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# A bypass transition in the Lamb-Oseen vortex

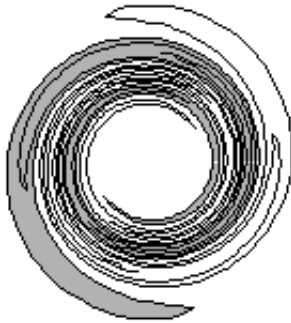
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APS – 65th Annual DFD Meeting  
Sunday, November 18th 2012

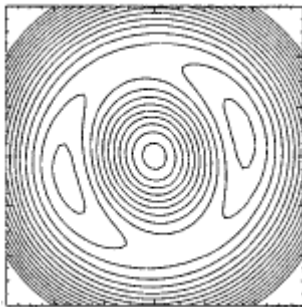
# Outline



**Observation of high energy transient amplifications** in the Lamb-Oseen vortex by means of the Linear Optimal Perturbation approach

a) Nonlinear Optimal Perturbation analysis?

Axisymmetrization process in the Lamb-Oseen monopole: a generic process?



**Emergence of tripole vortex observation in numerical simulations**

b) Nonlinear Optimal Perturbation: a potential path to a nonlinear bypass transition?

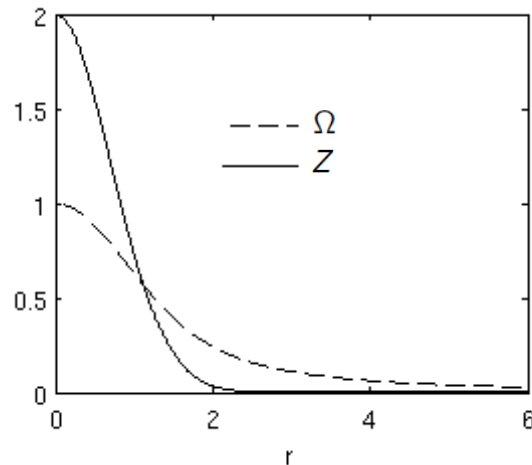
Rossi et al. 1997

# The Lamb-Oseen vortex

Reynolds number  $Re = \Omega_0 R_0 / \nu$

Axial vorticity  $Z(r, t) = 2 \exp(-r^2 / (1 + 4t / Re)) / (1 + 4t / Re)$

Angular velocity  $\Omega(r, t) = [1 - \exp(-r^2 / (1 + 4t / Re))] / r^2$



Linear stability analysis: one-signed vorticity gradient distribution is **linearly stable** at large times solution.

Shear-diffusion mechanism drives the axisymmetrization process (on the  $Re^{1/3}$  time scale for large Reynolds number flows - Bernoff and Lingeitch, 1994).

# Transient energy growth

Why?

Classical stability analysis:

- **small perturbations** in the base flow

$$\phi(r, \theta, z, t) = \Phi(r, \theta, z) + \phi'(r, \theta, z, t) \quad \phi' / \Phi \ll 1;$$

- **linearization** around the base state + modal decomposition

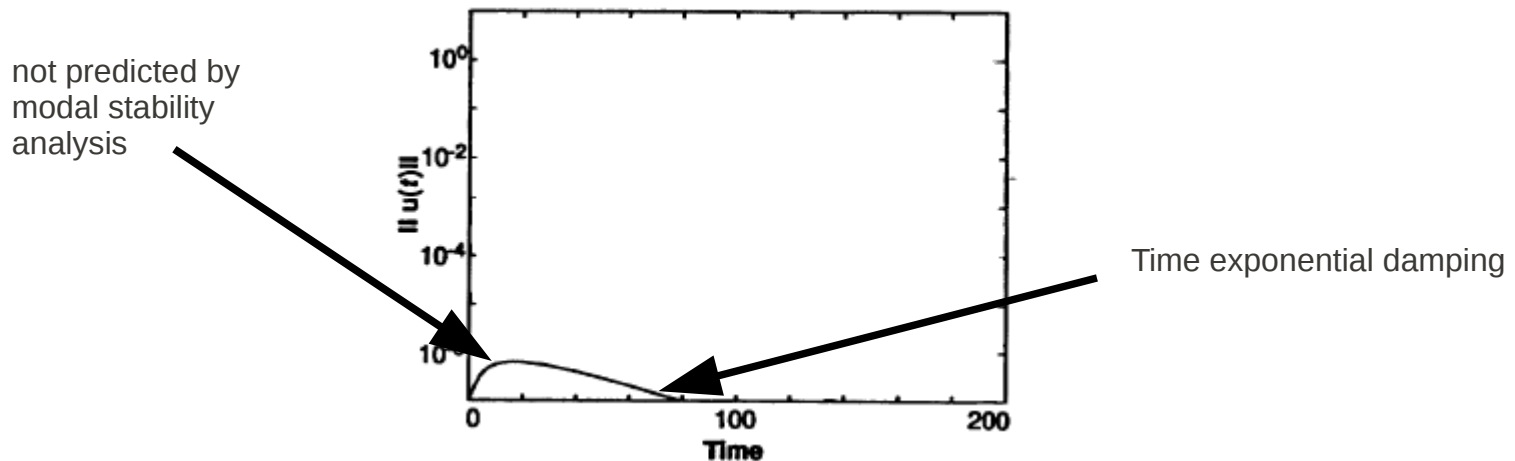
$$\phi'(r, \theta, z, t) = \hat{\phi}(r) \exp \{i(kz + m\theta - \omega t)\};$$

- eigenvalue analysis.

If all eigenvalues are in the stable complex half-plane,

**The flow is linearly stable.**

**But..** transient energy amplifications are possible if the governing system is not normal.



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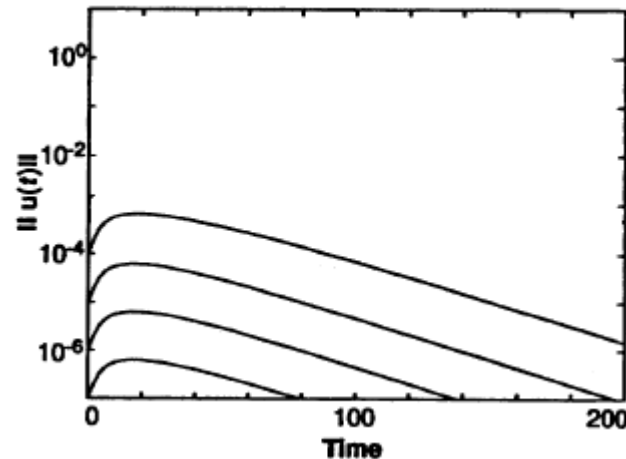
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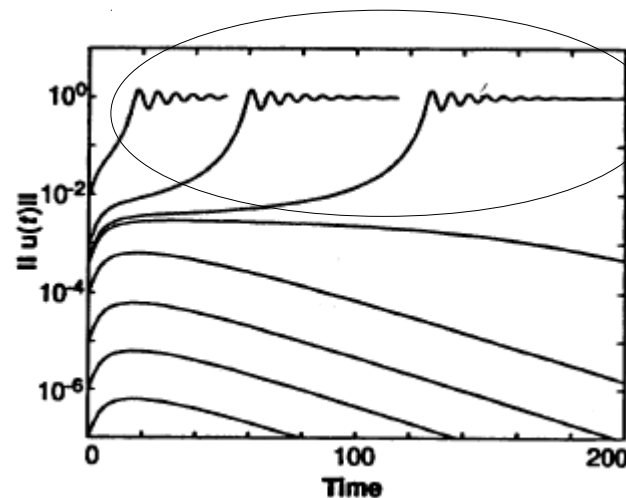
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**nonlinear bypass  
scenario**

# The Optimal Perturbation approach

## Analytical background

The “optimal perturbation” maximizes the gain at a given time (Farrell 1988).

Lagrangian multiplier technique to find Nonlinear Optimal Perturbation (Pringle & Kerswell 2010).

$$\mathcal{L} = E(\tau) - \langle \mathcal{F}(u), a \rangle - \lambda(E(0) - E_0)$$

$$\frac{\partial \mathcal{L}}{\partial \epsilon} \delta \epsilon = 0 \quad \Rightarrow$$

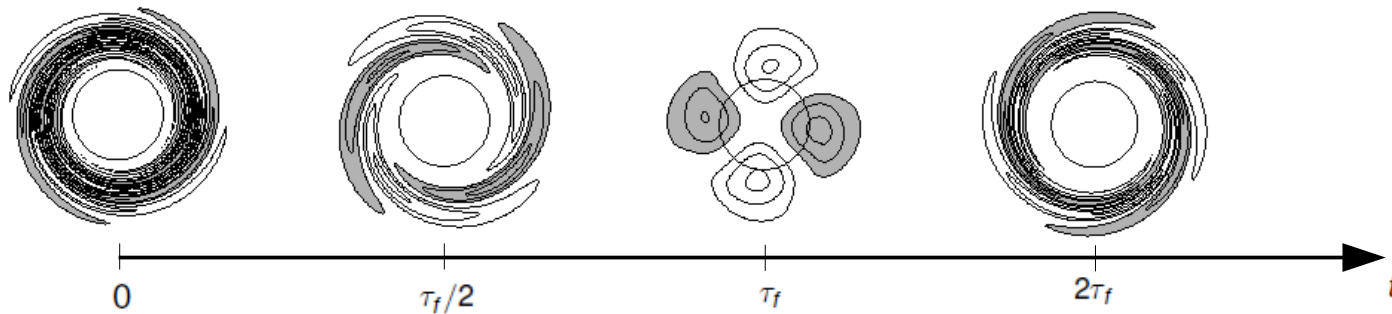
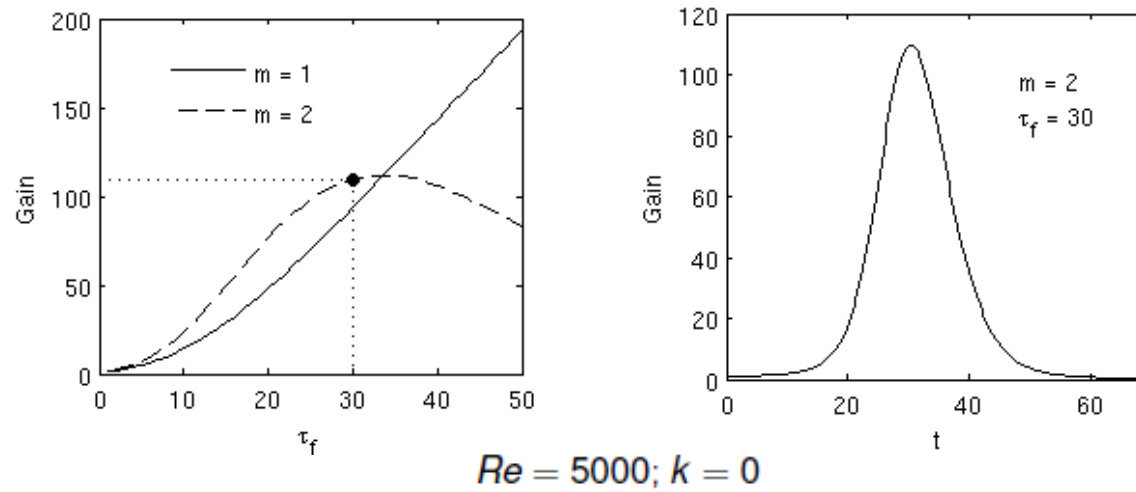
- Navier Stokes equations + b.c.
- Adjoint equations + b.c.
- compatibility conditions

... and a pseudo-spectral code!



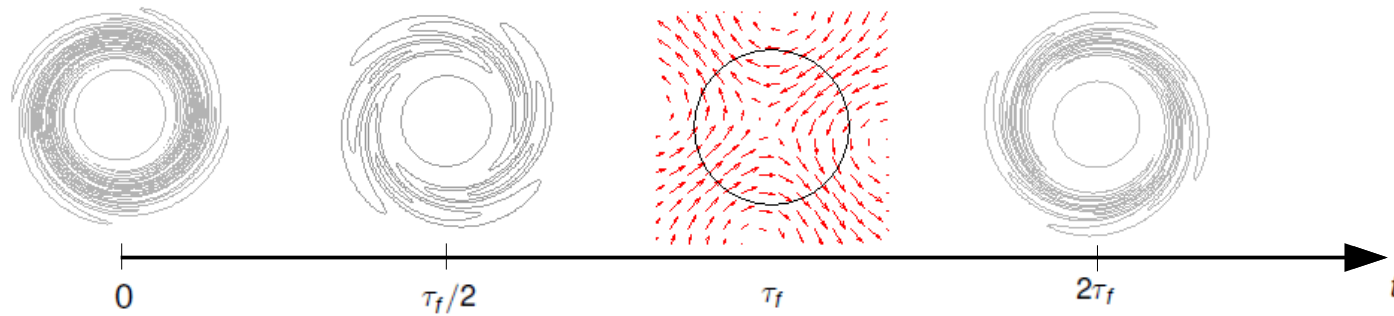
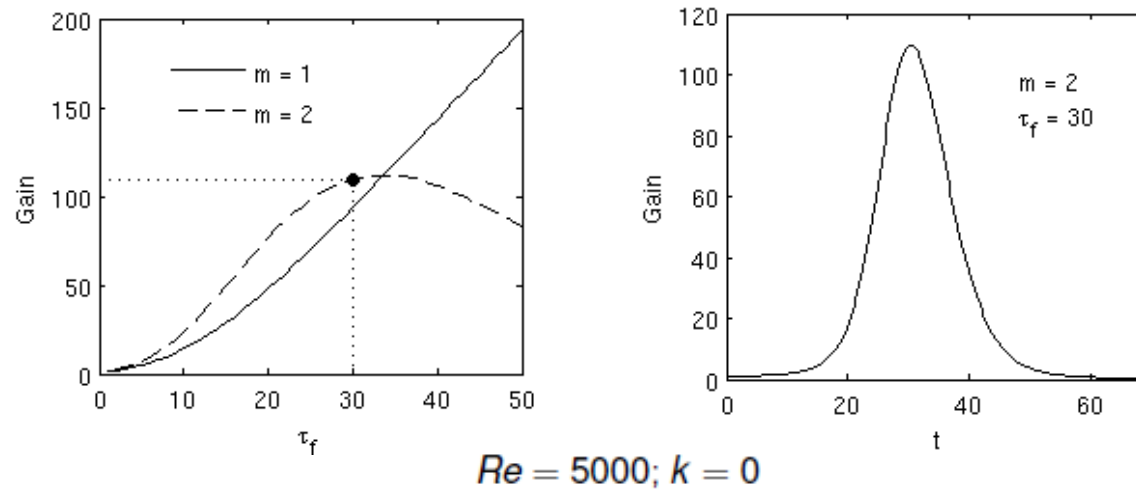
# Transient energy growth

## 2D Linear Optimal Perturbation



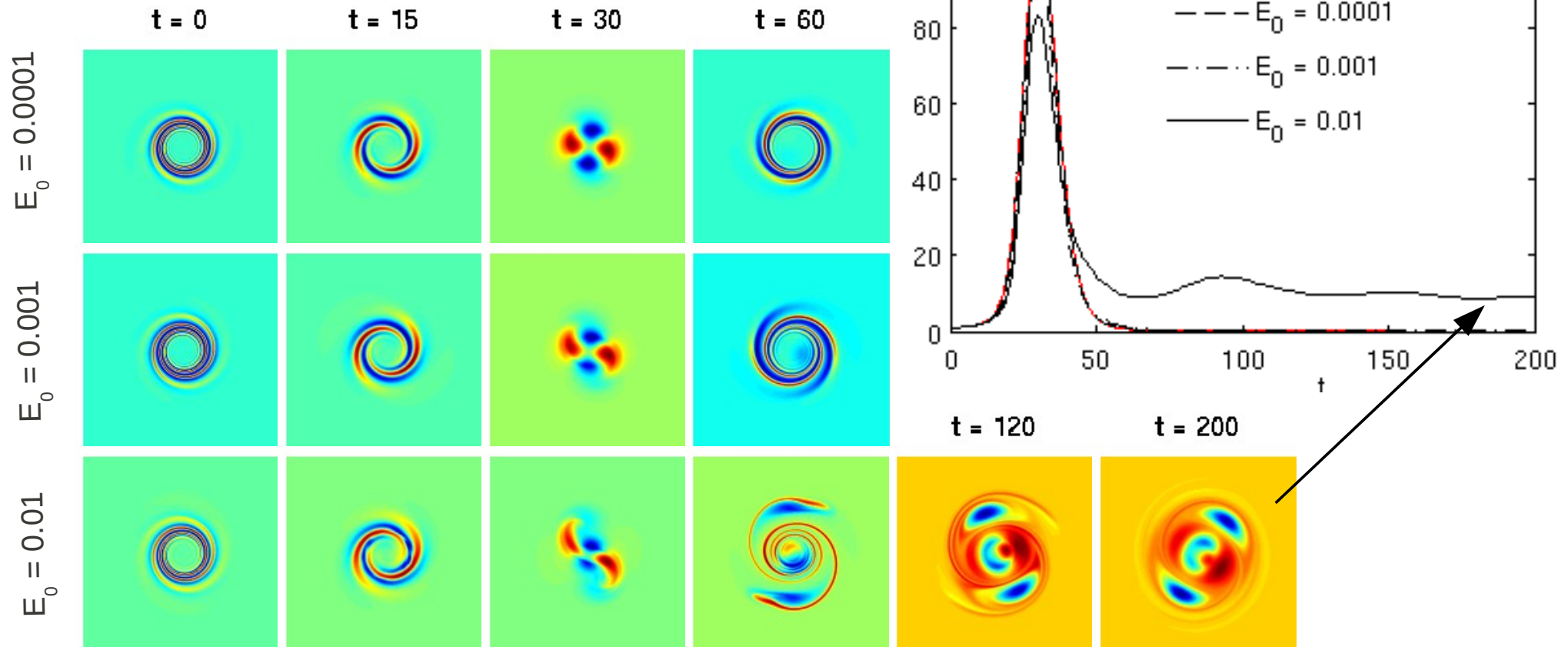
# Transient energy growth

## 2D Linear Optimal Perturbation



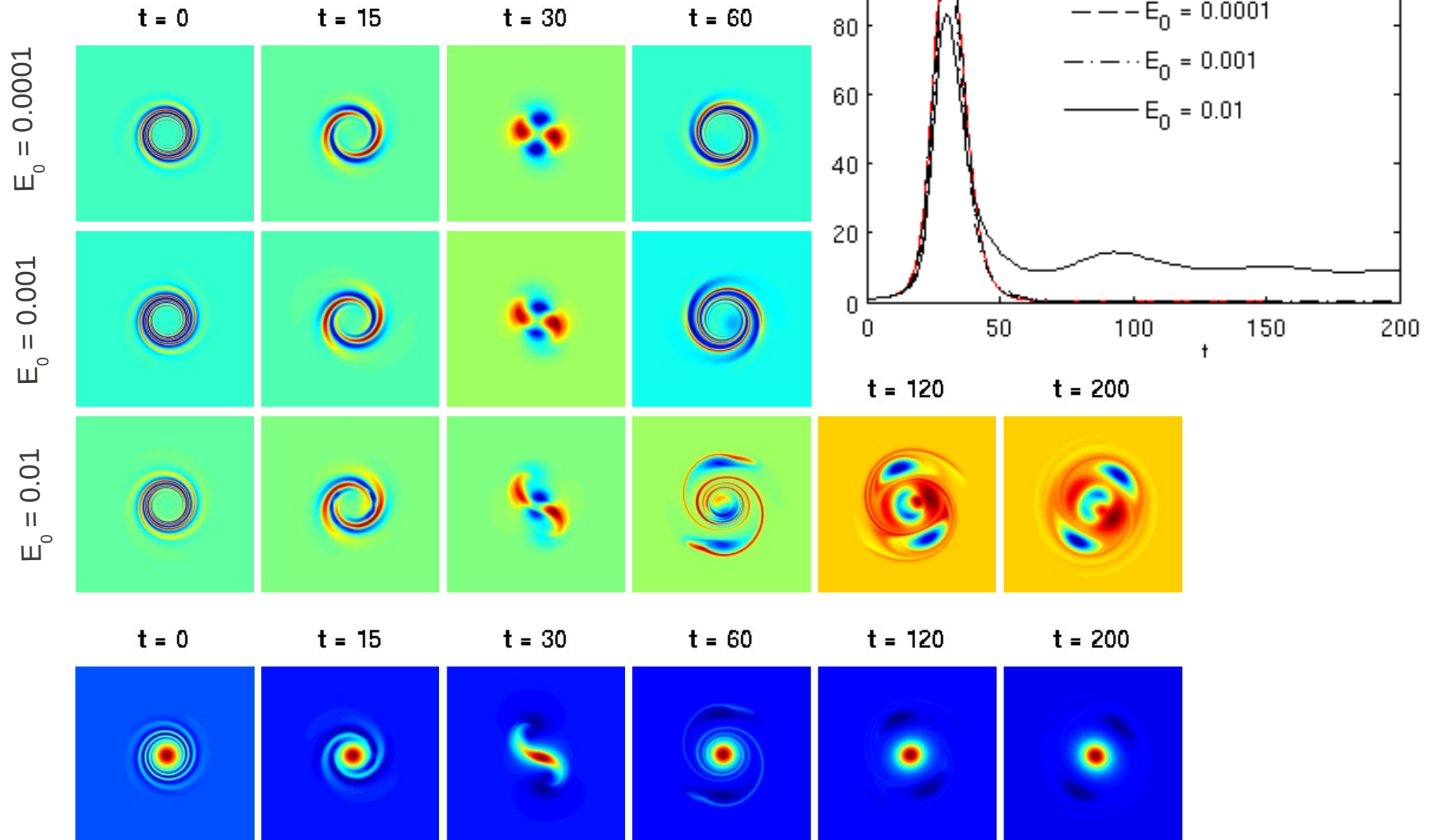
# Transient energy growth

2D Nonlinear Optimal Perturbation



# Transient energy growth

2D Nonlinear Optimal Perturbation



# Conclusions

- Nonlinear optimal perturbations: remarkable differences with respect to the linear case;
- Axisymmetrization is a systematic process only in the linear approach;
- High-energy tripole generation as a nonlinear bypass transition induced by a nonlinear transient growth mechanism revealed by a nonlinear optimal perturbation analysis;
- $\bar{E}_0$  'threshold' as  $(Re, \tau)$  function (in Rossi & al, 1997 and Barba & Leonard, 2007 but differences...);
- Kinematic energy gain: the most effective objective function to induce transition?