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Upscaling Fractured Heterogeneous Media: Permeability and Mass Exchange Coefficient

In order to optimize oil recuperation, to secure waste storage, CO_2 *sequestration and describe more precisely many environmental problems in the underground, we need to improve some homogenization methods that calculate petrophysical parameters. In this paper, we discuss the upscaling of fluid transport equations in fractured heterogeneous media consisting of the fractures themselves and a heterogeneous porous matrix. Our goal is to estimate precisely the fluid flow parameters like permeability and fracture/matrix exchange coefficient at large scale. Two approaches are possible. The first approach consists in calculating the large-scale equivalent properties in one upscaling step, starting with a single continuum flow model at the local scale. The second approach is to perform upscaling in two sequential steps: first, calculate the equivalent properties at an intermediate scale called the "unit scale," and, second, average the flow equations up to the large scale. We have implemented the two approaches and applied them to randomly distributed fractured systems. The results allowed us to obtain valuable information in terms of sizes of representative elementary volume associated to a given fracture distribution.*

Introduction

Many industrial and environmental problems involve flow in fractured porous media, like oil production, nuclear waste storage, and groundwater pollution. In this paper, we start from a fractured reservoir model as described by a geologist (Long et al. [1], Le Ravalec et al. [2]). We focus our study on the flow description at the scale of a grid-block in a numerical model (large scale). In this paper, we consider only characteristics associated with one-phase flow, such as the fracture permeability and fracture/matrix exchange coefficient.

We distinguish three scales illustrated in Fig. 1: (i) the local-scale characteristic of the fracture aperture; (ii) an intermediate scale called unit scale; and (iii) the large scale of the reservoir model also called a block scale. To describe the flow at the block scale, we have two main possible approaches. We present two approaches we have developed to identify large-scale parameters. The first approach consists of upscaling in one step (direct upscaling) from the local scale to the block scale and the second approach involves two stages (sequential upscaling) through the intermediate unit scale. We assume that the flow at the local scale is described by a simple Darcy-type equation. We also assume that the flow at the unit scale is described by a system of matrix-fracture equations according to the dual continuum model of Barenblatt and Zheltov [3]. This model was further developed by Warren and Root [4], and by many other contributors, e.g., Lough and Kamath [5] Quintard and Whitaker [6] In this paper, we use the general formulation developed theoretically by Quintard and Whitaker [6]

$$\phi_m c_m \frac{\partial}{\partial t}\{P_m\}^m = \nabla\left(\frac{1}{\mu}\mathbf{K}_{mm}\,.\,\nabla\{P_m\}^m + \frac{1}{\mu}\mathbf{K}_{mf}\,.\,\nabla\{P_f\}^f\right) - \frac{\alpha}{\mu}(\{P_m\}^m - \{P_f\}^f) \quad (1)$$

$$\phi_f c_f \frac{\partial}{\partial t}\{P_f\}^f = \nabla\left(\frac{1}{\mu}\mathbf{K}_{fm}\,.\,\nabla\{P_m\}^m + \frac{1}{\mu}\mathbf{K}_{ff}\,.\,\nabla\{P_f\}^f\right) - \frac{\alpha}{\mu}(\{P_f\}^f - \{P_m\}^m) \quad (2)$$

In this dual medium model, the first equation describes matrix flow, the second describes fracture flow, and the term with α models the fluid exchange between the matrix and fracture systems (Noetinger and Estébenet [7], Bourbiaux et al. [8]). Without going into details, let us emphasize that the upscaling from the unit scale to the block scale or the direct upscaling, may lead to (two) different classes of flow equations. If conditions are such that a single continuum model is valid, we only need to identify the equivalent permeability and the equivalent compressibility. But if a mechanical nonequilibrium model is required at the large scale, for instance in the form of a large-scale dual medium model similar to the one described by Eqs. (1) and (2), we need to identify a large-scale parameters that will be denoted by Kff^*, Kmm^*, α^*.

Direct Upscaling From the Local-Scale to the Block-Scale

By using the standard laboratory flow configuration, an imposed pressure drop and no flow boundaries called permeameter boundary conditions; we have developed an algorithm to calculate the full permeability tensor of heterogeneous anisotropic media or fractured systems [9]. For more details, by simulating a flow along the *ox* direction (resp. *oy*) for a square porous medium of dimension $L\times L$, the permeameter boundary conditions take this form

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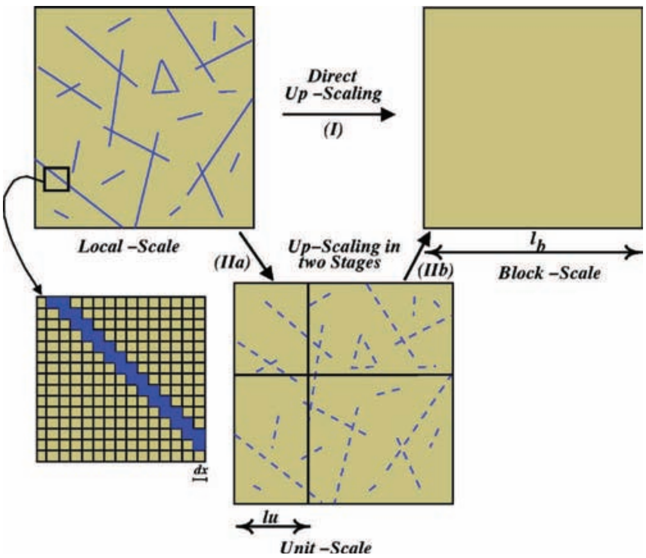

Fig. 1 Two different up-scaling paths: (I) Direct upscaling from the local-scale (dx) to the block-scale (L_b), and (II) Up-scaling in two stages passing through the intermediate unit-scale(l_u)

$$\begin{cases} \nabla p_{x,x} = \text{constant} \\ \displaystyle\int_{\Gamma_{yy}} \mathbf{n}\,.\,\mathbf{K}\,.\,\nabla p_{x,y=0,L} = 0 \end{cases} \quad \text{(Flow along the } ox \text{ direction)} \tag{3}$$

$$\begin{cases} \nabla p_{y,y} = \text{constant} \\ \displaystyle\int_{\Gamma_{xx}} \mathbf{n}\,.\,\mathbf{K}\,.\,\nabla p_{x=0,L,y} = 0 \end{cases} \quad \text{(Flow along the } oy \text{ direction)} \tag{4}$$

Where $p_{x,y=0,L}$ (resp. $p_{x=0,L,y}$) means the pressure calculated at the Γ_{yy} (resp. Γ_{xx}) faces for an imposed pressure gradient at the ox axis (resp. oy) quoted $\nabla p_{x,x}$ (resp. $\nabla p_{y,y}$). Where Γ_{yy} is perpendicular to the oy axis and Γ_{xx} is perpendicular to the ox axis (see Fig. 2). For a flow along the ox direction (resp. oy), the pressure on the impermeable edge will generate a transverse viscous force $\{\partial p/\partial y\}_x$ (resp. $\{\partial p/\partial x\}_y$) measurable numerically and experimentally. By measuring this additional information coupled to the fluxes measured on the ox, oy directions, we can calculate the full permeability tensor of the heterogeneous porous media

$$k_{xx} = \frac{q_{xx} - q_{yy}\dfrac{\delta x_y}{L}\left\{\dfrac{\partial p}{\partial y}\right\}_x}{1 - \left\{\dfrac{\partial p}{\partial x}\right\}_y \left\{\dfrac{\partial p}{\partial y}\right\}_x}; \quad k_{yy} = \frac{q_{yy} - q_{xx}\dfrac{\delta y_x}{L}\left\{\dfrac{\partial p}{\partial x}\right\}_y}{1 - \left\{\dfrac{\partial p}{\partial x}\right\}_y \left\{\dfrac{\partial p}{\partial y}\right\}_x} \tag{5}$$

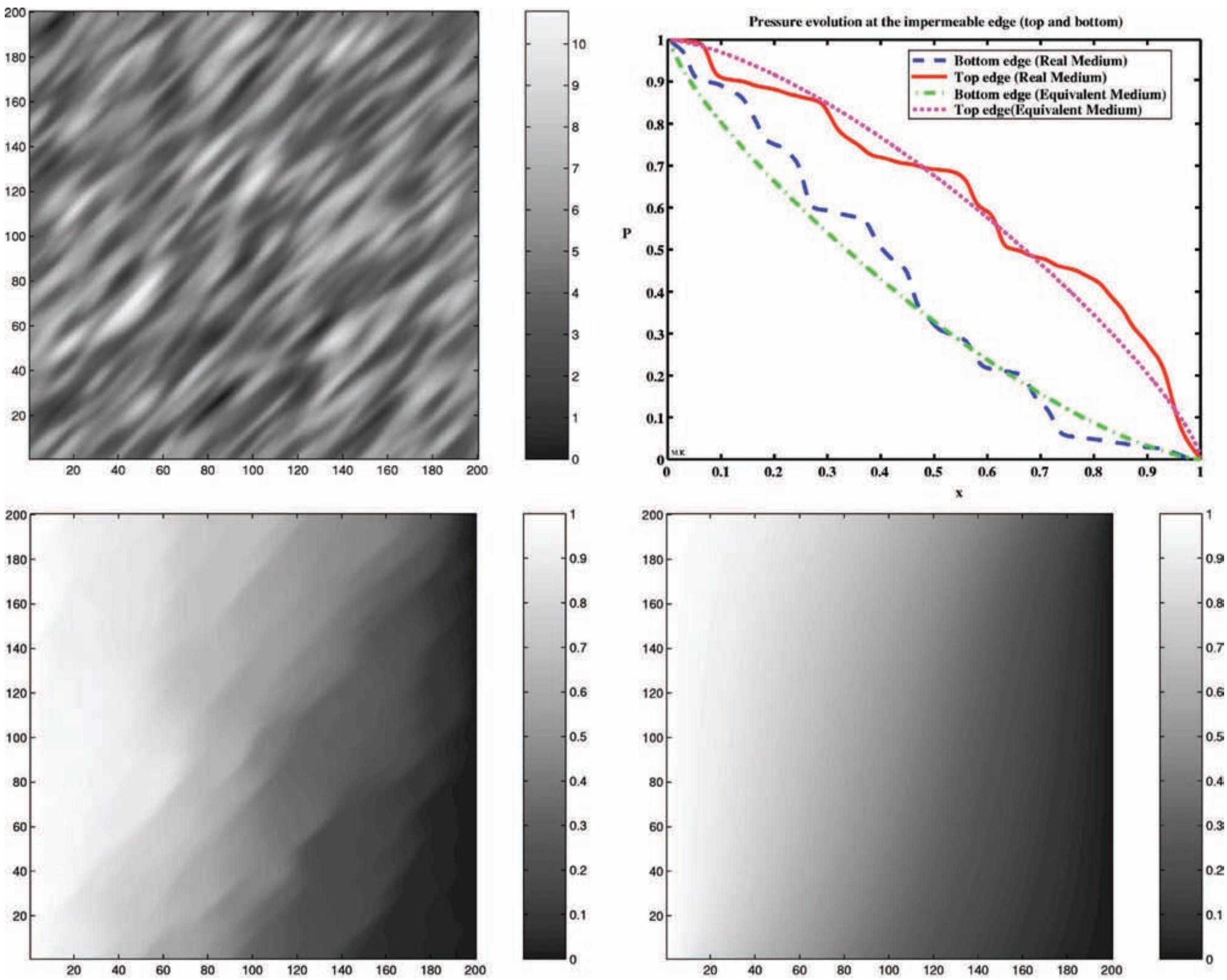

Fig. 2 This figure shows the faces quotation used in this paper for a square porous medium

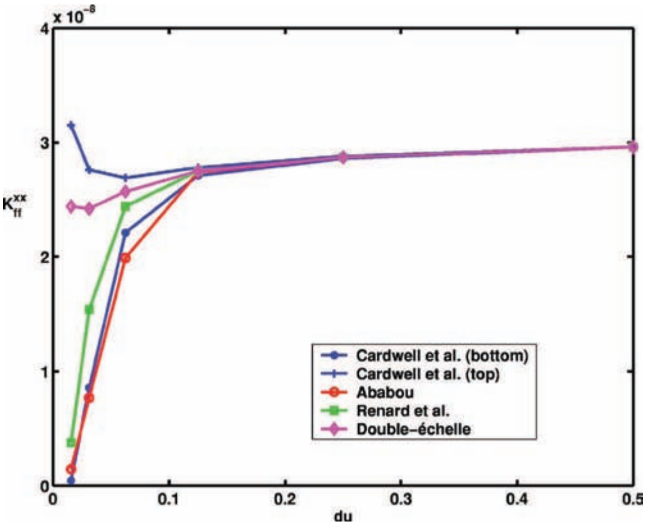

Fig. 3 Lognormal permeability map (Darcy) generated using FFTMA for 200×200 cells (at left) and the pressure (bar) evolution at the impermeable edges for a horizontal confined flow for the real medium and its anisotropic homogeneous equivalent (at right). At bottom, pressure maps for the real medium (at left) and its equivalent homogeneous medium (at right).

$$k_{xy} = -\left\{\frac{\partial p}{\partial x}\right\}_y k_{xx} + \frac{\delta x_y}{L} q_{yy}; \quad k_{yx} = -\left\{\frac{\partial p}{\partial y}\right\}_y k_{yy} + \frac{\delta y_x}{L} q_{xx} \quad (6)$$

Where δy_x (resp.δx_y) corresponds to the difference between the two flow barycentre at the outlet edge and the inlet edge of the sample for a flow imposed according to ox (respectively oy).

In Fig. 3 (top, left), we show a lognormal permeability map (k_g=121 Darcy; σ^2=2) generated with the stochastic code FFTMA [2]. Locally, the permeability is isotropic (scalar), but its correlation function (or variogram) is anisotropic. Here, it has an orientation of 44.46° with respect to the horizontal axis. Also we present in the same figure (Fig. 3, top, right) the pressure variations at the impermeable edges for a horizontal confined flow for the real medium and its anisotropic homogeneous equivalent medium determined by using the method in Ref. [9] and shortly described in this paper. In addition, we show, respectively, the pressure maps for the real medium (Fig. 3, bottom, left) and its equivalent medium (Fig. 3, bottom, right). For these calculations, the effective permeability tensor for the researched equivalent medium provided by this method is

$$\mathbf{K} = \begin{pmatrix} 122.05 & 58.08 \\ 58.08 & 119.86 \end{pmatrix} \quad (7)$$

The eigenvalues of this tensor are: k'_{xx}=179.05, k'_{yy}=62.86. The principal axes have an orientation of 44.46° with respect to the direction of the imposed flow.

Also, we have developed a numerical solver based on a two-dimensional (2D) finite volume scheme with a five point stencil for the "closure problems" presented in Quintard and Whitaker [6] Landereau et al. [10] which give all the required permeability tensors and the matrix-fracture exchange coefficient at the different large scales (unit scale or block scale). The flow is described by a dual continuum model. In Fig. 4, we show a zoom of a fractured porous media studied in this paper. The volume fraction of the 26,538 fractures present in the medium is around 9%, and the ratio of fracture permeability to matrix permeability at local-scale is around 1000. At local scale, we build a fine grid with 2048×2048 cells. We then choose to partition the domain into units of different sizes, leading to several possible partitions (4×4 units, 8×8 units, 16×16 units, and 32×32 units). For each partition, we have calculated a map of the matrix-fracture exchange coefficient α and of the first principal component of the equivalent permeability tensor $\mathbf{K}_{EQ}^{XX}$, and we have plotted their histograms in Fig. 4. The histogram for α shows a classical normal shape for all unit-scale partition. The results for $\mathbf{K}_{EQ}^{XX}$ show a more complicated structure. For fine unit-scale partition, we observe a bimodal histogram with a group of low values corresponding to nonpercolating units. This bimodal structure disappears for unit size "sufficiently" large. This word "sufficiently" will be associated in the discussion at the end of this paper to the size of a representative elementary volume (REV), important notion for practical applications.

Sequential Upscaling in Two Stages Through the Intermediate Unit-Scale

Once the permeability and the matrix/fracture exchange coefficient distributions (maps) are known at the intermediate scale of the "units," further averaging is required to obtain the flow behavior at the block scale. We have formulated a dual continuum model at the block scale starting from the Barenblatt et al. [3] model at the unit scale. We have obtained two systems of dual continuum equations in the matrix and fractured regions for averaged pressure (P_m or P_f) and for pressure deviations ($\tilde{P}_m$ or $\tilde{P}_f$). Without going into details, the equivalent permeability for the fractured region is obtained by solving the closure problem described below

$$\tilde{p}_f = \mathbf{b}_f \cdot \nabla \mathrm{P}_f \quad (8)$$

$$0 = \nabla(\mathbf{K}_{ff} \cdot \nabla \mathbf{b}_f) + \nabla \cdot \mathbf{K}_{ff} \quad (9)$$

$$\mathbf{b}_f(x + l_i) = \mathbf{b}_f(x) \quad (10)$$

$$\{\mathbf{b}_f\} = 0 \quad (11)$$

$$\mathbf{K}_{ff}^{*} = \{\mathbf{K}_{ff}\} + \{\mathbf{K}_{ff} \cdot \nabla \mathbf{b}_f\} \quad (12)$$

which is reminiscent of classical equations obtained for diffusion problem with heterogeneous diffusion coefficients (Saez et al. [11], Bourgeat et al. [12] Quintard and Whitaker [13]). We applied those formulas to the permeability map obtained at the unit scale for each partition of the block (see Fig. 4). The equivalent permeability of the fracture network is presented in Fig. 5 as a function of the unit size l_u. Our upscaled permeability called the "double scale" is compared to that obtained by using the Cardwell and Parsons [14] technique, Ababou [15] approach, Renard et al. [16] method. These results will be discussed below.

The determination of a large-scale mass exchange coefficient from the mapped α is not a trivial matter. Our approach is based on direct numerical simulation of the dual medium model at the unit scale and interpretation of the resulting block-scale fields. We studied the case $K_m \ll \phi_f K_f$. We have shown by numerical simulations that, when the exchange coefficient is large enough, the fracture pressure diffusion flux is negligible compared to the exchange flux. More precisely, this occurs when $\mu \alpha l_u^2 / K_f \gg 1$. Our simulation tests show that the asymptotic upscaled exchange coefficient α is the harmonic mean of the local coefficient α^*. On the other hand, when the exchange coefficient is small enough, the fracture pressure gradient becomes negligible because there is a strong diffusion which tends to homogenize the fracture pressure in space. More precisely, this occurs when $\mu \alpha l_u^2 / K_f \ll 1$. Our simulation tests show that the asymptotic upscaled exchange coefficient α is the minimum of the local coefficient α. Similar results were obtained with a stochastic method presented in the paper by Kfoury et al. [17].

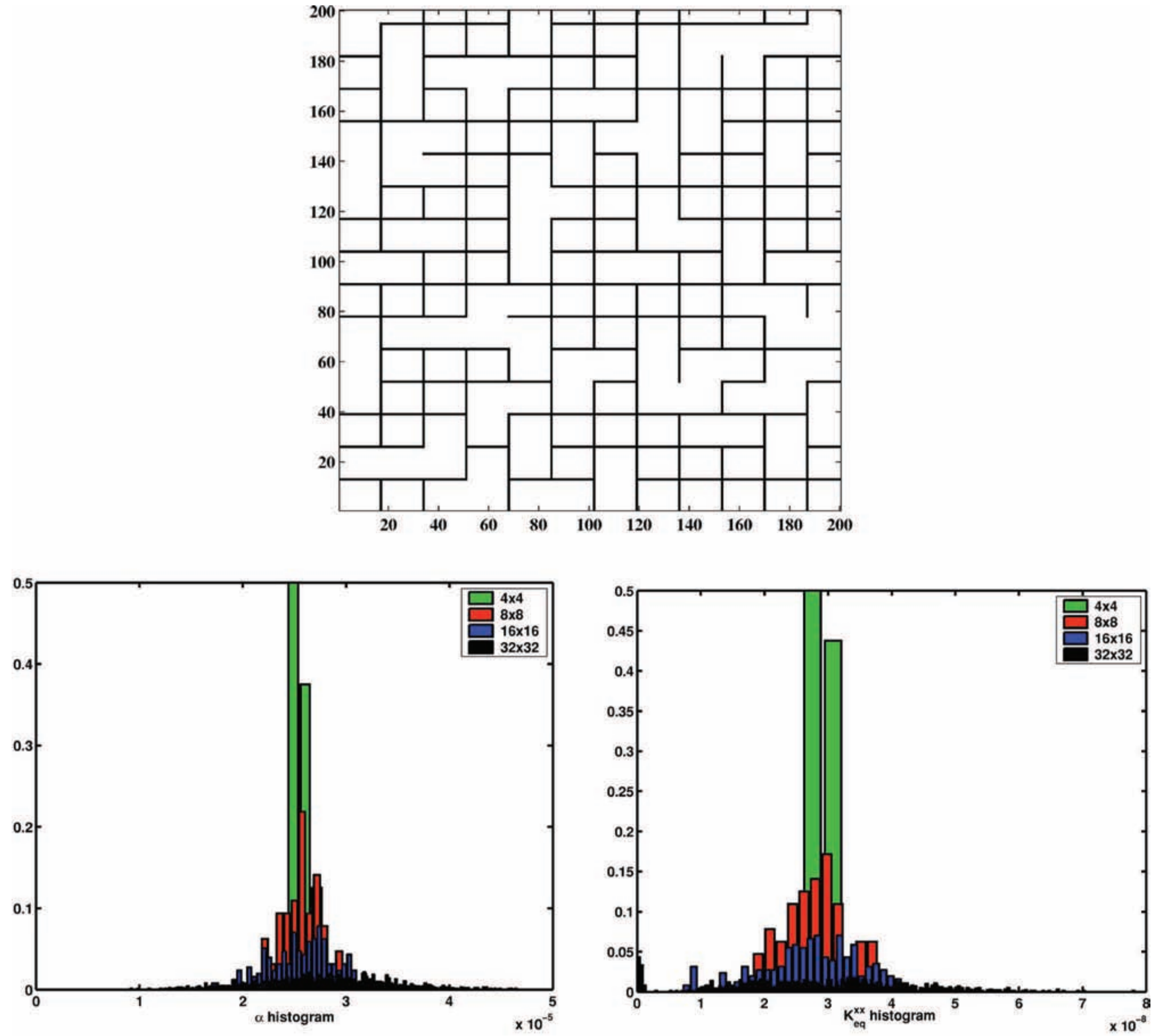

Fig. 4 Above: example of a fractured porous medium used in this work (200×200 cells). Below: histogram of matrix-fracture exchange coefficient α (left), and histogram of first component K_{EQ}^{XX} of equivalent permeability (right).

Representative Elementary Volume

While the choice of the block size by reservoir engineers is not in general truly determined by scientific considerations based on a detailed analysis of the lower-scale properties, it may be a safe engineering practice to have the block size a little bit larger than the spatial correlation length (or REV size) of the heterogeneous field under consideration within the grid block. The sequential upscaling technique presented in this paper offers some information about this important aspect. This is illustrated in Fig. 5. Results for $l_u \geqslant 1/8m$ are close to those obtained directly with a fine gridding of the Darcy-scale geometry. This result can also be confirmed by looking at the unit-scale repartition of K_{ff}. This is shown in Fig. 6 that represents the different values of K_{ff} as a function of ϕ_f, for each unit. We observe two populations, the one with a very low permeability being associated with nonpercolating fractures. We see that this nonpercolating cloud disappears for $l_u \geqslant 1/16m$, which is compatible with the REV size estimated from Fig. 5. The REV size can also be analyzed by looking at the distribution of α. Figure 7 represents the evolution of the arithmetic and harmonic mean of α, with respect to the unit-size l_u. We

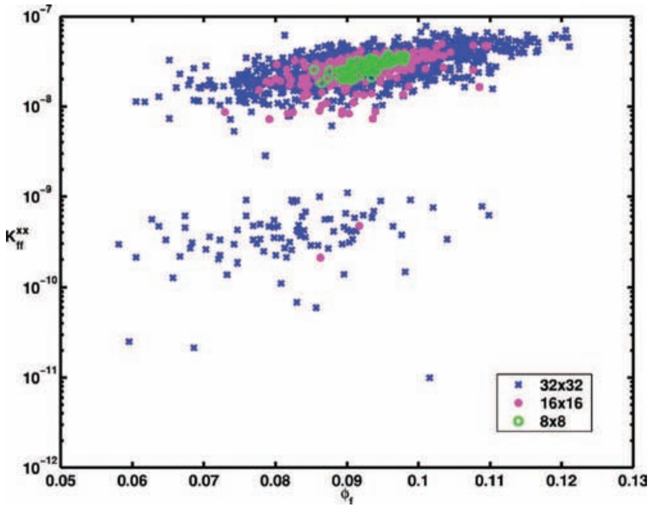

Fig. 5 First component of the equivalent permeability for a fracture network at the block-scale (k_{ff}^{xx}, m^2)

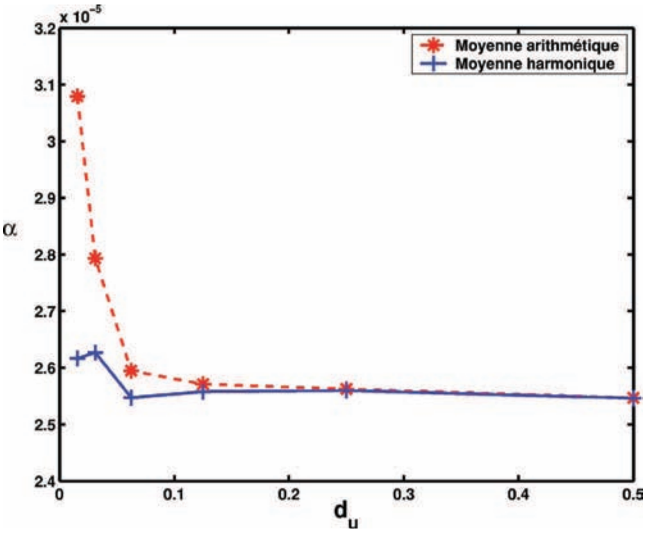

Fig. 6 Sequential upscaling: First component of fracture network permeability in each cell at the unit-scale for all partitions

remind the reader that these two values represent the two limiting behaviors of the system. We observe that the evolution of these two values is relatively small for $l_u \geqslant 1/8m$. This characteristic REV length scale is compatible with the estimate obtained from the analysis of the fracture permeability.

Conclusions

Direct and sequential approaches to upscaling flow properties in fractured heterogeneous porous media have been presented. Different methods to optimize the calculation of petrophysical parameters have been developed. We find that the idea of sequential upscaling in two steps is interesting in terms of computer effort (CPU time calculation) and of REV information.

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Nomenclature

$\mathbf{b}_f$ = Vector that map ∇P_f onto $\tilde{p}_f$ in the new double continuum model at the block-scale, m
c_f = Total compressibility in the fracture region, Pa^{-1}
c_m = Total compressibility in the matrix region, Pa^{-1}
K_g = Geometric average of the permeability distribution, Darcy
$\mathbf{K}_f$ = Darcy scale permeability tensor in the fracture region, m^2; 1 Darcy $\approx 10^{-12}$ m^2
$\mathbf{K}_m$ = Darcy scale permeability tensor in the matrix region, m^2
$\mathbf{K}_{ff}$ = Fracture region, unit-scale permeability tensor in the two-equation model, m^2
$\mathbf{K}_{mm}$ = Matrix region, unit-scale permeability tensor in the two-equation model, m^2
$\mathbf{K}_{mf}=\mathbf{K}_{fm}$ = Unit-scale cross-effect permeability tensor in the two-equation model, m^2
$\mathbf{K}^*_{ff}$ = Fracture region, block-scale permeability tensor, m^2
$\mathbf{K}^*_{mm}$ = Matrix region, block-scale permeability tensor, m^2
l_u = Unit size, m
$\{P_f\}^f$ = Intrinsic macroscopic pressure for the fracture region, Pa
$\{P_m\}^m$ = Intrinsic macroscopic pressure for the matrix region, Pa
$\tilde{P}_f$ = Large-scale pressure deviation associated with the fracture region, Pa
P_f = Superficial regional average pressure for the fracture region, Pa
q = Flux, m^2
V_∞ = Large-scale averaging volume, m^3
V_f = Volume of the fractured region contained within V_∞, m^3
V_m = Volume of the matrix region contained within V_∞, m^3
α = Exchange coefficient at the unit scale
α^* = Exchange coefficient at the block scale
$\phi_f = V_f/V_\infty$ = Volume fraction of the fracture region contained in the averaging volume
$\phi_m = V_m/V_\infty$ = Volume fraction of the matrix region contained in the averaging volume
σ^2 = Variance of the permeability distribution
μ = dynamic viscosity, N s/m^2

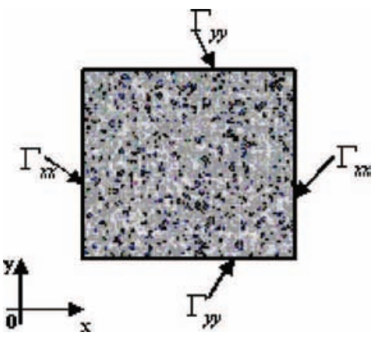

Fig. 7 Sequential upscaling: Arithmetic and harmonic values of exchange coefficient for each partition at the unit scale

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