To cite this version : Ababou , Rachid Stochastic PDE's for water flow, solute transport and wave propagation phenomena in heterogeneous geologic media. (2013) In: Mathematics and Geosciences: Global and Local Perspectives, 04 November 2013 - 08 November 2013 (Madrid, Spain)

MAthGeo 2013, Madrid 4th-8th November 2013
"Mathematics and Geosciences: Global and Local Perspectives" ***(ICMAT/UCM/UPM)***
Madrid, November 4th-8th, 2013. **http://www.icmat.es/congresos/mag2013/**

MAIN LECTURE

Stochastic PDE's for water flow, solute transport and wave propagation phenomena in heterogeneous geologic media.

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ABSTRACT

This lecture will present stochastic PDE's (Partial Differential Equations) to model various "transport" phenomena like water flow, solute transport and wave propagation, in heterogeneous geologic porous media. The material properties are represented by random functions of space F(x) (random fields). The resulting transport PDE's contain random field coefficients, and their solutions are stochastic (randomly heterogeneous).

In this framework, the objectives are to solve the stochastic PDE's, to analyze the behavior of the solutions (fluxes, velocities, pressures, stresses and strains), and to propose an upscaled version of the governing equations with effective or equivalent "macro-scale" coefficients that incorporate the effects of heterogeneity (e.g., anisotropic macro-permeability tensor Kij).

The methods used for obtaining the random solutions involve the so-called "*sigma-expansion*" method, where "*sigma*" stands for the standard deviation of the random parameters…or of their logarithms (it will be shown that this choice can make quite a difference regarding the robustness of the solution). The perturbation equations are then solved with Green's functions in space, and/or with spectral representation in the space of wave-vectors k (*Fourier-Wiener-Khinchin* representation with *Stieltjes* integrals).

Once the random fluctuations or their moments are known, the next task is to implement "relevant" averaging operations, and to study the behavior of averaged macro-scale quantities (mean flux, mean pressure gradient, mean displacement, etc.). Macro-scale governing PDE's can emerge from this upscaling step, e.g., one obtains a mass conservation equation div(Q) = 0 combined with a macro-scale Darcy equation Q = -K Grad(P) where macro-permeability K is a 2nd rank tensor embodying the geometric anisotropy of the geologic medium. However, due to nonlinear or stochastic interactions, the macro-scale PDE does not necessarily resemble the original "local scale" stochastic PDE…as will be shown.

Through this lecture, several types of phenomena of practical importance will be used as examples: hydrodynamic dispersion of a tracer; single phase flow (Darcy); two-phase flow (Darcy-Muskat); seismic elastic wave dispersion and attenuation....

KEY-WORDS: Stochastic PDE; Random fields; Random media; Geologic porous media; Perturbation methods; Fourier-Wiener-Khinchin; Homogenization; Upscaling; Darcy-Muskat; Elastic waves.