A bypass transition in the Lamb-Oseen vortex

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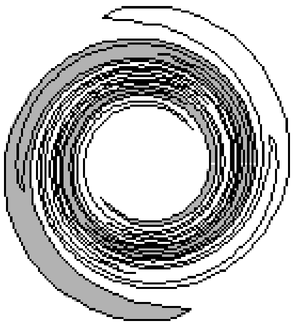
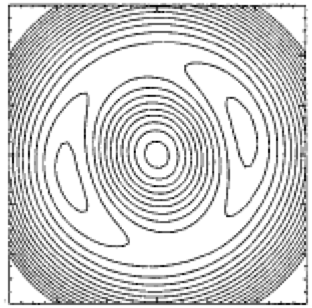
Outline



Observation of high energy transient amplifications in the Lamb-Oseen vortex by means of the Linear Optimal Perturbation approach

a) Nonlinear Optimal Perturbation analysis?

Axisymmetrization process in the Lamb-Oseen monopole: a generic process?



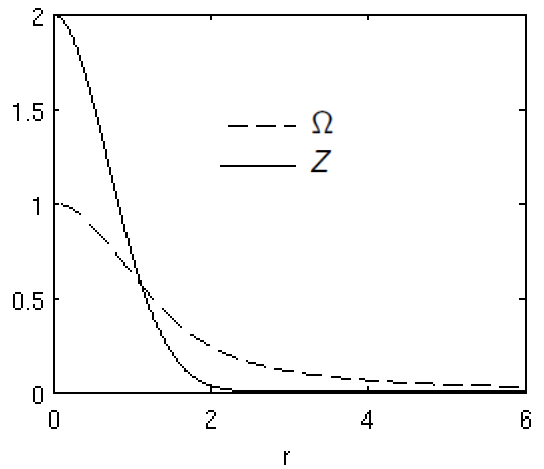
Rossi et al. 1997

Emergence of tripole vortex observation in numerical simulations

b) Nonlinear Optimal Perturbation: a potential path to a nonlinear bypass transition?

The Lamb-Oseen vortex

Reynolds number	$Re = \Omega_0 R_0 / \nu$
Axial vorticity	$Z(r,t) = 2\exp(-r^2/(1+4t/Re))/(1+4t/Re)$
Angular velocity	$\Omega(r,t) = [1-\exp(-r^2/(1+4t/Re))]/r^2$



Linear stability analysis: one-signed vorticity gradient distribution is **linearly stable** at large times solution.

Shear-diffusion mechanism drives the axysimmetrization process (on the $Re^{1/3}$ time scale for large Reynolds number flows - Bernoff and Lingevitch, 1994).

Transient energy growth

Why?

Classical stability analysis:

- **small perturbations** in the base flow

$$\phi(r,\theta,z,t) = \Phi(r,\theta,z) + \phi^{'}(r,\theta,z,t) \qquad \phi^{'}/\Phi << 1;$$

- **linearization** around the base state + modal decomposition

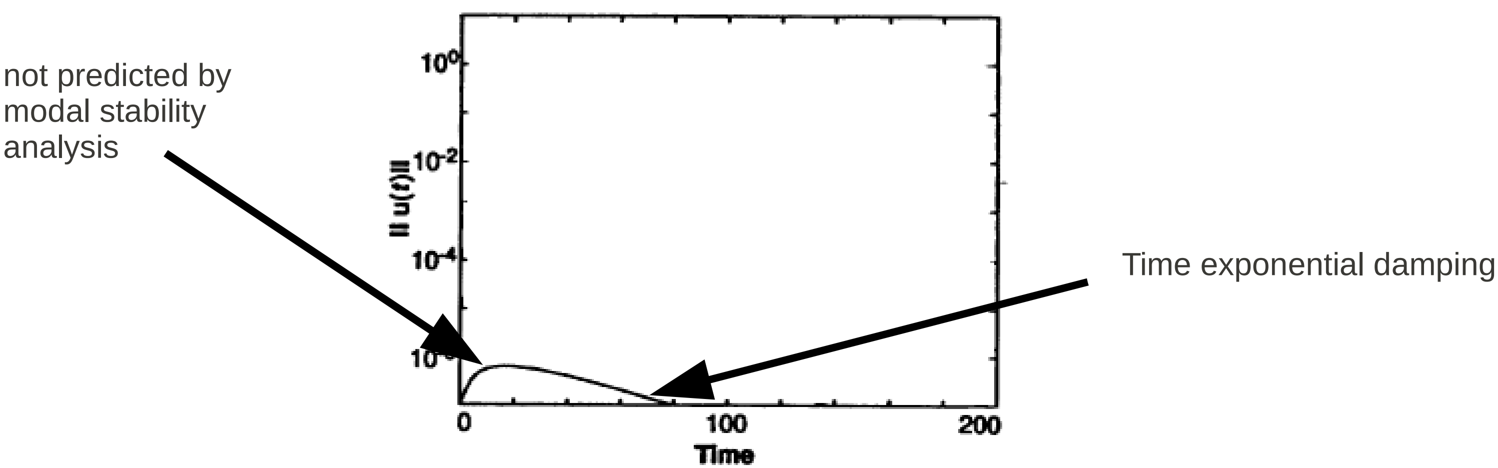
$$\phi^{'}(r,\theta,z,t) = \hat{\phi}(r)\exp\{i\,(kz + m\theta - \omega t)\}\,;$$

- eigenvalue analysis.

If all eigenvalues are in the stable complex half-plane,

The flow is linearly stable.

But.. transient energy amplifications are possible if the governing system is not normal.



Trefethen et al. 1993

Transient energy growth

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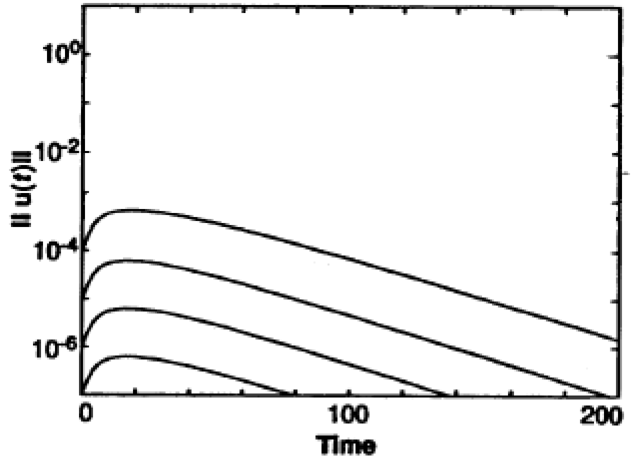

Trefethen et al. 1993

Transient energy growth

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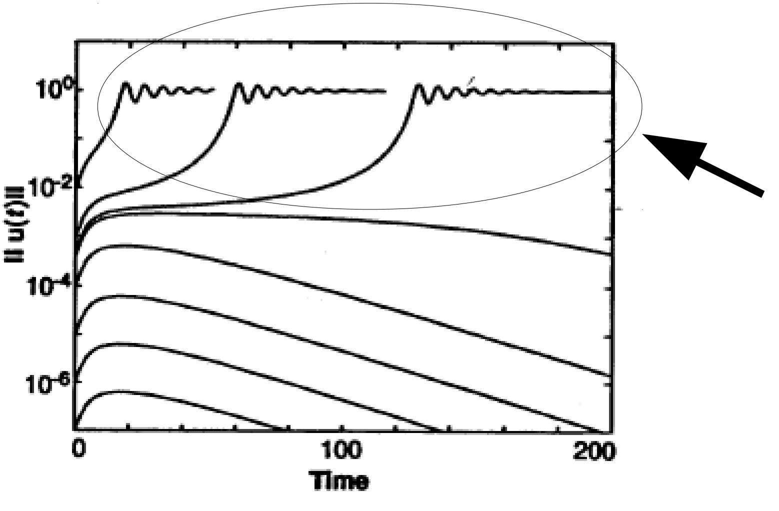


nonlinear bypass scenario

Trefethen et al. 1993

The Optimal Perturbation approach

Analytical background

The “optimal perturbation” maximizes the gain at a given time (Farrell 1988).

Lagrangian multiplier technique to find Nonlinear Optimal Perturbation (Pringle & Kerswell 2010).

$$\mathcal{L} = E(\tau) - \langle \mathcal{F}(u), a \rangle - \lambda (E(0) - E_0)$$

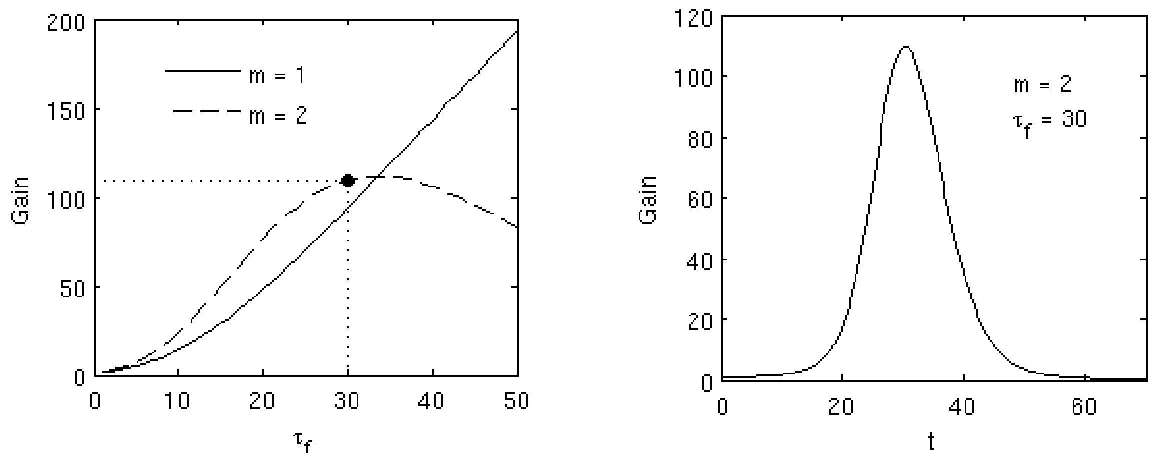
$$\frac{\partial \mathcal{L}}{\partial \epsilon} \delta \epsilon = 0 \quad \Longrightarrow$$

- Navier Stokes equations + b.c.
- Adjoint equations + b.c.
- compatibility conditions

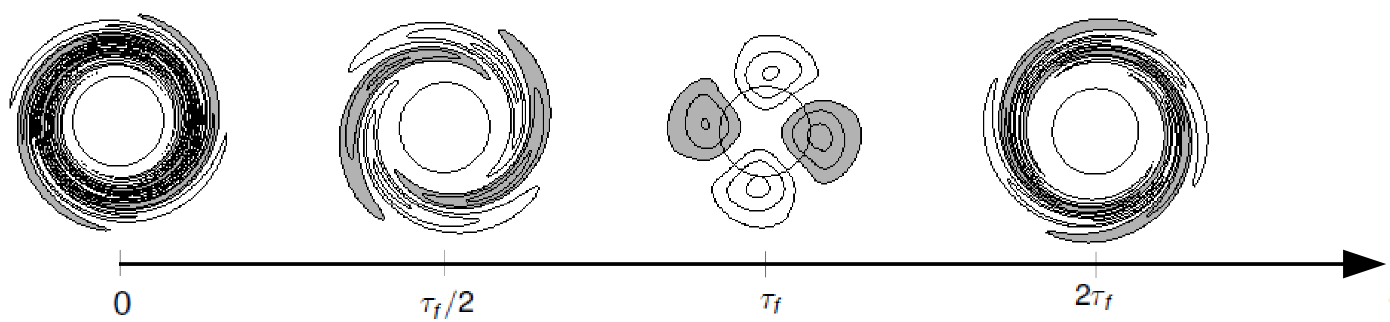
… and a pseudo-spectral code!

Transient energy growth

2D Linear Optimal Perturbation

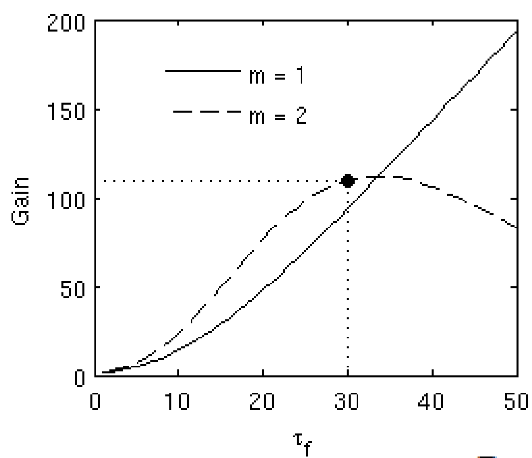


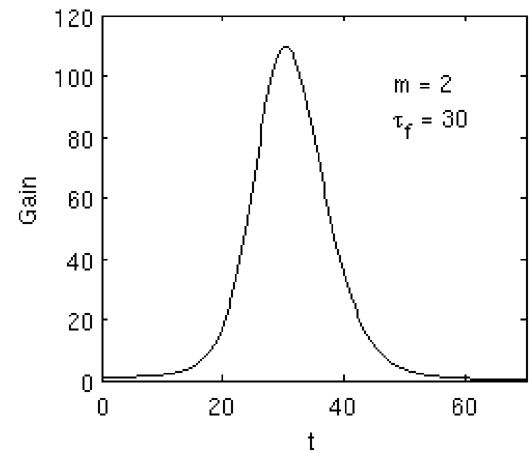
$Re = 5000; k = 0$



Transient energy growth

2D Linear Optimal Perturbation





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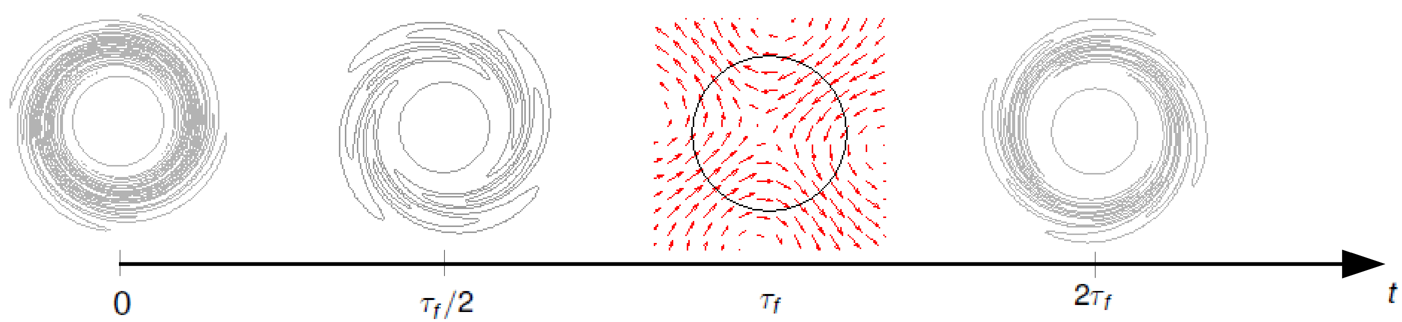
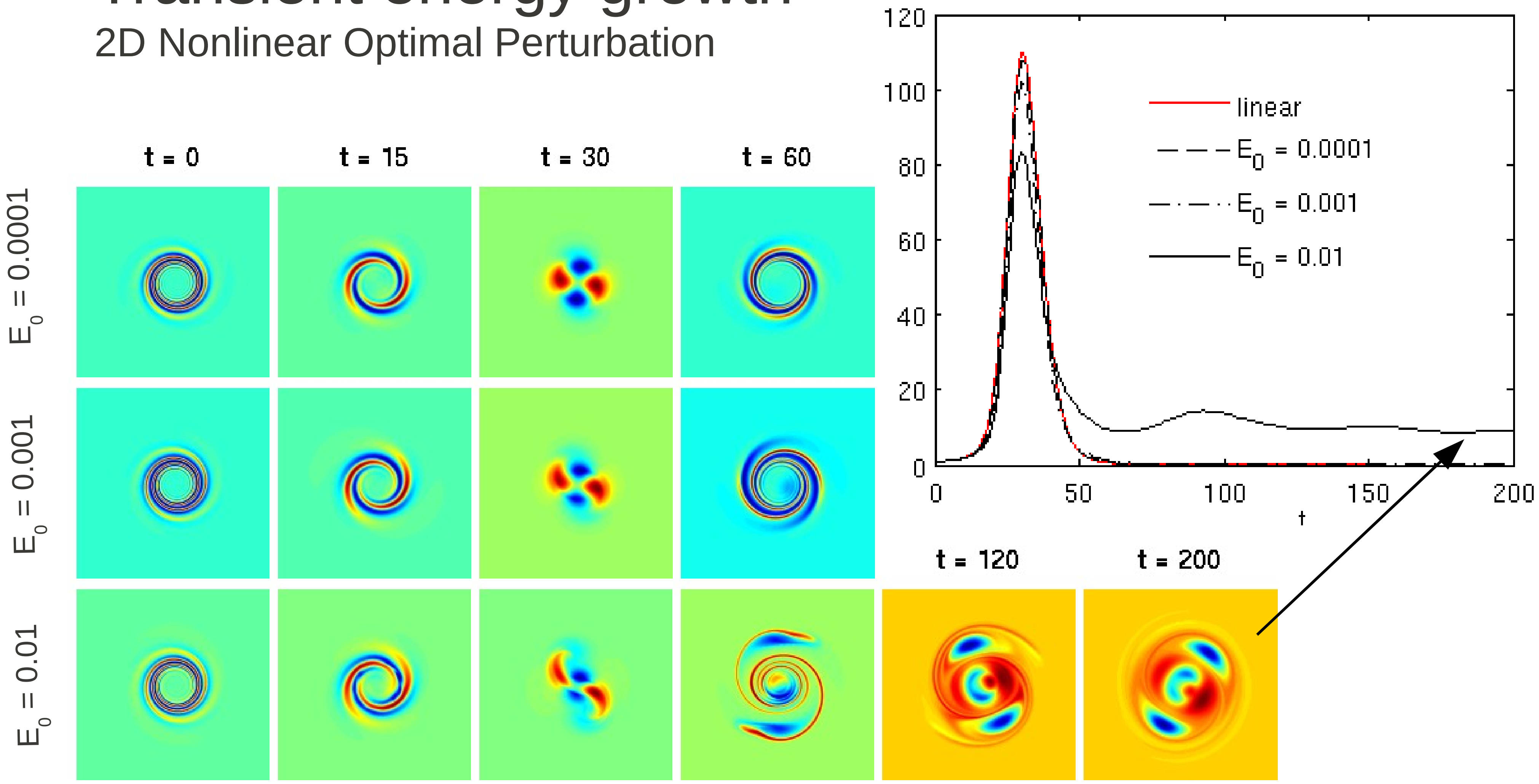


Transient energy growth

2D Nonlinear Optimal Perturbation



Transient energy growth

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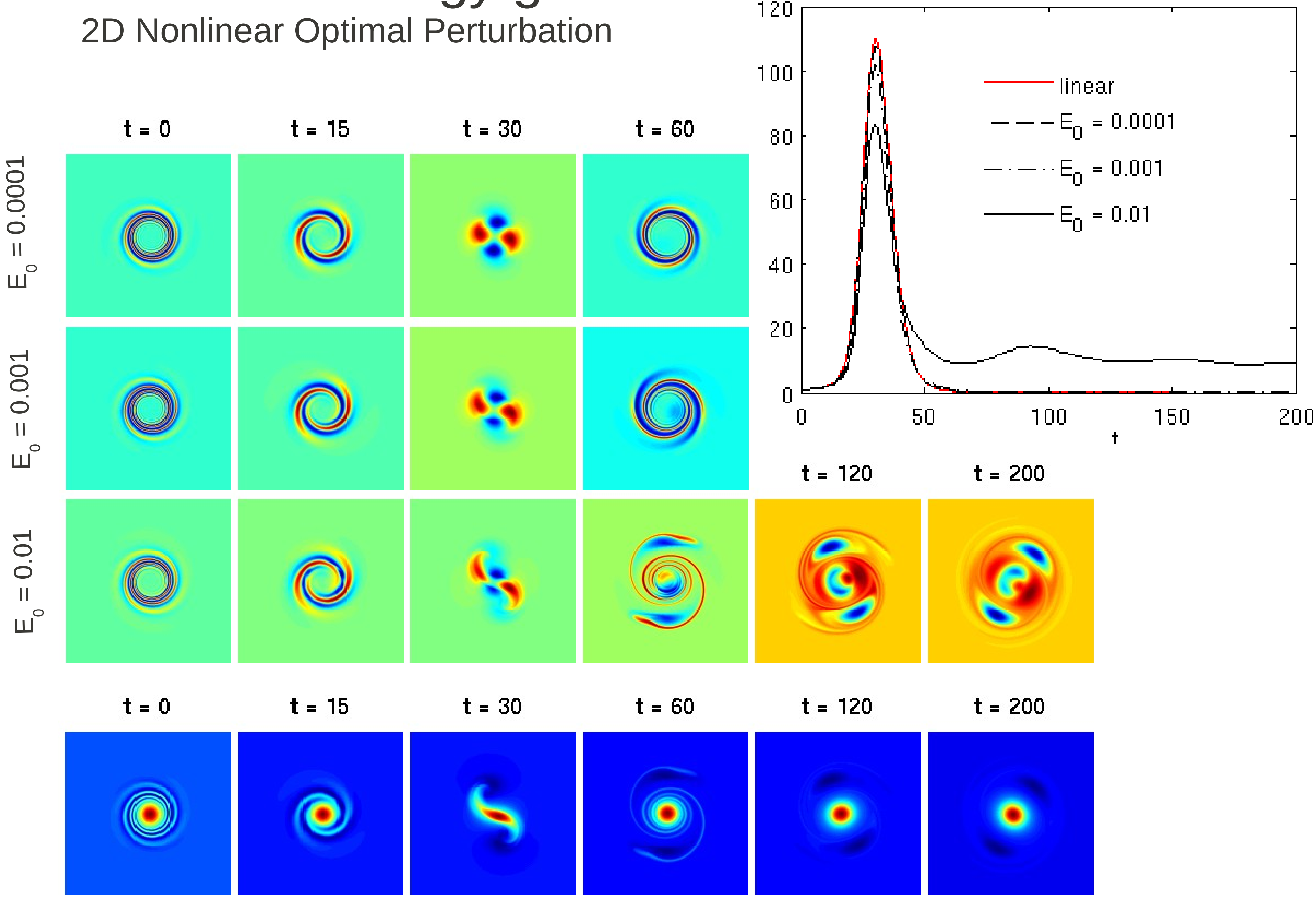

Conclusions

- Nonlinear optimal perturbations: remarkable differences with respect to the linear case;
- Axisymmetrization is a systematic process only in the linear approach;
- High-energy tripole generation as a nonlinear bypass transition induced by a nonlinear transient growth mechanism revealed by a nonlinear optimal perturbation analysis;
- $\bar{E}_0$ 'threshold' as (Re, τ) function (in Rossi & al, 1997 and Barba & Leonard, 2007 but differences...);
- Kinematic energy gain: the most effective objective function to induce transition?