REAL TIME CALCULATION OF THE HEAD RELATED TRANSFER FUNCTION BASED ON THE BOUNDARY ELEMENT METHOD

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ABSTRACT

In order to develop the 3D auditory display using a head phone which allow head movement, real time calculation of the head related transfer function (HRTF) is necessary. In the conventional studies, the calculations are performed approximately based on the mathematical model by regarding the head as the sphere shape. Although the boundary element method is also possible to calculate the transfer function by solving the wave equation from the accurate boundary condition of the head including the shape of the face and the ears measured by the 3D scanner, it is thought to be impossible to calculate it in real time because the amount of the calculation is too big. In this study, we discuss a new calculation method of the HRTF based on the reciprocity principle which enables significant speed-up of the calculation.

The demonstrations are available at the website:

http://acoust.archi.kyoto-u.ac.jp/HRTF/

1. INTRODUCTION

Generally, human obtains spatial informations through binaural listening, which is one of the functions of the phonoreception. Particularly, it is known that human perceives the direction of sound sources through the interaural sound level differences and time differences of the ear input signals [1].

The head-related transfer functions (HRTFs), which are the transfer functions between the sound source and listener's ears, contains such informations that the interaural sound level differences and time differences of the ear input signals. The spectral cues caused by these informations is said to be related to the sound localization. If a source signal and the HRTFs are convolved and the resulting signal can be precisely produced at both ears, it would be possible to let the listener perceive the sound source in that direction. In order to measure the HRTFs, we usually use a real head or a dummy head. However, because we can not measure the HRTFs in every directions, we have to predict the HRTFs in unmeasured direction. It is possible to interpolate HRTFs in the unmeasured direction from those in the measured directions [2]. However, if the HRTFs are needed for the other distance from the source, a number of measuring HRTFs are needed. Drastic approximation, such that the head shape is approximated to sphere or spheroids [3], is also realistic if we may ignore the difference between the front and the back.

On the other hand, it is possible to solve the wave equation strictly by using the finite/boundary element method [4, 5]. Although these methods can calculate HRTFs accurately, the calculation time is extremely long compared with the conventional calculation methods.

In this study, we describes the calculation method of the HRTFs by using the conventional BEM, the Chief method [6] which minimize the error caused at the internal eigen-frequencies and the reciprocity method which enables speeding up of the calculation. In the numerical study, these methods are compared and are discussed. Furthermore, we introduce the server system which calculate the HRTFs based on the BEM.

2. CALCULATING HRTFS BY USING THE BEM

The BEM which calculates the HRTFs is categorized as the external problem of the boundary integral equation. As the boundary condition, the boundary surface is given by the shape of the dummy head. By solving the integral equation, we obtain the acoustic pressure at ears when a sound source is located at a position considered.

2.1. Calculation of the acoustic pressure on the boundary surface

The BEM is a numerical method which solves the algebra equation by discretizing the boundary surface of the Kirchhoff Helmholtz integral equation expressed as follows:

$$\begin{aligned} C(\mathbf{s})p(\mathbf{s}) &= j\omega\rho\sum_{k=1}^{N} q_k G(\mathbf{r}'_k|\mathbf{s}) \\ &+ \iint_S G(\mathbf{r}|\mathbf{s})\frac{\partial p(\mathbf{r})}{\partial n} - p(\mathbf{r})\frac{\partial G(\mathbf{r}|\mathbf{s})}{\partial n} dS \\ C(\mathbf{s}) &= \begin{cases} 1/2 & (\mathbf{s}\in S) \\ 1 & (\mathbf{s}\in V) \\ 0 & (\mathbf{s}\notin V) \end{cases}, \end{aligned} \qquad (1)$$

where

S	:	Surface
V	:	Volume
$\mathbf{s}$	:	Position within the volume V
$\mathbf{r}'_k$	:	Point source position ($k = 1 \dots N$)
$\mathbf{r}$	:	Position on the surface S
p	:	acoustic pressure
G	:	the Green function
q_k	:	the volume velocity of the source k
ω	:	angular frequency
ρ	:	the air density
n	:	the normal vector.

In Eq.(1), the normal derivative is given as follows,

$$\frac{\partial p(\mathbf{r})}{\partial n} = -j\omega\rho v_n(\mathbf{r}) = -j\omega\rho y_n(\mathbf{r}),$$

where v_n and y_n are the particle velocity and the acoustic admittance in normal, respectively.

The boundary surface S is divided into M elements for discretization. In this study, we suppose the constant element which regard the physical parameters such as the acoustic pressure and the particle velocity are constant within the discretized element. Then ,if the j-th element is set to S_j and the center of S_j is defined as $\mathbf{r}_j$, Eq.(1) can be descretized as

$$\begin{aligned} &C(s)p(\mathbf{s}) \\ &= j\omega\rho\sum_{k=1}^{N} q_k G(\mathbf{r}'_k|\mathbf{s}) - \sum_{j=1}^{M} j\omega\rho v_n(\mathbf{r}_j)\iint_{S_j} G(\mathbf{r}|\mathbf{s})dS \\ &-\sum_{j=1}^{M}\left(\iint_{S_j}\frac{\partial G(\mathbf{r}|\mathbf{s})}{\partial n}dS + j\omega\rho y_n(\mathbf{r}_j)\iint_{S_j} G(\mathbf{r}|\mathbf{s})dS\right)p(\mathbf{r}_j), \end{aligned} \quad (2)$$

where

$$v_n(\mathbf{r}_j \notin S') = 0,$$
$$y_n(\mathbf{r}_j \in S') = 0.$$

Here, if the vector $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \cdots, \mathbf{r}_M$ which are located on the surface S is substituted for $\mathbf{s}$ in Eq.(2), we can solve them as simultaneous equations of many unknown $p\mathbf{r}_i (i = 1 \cdots M)$. As $\mathbf{r}_i$ is on the surface S, that is $C(\mathbf{s}) = 1/2$, Eq.(2) can be written as

$$\begin{aligned} \frac{1}{2}p(\mathbf{r}_i) &= j\omega\rho\sum_{k=1}^{N} q_k G(\mathbf{r}'_k|\mathbf{r}_i) - \sum_{j=1}^{M} j\omega\rho v_n(\mathbf{r}_j) G_{ij} \\ &- \sum_{j=1}^{M}\left(G^n_{ij} + j\omega\rho y_n(\mathbf{r}_j) G_{ij}\right) p(\mathbf{r}_j), \end{aligned}$$

where

$$\begin{aligned} G_{ij} &= \iint_{S_j} G(\mathbf{r}|\mathbf{r}_i) dS, \\ G^n_{ij} &= \iint_{S_j} \frac{\partial G(\mathbf{r}|\mathbf{r}_i)}{\partial n} dS. \end{aligned}$$

By transposing $p(\mathbf{r}_i)$ in the third term in the right side of Eq.(3), we can express simply as the matrix equation.

$$\left(\frac{1}{2}\mathbf{I}_M + \mathbf{G}_n + j\omega\rho\mathbf{G}\mathbf{Y}\right)\mathbf{P} = g\mathbf{A} - j\omega\rho\mathbf{G}\mathbf{V} \quad (3)$$

where

$$\begin{aligned} \mathbf{G}_n &= \left[G^n_{ij}\right] \in \mathbf{C}^{M\times M} \\ \mathbf{G} &= [G_{ij}] \in \mathbf{C}^{M\times M} \\ \mathbf{Y} &= diag(y_n(\mathbf{r}_1), y_n(\mathbf{r}_2), y_n(\mathbf{r}_3), \cdots, y_n(\mathbf{r}_M)) \in \mathbf{C}^{M\times M} \\ \mathbf{P} &= [p(\mathbf{r}_1), p(\mathbf{r}_2), p(\mathbf{r}_3), \cdots, p(\mathbf{r}_M)]^T \in \mathbf{C}^M \\ g &= \left[G(\mathbf{r}'_k|\mathbf{r}_i)\right] \in \mathbf{C}^{M\times N} \\ \mathbf{A} &= [A_1, A_2, A_3, \cdots, A_N]^T \in \mathbf{C}^N \\ \mathbf{V} &= [v(\mathbf{r}_1), v(\mathbf{r}_2), v(\mathbf{r}_3), \cdots, v(\mathbf{r}_M)]^T \in \mathbf{C}^M \end{aligned}$$

Therefore, if the boundary conditions such as the source amplitude, the vibration velocity and the acoustic admittance on the boundary surface are given and if $(\mathbf{I}_M/2 + \mathbf{G}_n + j\omega\rho\mathbf{G}\mathbf{Y})$ is the regular matrix, the acoustic pressure $\hat{\mathbf{P}}$ on the surface is given by

$$\hat{\mathbf{P}} = \left(\frac{1}{2}\mathbf{I}_M + \mathbf{G}_n + j\omega\rho\mathbf{G}\mathbf{Y}\right)^{-1}(g\mathbf{A} - j\omega\rho\mathbf{G}\mathbf{V}). \quad (4)$$

2.2. Calculation of the acoustic pressure within the volume

In the same manner of deriving Eq.(eq:discrete1), the acoustic pressure within the volume V ($p(\mathbf{s} \in V)$) can be written as

$$p(\mathbf{s}) = g_s\mathbf{A} - j\omega\rho\mathbf{G}_s\mathbf{V} - (\mathbf{G}_{ns} + j\omega\rho\mathbf{G}_s\mathbf{Y})\hat{\mathbf{P}} \quad (5)$$

where

$$\begin{aligned} g_s &= \left[G(\mathbf{r}'_k|\mathbf{s})\right] \in \mathbf{C}^N, \\ \mathbf{G}_s &= \left[\iint_{\mathbf{S}_j} G(\mathbf{r}_j|\mathbf{s})ds\right] \in \mathbf{C}^M, \\ \mathbf{G}_{ns} &= \left[\iint_{\mathbf{S}_j} \frac{\partial G(\mathbf{r}_j|\mathbf{s})}{\partial n}ds\right] \in \mathbf{C}^M. \end{aligned}$$

Therefore, after obtaining the acoustic pressure on the surface S, that is $\hat{\mathbf{P}}$, by solving Eq.(4), we can obtain the acoustic pressure at any position within the volume V.

2.3. Reciprocity principle

Suppose a sound fields driven by a sound source A with the volume velocity v located at $\mathbf{r}_a$ within the closed surface S, the sound field is expressed as the non-homogeneous wave equation as follows.

$$(\nabla^2 + k^2)p_a(\mathbf{r}) = -j\omega\rho_0 v \cdot \delta(\mathbf{r} - \mathbf{r}_a) \quad (6)$$

In the same manner, if we suppose another sound field driven by a sound source B with the volume velocity v located at $\mathbf{r}_b$ within the closed surface S, the sound field is also expressed as the non-homogeneous wave equation as follows.

$$(\nabla^2 + k^2)p_b(\mathbf{r}) = -j\omega\rho_0 v \cdot \delta(\mathbf{r} - \mathbf{r}_b) \quad (7)$$

By substituting $\mathbf{r}_b$ for $\mathbf{r}$ in Eq.(6) and $\mathbf{r}_a$ for $\mathbf{r}$ in Eq.(7) and comparing both, we can easily derive that $p_b(\mathbf{r}_a) = p_a(\mathbf{r}_b)$. The reciprocity principle in the wave equation can be transformed into the integral equation as

$$p_a(\mathbf{s}) = j\omega\rho v G(\mathbf{r}_a|\mathbf{s}) + G'(\mathbf{r}|\mathbf{s}), \quad (8)$$
$$p_b(\mathbf{s}) = j\omega\rho v G(\mathbf{r}_b|\mathbf{s}) + G'(\mathbf{r}|\mathbf{s}), \quad (9)$$

where

$$G'(\mathbf{r}|\mathbf{s}) = \iint_S G(\mathbf{r}|\mathbf{s})\frac{\partial p(\mathbf{r})}{\partial n} - p(\mathbf{r})\frac{\partial G(\mathbf{r}|\mathbf{s})}{\partial n}dS.$$

If $\mathbf{r}_a$ is the sound source position and $\mathbf{r}_b$ is the ear position, the HRTF can be obtained by calculating both $p_a(\mathbf{r}_b)$ and $p_b(\mathbf{r}_a)$. Because $p_b(\mathbf{r}_a)$ is the acoustic pressure at the source position $\mathbf{r}_a$ when the sound source is located at the ear position $\mathbf{r}_b$, HRTF can be solved by exchanging the sound source position and the ear position. This fact makes great merit in the HRTFs calculation. If we calculate the pressure signal $\hat{\mathbf{P}}$ with the sound source locate at the ear position $\mathbf{r}_b$ based on Eq. 4 once, the HRTF at any source position can be obtained by calculating Eq. 5.

In the calculation of the BEM, the most part of the calculation is occupied with the first calculation which calculates $\hat{\mathbf{P}}$ by solving Eq. 4. The second calculation finishes with a instant. However, because the source point is close to the boundary surface, the accuracy of solving $\hat{\mathbf{P}}$ can reduce. If the problem of the internal eigen-frequency mentioned below coincide it, further degradation of the accuracy is expected.

2.4. Cheif method

In solving Eq. (4), if $(1/2\mathbf{I}_M + \mathbf{G}_n + j\omega\rho\mathbf{GY})$ is the regular matrix, the pressure on the boundary surface, that is $\hat{\mathbf{P}}$, have an unique solution. It can, however, not be regular at all the frequencies. In particular, at the high frequencies, Eq.(4) tends to have many eigen-frequencies and not to have any unique solution. In the internal problem of the integral equation, it is physically natural because of the resonance inside of the closed volume. It is, however, unnatural to cause the same phenomena outside of the closed volume. We do not usually observe the resonance outside of the closed volume. This problem can be solved by changing the matrix. The Chief method is one of the solution by adding observation points where the acoustic pressure becomes zero, i.e. inside the volume in the external problem. In this method, first $\mathbf{s} \notin V$ are chosen and are substituted for $\mathbf{s}$ in Eq.(1). Next, the new equations are attached to Eq. (4). Because the equations are more than the number of the variable which is the acoustic pressure on the surface $S_j(j = 1 \cdots M)$, we must solve by using the least mean square method. If we choose $tildeM$ points of $\mathbf{s} \notin V$ and define as $\tilde{\mathbf{r}}_1, \tilde{\mathbf{r}}_2, , \cdots, \tilde{\mathbf{r}}_{\widetilde{M}}$, the equation is given as follows:

$$\begin{aligned} 0 &= j\omega\rho\sum_{k=1}^{N} q_k G(\mathbf{r}'_k|\tilde{\mathbf{r}}_i) - \sum_{j=1}^{M} j\omega\rho v_n(\mathbf{r}_j)\widetilde{G}_{ij} \\ &- \sum_{j=1}^{M}\left(\widetilde{G}^n_{ij} + j\omega\rho y_n(\mathbf{r}_j)\widetilde{G}_{ij}\right)p(\mathbf{r}_j), \end{aligned}$$

where

$$\begin{aligned} \widetilde{G}_{ij} &= \iint_{S_j} G(\mathbf{r}|\tilde{\mathbf{r}}_i)dS, \\ \widetilde{G}^n_{ij} &= \iint_{S_j} \frac{\partial G(\mathbf{r}|\tilde{\mathbf{r}}_i)}{\partial n}dS. \end{aligned}$$

By expressing the matrix equation, it becomes

$$\left(\widetilde{\mathbf{G}}_n + j\omega\rho\widetilde{\mathbf{G}}\mathbf{Y}\right)\mathbf{P} = \tilde{g}\mathbf{A} - j\omega\rho\widetilde{\mathbf{G}}\mathbf{V}, \tag{10}$$

where

$$\begin{aligned} \widetilde{\mathbf{G}}_n &= \left[\widetilde{G}^n_{ij}\right] \in \mathbf{C}^{\widetilde{M}\times M}, \\ \widetilde{\mathbf{G}} &= \left[\widetilde{G}_{ij}\right] \in \mathbf{C}^{\widetilde{M}\times M}, \\ \tilde{g} &= \left[G(\mathbf{r}'_k|\tilde{\mathbf{r}}_i)\right] \in \mathbf{C}^{M\times N}. \end{aligned} \tag{11}$$

By attaching Eq.(10) to Eq.(3), it is expressed as

$$\mathbf{HP} = \mathbf{L}, \tag{12}$$

where

$$\begin{aligned} \mathbf{H} &= \begin{bmatrix} \frac{1}{2}\mathbf{I}_M + \mathbf{G}_n + j\omega\rho\mathbf{GY} \\ \widetilde{\mathbf{G}}_n + j\omega\rho\widetilde{\mathbf{G}}\mathbf{Y} \end{bmatrix} \in \mathbf{C}^{(M+\widetilde{M})\times M}, \\ \mathbf{L} &= \begin{bmatrix} g \\ \tilde{g} \end{bmatrix}\mathbf{A} - j\omega\rho\begin{bmatrix} \mathbf{G} \\ \widetilde{\mathbf{G}} \end{bmatrix}\mathbf{V} \in \mathbf{C}^{(M+\widetilde{M})}. \end{aligned} \tag{13}$$

In this case, as Eq.(12) is the over-determined system, $\mathbf{P}$ does not have any solutions. However, we can obtain $\mathbf{P}$ which has the least error of the equation by solving

$$\hat{\mathbf{P}} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{L}. \tag{14}$$

3. COMPUTER SIMULATION

3.1. Boundary condition of head shape

We prepared triangle elements group which shapes the surface of the dummy head. First, the 3D coordinate data was measured by using 3D scanner (Cyberware 3030RGB/I). Next, the triangle mesh data was calculated by using the triangle mesh generator [7]. Although the measured coordinate data of the dummy head surface include the body of the head torso simulator, we use only the head part of the data in this study.

It is said that the element size should be less than 1/4 to 1/6 of the wave length to obtain enough accuracy in the calculation for the BEM. Therefore, we made upper limit of the element size when the surface is divided into the triangle element. Besides, because the ear pinna could affect the accuracy of the calculation, we controlled the upper limitation size near the ear pinna to be small enough. After determining these condition, the boundary elements are made as shown in Fig.1.

The number of the triangle element in this model is 5014. The average size of the element is about 6.8mm. Although the inlet of the external ear canal is closed in this model, it can be regarded that it is the same condition as the HRTF measured under the condition of the external ear canal closed which is said to include the spatial information [8].

The sound source is locate at an angle of front, right 30 degree and a distance of 1.4m from the center of the head torso. We calculate the acoustic pressure at the position of left and right ear. In order to confirm the accuracy of the computation, we calculate the acoustic pressure inside of the head, too.

3.2. HRTF using the conventional BEM

As the conventional method, we solve Eq.(4) first and Eq.(5) next to obtain the HRTFs. The frequency range of the calculation is 43-12015Hz at intervals of 43Hz. Fig.2 shows the HRTFs in the

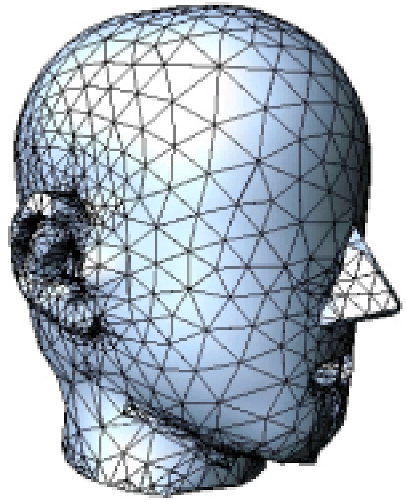

Figure 1: Computer model

frequency domain and the solid line and the dashed line indicate the S.P.L. at the right ear and left ear, respectively. The dotted line is the error level.

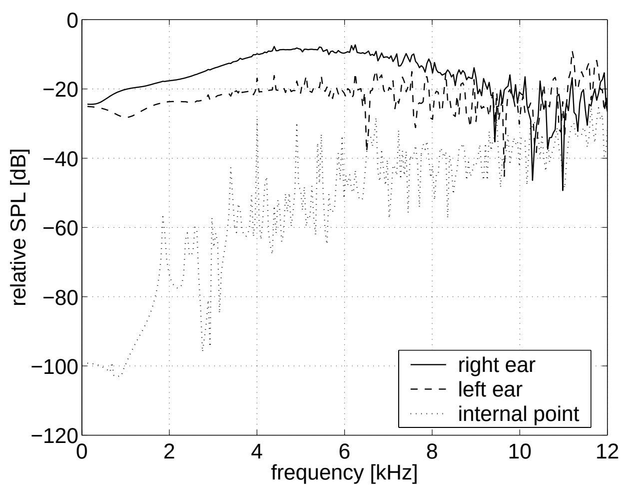

Figure 2: HRTF in the frequency domain by the conventional BEM

As shown in Fig.3, it can be seen that the amplitude of the right ear is higher than that of the left ear and the wave arrived at the right ear is earlier than that at the left ear. time is 45 hours using Xeon 2.5GHz.

3.3. HRTF calculation using the reciprocity principle

Fig.4 shows the HRTFs calculated by the method based on the reciprocity principle, that is $\mathbf{s}$ in Eq.(5) is set as the source position and $\mathbf{r}'_k$ in Eq.(4) is set as the ear position. Fig.5 is the HRTFs in the time domain, respectively.

The computation time for the first calculation is 45 hours and that for the second calculation is 2 seconds. The difference be-

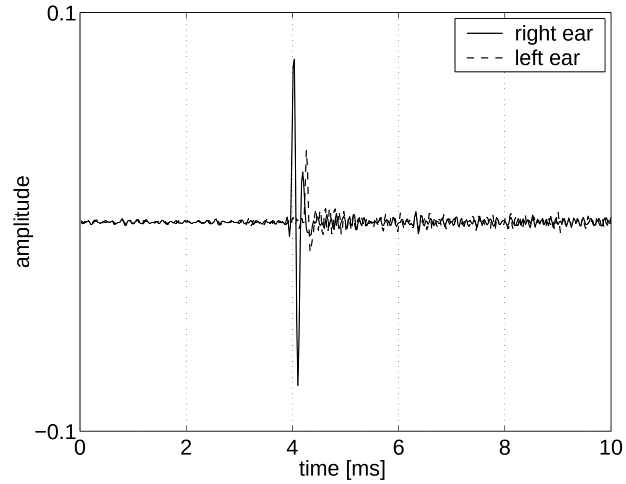

Figure 3: HRTF in the time domain by the conventional BEM

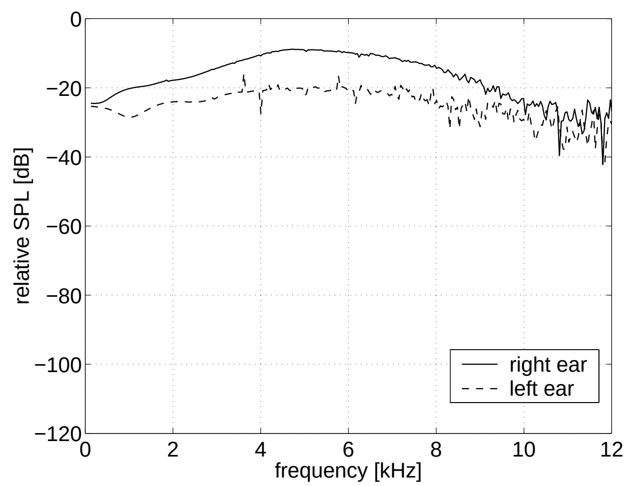

Figure 4: HRTF in the frequency domain by the reciprocity method

tween the peak and the dip in Fig.4 are less than the conventional method. The noise reduction can be seen before the peak of the HRTF waveform in Fig.5.

3.4. HRTF calculation using Chief method

Fig.6 shows the HRTFs calculated by the Chief method, that is $p(\mathbf{s})$ in Eq.(5) is calculated by substituting the acoustic pressure $\hat{\mathbf{P}}$ on the surface S given by Eq.(14).

The error level in the frequency domain in Fig.6 does not increases compared with the conventional method illustrated in Fig.2. The computation time is 116 hours using Xeon 2.5GHz.

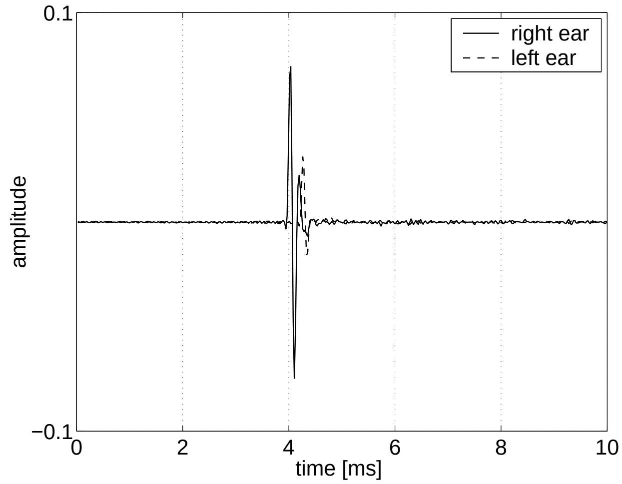

Figure 5: HRTF in the time domain by the reciprocity method

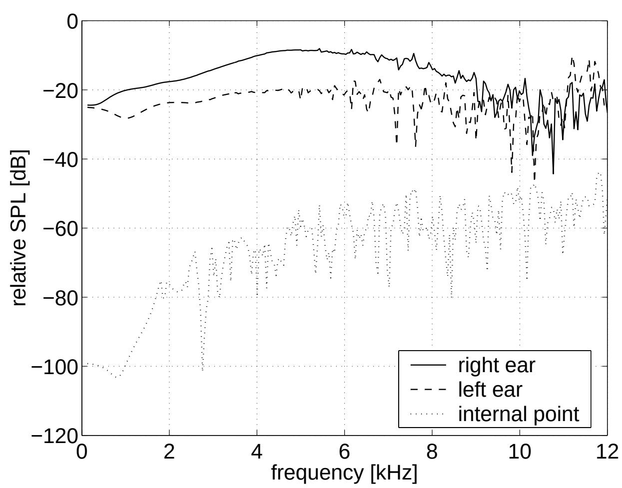

Figure 6: HRTF in the frequency domain by the Chief method

4. DEVELOPMENT OF REAL TIME HRTF SERVER

We developed the computation server as shown in Fig.8 which calculate the HRTFs quickly. The URL address of it is as follows:

http://acoust.archi.kyoto-u.ac.jp/HRTF/

The computation time is 2 seconds for one position of the sound source.

At the moment, it can not be said the perfect real time server, but comparing the computation time in the conventional BEM, that is 45 hours, it can be said that the revolutionary speed up of the calculation achieved. We are going to open this server for any application which will uses HRTF after confirming the copyright concerning the triangle mesh generator [7].

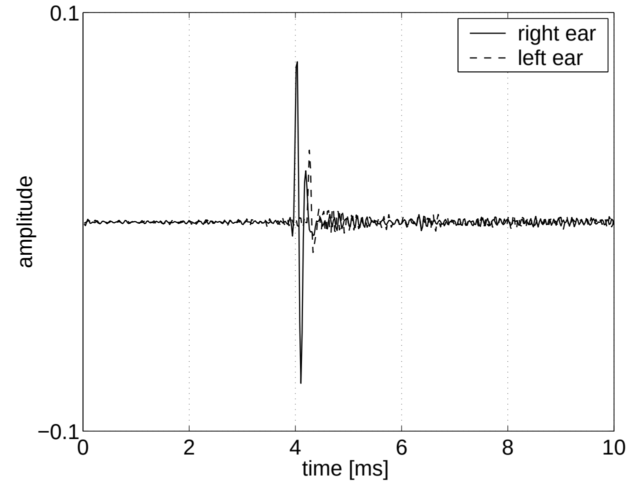

Figure 7: HRTF in the time domain by the Chief method

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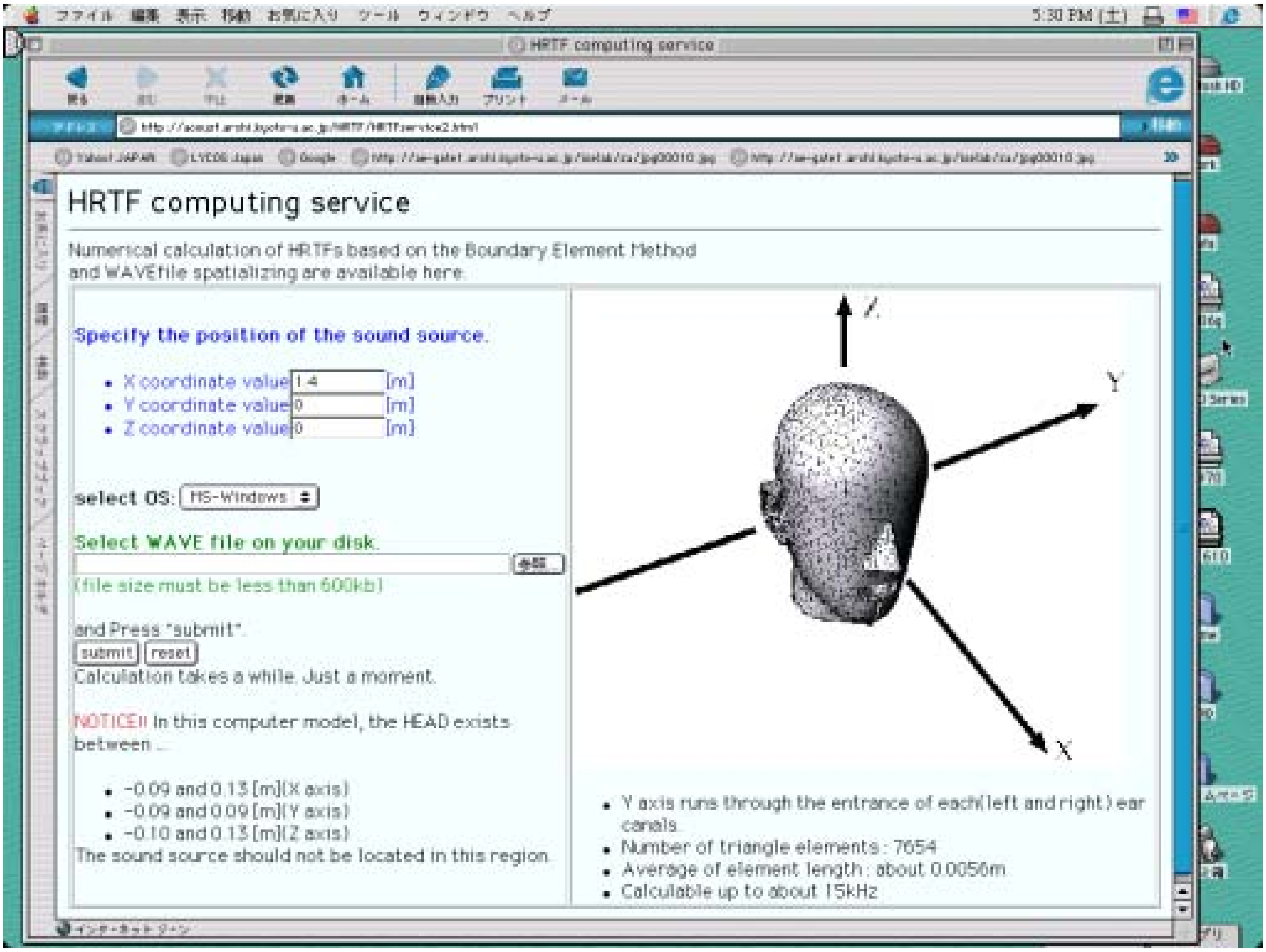

Figure 8: HRTF server