An Analysis of Buckling in Spherical Shells and its Design Implications for Hopper Poppers

by

Andrew D. Greenhut

SUBMITTED TO THE DEPARTMENT OF MECHANICAL ENGINEERING IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

BACHELOR OF SCIENCE

AT THE

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

JUNE 2006

Signature of Author...
Department of Mechanical Engineering
May 12, 2006

Certified by......................................
David Wallace
Associate Professor of Mechanical Engineering
Thesis Supervisor

Accepted by......
John H. Lienhard V
Professor of Mechanical Engineering
Chairman, Undergraduate Thesis Committee

An Analysis of Buckling in Spherical Shells and its Design Implications for Hopper Poppers

By

Andrew D. Greenhut

Submitted to the Department of Mechanical Engineering
On May 12, 2006 in Partial Fulfillment of the
Requirements for the Degree of Bachelor of Science in
Mechanical Engineering

ABSTRACT

The purpose of this project was to investigate a key design parameter of a hopper popper: the force it takes to "load," or invert, the popper. A hopper popper is an injection-molded rubber toy in the shape of a hemispherical shell that stores elastic potential energy when inverted. There is new interest for hopper poppers in projectile toys. However, the force it takes to invert the popper can easily exceed what child can produce.

In the course of the study, tests were conducted on ten different hopper poppers to measure the force and displacement of loading. Theoretical equations for the buckling of spherical shells were then correlated to the data. It was found that the equations accurately predicted the main variables in buckling, which are the Young's modulus of the material, the radius of the popper, and the thickness of the shell.

Furthermore, the ability of a hopper popper to be bi-stable (invert and stay inverted) was examined. It was found that the degree of curvature was the biggest factor in the stability of poppers; the closer the curvature was to 180 degrees, the more stable a popper was when inverted. Additionally, a more sophisticated brand of hopper poppers, known as Dropper Poppers, was examined to see what makes them more impressive than ordinary poppers. It was found that a hole of 0.14 inches in diameter helps these poppers stabilize when inverted even though their curvature is only 150 degrees. The lower angle was found to reduce inverting force because the normal force supplied to the bottom perimeter of the popper had a perpendicular component that helped stretch the popper out as it was being loaded.

Finally, this thesis presents ideas for the future of hopper poppers. One is a mechanism designed for a blaster, which uses the mechanical advantage of a lever arm to invert a popper. Another is a design for new hopper poppers which could take less force to invert than a normal hopper popper, but store the same elastic potential energy.

Thesis Supervisor: David Wallace
Title: Associate Professor of Mechanical Engineering

Contents

List of Figures

List of Tables

1 Introduction

1.1 Background

A hopper popper, or "popper," is an injection-molded rubber toy in the shape of a hemispherical shell (Figure 1-1). It stores elastic-potential energy when inverted (Figure 1-2). Certain hopper poppers are designed to be stable in their inverted state for a period of time. Hopper Poppers are not a patented technology because they are easily producible (cut a ball in half) and most physics professors are aware of their basic properties. There are several versions of hopper poppers on the market today; such as the version at herodads.com (Figure 1-3) and the Dropper Popper by Petra Toys, Inc.[7]

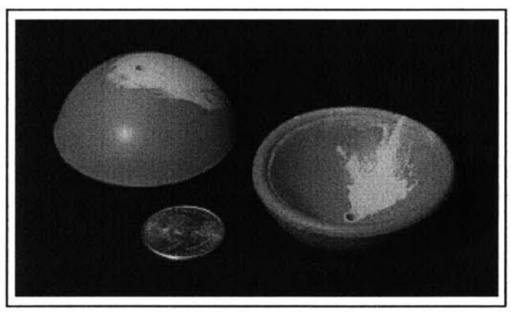


Figure A-1: The Dropper Popper in the Natural State [2, pp. 48]

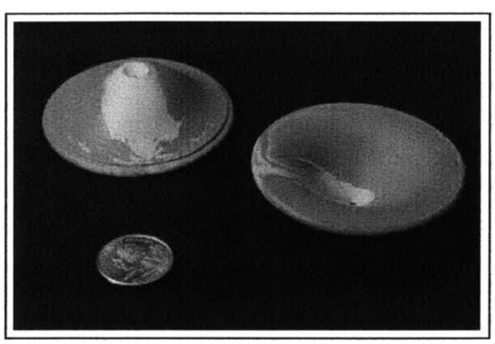


Figure 1-2: The Dropper Popper in the Inverted State [2, pp. 48]

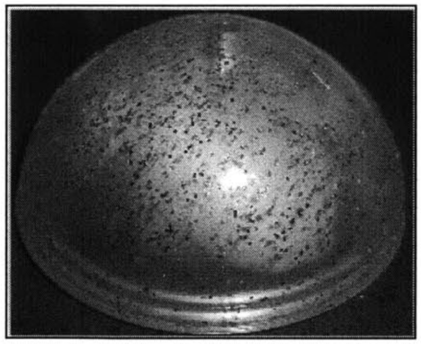


Figure 1-3: Hopper Popper from www.herodads.com

1.2 Objectives

William J. Fienup and Barry Kudrowitz documented a new use of the poppers in designing new projectile toys in their master's theses. They discovered that a popper could propel a round foam ball a considerable distance when the popper goes from its inverted state to its normal state (Figure 1-3).

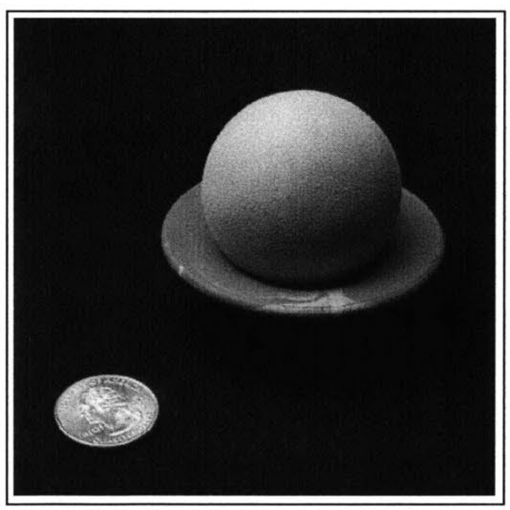


Figure 1-4: Foam Ball Resting in an Inverted Hopper Popper [2, pp. 49]

Fienup and Kudrowitz went further to design a device called the Hand Popper, which is currently being developed into a real product. A Hand Popper is a device that holds one Dropper Popper and allows a user to load it with a ball by pressing the ball into the popper.

The device has a trigger that lightly nudges the inverted popper back into its normal stage, thereby shooting the ball out. The device passed all of Hasbro®'s safety tests.

The Hand Popper is a novel product, but it has one key drawback. In the words of Fienup and Kudrowitz: "the greatest issue of concern was the force required to load the popper."[2, pp. 16] The loading force on early iterations of the Hand Popper was as high as 34lbs (Figure 1-4). The team found that high friction between the popper and the delrin housing caused the popper to resist inversion and created a high loading force. When they added talcum powder to lubricate the popper, it performed better, but still took 19lbs of force to invert.

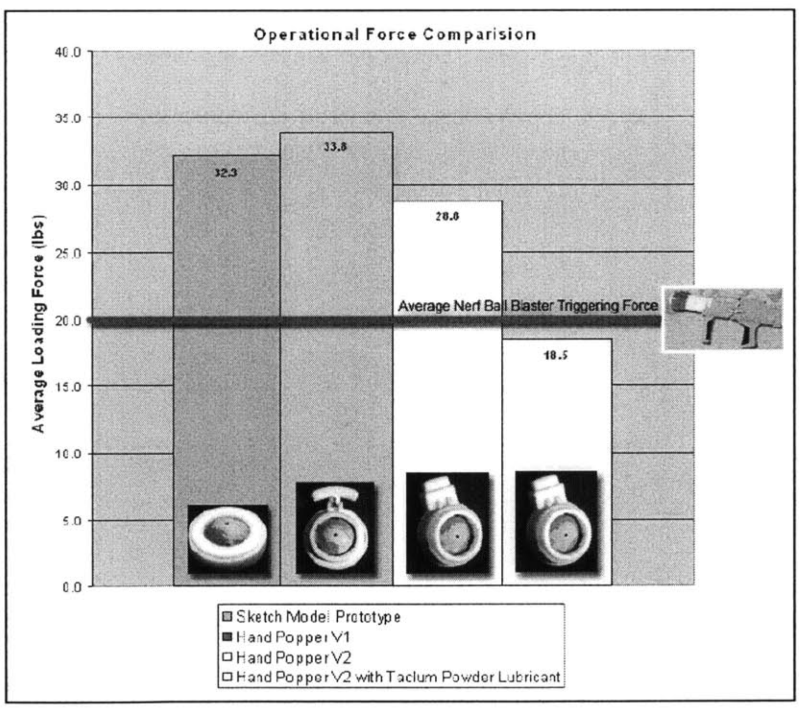


Figure 1-5: Operational Force Comparison [2, pp. 16]

The purpose of this thesis is to understand the loading force of a popper and what design parameters influence it. This thesis examines and analyzes different kinds of poppers – materials, shapes, and geometries – to see how loading force changes. Also, it analyzes theoretical models for spherical buckling, a similar process to inverting a popper, to discover whether the loading force can be predicted by equations.

1.3 Outcomes

The experiments done in this study reinforce the theoretical equations governing buckling in spherical shells. The design parameters predicted to influence the buckling force – that is Young's modulus, radius, and thickness of shell – were reflected in the measurements taken. It was also shown that the Dropper Popper has specific designs that make it perform better than the average hopper popper. This study should be useful to anyone interested in designing with hopper poppers.

2 Theory

2.1 Buckling of Spherical shells

For the elastic buckling of a uniformly compressed spherical shell under uniform pressure, the buckling point is [10, pp. 496]:

$$P_E = \frac{2Eh^2}{R^2\sqrt{3(1-v^2)}} \tag{1}$$

where E: Young's modulus

v : Poisson's ratio

h : shell thickness

R : radius to the mid-surface of the shell

However, empirical tests in literature have found this model to over predict the true buckling force by 400% [5]. A more accurate empirical formula for spherical shells is:

$$P_{cr} = 0.84\sqrt{E_s E_t}\left(\frac{h_a}{R_0}\right)^2 \tag{2}$$

where E_s, E_t : secant and tangent modulus

h_a : average thickness over a critical arc length L_c

R_0 : local radius to the outside surface of a shell over the critical arc length L_c

The tangent and secant modulus are special material properties. Figure 2-1 shows how they are defined.

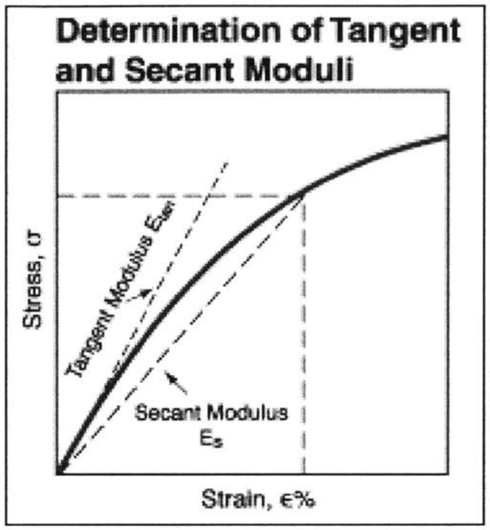


Figure 2-1: Stress-Strain Curve with Tangent and Secant Modulus [9]

The key parameters outlined in the theoretical equations are Young's modulus of the material, thickness of the shell, and radius of the shell. The critical pressure is proportional linearly to E, proportional to the square of h, and inversely proportional to the square of R.

2.2 Properties of Elastomers

Poppers are made of elastomers. Elastomers are a kind of material that can undergo very large strains and bounce back to its original shape because its glass transition temperature is below room temperature. There are some special properties of elastomers that should be reviewed before analyzing the poppers in detail.

A stress-strain curve for an elastomer is not linear like for metals. It follows a Neo-Hookean path, which means its Young's modulus varies with respect strain (Figure 2-1). Additionally, elastomers are rate dependant. That is, the speed of loading changes the stress-strain distribution. Figure 2-2 shows the load vs. elongation data for Vytaflex60 Liquid

Rubber with a 1/3 in/min load rate (solid line) and a 3 1/3 in/min load rate (dashed line). Notice how the rubber strains with less force when it is loaded faster.

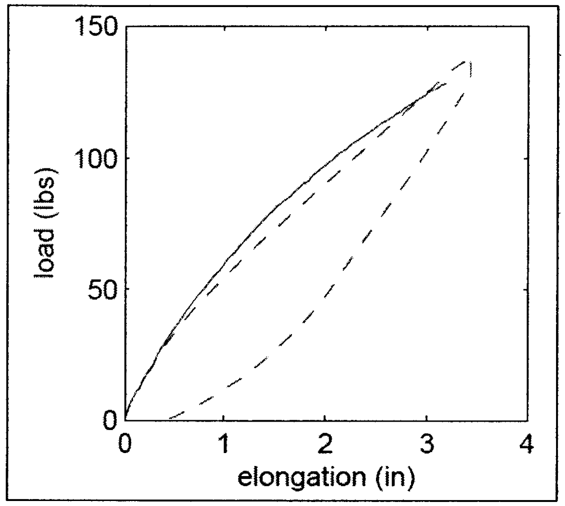


Figure 2-2: Load vs. Elongation of Smooth-On Vytaflex60 Liquid Rubber [1]

Another important elastomer property is the Mullin's effect. This is the stress softening of a material with multiple load cycles. For example, if you strain a rubber by 20% and it takes 40N, after a number of cycles the same strain will take less than 40N. The Mullin's effect is important to keep in mind for poppers because they are being used for cyclic loading. However, this is not very crucial because the poppers actually undergo little strain and high deformations, and it has been proven that the Mullin's effect is less pronounced in small strains.[4]

Elastomers also behave differently at different temperatures. However, for the purposes of this thesis, those effects are considered negligible and will therefore be ignored.

3 Experiment

3.1 Selected Poppers

Several different hopper poppers were selected for experimentation based on size and material. The first kind of poppers tested was the Dropper Poppers. There are two variants of Dropper Poppers: red and green. The red ones are slightly stiffer than their green counterparts, but they are both the same size. Figure 3-1 shows the dimensions of the Dropper Poppers. Note that the Dropper Poppers are highly specialized. They have approximately a 150 degree arc and a variable wall thickness that averages to 0.2 inches. There is also a hole in the middle of the popper whose effects will be discussed later.

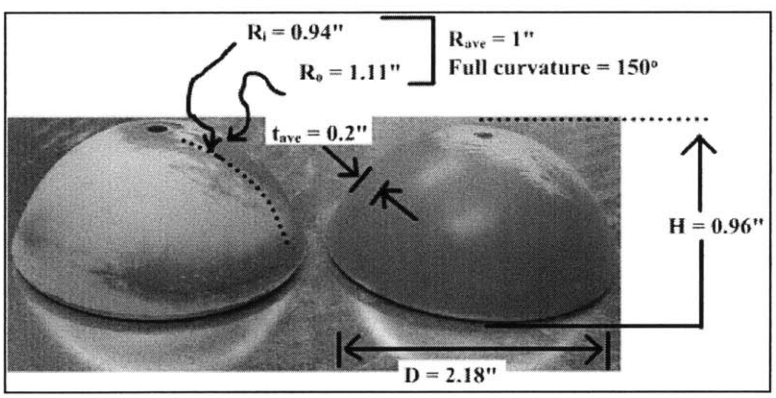


Figure 3-1: Dropper Popper Dimensions

The next popper was made by cutting a racquet ball in half (Figure 3-2). It is about as thick as the Dropper Poppers, but has a larger radius of curvature and is exactly 180 degrees.

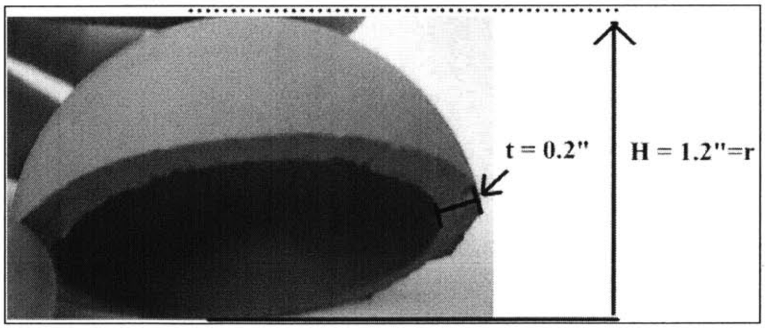


Figure 3-2: Home-made Popper

The last family of poppers is made from Smooth-On PMC-870 polyurethane rubber. This material was originally chosen because it had the same durometer rating as the Dropper Poppers. However, the durometer rating turned out to be an insufficient parameter for identifying the material. Instead, a completely new kind of popper was made with completely different properties. Although this was not what I was aiming for, the new poppers behaved well and gave me good data.

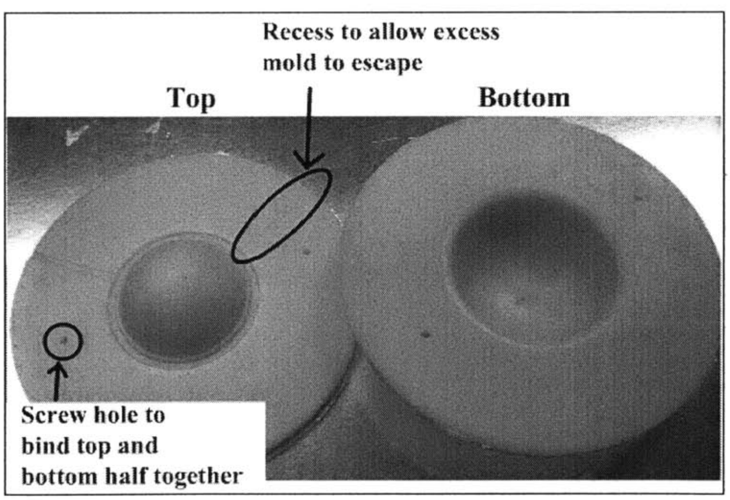


Figure 3-3: Normal Dropper Popper Mold

To make the polyurethane poppers, a mold was designed in SolidWorks using a model of a Dropper Popper (see Appendix A). The Mold (seen in Figure 3-3) was rapid prototyped in a stereo lithography printer. It features grooves to allow excess mold to escape and screw holes to clamp the top and bottom together.

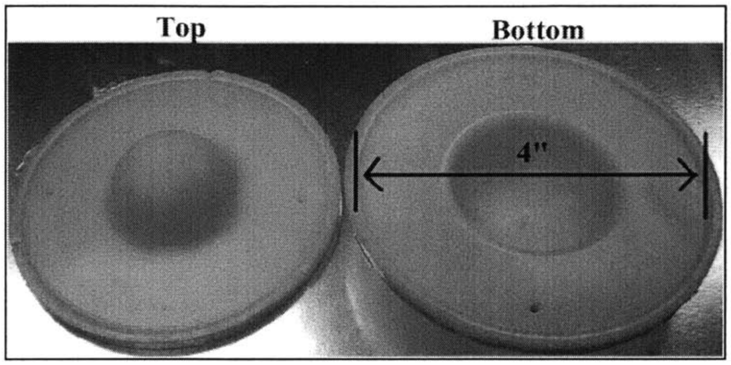


Figure 3-4: Winged Dropper Popper Mold

Additionally, a mold for a winged hopper popper was made to test the idea proposed by Fienup, W. J. and Kudrowitz, B [2, pp. 93] (Figure 3-4). The idea is that a popper with wings, or a skirt, will give the user a mechanical advantage in loading as a lower force is applied over a longer distance.

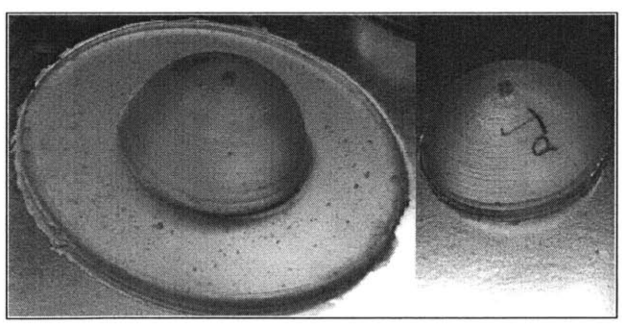


Figure 3-5: First Group of Polyurethane Poppers (1a right, 1b left)

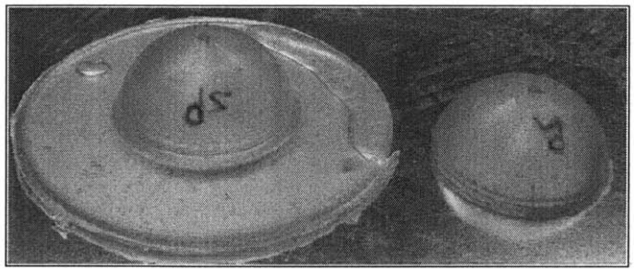


Figure 3-6: Second Group of Polyurethane Poppers (2a right, 2b left)

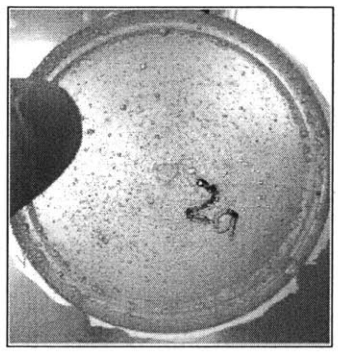


Figure 3-7: Close-up of Bubbles in Sample 2a

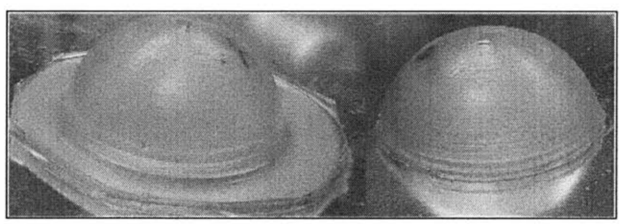


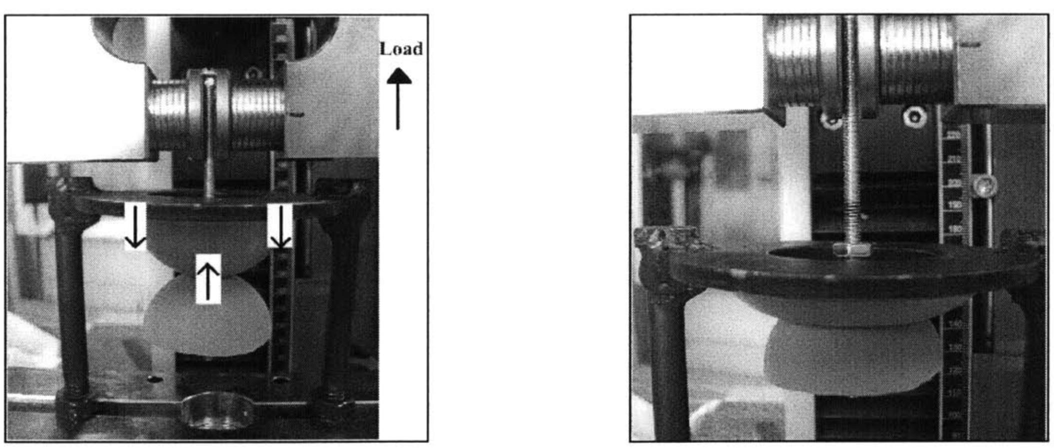
Figure 3-8: Third Group of Polyurethane Poppers (3a right, 3b left)

Figures 3-5, 3-6, and 3-8 show the three iterations of popper molds respectively. Each batch is slightly different. The first group (1a and 1b) came out thicker than a normal hopper popper because it was not clamped well and the top and bottom molds separated slightly while curing. The second group (2a and 2b) came out somewhat weaker. The polyurethane contained bubbles from air entrained while mixing and those bubbles led to significant gaps in material in the popper. Figure 3-7 shows a close-up of those bubbles. Vacuum bagging during the curing process should remove the air bubbles. Finally, the third group came out closest to the real Dropper Poppers.

3.2 Methods

Each popper was tested to see how much force it takes to invert it with a foam rubber ball. The basic test setup is shown in Figure 3-9. A screw was fed through the hole in the popper and then through half of a foam ball of radius 1.7in. Washers were put on the end of the screw to help distribute the force and keep the screw from ripping through the foam ball. A support was made from a steel ring and two nuts to oppose the motion of the popper (see Figure 3-9).

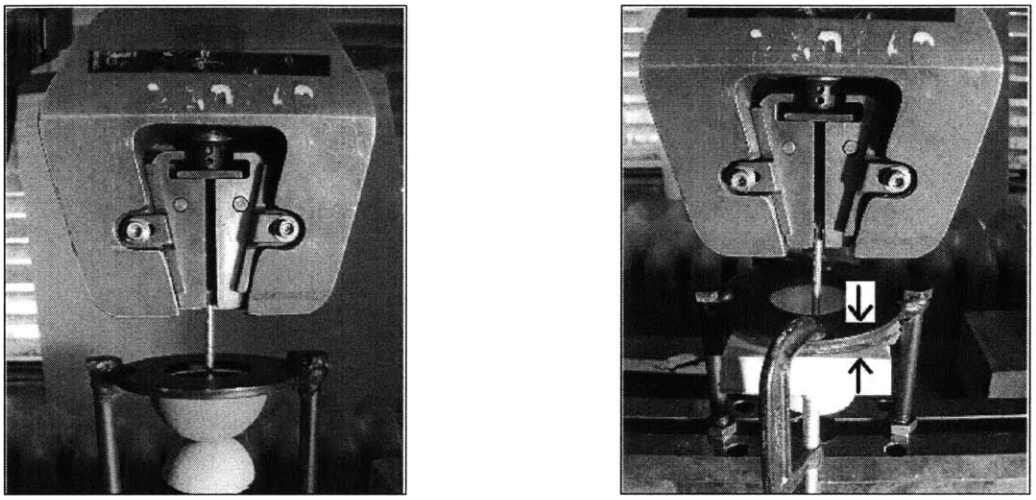
The first set of tests was done on the Zwick/Roell Z2.5 machine. As you can see, two arms were clamped to the screw, and then the machine pulled up to load the popper. The loading rate was 50mm/min. The loading rate was not ideal because in practice poppers would be loaded dynamically and we have seen that the rate of load affects the behavior of elastomers. The load and displacement of the machine were measured by the load cell.



(a) (b)

Figure 3-9: Popper Test Apparatus Before Loading (a) and During Loading (b)

Unfortunately, after half the tests were completed, the machine broke down. The setup was then moved to the Instron 1125 machine to complete the tests (Figure 3-10). The data from both machines are equally valid because the set-up was the same.



(a) (b)

Figure 3-10: Second Test Apparatus with Home-made Popper (a) and Winged Popper (b)

3.3 Results

Figure 3-11 shows the results of the ten tests conducted. The data points are plotted and adjusted so that the displacements all start at zero. As you can see, it took more and more force to invert the poppers up to a certain point after which the necessary force decreased. The necessary force continued to decrease until the popper was completely inverted. A breakdown of each plot individually can be seen in Appendix B.

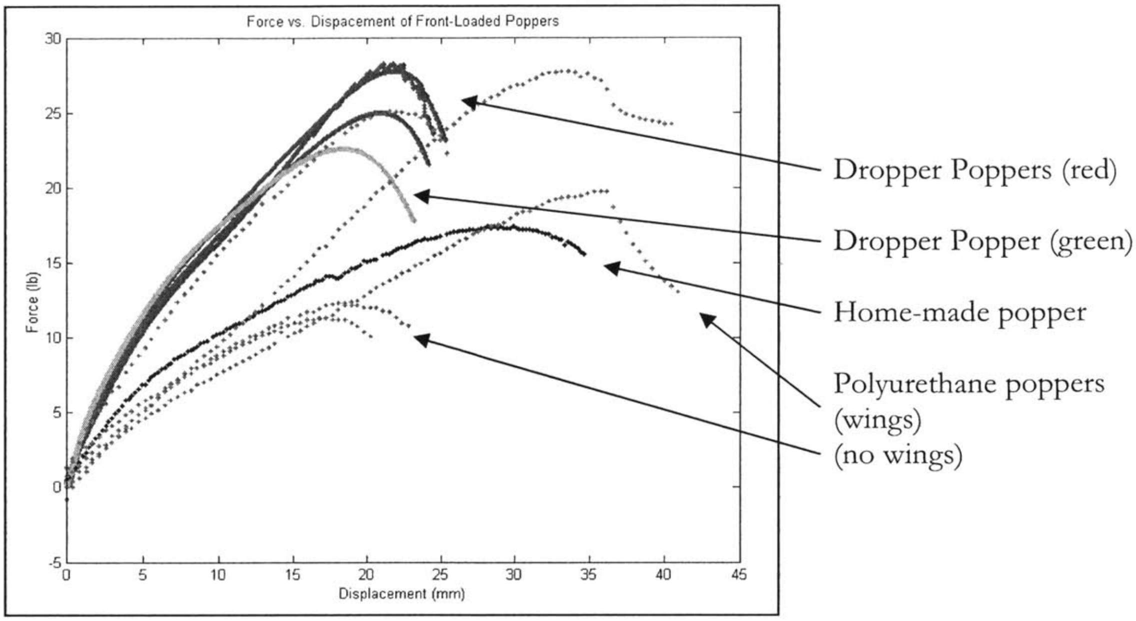


Figure 3-11: Force vs. Displacement of Front-Loaded Poppers

In the first test, which was one of the red Dropper Poppers, it was placed it in a popper holder to simulate real conditions (Figure 3-12). However, for subsequent tests the popper was placed straight against the steel ring. The first test generated more jagged data around the point of buckling, because the friction between the popper and the holder was very great and resisted the inversion. When the force was great enough to counter the friction, the popper would slip, and the load would quickly decrease.

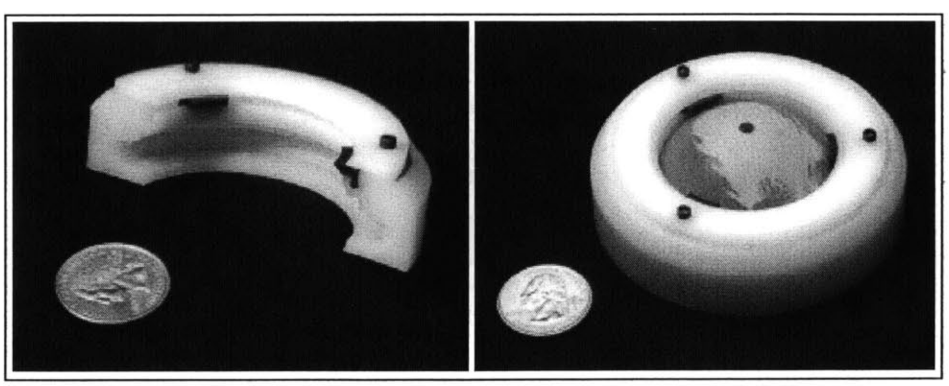


Figure 3-12: The Dropper Popper Holder [2, pp. 50]

The maximum forces measured and their corresponding displacements are listed in Table 3-1. Additionally, the forces and displacements are ranked in Table 3-2.

Table 3-1: Maximum Buckling Force and its Corresponding Displacement

Hopper Popper	Max. force (lb)	displacement (mm)
Red popper1	28.29	21.83
Red popper2	27.75	21.87
Red popper3	24.97	20.85
Green popper	22.58	18.4
Home-made popper	17.4	29
Polyurethane-1a	25.1	21.5
Polyurethane-2a	11.35	16.8
Polyurethane-3a	12.3	18.5
Polyurethane-1b	27.7	33.5
Polyurethane-3b	19.75	35.5

Table 3-2: Ranking of Data Points

	Ranking of Max Force (easiest to hardest)	**Ranking of Displacement (smallest to largest)**
1	Polyurethane-2a	Polyurethane-2a
2	Polyurethane-3a	Green popper
3	Home-made popper	Polyurethane-3a
4	Polyurethane-3b	Red popper3
5	Green popper	Polyurethane-1a
6	Red popper3	Red popper1
7	Polyurethane-1a	Red popper2
8	Polyurethane-1b	Home-made popper
9	Red popper2	Polyurethane-1b
10	Red popper1	Polyurethane-3b

As seen above, the maximum force required to buckle the poppers ranged from 12.3lbs to 28.29lbs. The rankings show that the first red popper sample took the most force to invert, and the polyurethane-2a popper took the least force to invert. The displacement at maximum buckling force occurred in one of three areas. This was due to the geometric differences between the poppers. These results will be explained further in the next section.

4 Discussion

4.1 What did the Theory Predict?

The theoretical equations for spherical shell buckling generally predicted 4-5 times higher buckling loads than were recorded. The reason for this discrepancy is mostly due to the fact that hopper poppers are made of elastomers, and so the material properties are hard to predict under these conditions. Furthermore, the equations of Timoshenko [10] and others have been found to over-predict the buckling load, as discussed previously in Section 2.1.

In calculating the buckling loads, an approximate Young's Modulus was used. According to Finite Element Analysis handbooks, the Young's modulus of rubber generally varies from 0.76 to 7.6 MPa. Assuming the load was over the cross sectional area of the foam ball, equation (2) predicts the loading for the Dropper Popper to be at 37lbs of force at a minimum and 374lbs of force at a maximum. Given this relatively large range, it is clear that the exact Young's modulus behavior is critical to getting a more accurate approximation.

4.2 Key Parameters

Even though the exact maximum buckling loads cannot be predicted precisely from these equations, three key parameters can be identified and backed up with the data. These are the Young's modulus, radius of curvature, and thickness of the popper. As mentioned earlier, the critical pressure is proportional linearly to E, proportional to the square of h, and inversely proportional to the square of R. For Young's Modulus, we see that in general the polyurethane poppers took less force to invert than Dropper Poppers. We can attribute this discrepancy to the fact that the polyurethane was noticeably softer than the Dropper Poppers, hence a lower Young's modulus and lower buckling load point.

The only popper with a different wall thickness was the 1a and 1b sample of polyurethane poppers. The equations predict the loading force to be proportional to the square of h, and we see here that the thicker poppers were approximately 40% harder to invert than the other polyurethane poppers. In fact, the thicker poppers were 20% thicker, or 0.24in.

Finally, the radius of the home-made popper was 0.2in greater than other poppers tested. Although the exact Young's modulus is not known, by physical inspection, it was similar to the Dropper Popper's. We see that for the home-made popper, the inverting force was less that all the Dropper Poppers. This could be because it had a larger radius.

4.3 Detailed Look at Dropper Poppers

The Dropper Poppers are designed somewhat uniquely compared to the general hopper popper. This section will look at distinct design aspects of the Dropper Poppers and how they contribute to its stability and high performance.

First, Dropper Poppers have a hole of 0.14in in diameter through their center. It is believed that this hole plays a role in the bi-stability of the popper. When the popper inverts, the inside (now the outside) rubber around the hole stretches radially. This allows the rubber to relax and therefore stabilize more. If the hole was not there, the rubber would pull itself inward and be more likely to invert. In fact, Fienup and Kudowitz filled the hole of a Dropper Popper in previous tests and found that the Dropper Poppers would no longer stabilize in the inverted position.

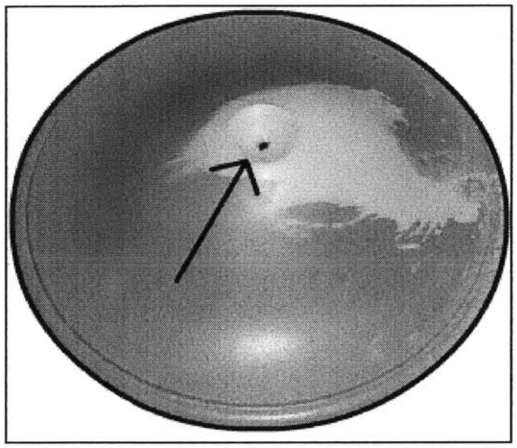


Figure 4-1: Highlight of Stretched Hole for an Inverted Dropper Popper

Stability proved to be an interesting topic because it affects the predictability in applications, and so it was explored further. From experimenting with the poppers, it was found that thickness, arc length, and material stiffness played a large role in popper bi-stability. The greater the arc length, the more stable a popper would be in its inverted state. This was seen in the home-made popper, which was completely stable in either normal or inverted positions. Also, the thicker the popper, the less likely it was to be bi-stable. For example, the 1a sample of polyurethane poppers was not stable at all when inverted. Finally, it was found that the stiffer the material, the more stable it was in the inverted position. It is believed that stiffer materials can resist pop-back buckling better than less stiff materials. Overall, it is believed the radius does not affect stability and is just a scaling factor for poppers.

Two other design aspects of the Dropper Popper are its 150 degree radius of curvature and its round bottom that contacts the surface as seen in Figure 4-2. We can see that because the radius of curvature for Dropper Poppers is less than 180 degrees, some of the normal force from the surface upon loading is in the outward normal direction. This helps invert the popper because stretching the material decreases the local elastic modulus and facilitates buckling. That is also why it is easier to invert a Dropper Popper by hand. The 180 degree counterpart does not have that advantage because none of the normal force is transmitted outward.

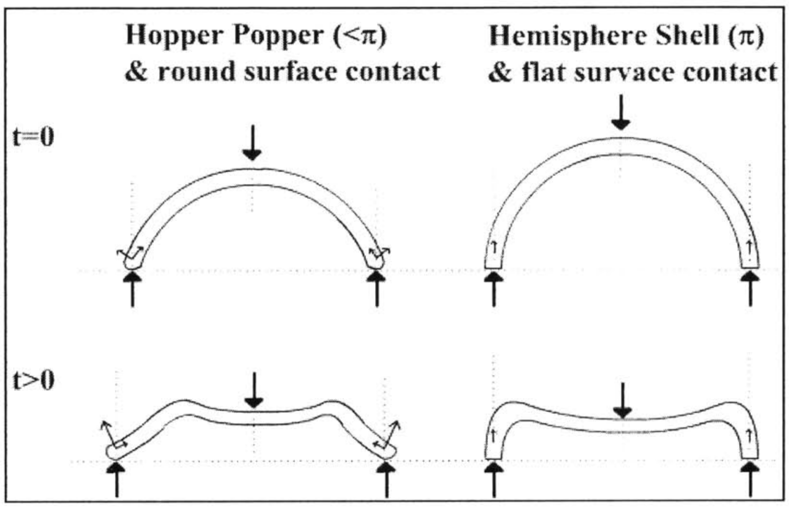


Figure 4-2: Force Vectors for Poppers of Different Arc Lengths and Surface Contacts

Furthermore, the round bottom of the Dropper Popper allows the surface contact to roll and move the popper outward, thus increasing the component of normal force in the outward direction. For the flat surface counterpart, we see that it cannot roll and that friction opposes outward sliding between the popper and surface. Friction is also an issue with the Dropper Popper, but to a lesser extent.

4.4 New Ideas

The polyurethane popper with wings was one new idea explored. This did not turn out to be as good a performer as hypothesized. The poppers with wings took 50-100% more force to invert than their non-winged counterparts. It is believed that grabbing the popper by a skirt restricts outward motion and gives it properties similar to those of the 180 degree shell in Figure 4-2. Additionally, grabbing the popper by the skirt provides less normal force than straight surface contact because a percentage of resistance to the load force is going into deforming the material in the skirt right around the circumference of the popper body.

Another new idea explored was a loading mechanism that could be integrated into Nerf® products. Figure 4-3 shows one current blaster on the market.

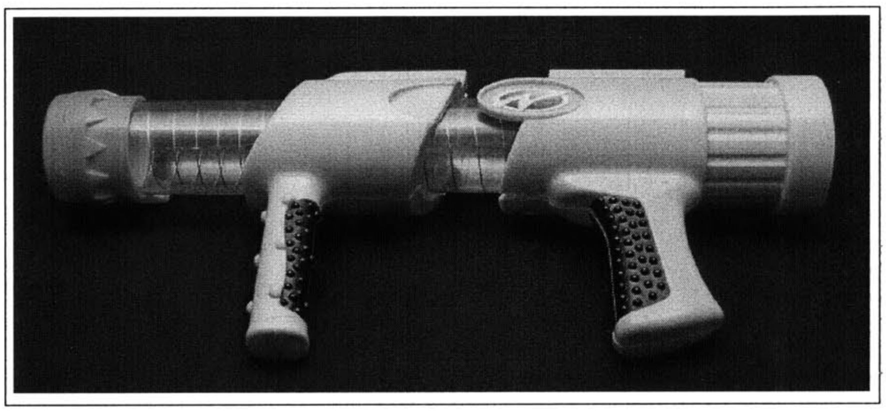


Figure 4-3: The Nerf® Ball Blaster™ [2, pp. 99]

The proposed new loading mechanism consists of three steps: (1) initial configuration, (2) inverting the popper, (3) hitting the trigger and destabilizing the popper (Figure 4-4). This design has not been tested, but here is how it would work. A long rod would be attached to the hopper popper via the hole in the middle. A loader on the gun would be rotated, such that on rotating, it would pull back the hopper popper and invert it. The loader would have a torsion spring reset itself. When the popper is inverted, a trigger would internally push the popper and supply enough force to destabilize it and release the elastic potential energy. This design allows you to use a hopper popper that takes a lot of force to invert because you get the mechanical advantage of the loader. This means you can use stronger poppers that shoot the ball further. Also, you can use completely bi-stable poppers because you can provide a decent restoring force from your trigger.

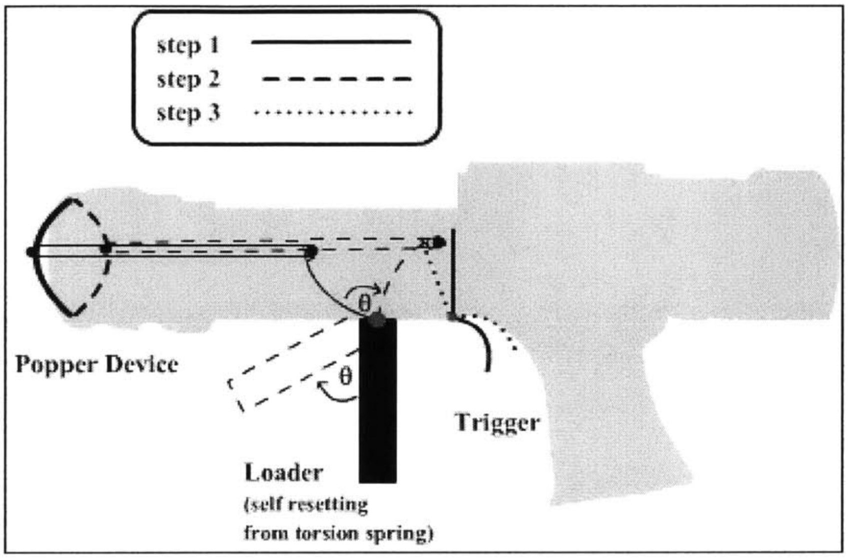


Figure 4-4: Sample Blaster with New Loading Mechanism

5 Conclusions

5.1 Outcomes

Overall, both theoretical equations were found to be insufficient at predicting exact loading force, but good at predicting dimensional analysis based on three parameters: Young's modulus, the radius of the popper, and the shell thickness. With detailed material data and more tests, one can find ways to fit the equations to the data and use it to predict the loading force for a family of future hopper poppers.

The experiments in this study correlated with theoretical predictions. This gives credence to the model. In addition, finer design elements of Dropper Poppers were examined to see how they affected bucking load and bi-stability. This study met the original objectives, which were to understand the key parameters in inverting a hopper popper.

5.2 Future Considerations

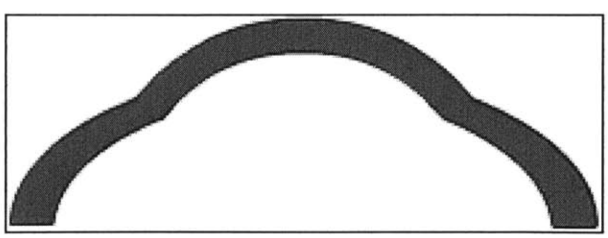


Figure 5-1: Future Popper Design to Try

Hopper poppers have a lot of potential in projectile toys. It is time to design poppers of specific kinetic energy densities. Also, future work should be done on popper loading devices that give users a mechanical advantage. This way, poppers that store a lot of elastic potential energy and are hard to invert can be used. Additionally, tests should be conducted on poppers that have a complex curvature. How would a popper behave if it was made of the cross-section seen in Figure 5-1? Would it be easier to invert? Would it pop back with the same force as a normal hopper popper?

Acknowledgements

I would like to thank Bill Fienup and Barry Kudrowitz for discovering the hopper popper and inspiring me with this curious toy. Big thanks to Professor David Wallace for everything he has taught me about product design and giving me this opportunity to write about hopper poppers. I am most indebted to Pierce Hayward for all his help in the 2.002 lab. Additionally, I would like to thank Hasbro® for 3D printing my molds and James Penn for his help with everything from solid modeling to mold making. Finally, I would like to thank Gita Srivastava for her support during this project.

Appendix A: SolidWorks Models

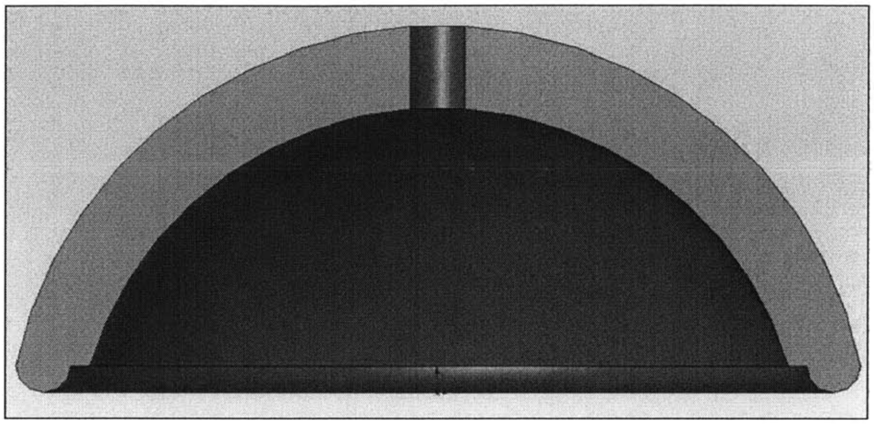


Figure A-1: Cross-Section Model of Dropper Popper

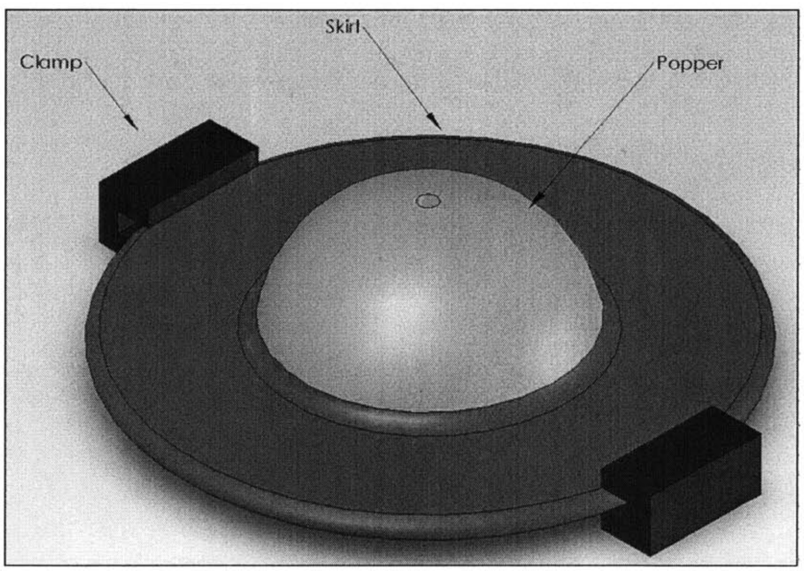


Figure A-2: Model of Dropper Popper with Skirt [2, pp. 93]

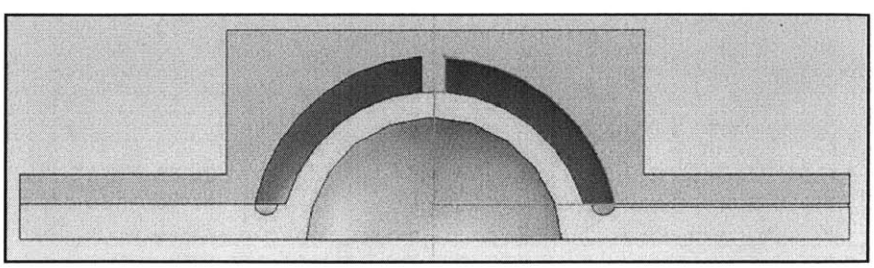


Figure A-3: Cross-Section of Normal Dropper Popper Mold

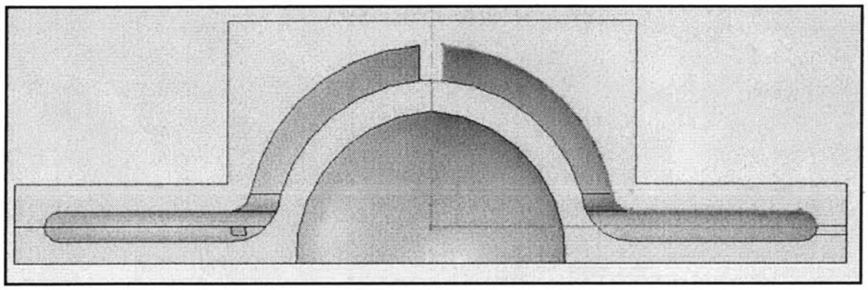


Figure A-4: Cross-Section of Winged Dropper Popper Mold

Appendix B: Additional Plots

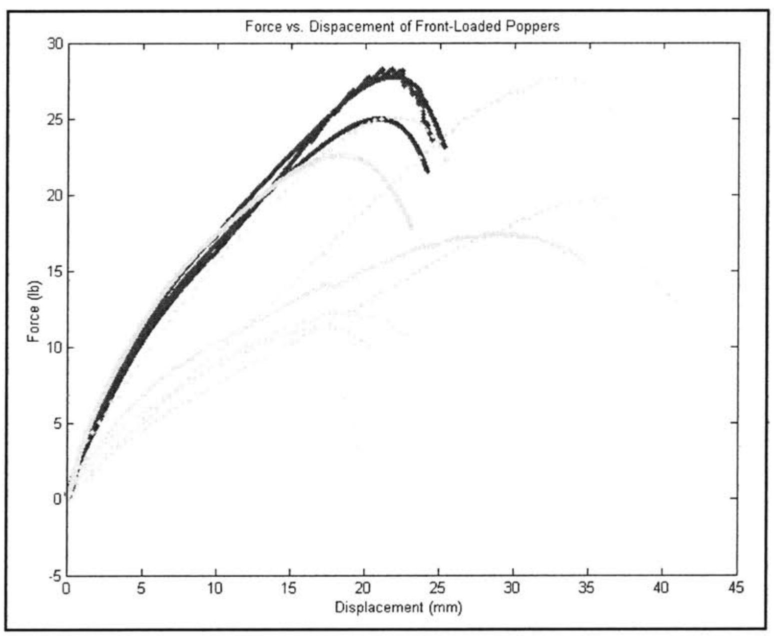


Figure B-1: Force vs. Displacement Curve Highlighting the Red Poppers

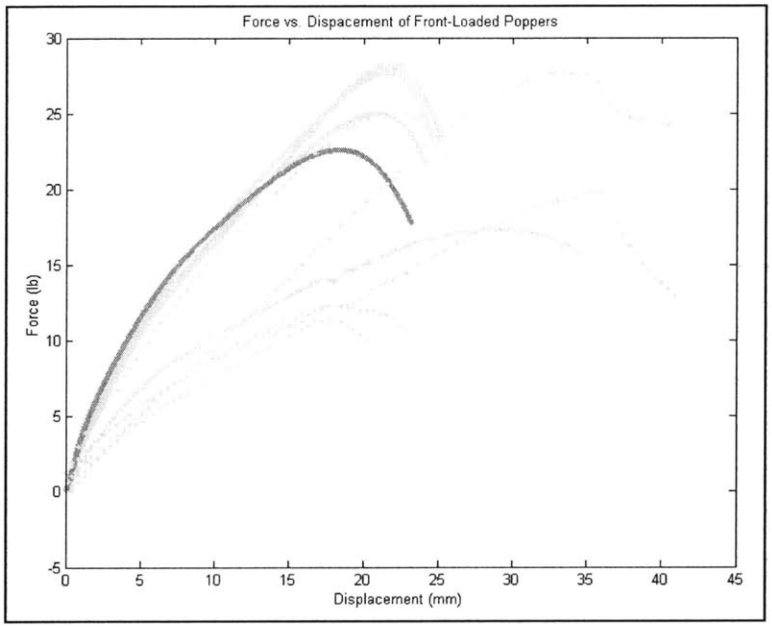


Figure B-2: Force vs. Displacement Curve Highlighting the Green Popper

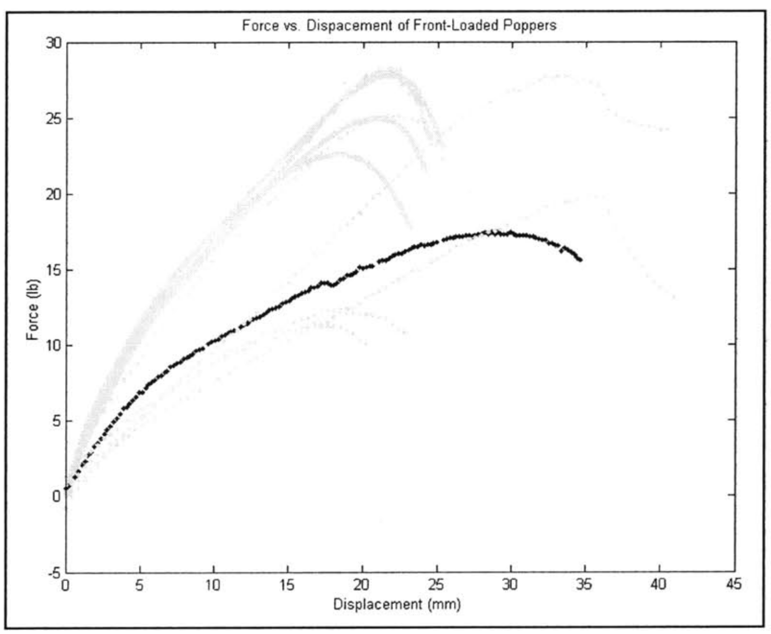


Figure B-3: Force vs. Displacement Curve Highlighting the Home-made Popper

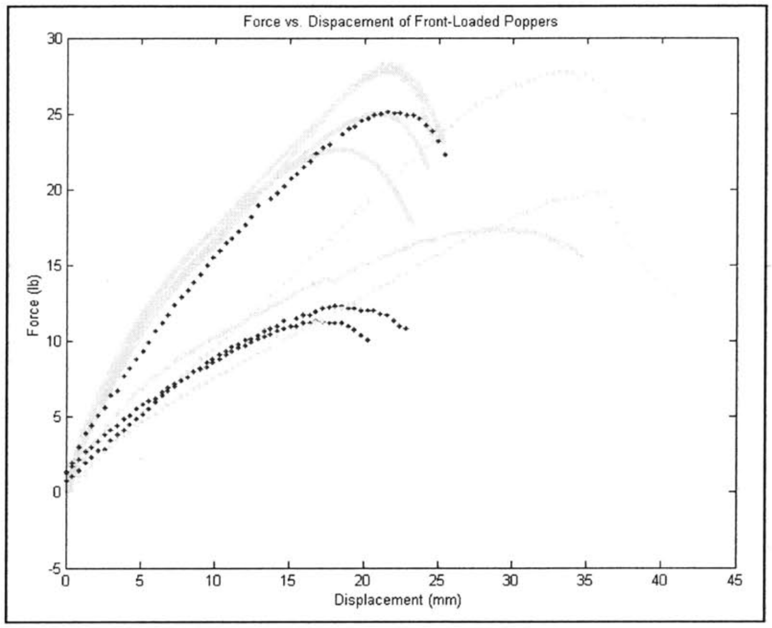


Figure B-4: Force vs. Displacement Curve Highlighting the Normal Polyurethane Poppers (1a is top)

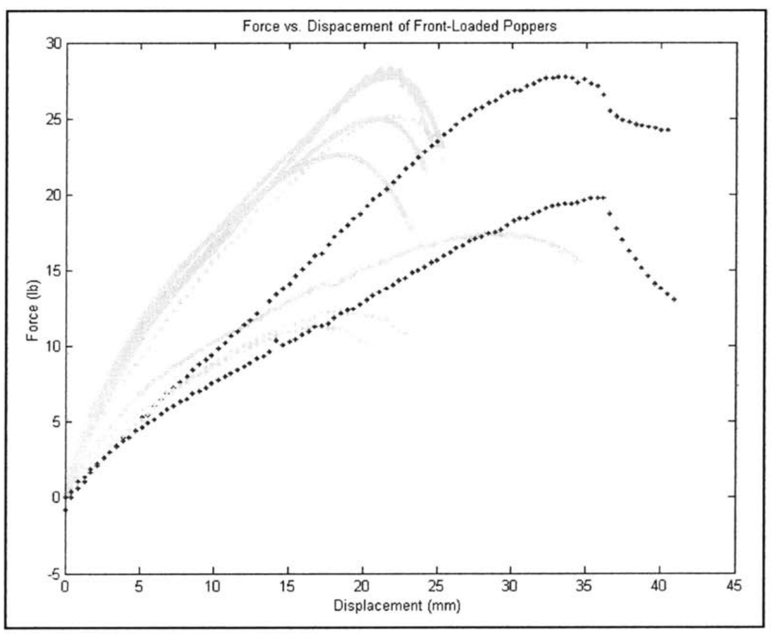


Figure B-5: Force vs. Displacement Curve Highlighting the Winged Polyurethane Poppers (1b is top)

Bibliography

[1] Arwade, S. R., 2006, "Material Property Tests of Smooth-On Vytaflex60 Liquid Rubber," JHU, Baltimore, USA, http://www.ce.jhu.edu/sarwade/models/rubber/vytaflex90_notes.pdf

[2] Fienup, W., Kudrowitz, B., 2005, "The Exploration of Concepts for Projectile Toys," MIT, Cambridge, USA.

[3] Galiev, S. U., Blachut, J., Skurlatov, E. D., Panova, O.P., 2004, "Experimental and Theoretical Design Methodology of Hemisperical Shells Under Extreme Static Loading," Strength of Materials, 36(5).

[4] Joyce, K. P., 2001, "Finite Element Analysis of Durometer Hardness Test and Correlation Between Durometer Hardness and Elastomer Stress-Strain Behavior," MIT, Cambridge, USA.

[5] Luo, P., Luo H., and Tong F., 1991, "The Influence of Prebuckling Deformations and Stresses on the Buckling of the Spherical Shell," International Journal of Offshore and Polar Engineering, 1(4).

[6] MSC.Software Corp., 2000, "Nonlinear Finite Element Analysis of Elastomers," http://www.axelproducts.com/downloads/MARC_FEA_ELASTOMERS_2000.pdf

[7] Petra Toys, Inc., http://www.petratoysusa.com/shop/creative_playthings_collectibles/dropper_popper

[8] Smooth-On, Inc., PMC-870 Industrial Liquid Rubber Compound, http://www.smooth-on.com/PDF/870.pdf

[9] The Dow Chemical Company, 2006, "Tensile Stress-Strain Behavior," http://www.dow.com/sal/design/guide/tensile.htm

[10] Timoshenko, S., 1936, *Theory of Elastic Stability*, McGraw-Hill Book Company, Inc., New York and London.