, as is usual, but is implicit in this next frame material. It is identifiable by being printed in upper case lettering. Thus the reinforcement is given and the student is beginning to read the next frame almost before he realises what is happening.

Here is a short sequence showing the technique.

1. When light changes direction after striking a mirror, we say that it has been ....

2. This REFLECTED or DEVIATED light is still travelling in the same .... as before.

3. The statement that "a ray of light remains in the same PLANE before and after reflection" is one of the laws of ....
4. Another part of this law of REFLECTION, is that the normal to the mirror at the point of reflection also lies in the same ....
5. This PLANE, containing the ray of light and the normal to the mirror, is called the Plane of .... etc.

A common feature of all of these techniques is the call for great care in deciding exactly what is to be taught. The various methods described each have a different emphasis on the way in which a programme is built up after this.

It has been pointed out that one cannot dogmatise on programming methods and provided that one uses the three principles of small steps, active responding and feedback to the student, any of the methods will teach. (Markle 1964).

CHAPTER THREE

THE COMPILING AND PRELIMINARY TESTING OF THE PROGRAMME

1965

The original aim of this programme was to present the topic of complex numbers to first year Higher National Certificate Electrical Engineers. These students were, in general, highly motivated to learn the topic, as they used it in their Electrical Technology subjects a few weeks after it had been taught in mathematics. It was thought by using the topic of complex numbers, it would be easier to compare the results of programmed and conventional teaching, as the previous knowledge of the subject was nil for most students. All learning by the programmed learning group would be due to the programme, and thus a valid comparison could be made.

The programme was to teach the concept and manipulation of complex numbers, covered normally by three 2 hour lecture/study periods. An analysis of the essentials required showed that the programme would have to teach the students nine stages. These were:-

1. To define the symbol 'j'.
2. To define and identify real, imaginary and complex numbers.
3. To evaluate exponents of 'j'.
4. To solve quadratic equations which have complex roots.
5. To add and subtract complex numbers.
6. To multiply complex numbers.
7. To define complex conjugates.
8. To use complex conjugates to rationalise a complex quotient.

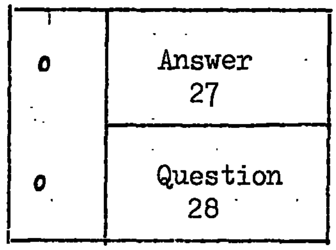
9. To recognise real and imaginary parts of complex expressions. The students were to be told that 'j' was simply a shorthand symbol for $\sqrt{-1}$.

No machines were available so that programmed texts would have to be used, but this was not implying any disadvantage, as these have been found to perform as well as machine programmes (Eigen 1964). Branching would be difficult in this case without resorting to 'scrambling' (Tutor Text), so it would have to be a linear programme.

The sequence of the programme would have to be in the same order as the stages listed above, as there is a definite hierarchy involved.

Using an article by Klaus as a guide, in which he lists 12 'rules' of programming (Klaus 1961), a programme of 50 frames was written with a view to trying it out on several courses of students. No special technique was used to produce the frames. A sequence of frames was built onto the bare bones of the nine stages so that there seemed to be a natural flow. It was thought that very short frames would seem trivial to these students, who had already covered some sophisticated mathematics in their previous courses. The frames were a good deal less verbal than most of Klaus' examples and often merited numerical answers. A mere number is of little value to a student working through a problem if he arrives at the wrong answer. To guide these students, it was decided that, as far as possible, the working of the problem should be given as the answer (Keurst 1964). This could be faded as further examples were given.

As the intended method of presentation was to be loose leaf pages in a ring binder, this led to the question of 'cheating', easily done by lifting the page.



However it can be argued that if a student turns over and reads the worked answer, he is going to learn something if he makes some effort to follow it. It is rather like a case of classical conditioning (Zeaman 1962), he learns to associate the answer (response) with the appropriate question (stimulus) and would, after several trials, respond accordingly. It was decided not to attempt any direct prevention of cheating.

These students would have to be able to incorporate this work in their electrical technology subjects and write it down. An overt response of writing out the working of the problems in a booklet was to be required of them. These could be checked when required, and a study of the errors made.

A post test was made out, consisting of questions directly testing the nine stages of the programme. As the topic of complex numbers was also tested directly in a sessional examination the criterion was set accordingly. A short pre programme check was also made to find out those students who had previous knowledge of complex numbers (a few electronic engineers and course repeats) with a view to ignoring their work in any attempt which was to be made in evaluating the programme.

At this stage, a change of teaching post occurred, and the engineers were replaced by mature students in a College of Education. These were of a much wider range of mathematical ability, the majority being of a much lower standard. The Curriculum (Professional) Course in Mathematics for the second year students at the college had no specified syllabus, the aim being to introduce the students to new branches of mathematics, and show them how they could lead on to some interesting ideas.

None of the frames in the prepared programme had been shown to students, so it was decided to try the programme with 81 of these students during the Summer Term. They were told that it was an experimental programme, still in its early stages, and that if things became too difficult, it was not their fault, but that of the programme. This was a novel approach to all of the students and they agreed to cooperate.

As there were also some Main Course Mathematics students in the groups, it was decided to change the 'j' operator (as commonly used by Electrical Engineers) to the 'i' of Pure Mathematics texts.

The motivation of these students was very much less than that of the engineers. It would be even less if they had been allowed to try the programme in its current form as it had been designed for students who were competent in the use of slide rules and could evaluate square roots and clumsy quotients without trouble. Only two (ex engineers!)

of the present students could use a slide rule and some of the frames in the prepared programme contained some formidable arithmetic if slide rules or the use of logarithmic tables were to be avoided. In view of these difficulties it was thought prudent to rewrite the programme, ensuring that wherever possible, arithmetic should be exact.

With these students of such varying mathematical ability, it was thought that they should be given the necessary help to enable them to carry out the algebraic techniques needed for the completion of the programme. These techniques were found to be

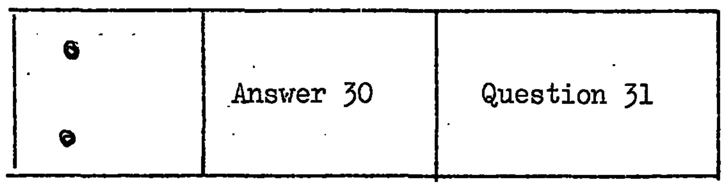
1. The manipulation of indices (including square roots).
2. Multiplying binomial expression.
3. Solving a quadratic equation by using the formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2^a}$

A questionaire was prepared (Pre programme check) Appendix I to test the students' knowledge of these, and it included three questions to see whether they had encountered complex numbers previously. These were put in so that the results of students who had met them before could be omitted from experimental conclusions.

The results of the questionaire, administed at the beginning of the term, showed that it was necessary to revise or instruct in all three of the above topics with most of the students. This seriously affected the amount of time that the students were on the programme.

The programme was then re-edited, inserting extra frames in several places, and amplifying the answers to give encouragement and support to weaker students. It was then 68 frames long. The form of the programme can be seen from Table I (page 28).

Insufficient ring binders were available for presentation of the frames as planned so after considering some alternative methods (Feldhuson and Birt 1962) it was decided to present the programme as three duplicated booklets, 8 inches wide by 2 inches deep.



The pages in each were fastened together by split brass fasteners so that they could be taken apart easily for revision and rearrangement of the programme. On each page of the booklet, the right hand half contained the given facts and questions, whilst the left hand half gave the answers and working of the previous page. There was no device incorporated to prevent cheating i.e. looking ahead in the booklets, but it was felt that the deliberate turning of a page - or not, was sufficient discouragement. In any case, a good answer should be self evident to the student, and he should be able to learn even if he did look ahead to the occasional answer.

An answer booklet for the students was made by duplicating horizontal lines on both sides of blank foolscap pages at two inch intervals, folding these down the centre, and then stapling three pages together. This gave 72 answer spaces approximately 4 inches wide and 2 inches deep, i.e. there were sufficient spaces for the working of the whole programme.

Table 1. Content of 1965 Programme

Frame No.	No. of Parts	Content
1	1	Definition of a real number.
2	3	Revision of square roots.
3	1	Introduction to square root of -ve no.
4	3	Introduction of $\sqrt{-1}$ as common factor.
5	1	Definition of i. Practice in its use.
6	6	Discrimination between real and other numbers needing 'i' notation. Practice in i.
7	6	Definition of imaginary number. Identification of imaginary numbers.
8	1	Evaluation of i^2.
9	1	Discrimination between real and imaginary.
10	1	Use of $i^2 = -1$. Simplification of i^3.
11	1	Discrimination between real and imaginary.
12	3	Simplification of exponents of i.
13	3	Further practice in above.
14	1	Generalisation from previous two frames. $i^n = \pm 1$ ($\therefore$ n even)
15	1	" " " " " " $i^n \doteqdot \pm i$ ($\therefore$ n odd)
16	2	Simplification and identification of real numbers.
17	2	Simplification and identification of imaginary numbers.
18	1	Impossibility of combination of real and imaginary numbers.
19	1	Definition of a complex number. Practice in formation of C.N.
20	3	Identification of complex number.
21	1	Revision of solution of quadratic equation using the formula.
22	2	Solution of a quadratic with complex roots establishing the need to use 'i'.
23	2	Use of i in previous answer, practice identification of complex number.

Continued ......

Table 1. Content of 1965 Programme (continued)

Frame No.	No. of Parts	Content
24	1	Introduction of 'Complex roots'. Further practice in solution of quadratic with c roots.
25	2	Practice in complex roots. Imaginary part of complex number.
26	1	Method of addition of complex numbers.
27	1	Practice in addition of C.N.
28	1	Method of subtraction of C.N.
29	1	Practicemin subtraction of C.N.
30	2	Further combined practice of addition and subtraction. Identification of real no.
31	1	Revision of $i^2 = -1$.
32	1	Introduction of product involving i^2.
33	1	Practice in simplifying product involving i^2.
34	1	Multiple of i.
35	2	Method of multiplying complex nos. Identification of complex no.
36	1	Practice in multiplying C.N. Use of word 'product'.
37	1	" " " "
38	2	" " " ('squaring). Identification of complex no.
39	2	Practice in multiplying C.N. Identification of - non C.N.
40	1	Formation of complex conjugate by direct instruction.
41	2	Formation of complex conjugate by direct instruction followed by multiplication of the conjugate.
42	1	Practice in multiplying C.N. Introduction of words 'Complex Conjugates'.
43	1	Practice in formation of c conjugate.
44	2	Practice in formation of c conjugate followed by multiplication.
45	3	" " " " " " " " "
46	1	Identification of real number, Rule from Product of Complex Conjugates.

Continued ....

Table 1. Content of 1965 Programme (continued)

Frame No.	No. of Parts	Content
47	2	Addition of complex conjugates. Identification of real number.
48	6	Further addition of complex conjugates. Identification of real number.
49	1	Identification of real number. Rule from addition of complex conjugates.
50	1	Instruction for rationalisation process.
51	1	Instruction for rationalisation process. Use of word 'Rationalise'.
52	2	Practice of rationalisation and simplification.
53	1	Reason for rationalising denominator.
54	1	Reason for rationalising both numerator and denominator.
55	2	Formation of quotient. Practice in rationalisation.
56	1	" " " " " " " " " Use of word 'divide'.
57	3	Revision of product. Use of division. Practice in rationalisation.
58	2	Evaluation of a squared quotient. Practice in rationalisation.
59	2	Revision of product (twice). " " " "
60	2	Practice in rationalisation. Identification of real part of C.N.
61	3	Revision of squaring C.N. Practice in rationalisation. Identification of real part of C.N.
62	2	Revision of product. Identification of imaginary part of C.N.
63	3	Combination of two quotients into 1 fraction. Practice in rationalisation. Identification of imaginary part of C.N.
64	1	Revision of product, more difficult e.g.
65	1	Identification of real and imaginary parts of C.N.
66	2	Revision of product. Identification of real and imaginary parts of C.N.
67	1	Revision of rationalisation.
68	1	Identification of real and imaginary parts of C.N.
	120	

A post-programme test was assembled to test the nine specific points in the aims of the programme (Appendix II).

At the beginning of the Summer Term 6 weeks of lecture time (1 per week) were devoted to instruction and revision of the necessary fundamental topics, during which it was realised that the spread of ability was even wider (to a lower level) than previously supposed. However, the programme was begun with a half hour session, during which nearly every student came to grief on frame 8. This did not help the motivation, and about ten students gave up at frame 21 a fortnight later.

Whilst working through the programme, the students were told to check their answers by turning over the page in the booklet and putting a tick (✓) beside correct answers, and a cross (✗) beside wrong ones. Wrong answers were to be checked for

1. Misreading of the question.
2. Arithmetic mistakes.
3. Algebraic mistakes.

If after these checks a correct answer was obtained, a ring was to be put round the cross (⊗) before proceeding. If the answer was still incorrect, it was to be left crossed, and the student was advised to look back into the booklet for relevant help. If a particular frame helped them, they were asked to place the number in a ring beside the cross. This was for the purpose of revision of the programme, so that it could be seen where extra practise frames or review frames were necessary. If this still left the student in difficulties the

Table 2. Number of errors occuring at each frame

N is the number of students who tried the frame during this experiment.

P is the percentage of the whole group who reached as far as the particular frame.

Frame No.	N	No. of Errors	P	Frame No.	N	No. of Errors	P
1	81	0		35	54	0	
2	81	0		36	54	0	
3	81	3		37	51	0	
4	81	0		38	51	0	
5	81	0		39	50	0	
6	81	0		40	49	0	
7	81	1		41	49	1	
8	81	20	75%	42	49	2	
9	81	23		43	48	6	
10	81	12		44	47	2	
11	81	7		45	47	1	
12	81	7		46	46	0	
13	81	8		47	45	2	
14	79	0		48	45	0	
15	79	7		49	44	1	
16	79	0		50	43	2	
17	79	2		51	40	1	50%
18	79	3		52	38	0	
19	78	0		53	36	1	
20	77	5		54	35	2	
21	77	6		55	33	0	
22	71	4		56	32	1	
23	68	4		57	29	0	
24	68	0		58	27	1	
25	64	4		59	22	0	
26	58	0		60	21	0	
27	58	0		61	21	2	
28	57	0		62	20	0	25%
29	57	0		63	18	0	
30	57	1		64	17	1	
31	55	0		65	16	2	
32	55	9		66	16	1	
33	55	1		67	15	0	
34	55	2		68	15	0	18½%

Table 3. Results of Post Programme Test

Student	Question Number								Score	Time on Prog. (min.)	Main Course Maths.
	1	2	3	4	5	6	7	8			
D.B.	X	✓	X	✓	✓	✓	✓	X	5	183	
A.C.	✓	✓	✓	X	X	✓	✗	X	4	153	✓
E.D.(1)	✓	✓	✓	✓	✓	✓	✓	✓	8	150	✓
E.D.(2)	✓	✓	✓	✓	X	✓	✓	✓	7	134	✓
M.F. *										91	✓
F.G.	✓	✓	✓	✓	✓	X	✓	X	6	87	✓
F.T.J. *										145	
N.J. *										91	✓
F.M.	✓	✓	✓	✓	✓	✓	✓	✓	8	124	✓
D.P. *										148	✓
E.R.	✓	✓	X	✓	X	✓	✓	X	5	65	✓
N.R.	X	✓	✓	✓	✓	✓	✓	✓	7	126	✓
E.S.	✓	✓	X	✓	✓	✓	✓	✓	7	148	✓
J.S.	✓	✓	✓	✓	X	✓	✓	X	6	105	✓
V.W. *										166	
No. of errors	2	0	3	1	4	1	1	5			

* These students had completed the programme but had not time to complete the POST PROGRAMME TEST.

Mean Score (N = 10) 6.3

Mean Time (N = 15) 127.7 min.

supervisor would assist them.

The programme was given in periods of 40 mins to 45 mins at a time, as it was considered to require considerable concentration from most of the students. The few students who finished the programme were given the post test immediately. The majority of the students did not have sufficient time to complete the programme before the end of the term.

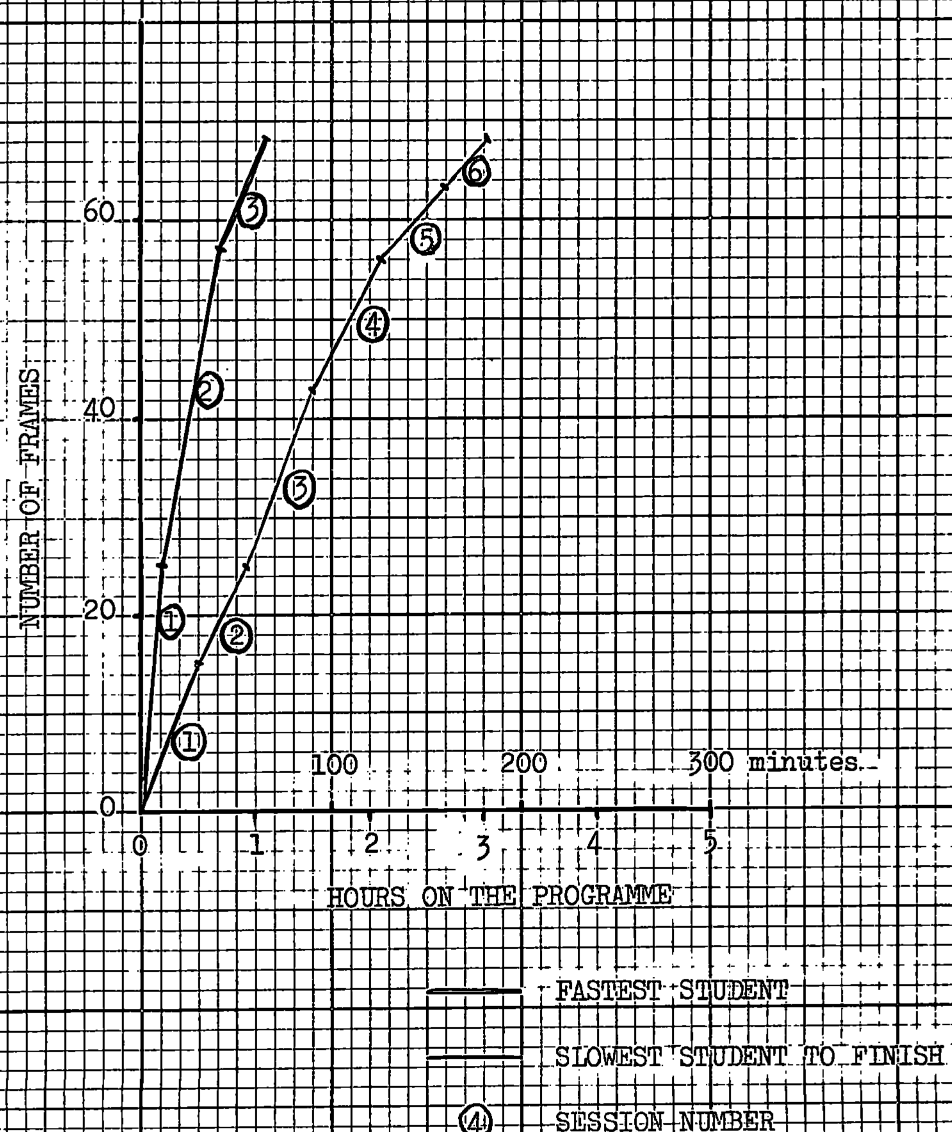
Results of this trial

The checking of the frames by the students worked well and enabled misreading/arithmetic/algebra mistakes (by far the majority) to be sorted from genuine errors in principle. A table is drawn up of the number of students who attempted each frame and the number of errors occuring at each. Table 2, page32.

15 students managed to complete the programme, but only 10 of them were able to take the post test, and again, only 1 of these was able to take a retest a week later. Only 3 students who were not taking Mathematics as a 'Main' subject, finished. Table 3, page 33 , summarises these results. This was purely a trial and no conclusions are drawn from these.

A progress diagram (page 35) was drawn for the fastest student and the slowest student to finish. This showed that highest rate of working through the programme was about 2.75 frames per minute and the lowest 0.2 frames per minute for students who finished the programme. The most rearward student had completed 20 frames in 2 hours of his available time, but most students were between frame40 and the end of

PROGRESS DIAGRAM 1965



the programme after 2 to 3 hours work. All students showed a slowing down near frame 50, and this is a reflection of the large increase in step size which occurs at this frame.

It seems clear that the students needed more thorough preparation for the programme, and that the programme in that form was slightly too difficult for the majority of students.

The Post Programme Test (Appendix II) was not satisfactory as it did not test all the points required to be taught by the programme. Real and imaginary numbers did not appear in it at all. The first question should have tested the knowledge of $\sqrt{-1}$, not given the students the information, and the important $i^2 = -1$ was not called for clearly. It did occur in questions 5 to 8, but only incidentally in another process. In question 6, the complex conjugate of a number was required, but the issue was clouded by requiring a product as well. Similarly question 8 required a rationalisation of the quotient, but it was intended to test the ability to identify the imaginary part of a complex number and equate it to zero.

Revision of the Programme

The programme had obviously broken down at frame 8 as the students were not prepared for this step. The topic was that of evaluation of exponents of i as far as frame 15, and due to the doubt about $i^2 = -1$, the errors in the intermediate frames were high. This part of the programme needed complete rewriting. In the end 8 new frames preceeded the one which caused so much trouble.

Frame 20 showed (Table 2, page 32) that $^5/_{77}$ students failed to distinguish between complex numbers and other algebraic expressions.

Extra practice frames and a better definition in frame 19 were inserted. Question 19 did not say anything about differences between real and imaginary numbers, and yet one answer given was 7 - i4. This was deleted, and another example frame used to show a difference.

Answer 21 was unfamiliar to some students, as it used the dot multiplication sign. This was changed to the more common x sign. Question 22 was broken down into 3 separate steps and Question 23 halved. A formal definition of complex roots was introduced and questions 24 and 25 slightly reworded.

The answers to question 32 showed a lack of appreciation that $i^2 = -1$, but on the general reshaping of the programme this should be remedied. A slightly altered wording was thought to be sufficient.

Frame 43 showed that the definition of a complex conjugate was not clear from 42 alone. Two example frames and a generalisation frame were written into this section.

On rereading the section on rationalisation (frames 50 to 55) it was seen that no specific information on this process was given. It was decided to introduce a defining frame. Nowhere was it pointed out to the student that rationalisation always produces a real number in the denominator and hence that normal division of the numerator may be carried out, although examples of this were given. Two new frames covered this omission.

An extra practice frame on real and imaginary parts was put into the last section as it was not scored as well as other sections in the post test.

Table 4. Content of 1966 Programme

M Indicates a modified or reworded frame.

NF Indicates a new frame.

Frame No.	Old Frame No.	No. of Parts	Content
1	1	1	Definition of a real number.
2	2	3	Revision of square roots.
3	3	1	Introduction to square root of negative no.
4	4	3	Introduction of $\sqrt{-1}$ as common factor.
5	N.F.	1	Definition of i. Identification as -1 by student.
6	N.F.	1	Demonstration of i notation. Practice in use of i.
7	N.F.	6	Discrimination between real and other numbers needing i notation. Practice in i.
8	N.F.	1	Definition of imaginary numbers. Identification of imaginary number.
9	N.F.	1	Position of i defined in imaginary number. Identification of imaginary number.
10	N.F.	6	Practice identification of imaginary numbers.
11	N.F.	1	Revision of general square root notation.
12	N.F.	1	Use of square of square root, when number is negative.
13	8	1	Evaluation of i^2.
14	N.F.	1	Identification of -1 as real number.
15	N.F.	1	Cued frame to identify i^2 as real.
16	10	1	Use of $i^2 = -1$. Simplification of i^3.
17	11	1	Discrimination between real and imaginary.
M 18	12	3	Simplification of exponents of i.
19	13	3	Further practice in above.
20	14	1	Generalisation from previous two frames $i^n = \pm 1$ (n even)
M 21	15	1	Ditto $i^n = \pm i$ (n odd).

Continued ....

Table 4. Content of 1966 Programme (continued)

Frame No.	Old Frame No.	No. of Parts	Content
M 22	16	2	Simplification and identification of real nos.
M 23	17	2	Simplification and identification of imaginary numbers.
M 24	18	1	Impossibility of combination of real and imaginary numbers.
25	19	1	Definition of c complex number. Practice in formation of C.N.
26	N.F.	1	Cued frame to identify complex number.
27	20	3	Identification of complex numbers.
28	21	1	Revision of solution of quadratic equation using the formula.
M 29	22	1	Solution of quadratic with complex roots - establishing the need to use i.
30	N.F.	1	Identification of imaginary number.
M 31	23	2	Use of i in previous answer. Complete arithmetic.
32	N.F.	1	Identification of complex number.
33	N.F.	1	Cued frame introducing complex roots.
M 34	24	1	Use of complex roots. Practice in solution of an equation with complex roots.
M 35	25	2	Practice in complex roots. Imaginary part of complex number.
36	26	1	Method of addition of complex numbers.
37	27	1	Practice in addition of complex numbers.
38	28	1	Method of subtraction of C.N.
39	29	1	Practice in subtraction of C.N.
40	30	2	Further combined practice of addition and subtraction of C.N. Identification of real no.
M 41	31	1	Revision of $i^2 = -1$.
M 42	32	1	Introduction of product involving i^2.

Table 4. Content of 1966 Programme (continued)

Frame No.	Old Frame No.	No. of Parts	Content
43	33	1	Practice in simplifying product involving i^2.
44	34	1	Multiple of i.
M 45	35	2	Method for multiplying complex numbers. Identification of complex number.
46	36	1	Practice in multiplying complex numbers. Use of word 'product'.
47	37	1	" " " " " " "
48	38	2	Practice in multiplying complex numbers (squaring). Identification of complex no.
49	39	2	Ditto. Identification of non complex no.
50	40	1	Formation of complex conjugate by direct instruction.
51	41	2	Ditto. followed by multiplcation of the conjugates.
52	42	1	Practice in multiplying complex nos. Introduction of words Complex Conjugates.
53	N.F.	1	Cued formation of complex conjugates (positive sign).
54	N.F.	1	Ditto. (negative sign).
55	N.F.	1	Generalisation of rule from previous three frames.
M 56	43	1	Practice in writing complex conjugates.
57	44	2	Ditto. followed by multiplication.
58	N.F.	2	Ditto. Ditto. Use of product.
59	45	3	Ditto. followed by multiplication.
60	46	1	Identification of real number. Rule from product of complex conjugates.
61	47	2	Addition of complex conjugates. Identification of real number.
62	48	6	Further addition of complex conjugates.
63	49	1	Identification of real number. Formation of rule for addition of complex numbers.

Table 4. Content of 1966 Programme (continued)

Frame	Old Frame No.	No. of Parts	Content
64	50	1	Instruction for rationalisation process.
65	N.F.	1	Recognition of multiplying by complex conjugate of denominator. Use of word rationalisation.
M 66	51	1	Instruction for rationalisation process.
67	52	2	Practice of rationalisation and simplification.
68	N.F.	1	Identification of real number in denominator after rationalisation.
69	N.F.	1	Use of real denominator.
70	53	1	Reason for rationalising the denominator.
71	54	1	Reason for rationalising both the numerator and denominator.
72	55	2	Formation of quotient. Practice in rationalisation
73	56	1	Ditto. Ditto. Use of word 'divide'.
74	57	3	Revision of product. Use of division. Practice in rationalisation.
75	58	2	Evaluation of a squared quotient. Ditto.
76	59	2	Revision of product (twice). Ditto.
77	60	2	Practice in rationalisation. Identification of real part of C.N.
78	61	3	Revision of squaring C.N. Practice in rationalisation. Identification of real part of C.N.
79	62	2	Revision of product. Identification of imaginary part of C.N.
80	63	3	Combination of two quotients into one fraction. Ditto. Practice in rationalisation.
81	64	1	Revision of product (more difficult).
M 82	65	1	Identification of imaginary number.
83	N.F.	2	Revision of product. Putting real part = 0.
84	66	2	" " " imaginary part = 0.
85	67	1	Revision of rationalisation.
M 86	68	1	Putting real part = 0.
		139	

The contents of the programme frames are shown in Table 4 (pages 38, 39, 40, 41) and the actual programme is reproduced in Appendix III. The length of the programme is now 86 frames, representing an increase of approximately 25% of the original length.

CHAPTER FOUR

THE FIRST TRIAL OF THE PROGRAMME - 1966

This experiment was carried out on another second year group of 50 students in the College of Education.

The aims of these were:-

1. To carry out another trial of the programme preparatory to a further revision.
2. To compare its use with formal teaching.

The students were divided into three groups for college purposes, viz.

Group 2. An all female group of 12, training to be primary teachers.

Group 3. A mainly male group of 16, training to be primary teachers.

Group 4. A mixed group of 22, training to be secondary stage teachers. Some of these were specialising in art subjects, and had very little mathematical ability.

Only 3 students were taking a 'Main' course in Mathematics. Group 1, who were training to be Infant teachers were not used in the experiment. In the previous years trial 9 out of 10 who abandoned the programme were in this category, and it was thought that they would require much more preparation time than any of the other groups.

To allow as many students as possible to try the programme, it was decided to teach the smallest group (2) by conventional methods.

The most convenient time for the experiment was during the summer term. The three groups were given Precheck I (Appendix I) to complete in January, and this was marked and analysed. Performance was generally poor, (8 passes) and it was decided that the revision course should last half a term to improve

(a) Manipulation of surds and indices.

(b) Binomial multiplication.

(c) Solution of quadratic equations.

(d) Manipulation of algebraic fractions.

The course was prepared and started at the beginning of the summer term. It was necessarily hurried as some of the students really required complete reteaching. In view of the previous years experience, it was decided to check the progress of the students by administering the January test again as Pre Check II at half term. 14 students who could not achieve a satisfactory performance on this should not really have started on the programme, but as participating students were so few, all were allowed to begin. A Pre Test (Appendix IV) had been prepared, and this was identical in form to the revised Post Test (Appendix V) apart from numerical values. This was administered at the same time as Pre Check II. The three groups then began to study complex numbers in the two different ways.

The same instructions were given to the programmed learning groups as in 1965, to tick correct answers and put a cross beside wrong ones. The same checking system was used to sort errors of principles from the common misreading, arithmetic and algebraic errors.

The time of starting and finishing each session of the work was to be noted so that the total could be obtained. At the start of each session the students could have a few minutes (counted in the total time) to look back into the programme and answer booklets for revision where it was required.

These groups were asked to work only by themselves and not to use reference books or discuss the process after class. This was well adherred to. The other group was allowed to use any aids they felt they needed and, though no one actually referred to text books, they did discuss, and teach each other, all through the six weeks available.

After the first 3 weeks it was clear that not all of the two groups on the programme would be able to finish it, if they followed the schedule of 1 session per week. The students volunteered to put in 2 per week instead, and towards the end some put in 3. Even so, 5 were not able to finish the programme; two of them had given up after 2 sessions.

The Post Test had been revised as a result of the 1965 trial and consisted of 12 questions instead of 8. This was given to each student as soon as he had completed the three programme booklets, and where possible, a week later as a retention test. (Post Test II). (Appendix V). Group 2 had the Post Test during the last half hour of their lecture time, and there was no time for a retention test.

The marking of the Pre Test and Post Test had to be refined to provide proper discrimination between students answers, and it was necessary to use half marks. The balance of marks was decided empirically, allowing 3, ($6 \times \frac{1}{2}$) for question 6, 2 marks ($4 \times \frac{1}{2}$) for each of 9 and 11 and 1 each for the remainder, though 5, 7 and 8 could be halved. This effectively gave 26 units for allocation, rather than the 16 marks one by one.

The results for the 3 graphs are given in tables 5, 6 and 7, pages 47, 48 and 49 . A table giving the number of errors for each frame was also drawn up (Table 8), and using this, a further table was made out for frames having 4 or more errors (12% or more), showing what errors occurred (Table 9).

Progress diagrams were again drawn for the fastest and slowest students, and the highest rate on the programme was 1.3 frames per minute, the lowest being .12 frames per minute. Again it is clear that a slowing down took place for all students from frame 64 onwards due to an increase in the step size. (page 54).

Table 10, page 53 , shows the number and percentages of students who answered each question correctly in Post Test I. As each question is testing a particular concept, this is an indication of how well the programme (or teacher) has performed in the teaching of each concept.

Table 5. Data for Group 2 (Conventional Teaching)

Students	Sex	Age	'0' Level Maths.	Pre Check		Pre Test (16)	Post Test (16)
				I	II		
G.B.	F	41	No	F	P	0	$12\frac{1}{2}$
M.D.	F	44	Yes	F	P	2	11
E.H.	F	38	Yes	F	P	1	11
J.J.	F	40	No	F	F	0	12
B.M.	F	39	No	F	P	0	$11\frac{1}{2}$
J.M.	F	42	Yes	F	P	1	10
R.P.	F	42	No	F	P	0	11
M.S.	F	26	No	F	P	0	$11\frac{1}{2}$
J.T.	F	29	No	F	P	1	$11\frac{1}{2}$
V.W.(1)	F	32	Yes	F	P	1	14
V.W.(2)	F	31	Yes	F	P	1	$10\frac{1}{2}$
D.W.	F	42	Yes	F	P	1	$12\frac{1}{2}$

Mean Age 37.2 N = 12

Total Time 5 x 45 + 1 x 15 = 240 min.

Mean Gain Score (Post - Pre Test) = 10.9 = 68%.

Table 6. Data for Group 3 (Programmed Learning)

Students	Sex	Age	'O' Level Maths.	Pre Check		Pre Test (16)	Post Test		Time on Programme (min.)
				I	II		I	II 1 week	
G.B.	F	43	Yes	F	P	1	10½		180
G.D.	M	44	Yes	F	P	1	14		270
J.G.	M	30	No	F	P	1	10	9	247
B.H.G.	M	40	No	F	F	0	13		375
D.F.G.	M	46	Yes	F	P	0	15½	15½	275
S.G.	M	29	No	F	P	0	16	16	220
J.C.G.	M	45	Yes	F	P	4	16		200
A.H.	M	33	Yes	F	P	4	15½	15	219
D.H.	M	38	Yes	F	P	4	12½	12½	235
J.D.I.	M	34	No	F	F	0	12		375
J.R.L.	M	43	No	F	F	0	9½		300
A.L.	M	45	Yes	P	P	3½	16	15½	200
H.N.	M	40	Yes	P	P	2	16	15	155
N.P.	F	31	Yes	F	P	0	13½		245
D.M.	M	21	Yes	F	P	0	14		185
M.T.	F	46	Yes	F	F	0	13½		215

Mean Age 38 N = 16.

Mean Time on Programme 241 min.

Mean Gain Score = 12.3 = 77%.

Table 7. Data for Group 4 (Programmed Learning)

Students	Sex	Age	'O' Level Maths.	Pre Check I	Pre Check II	Pre Test (16)	Post Test I	Post Test II 1 week	Time on Programme (min.)
T.B.	M	43	Yes	P	P	10	15½		245
J.B.	M	44	Yes	P	P	2	10	9½	156
/D.C.	F	42	No	F	F	1	13½		330
G.C.	M	42	Yes	F	F	0	13½		280
A.E.H.	F	46	Yes	F	P	0	10½		260
B.H.	F	40	Yes	F	P	2	13		315
W.H.	M	30	Yes	P	P	3	15½	15½	135
H.H.	F	33	No	F	P	2	11½		217
A.F.H.	M	34	Yes	F	P	2½	13½		235
M.S.J.	F	39	No	F	P	½	15	15	295
J.M.	M	29	Yes	P	P	2½	14½		115
M.M.	F	39	Yes	F	F	0	10½		315
R.C.M.	M	37	Yes	F	P	0	13½		245
D.O.	M	44	Yes	P	P	8½	13½		127
A.S.	M	39	Yes	F	F	2½	13		205
R.T.	F	36	No	F	P	1½	14		220
S.G.W.	M	35	No	F	P	0	9½		275

Mean Age 37½ N = 17.

Mean Time on Programme 228 min.

Mean Gain Score = 10.7 = 67%.

Table 8. Number of errors occuring at a frame

Frame No.	No. of Errors	%	Frame No.	No. of Errors	%	Frame No.	No. of Errors	%	Frame No.	No. of Errors	%
1	0	0	23	1	3	45	1	3	67	0	0
2	1	3	24	0	0	46	3	9	68	0	0
3	3	10	25	0	0	47	1	3	69	0	0
4	2	6	26	7	21	48	0	0	70	3	9
5	0	0	27	0	0	49	1	3	71	1	3
6	0	0	28	0	0	50	1	3	72	3	9
7	0	0	29	1	3	51	1	3	73	0	0
8	0	0	30	4	12	52	0	0	74	2	6
9	1	3	31	5	15	53	0	0	75	0	0
10	0	0	32	3	9	54	0	0	76	0	0
11	1	3	33	3	9	55	0	0	77	0	0
12	11	33	34	0	0	56	0	0	78	0	0
13	2	6	35	1	3	57	0	0	79	0	0
14	6	18	36	0	0	58	0	0	80	12	36
15	1	2	37	2	6	59	0	0	81	0	0
16	7	21	38	0	0	60	0	0	82	4	12
17	10	30	39	0	0	61	0	0	83	0	0
18	0	0	40	0	0	62	0	0	84	3	9
19	2	6	41	1	3	63	0	0	85	1	3
20	0	0	42	7	21	64	1	3	86	6	18
21	5	15	43	5	15	65	7	21			
22	0	0	44	6	18	66	1	3			

Total no. of programmes = 33.

Table 9. <u>Analysis of errors</u>

Frame Number	Number of Errors	Errors	Possible Reason for Error
12	11	$\sqrt{-a} \times \sqrt{-a} = a$	Definition of square root not sufficiently well known. Insufficient practice with surds.
14	6	-1 identified as imaginary.	Lack of practice in identifying real numbers.
16	7	Failure to carry out the instruction.	Lack of prompting - show how i^2 can be used.
17	10	-i identified as real.	Lack of practice at discriminating between real and imaginary numbers.
21	5	± 1 instead of $\pm i$	Insufficient examples of n where n is odd before asking for a generalisation.
26	7	'imaginary' instead of 'complex' number.	Lack of practice in identifying imaginary numbers.
30	4	Failure to recognise imaginary number.	" " " " "
31	5	Failure to use i notation properly. $\frac{2 \pm i2}{2} = 1 \pm i2$	Review of frame 6 needed just before this. Lack of practice with fractions.
42	7	$1^2 = i$	More review of exponents of i needed.
43	5	$-i \times i = -i$	More review of exponents of i needed.
44	6	*Comparison of i with i^2	" " " " "
65	7	Common answer "Complex number".	Insufficient prompting of the connection with complex conjugates of the denominator. Generalisation too soon.
80	12	Failure to combine fractions properly. Forgot about complex nonjugates.	Too much material in 1 frame.

Erratum * Confusion

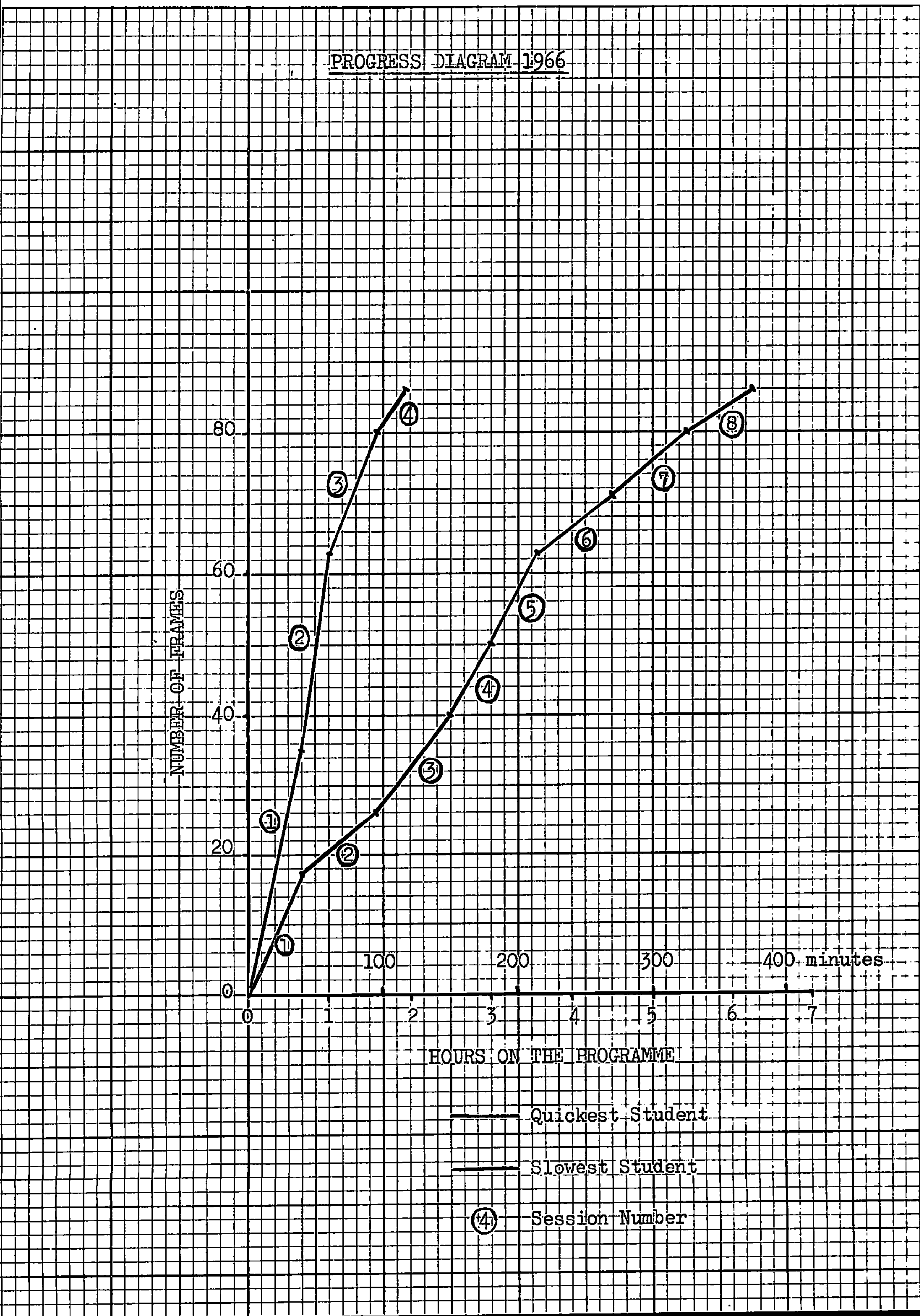
Continued...

Table 9. Analysis of errors (continued)

Frame number	Number of Errors	Errors	Possible Reason for Error
82	4	Failure to identify answer as imaginary.	Identification of imaginary numbers needs review.
86	6	Failure to recognise the real part of the numerator.	Students do not realise that an expression can be real.

Table 10. Percentage Success on each question in POST TEST I

Question No.	Group 2 (Control)		Group 3 (On Programme)		Group 4 (On Programme)	
	12	%	16	%	17	%
1	10	83	13	81	17	100
2	12	100	16	100	17	100
3	12	100	16	100	16	94
4	12	100	16	100	15	88
5	12	100	16	100	16	94
6	8	67	9	56	13	77
7	8	67	11	69	14	82
8	10	83	11	69	15	88
9	5	42	14	88	12	71
10	12	100	16	100	17	100
11	11	92	16	100	14	82
12	7	58	10	62	9	53


PROGRESS DIAGRAM 1966
NUMBER OF FRAMES
0
20
40
60
80
100
200
300
400 minutes
0
1
2
3
4
5
6
7
HOURS ON THE PROGRAMME
Quickest Student
Slowest Student
Session Number

CHAPTER FIVE

STATISTICAL TESTS

1. It was required to test whether groups 3 and 4 could be considered as a homogeneous group. The Wilcoxon test (a rank sum test) is a sensitive test for comparing locations of groups of results (Lindgren and McElrath) and this was applied to the gain scores of the two groups

	N	Means	Rank Sums
Group 3	16	12.3	316½
Group 4	17	10.7	244½

Using the formula in Garrett
$$Z = \frac{2R_1 - N_1 (N + 1)}{\sqrt{\frac{N_1 N_2 (N + 1)}{3}}}$$

this gave the Z scores as $\pm$ 1.6

As $N_1 > 8$ and $N_2 > 8$, the Z statistic is normally distributed.

The null hypothesis was that the means were equal.

The test hypothesis was that the means were not equal.

The critical value of the Z statistic at the .01 level for a two tailed test was $\pm$ 2.58.

As $- 2.58 < Z = \pm 1.6 < 2.5$ the null hypothesis was accepted and we could assume that the means were equal i.e. the two groups come from the same population.

2. As a result of the previous test, the scores of the groups 3 and 4

were combined in alphabetical order. The mean of the combined group was to be compared with the mean of group 2. To eliminate the effects of the Pre Test, the group 2 students were matched with students in the combined group having the same pre test score. These selections were carried out alphabetically, which may be considered as random in this case. The Post Test scores of these students were then compared, again using a rank sum test.

	N	Means	Rank Sums
Group 2	12	11.58	126
Combined Group	12	12.95	174

Calculating Z in the same way as before, this was found to be $\pm$ 1.38.
Again, the null hypothesis was that the means were equal,

the test hypothesis was that the means were unequal.

This required a two tailed test, the critical value of the statistic at the .01 level being $\pm$ 2.58.

As $-2.58 < Z = \pm 1.38 < 2.58$ the null hypothesis was accepted. There was no significant difference between the means of the two groups. This test assumed no underlying distribution of scores within the groups, but the scores may well have been normally distributed, as are the results of many tests.

3. A confirmatory test was made on this assumption, using the t statistic, which has Student's distribution for small values of N.

	N	Means	S.D.
Group 2	12	11.58	1.017
Combined Group	12	12.95	2.09

The combined S.D. was evaluated as 1.72 and the resulting value of t as 1.95. The number of degrees of freedom = 12 + 12 - 2 = 22. The null hypothesis was that the means were equal, and the test hypothesis was that the means were not equal. The critical value of the t statistic at the .01 level for a two tailed test with 22 d.f. was $\pm$ 2.82. As $-2.82 < t = 1.95 < 2.82$, the null hypothesis was retained, confirming the result of test 2.

4. 10 students completed a second Post Test one week after the first. It was required to find whether there was any significant fall in the score from one week to the next. A small sample modified t test (Freund) was used.

	N	Mean	S.D.
Post Test I	10	14.2	
Post Test II	10	13.85	2.49

$$t = \frac{X - M \sqrt{N - 1}}{\sigma_s}$$

This tests the significance of the difference between the sample mean (M) and some given value (X). Taking X to be the Post Test I Mean, this gave t = .424 with 9 d.f.

The null hypothesis was that there was no difference between the two means, and the test hypothesis was that the Post Test II mean was less than the Post Test I Mean. This required a one tailed test. The critical value of the t statistic at the .01 level for a one tailed test with 9 d.f. was $\pm$ 2.82.

As $-2.82 < t = .424 < 2.82$ the null hypothesis was retained. Thus the means are not significantly different at the .01 level.

5. The combined group had a mean time on the programme of 233 minutes, and the lecture time given to group 2 was 240 minutes. It was required to test the difference again to see if this was significant.

	N	Time (min.)	S.D.
Group 2	12	240	0
Combined Group	33	233	65.2

No. of degrees of freedom = 33 - 1 = 32

Using the same t test as previously t = -0.61. The null hypothesis was that the times taken were equal and the test hypothesis was that the combined group took less than Group 2.

The critical value of the t statistic at the .01 level for a one tailed test with 32 degrees of freedom was $\pm$ 2.46.

As $-2.46 < t = -0.61 < 2.46$ we retain the null hypothesis.

Thus we may say that the time taken by the group on the programme was not less than that taken by Group 2 at a .01 level of significance.

6. A coefficient of correlation for time on the programme with gain scores for the whole combined group was evaluated at $r = .115$. A t test on this value gave $t = .6446$ (Spiegel).

The null hypothesis was that r was not greater than zero, the test hypothesis was that t was greater than zero.

The critical value of the t statistic at the .05 level for a one tailed test with 31 degrees of freedom was ± 1.7.

As $-1.7 < t < 1.7$ we accept the null hypothesis, i.e. r is not significantly different from zero and there is no evidence of linear correlation between time on the programme and the gain scores.

7. It was decided to test if the passing of a G.C.E. 'O' level Mathematics examination (up to 20 years previously!) produced any noticeable effect on the results of the programme. A correlation of whether or not a student possessed an 'O' level with the Gains Score should show this. A point biserial correlation was thought to be the most appropriate as it could not be said that possession/non possession of certificate could be normally distributed, if some of the students had not sat the examination.

	N	Mean Gain Score	S.D. of all Scores
Passed 'O' level maths.	23	11.8	2.51
No certific.	10	11.8	2.51

This gave a value of $r_{pr.\ bis.} = 0$. This was accepted as negligible without further tests.

8. A group of 8 students failed the Pre Check II test before starting on the programme, and this indicated that they had not the algebraic facility to be able to complete it satisfactorily. This failure of Pre Check II was correlated with Gain Score to test their performance.

	N	MG Score	SD of all G Scores
Passed PC 2	25	11.75	2.51
Failed PC 2	8	11.85	2.51

This gave a value of $r_{pr.\ bis.} = .017$.

This was accepted as negligible without further tests.

9. The 8 students referred to in test 8 would be expected to take a longer time in working through the programme. A point biserial coefficient of correlation was calculated for Pre Check II with time spent on the programme.

	N	Mean Time	SD of all Times
Passed PC 2	25	215	65.2
Failed PC 2	8	307	65.2

The value of $r_{pr.\ bis.}$ was + .61 and this was significant at the .01 level (Garrett). The 99% upper and lower limits of r were calculated as .82 and .22 respectively (Spiegel).

10. The performance of the linear programme as a teacher could be compared with the teacher by correlation of the test scores for each question between the various groups.

Using a linear formula $r = \frac{\sum xy}{\sqrt{\sum x^2 + \sum y^2}}$ (Garrett).

This was done between groups 2 and 3 and 2 and 4. These are the results:-

Group	Mean No. of Students Passing each question	Correlation
2	9.9	
3	13.7	2/3 = .69
4	14.6	2/4 = .79

CHAPTER SIX

COMMENTS ON THE RESULTS, AND POSSIBLE FUTURE DEVELOPMENT OF THE PROGRAMME

It would appear that the programme is successful, in that it will enable students to work through it and pass a criterion (Mean = 12.9/16).

The result of the first statistical test is not surprising as the students in group 3 and the 17 remaining in group 4 were a good cross section of all students. The 5 who dropped out of the experiment all belonged to group 4, and had they completed the programme and tests, it is highly probable that they would have had a pronounced effect on the results as their mathematical ability seemed low.

When the Post Test mean of the group 2 students and 12 students from the combined group 3 and 4 with matching pre test scores were compared (Test 2) there was found to be no significant difference. Thus the programmed learning has performed as well as conventional teaching. This result agrees with most studies in this respect.

The results of the 10 students who were able to take a post test one week after their first are shown by test 4 to have no significant difference. Thus nearly all the material learned by these students has been retained over the period of one week. This is not surprising when it is considered that these ten students were the first finished out of the whole group of 50. They probably have the highest mathematical abilities in the group.

Test 5 showed that the mean time for the whole programmed learning group was not significantly different from that given to the conventional teaching group. This agrees with an experiment carried out by Unwin at Loughborough College of Technology (Unwin, 1966) using similar material (determinants) with 1st year undergraduates. His programme of 100 frames took 10 hours (or 1/6 frame per minute) which suggests that his steps were similar in size to those of frame 64 onwards in this programme. The lecture group were given 10 x 1 hour periods and there was no significant difference between post test results for the groups.

A U.S. Navy experiment using a science programme found a 70% saving in time using programmed material (Mayo and Longo 1966) but there was no significant difference in performance. The subjects were navy recruits, but although age was not reported it can be assumed that they were young adults. There does not seem to be any evidence that older students (circa 40 years) perform any worse in a programmed learning situation than younger adults (Belbin and Downs 1966).

The range of time taken by the students on this programme is large (115 min to 375 mins.). Was this due to more thorough working during the programme or not? More thorough working would be supported by better gains on the programme, and test 6 (correlation time on programme v. gain score) was to verify this. As there was no significant correlation this idea must be rejected. The longer time taken is probably due to lack of practice in algebraic manipulation by those students who have not recently studied mathematics.

One might also hypothesise from this, that a student possessing an 'O' level pass in Mathematics would perform better on the programme than one without it. The correlation of Mean Gain Scores v '0' level maths is found to be zero (test 7) which rules out the hypothesis.

If a student has not a great deal of algebraic facility this should be shown in the Pre Check II test by a failure, and this handicap would reasonably lead to a poorer gain score on the programme. Test 8 found negligible correlation between Mean Gain scores and Pre Check II results. Thus we cannot distinguish between the students who passed or failed on Pre Check II by looking at the Gain Scores. This would suggest that (a) the Pre Check II is unnecessary, and (b) the worked answers on the programme are helping just those students for whom they were written.

The careful reading and working through of these answers must slow down these students, and we would expect them to take longer to work right through the programme. This is borne out by test 9 which gives a significant correlation of + 0.61 between Pre Check II results and Mean Time.

As these weak students seem to fare no worse on the programme, it could be used for individual tuition without a supervisor. Provided he is given sufficient time, the student should achieve similar scores to those working through more quickly.

Looking at table 10, page 53, enables a comparison to be made between the post test scores for each question in each group. A correlation of these scores, between groups, should enable a comparison of the efficiency of each method of teaching to be made. Test 10 in the Statistical Tests (page 61) shows that there is high correlation between the numbers of students who passed in each question between the control and experimental groups. This would indicate that the programme can perform as well as the teacher, but not better. It is worth noting at this point that the teacher in this case is the same as the programmer.

As there seems to be little to choose between this programme and conventional teaching we must ask whether it can be improved in any way and made more efficient.

Experimenters in the field of Programmed Learning have tried to find some index of efficiency for a programme. One quoted (Poppleton and Austwick 1964) is

$$\frac{\text{Mean Post Test Score}}{\text{Mean Time Taken (min.)}} \times 100.$$

As the possible Post Test Score varies between programmes, this could really only be used to compare the results of the use of the same programme at different times.

A gain ratio is suggested by McGuigan (1963) in the form

$$\frac{m_2 - m_1}{p - m_1}$$

where m_2 = Mean Post Test score

m_1 = Mean Pre Test score

p = Maximum Test score.

This has a maximum value of 1 when $m_1 = 0$ and $m_2 = p$. However, it is pointed out (Blake 1966) that this will have a value of 1 whenever $m_2 = p$, whatever value m_1 has. Thus this does not really take pre learning into account and Blake suggests the addition of a term giving

$$G.R. = \frac{m_2 - m}{p - m_1} + \frac{m_2 - m_1}{p}$$

This has a maximum value of 2.

Blake has found that his programmes seem to be satisfactory with a G.R. > 1.2.

For this programme, using $m_1 = 1.79$, $m_2 = 13.25$ and $p = 16$ the G.R. = 1.52, which would seem to comply with Blakes' requirements.

A decrease in step size could be made in some places, as there are still a number of frames with a high (> 10%) error rate (see table 8, p.50). Smith and Moore (1962) found that the efficiency of a spelling programme did not alter with larger step size, but whether those steps can be compared with steps on this programme is questionable. One is disturbed by an error rate of 12/33 for frame 80 however. This is a long frame, and it seems that the remedy would be to cut it into parts. A suggested expansion into 4 frames for this is given in Table 11, p.67.

As this last section of the programme (frame 76 onwards) has only been tested previously with 15 capable students (1965) it would be as well to reduce the step size for a number of these frames (82, 84, 86) and introduce another 3 practice frames.

The high error rate for frame 12, indicates a lack of appreciation of the definition of a square root, and this should be clarified by the insertion of a few frames before 12, e.g. 11, 12, 13, 14 of the suggested loop sequence in Table 12, pages 72, 73, 74. Frame 14 requires that some previous encounter with real numbers in the programme is required, other than in the first frame. A frame below 11 could be inserted in the form:-

Table 11. Frame 80 Revision

	Q.80. Combine the complex fractions over a common denominator of $(1 - i)(-1 + i7)$ and simplify the three products which you make $\frac{(2 + i)}{(1 - i)} + \frac{(3 - i)}{(-1 + i7)}$
A.80. $\frac{(2 + i)(-1 + i7) + (3 - i)(1 - i)}{(1 - i)(-1 + i7)}$ $= \frac{(-0 + i13) + (2 - i4)}{(6 + i8)}$ $= \frac{-7 + i9}{6 + i8}$	**Q.81** Rationalise A.80 and simplify the result.
A.81. $\frac{-7 + i9}{6 + i8} - \frac{(6 - i8)}{(6 - i8)}$ $= \frac{30 + i110}{36 + 64} = 0.3 + i1.1.$	**Q.82.** If $(a + ib)$ is identical with $(0.3 + i1.1)$, what is the value of b?
A.82. If the two complex numbers are identical their real and imaginary parts must match. $(a \quad + \quad ib)$ $(0.3 + i1.1)$ so $b = 1.1$.	**Q.83.** Rationalise and simplify $\frac{(2 + i5)}{(1 - i3)}$ If the answer is identical to the complex number $x + iy$, what value must x have?
A.83. $\frac{(2 + i5)}{(1 - i3)} \cdot \frac{(1 + i3)}{(1 + i3)} = \frac{-13 + i11}{10}$ $= -1.3 + i1.1$ x must be the real part of the answer $\therefore x = -1.3$	

Q.10.	A.10.
What kind of numbers are all of these? $1, 3, -6, -1.7, \frac{24}{25}, -1\frac{9}{11}$ 2.731, 4,000.	Real numbers.

Frame 16 and 17 seem to have been badly worded with insufficient prompting. These could be written.

Q.16.	A.16.
Write out i^3 in full and then simplify it using the fact that $i^2 = -1$.	$i^3 = i \times i \times i$ $= i^2 \times i$ $= -1 \times i$ $= -i$.
Q.17.	**A.17.**
If i^3 is the same as $-i$ is i^3 real or imaginary?	Imaginary. It is a multiple $(-1x)$ of i.

Question 26 was insufficiently prompted, as the words 'complex number' did not carry over from the previous frame. The letters c..... and n..... could be used as additional prompts, though this might not be improving efficiency, as there are no errors on the following frame involving identification of complex numbers. Frame 42 had a high error in 1965, and was left unchanged in the hope that students would have a better appreciation of $i^2 = -1$ this time. It appears to be insufficiently prompted. It could be replaced by

<u>Q.42.</u>	<u>A.42.</u>
If $i \times i = i^2 = -1$	$i \times i4 = i^2 \times 4$
what is $i \times i4$?	$= -1 \times 4 = -4.$

and this could be followed by the present frame 42 as it stands.
Question 43 also needs some preliminary work without the complication of numbers. This could be inserted before it

<u>Q.43.</u>	<u>A.43.</u>
What is the value of $-i \times i$?	$-i \times i = -i^2$
	$= -(-1) = 1.$

The students who obtained the wrong answer for frame 44 did not appreciate that multiplication is commutative and that the numbers must be taken together and the i left alone.. A simpler frame might be

<u>Q.44.</u>	<u>A.44.</u>
What is $2 \times i3$?	$2 \times i3 = 2 \times i \times 3 = i \times 2 \times 3$
	$= i6.$

followed by

What is $2 \times -i5$?	$2 \times -i5 = -i10.$

followed by 44 as it is.

It would seem from frame 65, that the generalisation is called for too soon. An interchange of 65 and 66 with a slight rewarding might accomplish the extra practice needed.

Frame 64 is straightforward instruction, 65 could be

Q.65.

Simplify this quotient by multiplying by the quantity inside the large brackets

$$\frac{(4 + i)}{(2 - i6)} \times \left[\frac{2 + i6}{2 + i6}\right]$$

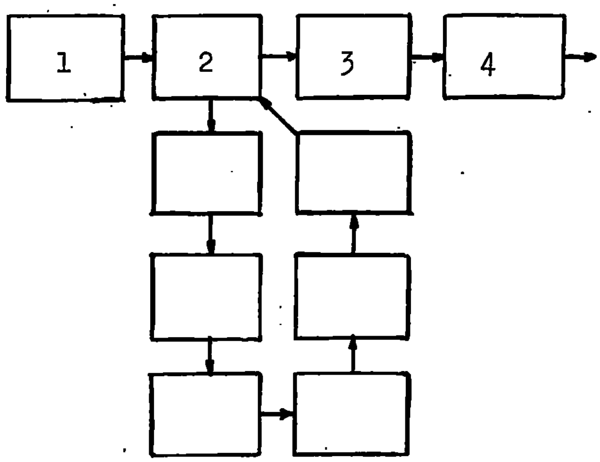
A.65.

$$\frac{(4 + i)\ (2 + i6)}{(2 - i6)\ (2 + i)} = \frac{2 + i26}{4 + 36}$$

$$= \frac{2 + i26}{40} = (0.05 + i0.65)$$

Frame 65 would then become the new frame 66, with the last line altered to 'What must this quantity be?'. These measures would almost certainly reduce the error rates in the frames mentioned, but it is questionable whether the overall efficiency would be increased (Austwick, 1965).

An alternative approach would be to introduce 'wash back' loops. The wrong answers to these frames nearly all indicate some lack of specific knowledge at that point. A loop sequence could be inserted where difficulties occur (Bjerstedt 1965).



This diagram illustrates the technique. A failure at a particular frame would lead to the loop (still linear) sequence which finishes at the same frame. Such loops could be printed on yellow paper following the particular frame, and students who obtained a correct answer could be told to omit the yellow pages. This coloured paper approach has been carried out at two levels on a 'skip' branching programme with degree level electronics for army officers at R.M.C. of Science (Duncan 1965). It was found in that experiment that Programmed Learning performed no better than Conventional lecturing. A 'bypassing' experiment (Campbell 1963) allowed pupils in one group to 'skip' after certain frames if they were correct, and had to go through a loop sequence if they were not. These were compared with another group who worked through all the frames, including the loops. Learning was no more efficient either way.

A skip branching programme on logarithms (Hartley 1965) where the main programme was supported by remedial linear sequences was thought to be more effective than a linear programme for 13 year old Secondary Modern School girls who were "not homogenous in sophistication, ability or pre-knowledge".

The proposal here however is almost in the same category. The linear programme, as a whole, teaches, but it could become more effective for less able students with several remedial loops.

For example, the most common error occuring on frame 12 was to confuse $\sqrt{-a} \times \sqrt{-a}$ with $-\sqrt{a} \times -\sqrt{a}$, leading to answers of a or a^2. The student needs to be shown that the product of two identical square roots always gives the number under the $\sqrt{\ }$ sign.

A specimen loop is given in Table 12, pages 72 - 74, and this, of course would follow question 12. This has not yet been tried with students.

Table 12. Specimen Loop Sequences

A.12. -a If you obtained the wrong answer for this, or are not satisfied as to why this is the answer, work through the yellow pages following. If you were correct, omit the yellow pages and go directly to Q.13.	**L.Q.1.** What are the square roots of 16?
L.A.1. +4 or -4 i.e. $+4 \times +4 = 16$. and $-4 \times -4 = 16$.	**L.Q.2.** Using the special $\sqrt{\ }$ sign, we may say that $+4 = \sqrt{16}$ and $-4 = \sqrt{16}$ What is $\sqrt{25}$?
L.A.2. $\sqrt{25} = +5$ or -5.	**L.Q.3.** How can we check that $17 = \sqrt{289}$?
L.A.3. By multiplying 17 by itself i.e. $17 \times 17 = 289$.	**L.Q.4.** What number has a square root of 15 ?
L.A.4. $15 \times 15 = 225$.	**L.Q.5.** If $12 = \sqrt{x}$, what value has x ?

L.A.5.

The equation states that 12 is a square root of x, so x must be 12 x 12

$= 144$.

L.Q.6.

If $3 = \sqrt{p}$

What is p ?

L.A.6.

$p = 3 \times 3 = 9$.

L.Q.7.

If $3 = \sqrt{p}$

What is $\sqrt{p} \times \sqrt{p}$?

L.A.7.

$\sqrt{p} \times \sqrt{p} = 3 \times 3 = 9$.

This could be written as

$(\sqrt{p})^2 = 3^2 = 9$.

L.Q.8.

Look at L.A.6. and L.A.7.

Does $p = (\sqrt{p})^2$?

L.A.8.

Yes, they are both equal to 9.

$p = 9$

$(\sqrt{p})^2 = 9$

L.Q.9.

Is $(\sqrt{t})^2 = t$?

i.e. $\sqrt{t} \times \sqrt{t} = t$?

L.A.9.

Yes, if it is true for p, then it is true for any symbol, letter, or number instead of p.

L.Q.10.

If $(\sqrt{x})^2 = x$

does $(\sqrt{7})^2 = 7$?

L.A.10. Yes, we have merely replaced x by 7.	**L.Q.11.** What is $(\sqrt{10})^2$?
L.A.11. 10.	**L.Q.12.** What is $\sqrt{8} \times \sqrt{8}$? i.e. $(\sqrt{8})^2$?
L.A.12. $\sqrt{8} \times \sqrt{8} = (\sqrt{8})^2 = 8.$ This says that the square root of 8, multiplied by the square root of 8, is 8.	**L.Q.13.** What is $(\sqrt{-4})^2$? or $\sqrt{-4} \times \sqrt{-4}$?
L.A.13. -4 We are replacing the positive number by a negative number under the $\sqrt{}$ sign. The product of two square roots is still the number under the $\sqrt{}$ sign.	**L.Q.14.** What is $(\sqrt{-a})^2$? or $\sqrt{-a} \times \sqrt{-a}$?
L.A.14. $-a$	TURN OVER and continue with the white paper in the booklet.

There are a number of frames in the programme which call for multiple responses (10, 18, 19, 22 etc.). In scoring these for errors, they were counted correct if the majority of the answers in the frame were acceptable. Thus a student who found difficulty in the first part, may have turned the page to find the confirmation, or otherwise, of his answer. Seeing the correct method, he would then tend to score correctly on the remaining parts of the frame. This appears to have happened in a number of cases and the first error is thus concealed. Multiple responses were used in the programme to provide practice in a particular process and save space. It would seem that these should only be used after adequate instruction in the process has been given and are truly for practice. This delay in the knowledge of the results should act as a good reinforcer to a successful student.

General Comment

Most of the students expressed great interest in the experiment, not having come into contact with Programmed Instruction before. Several 'non-mathematical' students regarded it in the same light as a crossword puzzle and showed lack of tension, even though they had to think consciously about signs and rules of algebra. The fact that "no one else sees your daft mistakes" was mentioned specifically by eight students as a reason for their enthusiasm. The 'Hawthorne' effect was certainly present in both groups, as the control group knew they were being compared with the programmed groups (Austwick 1966). Most showed a determination to finish the programme, even though it was of no relevance to their studies, and regarded it as a demonstration of teaching method, rather than mathematics. The introduction of Programmed Learning into the curriculum of a College of Education is advocated (Curr 1963) for three reasons. First, a student constructing programmes is acquiring a skill which can be brought into everyday use later. Second, the painstaking analysis required, followed by the building of programmes leads to a better understanding of theories of learning. Thirdly, students would acquire standards which would enable them to evaluate critically the commercial programmes with which they may come into contact.

After having written the programme, it was difficult to present the material to the conventional teaching group without incorporating the programmed approach. It would have been more useful, though not possible in this situation, to have had an independent teacher who had not seen the programme at all. This would have given a more reliable comparison of the two methods of teaching.

During the administration of the programme, students complained that the amount of space allocated for an answer in their booklet was not adequate in many cases. They were told to ignore the horizontal duplicated lines if necessary but it does seem that a larger answer booklet is needed.

In the typing of the programme it was found difficult to accommodate some of the frame material and answers in the small $3\frac{1}{2}$ in. x $2\frac{1}{2}$ in. format of the booklet (e.g. Q.9., A.75, A.80). An increase in depth of $\frac{1}{2}$ inch would probably be sufficient to accommodate even such large frames as these. A clear layout of material helps the students, whereas a crowded frame which has no distinctive layout tends to make him uncertain where to begin, even though common sense tells him to follow the sequence from the beginning.

The students also found that the programme booklets would not lie flat and remain open at a required page. It was rather frustrating to be constantly losing ones place. This criticism could be met by reverting to the original plan of using loose leaf pages in a ring binder.

A further criticism in the administration of the programme, is that it was not divided into groups of frames, sufficient for a lesson. A student made a note of where he stopped and went on from there next time. This often entailed a few minutes wasted time looking back into the programme for a particular frame for revision purposes. It would have been better to have had the programme in about six sections, the last five beginning with a number of review frames, so that time was spent in directed revision, rather than haphazard searching for help.

LINEAR PROGRAMME ON COMPLEX NUMBERS.

PRE PROGRAMME CHECK.

NAME .... YEAR...................

GROUP............... DATE...................

Please attempt all the questions. If you cannot answer any question put a dash beside it.

1. Have you any previous knowledge of complex numbers?..................
2. Do you attach any significance to i ? ....
3. If so, what is i^2 ? ....
4. What is $\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2}$? ....
5. What is $(\sqrt{3})^5$? ....
6. What is $\sqrt{a} \times \sqrt{a} \times \sqrt{a}$? ....
7. What is $(\sqrt{p})^7$? ....
8. If $d^2 = -1$, what is d^6 ? ....
9. What is the product of $(3a + 2b)$ and $(5a + 3b)$? ....

   ....
10. What is the product of $(4a + b)$ and $(4a \div b)$? ....

    ....
11. Solve the quadratic equation $(3a - 1)\ (2a + 5) = 0$ ....

    ....
12. Do you know a formula for solving a quadratic equation ? ....
13. If so, quote it ....
14. Use the formula to solve the quadratic equation $x^2 - 3x + 6 = 0$........

    $(\sqrt{15} = 3.87)$

APPENDIX II

LINEAR TEACHING PROGRAMME ON COMPLEX NUMBERS.

(POST PROGRAMME TEST)

NAME.................................... YEAR.....................

GROUP....................Training for INFANT / PRIMARY / SECONDARY (Please cross out the two which do not apply)

	ANSWER	LEAVE BLANK
Q 1. If $i^2 = -1$ what are a) i^3 ? b) i^6 ? c) i^9 ?		
Q 2. Which of these is a complex number? a) 3a + 4b b) 27g c) 4n + ib d) p - 3g		
Q 3. Use the formula to find the complex root of the equation $x^2 - 4x + 13 = 0$		
Q 4. Simplify $(3R - iw) + (5R + i3w) - (i6w + 2R)$		
Q 5. Find the product $(4R - iwL)(7R + i\,2wL)$		
Q 6. Write down the complex conjugate of $(3p - i4q)$ and find the product of the two numbers.		
Q 7. Rationalise and simplify $\frac{(10 - i7)}{(4 + i3)}$		
Q 8. Simplify $\frac{(3a + i\,4b)}{(b + ia)}$ If the result is to be wholly real, what must be the connection between a and b ?		

LINEAR TEACHING PROGRAMME

COMPLEX NUMBERS

PART 1

	Q.1. A real number is any one (positive or negative) which you have used before in calculations. Write any three real numbers.
A.1. Your answer is bound to be correct !	**Q.2.** What are the values of these real numbers ? (a) $\sqrt{36}$ (b) $\sqrt{25}$ (c) $-\sqrt{25}$
A.2. (a) ± 6 (b) ± 5 (c) ± 5	**Q.3.** Can we evaluate $\sqrt{-16}$?
A.3. No. Squaring any real number never gives a negative result.	**Q.4.** $\sqrt{-16}$ can be written as $\sqrt{16} \times \sqrt{-1}$. Write these numbers in the same way. (a) $\sqrt{-24}$ (b) $\sqrt{-49}$ (c) $\sqrt{-100}$

A.4.

(a) $\sqrt{24} \times \sqrt{-1}$

(b) $\sqrt{49} \times \sqrt{-1}$

(c) $\sqrt{100} \times \sqrt{-1}$

Q.5.

As writing $\sqrt{-1}$ continually becomes tedious after a while, we denote it by a special symbol which is quicker to write. This is 'i'.

What does 'i' equal ?

A.5.

$\sqrt{-1}$

Q.6.

Using the 'i' notation,

$\sqrt{-64} = \sqrt{64} \times \sqrt{-1} = \pm 8 \times i$

$= \pm 8i.$

Rewrite $\sqrt{-100}$ in the same way.

A.6.

± 10

Q.7.

In the following, evaluate the real numbers and rewrite the others using the 'i' notation.

(a) $\sqrt{-49}$ (b) $\sqrt{25}$ (c) $\sqrt{-144}$

(d) $-\sqrt{16}$ (e) $\sqrt{-25}$ (f) $\sqrt{-1.44}$

A.7.

(a) $\pm 7i$ (b) ± 5 (c) $\pm 12i$

(d) ± 4 (e) $\pm 5i$ (f) $\pm 1.2i$

Q.8.

As $\sqrt{-1}$ has no real number value i is called an imaginary quantity, and any multiple of it is taken to be imaginary too.

Is 7i imaginary ?

A.8.

Yes, it is a multiple of

i and therefore imaginary.

Q.9.

To distinguish imaginary quantities from real ones it is usual to write the 'i' first to avoid confusion. Thus, 7x would be taken to represent a real number, but i7x is ....

Write the missing word on your answer sheet.

A.9.

Imaginary

Q.10.

Which of these numbers is a multiple of i, and thus imaginary ?

(a) $\sqrt{-11}$ (b) $-\sqrt{13}$ (c) i24

(d) 15 y (e) $i\sqrt{19}$ (f) i3.67.

A.10.

(a)
(c)
(e)
(f)

Q.11.

What is $\sqrt{a} \times \sqrt{a}$?

or $(\sqrt{a})^2$

A.11.

a

Q.12.

What is $\sqrt{-a} \times \sqrt{-a}$?

or $(\sqrt{-a})^2$

A.12. −a	**Q.13.** What is the value of i^2 ? (or $\sqrt{-1}$ x $\sqrt{-1}$)
A.13. −1	**Q.14.** What kind of number is −1 ?
A.14. Real	**Q.15.** As $i^2 = -1$, then i^2 is also a ...... quantity. Supply the missing word.
A.15. Real i^2 is not a multiple of i. It is a power or exponent of i.	**Q.16.** What is the value of i^3 ? (in terms of i)

A.16. $i^3 = i \times i \times i$ $= i^2 \times i$ $= -1 \times i$ $= -i.$	**Q.17.** Is i^3 real or imaginary ?
A.17. Imaginary	**Q.18.** Evaluate these exponents of i in terms of 1 or i, (a) i^4 (treat this as $i^2 \times i^2$) (b) i^5 (c) i^6
A.18. (a) $i^4 = i^2 \times i^2 = -1 \times -1$ $= 1.$ In a similar way, (b) $i^5 = i.$ (c) $i^6 = -1.$	**Q.19.** Evaluate, in terms of 1 or i, (a) $i^3 \times i^6$ (b) $i^5 \times i^7$ (c) $i^3 \times i^4 \times i^8$.
A.19. (a) $i^3 \times i^6 = -i \times -1 = i$ (b) 1 (c) -i	**Q.20.** What are the two possible values of i^n if n is an even number ?

A.20. +1 or -1	**Q.21.** What are the two possible values of i^n if n is odd ?
A.21. +i	**Q.22.** Simplify 6 + 7. What kind of numbers are these ?
A.22. 13 Real numbers.	**Q.23.** Simplify i5 + i9. What kind of numbers are these ?
A.23. i14 Imaginary numbers.	**Q.24.** Why can we not simplify 3 + i6 ?

A.24. One number (3) is real, and the other (i6) is imaginary. These will not combine to form one kind of number.	**Q.25.** A number (3 + i6) representing the sum of a real and an imaginary number is called a COMPLEX NUMBER. Form a complex number from 7 and i4.
A.25. 7 + i4	**Q.26.** (8 - i3) is the sum of 8 and -i3. Thus (8 - i3) is also a ....
A.26. Complex number.	**Q.27.** Which of these are complex numbers ? (a) 20 - 3y. (b) 2 + i7. (c) a - ib.

A.27.

(b) and (c)

Q.28.

Solve the quadratic equation $x^2 - 2x - 15 = 0$ using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

A.28.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times (-15)}}{2 \times 1}$$

from which x = 5 or -3

Q.29.

Use the formula to solve the equation $x^2 + 2x + 2 = 0$ as far as you can.

A.29.

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$= \frac{-2 \pm \sqrt{-4}}{2}$$

Q.30.

The answer to Q.29. involved the term $\sqrt{-4}$. What kind of number is this ?

A.30.

Imaginary.

Q.31.

Use the 'i' notation for the imaginary part of A.29 and then complete the Arithmetic.

LINEAR TEACHING PROGRAMME

COMPLEX NUMBERS

PART 2

<u>A.31.</u>

$$x = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm i\sqrt{4}}{2}$$

$$= \frac{-2 \pm i2}{2} = -1 \pm i$$

<u>Q.32.</u>

What kind of numbers are represented by $(-1 \pm i)$?

<u>A.32.</u>

Complex Numbers.

<u>Q.33.</u>

The values of x obtained from an equation are called the roots of the equation. If the values obtained are complex numbers, the equation may be said to have .... roots.

<u>A.33.</u>

Complex.

<u>Q.34.</u>

Use the formula to find the complex roots of the equation $x^2 + 4x + 5 = 0$.

<u>A.34.</u>

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1}$$

from which $x = -2 + i$

or $-2 - i$

<u>Q.35.</u>

Find the complex roots of the equation $x^2 + x + 1 = 0$ and then underline the imaginary parts of these. ($\sqrt{3} = 1.732$)

A.35. The complex roots are $-0.5 \pm i0.866$	**Q.36.** Add (3 + i4) to (7 + i2) by adding the real and imaginary parts separately.
A.36. Real Parts 3 + 7 = 10. Imaginary Parts i4 + 12 = i6 The complex number formed is (10 + i6)	**Q.37.** What is the sum of (2 + i3.5) (3 - i0.5) and (-1 + i) ?
A.37. (4 + i4)	**Q.38.** In the same way, subtract (2 + i9) from (8 + i10)
A.38. (6 + i)	**Q.39.** Subtract (3 + i8) from 1 + i2)

A.39. (-2 - i6)	**Q.40.** Evaluate (2 + i3) + (4 - i6) - (4 - i3) Is the answer a complex number ?
A.40. 2 No, the answer does not contain an 'i' term, the the number is real.	**Q.41.** What is the real number value of i^2 ?
A.41. -1	**Q.42.** Evaluate i2 x i6
A.42. i2 x i6 = i^2 12 = -1 x 12 - -12	**Q.43.** What is -i2 x i4 ?

A.43.

$-i2 \times i4 = -i^2 \times 8$

$= -(-1) \times 8$

$= +1 \times 8 = 8$

Q.44.

What is $-i3 \times 2.1$?

A.44.

$-i6.3$

Q.45.

Expand $(3 + i2)(4 + i3)$ by the ordinary method for binomials and simplify the answer.

What kind of number is the answer?

A.45.

$(3 + i2)(4 + i3)$

$= 3\ (4 + i3) + i2\ (4 + i3)$

$= 12 + i9 + i8 + i^2 6$

$= 12 + i17 - 6$

$= (6 + i17)$ This is a complex number.

Q.46.

Find the product of $(2 + i6)$

and $3 - i2)$

A.46.

$(18 + i14)$

Q.47.

What complex number is given

by $(4 - i)(2 - i4)$?

A.47. (4 - i18)	**Q.48.** What is the square of (3 - i) i.e. (3 - i)(3 - i) Is the answer complex?
A.48. (8 - i6) Yes, this is complex.	**Q.49.** Evaluate (4 - i2)(4 + i2) Is the result a complex number?
A.49. 20 This is not a complex number.	**Q.50.** Change the sign of the imaginary part of the complex number (3 + i4) to the opposite one.
A.50. (3 - i4)	**Q.51.** Multiply (2 - i3) by the complex number which has the same real part, but the imaginary part has the opposite sign.

A.51.

$(2 - i3)(2 + i3)$
$= 2(2 + i3) - i3(2 + i3)$
$= 4 + i6 - i6 - i^2 9$
$= 4 - (-1 \times 9)$
$= 4 + 9$
$= 13.$

Q.52.

What is the product of the

COMPLEX CONJUGATES

$(a - ib)$ and $(a + ib)$?

A.52.

$a^2 + b^2$

Q.53.

The complex conjugates of

$(3 + i2)$ is $(3 - i2)$.

Write the complex conjugates of

$(9 + i4)$.

A.53.

$(9 - i4)$

Q.54.

The complex conjugates of

$(2 - i5)$ is $(2 + i5)$.

Write the complex conjugates of

$(x - iy)$.

A.54.

$(x + iy)$

Q.55.

How do we obtain the complex

conjugates of a complex number ?

<u>A.54.</u> $(x + iy)$	<u>Q.55.</u> How do we obtain the complex conjugates of a complex number ?
<u>A.55.</u> By changing the sign of the imaginary part of the complex number.	<u>Q.56.</u> Write the complex conjugates of $(-7 - i6)$.
<u>A.56.</u> $(-7 + i6)$	<u>Q.57.</u> Multiply $(6 - i3)$ by its complex conjugates.
<u>A.57.</u> $(6 - i3)(6 + i3) = 6^2 + 3^2$ $= 45.$	<u>Q.58.</u> What is the product of $(-2 + i3)$ and its complex conjugate ?

<u>A.58.</u>

$(-2 + i3)(-2 - i3)$

$= (-2)^2 + 3^2$

$= 13.$

<u>Q.59.</u>

Multiply each number by its complex conjugate.

(a) $(3 - i4)$

(b) $(-2 + i5)$

(c) $(0.4 + i0.7)$.

<u>A.59.</u>

(a) $(3 - i4)(3 + i4)$
$= 3^2 + 4^2 = 25.$

(b) $(-2 + i5)(-2 - i5)$
$= 2^2 + 5^2 = 29.$

(c) $(0.4 + i0.7)(0.4 - i0.7)$
$= 0.4^2 + 0.7^2$
$= 0.16 + 0.49 = 0.65.$

<u>Q.60.</u>

What kind of number does the product of complex conjugates always produce ?

<u>A.60.</u>

A real one.

<u>Q.61.</u>

Add $(3 + i4)$ to its complex conjugate.

Is the answer real, complex or imaginary?

<u>A.61.</u>

$(3 + i4) + (3 - i4) = 6$

This is real.

<u>Q.62.</u>

Add each of the following to its complex conjugate.

(a) $(1.2 - i0.6)$ (b) $(-2.5 - i\,1.6)$

(c) $(a + ib)$ (d) $(0 + i4)$

(e) $(R + i\omega L)$ (f) $- i7$

LINEAR TEACHING PROGRAMME

COMPLEX NUMBERS

PART 3

A.62.

(a)	2.4	(b)	-5
(c)	2a	(d)	0
(e)	2R	(f)	0

Q.63.

What kind of number is produced whenever complex conjugates are added together ?

A.63.

A real one.

Q.64.

Multiply the numerator and denominator of this quotient by the complex conjugates of the denominator

$$\frac{(-7 + i8)}{(1 + i3)}$$

A.64.

$$\frac{(-7 + i8)(1 - i3)}{(1 + i3)(1 - i3)}$$

$$= \frac{17 + i29}{10}$$

$$= (1.7 + i2.9)$$

Q.65.

This process of multiplying the numerator and denominator by the same quantity is called RATIONALISATION.

What must the quantity be?

A.65.

The complex conjugates of the denominator.

Q.66.

Rationalise this quotient by multiplying by the quantity inside the large brackets

$$\frac{(4 + i)}{(2 - i6)} \left[\frac{2 + i6}{2 + i6}\right]$$

A.66.

$$\frac{(4 + i)}{(2 - i6)} \cdot \frac{(2 + i6)}{(2 + i6)}$$

$$= \frac{2 + i26}{4 + 36} = (0.05 + i0.65)$$

Q.67.

Rationalise this quotient and then evaluate the result.

$$\frac{(4 - i3)}{(5 + i5)}$$

A.67.

$$\frac{(4 - i3)(5 - i5)}{(5 + i5)(5 - i5)}$$

$$= \frac{5 - i35}{25 + 25} = (0.1 - i0.7)$$

Q.68.

What always happens to the denominator of a complex quotient after it has been rationalised ?

A.68.

It becomes a real number.

Q.69.

What can we do with the numerator because the denominator is real ?

A.69.

We can divide it by the real number and the expression is simplified.

Q.70.

Can you see why we have to rationalise a complex quotient ?

A.70.

To make the denominator a real number. We cannot divide by a complex number, but we can divide by a real number.

Q.71.

Why are <u>both</u> numerator and denominator of a complex quotient multiplied by the complex conjugate of its denominator?

A.71.

The ratio of the multiplying factor is 1, and this does not affect the total value of the quotient.

Q.72.

(3 - i2) divided by (7 + i) may be written in the form of a quotient.

Rationalise and evaluate this quotient.

A.72.

$$\frac{(3 - i2)}{(7 + i)} \cdot \frac{(7 - i)}{(7 - i)}$$

$$= \frac{19 - i17}{49 + 1} = (0.38 - i0.34)$$

Q.73.

Divide (7 - i4) by (2 + i) in the same way.

A.73.

$$\frac{(7 - i4)}{(2 + i)} \cdot \frac{(2 - i)}{(2 - i)}$$

$$= (2 - i3)$$

Q.74.

Find the product of (6 + i5) and (3 - i2) and divide the result by (2 + i2)

A.74.

$$\frac{(6 + i5)(3 - i2)}{(2 + i2)} \cdot \frac{(2 - i2)}{(2 - i2)}$$

$$= \frac{62 - i50}{8}$$

$$= (7.75 - i6.25)$$

Q.75.

Evaluate $\left[\frac{2 + i}{1 + i2}\right]^2$

A.75.

$$\frac{(2+i)^2}{(1+i2)^2} = \frac{(3+i4)}{(-3+i4)}$$

$$= \frac{(3+i4)}{(-3+i4)} \cdot \frac{(-3-i4)}{(-3-i4)}$$

$$= \frac{7-i24}{3^2+4^2} = \frac{7-i24}{25}$$

$$= (0.28 - i\,0.96)$$

Q.76.

Evaluate

$$\frac{(-3+i3)(-2+i2)}{(3+i)(2+i)}$$

A.76.

$$\frac{(-3+i3)(-2+i2)}{(3+i)(2+i)} = \frac{-i\,12}{(5+i5)}$$

$$= \frac{-i12\,(5-i5)}{(5+i5)(5-i5)} = \frac{-60-i60}{50}$$

$$= (-1.2 - i\,1.2)$$

Q.77.

Evaluate the quotient $\frac{(10+i4)}{(1-i)}$ and write down the real part of the answer.

A.77.

The evaluation is $(3 + i7)$

The real part of this complex number is 3.

Q.78.

What is the real part of

$$\frac{(7+i)^2}{(4+i2)} \quad ?$$

A.78.

$$\frac{(7+i)^2}{(4+i2)} = \frac{(48+i\,14)}{(4+i2)}$$

$$= \frac{(48+i14)}{(4+i2)}\,\frac{(4-i2)}{(4-i2)}$$

$$= \frac{220-i\,40}{4^2+2^2} = 11 - i2$$

The real part is 11.

Q.79.

Find the product of $(2 - i4)$ and $(1.5 + i2.5)$, and write down the imaginary part of the answer.

A.79.

The product is (13 - i)

The imaginary part of this is - i.

Q.80.

If (a + ib) is identical to

$$\frac{(2 + i)}{(1 - i)} + \frac{(3 - i)}{(-1 + i7)}$$

what is the value of b ?

Put the fraction on a common denominator of (1 - i)(- 1 + i7)

A.80.

$$\frac{(2 + i)(-1 + i7) + (3 - i)(1 - i)}{(1 - i)(-1 + i7)}$$

$$= \frac{(-7 + i9)}{(6 + i8)}$$ (Rationalise)

= 0.3 + i1.1 This is identical

with (a + ib) if a= 0.3

and b= 1.1

Q.81.

Evaluate the product

(a + ib)(3a - i4b)

A.81.

$(3a^2 + 4b^2) - iab$

Q.82.

If in A 81, a and b have values such that $3a^2 + 4b^2 = 0$, what kind of number is left ?

A.82.

The answer becomes (0 - iab) which is entirely imaginary.

Q.83.

Find the product of (2x + iy) and (5x - i6y), and then write the connection between x and y so that the answer is imaginary.

A.83.

The product is $10x^2 + 6y^2 - i7xy$.
If $10x^2 + 6y^2 = 0$, only $- i7xy$ is left, and this is Imaginary.

Q.84.

If $(m + i2n)(5n + i3m)$ is entirely real, what part of it must equal 0 ?

A.84.

The product is $-mn + i(10n^2 + 3m^2)$
This is real if the imaginary part is 0.

i.e. $10n^2 + 3m^2 = 0$.

Q.85.

Rationalise:-

$$\frac{(t + is)}{(3t - i4s)}$$

A.85.

$$\frac{3t^2 - 4s^2 + i7st}{9t^2 + 16s^2}$$

Q.86.

If this answer (A 85) is to be wholly imaginary, what relationship must exist between s and t ?

A.86.

The real part must be 0 if the whole expression is imaginary.

i.e. $3t^2 - 4s^2 = 0$

from which $t = \frac{\pm 2s}{\sqrt{3}}$ or $s \pm \frac{\sqrt{3}t}{2}$

LINEAR TEACHING PROGRAMME ON COMPLEX NUMBERS

PRE TEST This test must be taken before the programme.

NAME .... **YEAR IN COLLEGE** ....

GROUP .... Training for INFANT / PRIMARY / SECONDARY (Please cross out the two which do not apply)

SCORE

QUESTION	ANSWER	LEAVE BLANK
1. When 'i' is used in imaginary numbers what does it represent ?		
2. What is i^a ?		
3. What kind of number is i7 ?		
4. What kind of number is 24 ?		
5. Which of these is a complex number ? a) 7g + 2s : b) 2 + 3p c) 4p - i3q : d) $x^2 + y^2$		
6. Use the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find the complex roots of the equation $x^2 - 6x + 10 = 0$		
7. Simplify (x - i4y) + (9x - i2y)		
8. Subtract (2p + iq) from (7p - iq)		
9. Find the product (5p - i3ab)(2p + i4ab)		
10. Write the complex conjugate of (2a + i3b)		
11. Rationalise the quotient $\frac{(8 + i3)}{(3 + i)}$ and simplify your answer as far as possible.		
12. If $27a^2 + i3b + 8b^2 - i9a$ has to be a completely real quantity, what is the connection between a and b ?		

LINEAR TEACHING PROGRAMME ON COMPLEX NUMBERS

POST TEST This test must be taken **after** the programme.

NAME .... YEAR IN COLLEGE ....

GROUP .... Training for INFANT / PRIMARY / SECONDARY (Please cross out the two which do not apply)

Score

QUESTION	ANSWER	LEAVE BLANK
1. What does 'i' represent when it is used in connection with imaginary numbers ?		
2. What is i^2 ?		
3. What kind of number is i6 ?		
4. What kind of number is 13 ?		
5. Which of these is a complex number ? a) 3a + 4b b) 5 - i7y c) p - 3g d) 2.7x + 3.9y		
6. Use the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find the complex roots of the equation $x^2 - 4x + 13 = 0$.		
7. Simplify (3R - iw) + (5R + i3w)		
8. Subtract (2x - i5y) from 5x - i2y)		
9. What is the product (4R - iwL)(7R - i2wL)?		
10. Write the complex conjugate of (3p - i4q)		
11. Rationalise the quotient $\frac{(10 - i7)}{(4 + i3)}$ and simplify the answer as far as possible.		
12. If $\frac{7ab + i(4b^2 - 3a^2)}{a^2 + b^2}$ has to be a completely real quantity, what is the connection between a and b ?		

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