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# Identification of the Elastic Properties of Composite Materials

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**Abstract:** The mechanical characterization of composite materials is a research axis of scientific and economic importance. In fact, it is essential to measure the elastic constants of a material with accuracy in order to realize structural analysis and to optimize the design. Among the developed methods, the static methods based on strain measurement present some disadvantages: destructive evaluation and difficulties of measuring the out-of-plane elastic modulus because of the thinness. These disadvantages could be avoided by the use of non-destructive dynamic methods such as modal analysis or ultrasonic waves evaluation. Among the non-destructive methods, ultrasounds are efficient. Besides, because of the accuracy of the results as well as the repeatability of the measures, scientists are generalizing this method. This paper presents a way to determine the mechanical properties of carbon fiber composite materials. This work underlines the results obtained of phase velocity at different incident angles with a dynamic characterization method which is developed as part of non-destructive evaluation by ultrasonic waves.

**Keywords:** Ultrasonic immersion testing; Elastic constants measurement; Material characterization; Orthotropic laminated composite

## 1. Introduction

The use of composites in the aerospace industry has increased dramatically since the 1970s [1]. The primary benefits that composite components can offer are the reduction of weight and the simplification of assembly. Composite materials such as carbon fiber-reinforced composites are very used for many structural applications. The determination of mechanical properties is critical to provide performance to the structure. In addition, the knowledge of complete elastic stiffness matrix is important for modeling and evaluating the mechanical behavior of composite materials under severe loading conditions [2].

The design of most composites is based on their stiffness, and methods for static measurement of stiffness are in wide use [3]. The disadvantages of these methods are: some engineering constants of anisotropic materials are difficult to measure, they are destructive in nature, high costs involved in producing samples of desired shape and size, and in situ measurements are difficult [4,5]. There exists a classical ultrasonic technique for determining elastic constants of composites by cutting samples from the composites; then the elastic constants can be determined when the number of measurements is equal to the number of unknown independent static constants [2,3,6]. However, these methods are destructive, requiring cutting the samples in specific directions. It is therefore important to have another technique to evaluate the mechanical properties of composite materials after manufacturing and in service in order to make the correct maintenance or replacement [7].

Ultrasonic techniques are qualified for non-destructive measurement of the elastic constants in such materials. Elastic constants are determined by measuring ultrasonic wave velocities, which are related to the material properties [8-12]. To measure the elastic constants non-destructively, several ultrasonic bulk wave methods have been developed to make phase velocity measurements in anisotropic plates when only one or two sides of the sample are accessible [8]. This technique allows to get phase velocity and thus the elastic constants can be determined from the measured phase velocity data by inverting the Christoffel equation [7].

There are two possible methods to make the experiment: through-transmission and back-reflection techniques [5]. Through-transmission method requires two transducers, one for send the wave through the sample, and the second transducer to receive the transmitted wave. As the ultrasonic wave travels through the test sample, the wave is reflected in part as it encounters a medium of different acoustic impedance. Then the reflected wave is received by the transducer and displayed or stored for analyses. In the back-reflection technique, instead of the receiving transducer, a large flat reflector is used which is oriented parallel to the transmitter. The major advantage of this method in respect to through-transmission technique is that when the sample angle is changed, the position of the incident beam on the back reflector changes; but this does not affect the position of the beam arriving at the receiver transducer, thus eliminating the necessity of moving either the reflector or the transmitter/receiver transducer. The back-reflected wave travels along exactly the same path as the incident wave in the opposite direction [4] and arrives at the transducer which works in pulse/echo mode.

For a transversely isotropic composite, measurements in symmetry planes are sufficient to determine the five independent elastic constants [8]. However, even for a unidirectional composite, orthotropy may appear due to uneven fiber distribution or matrix texture. For an orthotropic composite, it is possible to determine seven of its nine elastic constants from ultrasonic bulk wave velocity measurements in two accessible symmetry planes. Chu et al. [8] and Baste et al. [13] demonstrated that the two remaining elastic constants,  $C_{12}$  and  $C_{66}$ , are not measurable using an immersion method in these two planes of symmetry. This limitation is removed when velocities are measured in a non-symmetry plane since three wave types (one quasi-longitudinal and two quasi-transverse) can be excited in an immersion experiment [8,13].

In this work, phase velocity is calculated from the measured time of flight at different incident angles using numerical methods [4] from the ultrasonic immersion technique. The phase velocity information can be then related to the material stiffness constant using commercial parametric identification software [14]. The determination of the elastic constants is in progress.

## 2. Theoretical background

The elastic constants  $C_{ijkl}$  of a generally anisotropic medium and the phase velocities  $V_p$  are related by the Christoffel equation (equation 1) [6].

$$|C_{ijkl}n_i n_l - \delta_{jm} \rho V_p^2| = 0 \quad (1)$$

where  $n_i$  is a unit vector in the wave propagation direction and  $\rho$  is the density. Equation 1 allows to determine the elastic constants from the phase velocity in particular propagation directions using the inverse model [4,5]. From the experimental values for each incident angle and from mathematical analysis (Equation 1), it is possible to get the nine elastic constants, through a non-linear optimization technique [4].

The elastic stiffness matrix for an orthotropic material is given by equation 2.

$$[C_{ij}] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \quad (2)$$

To calculate the velocity of propagation in the medium, it is required to find exact time of travel in the specimen for the given angle of propagation. Exact time of travel is determined using the cross-correlation technique [15]. It requires a reference signal, and in this case the signal recorded without the sample (only water) is the reference signal. If the time of flight along the reference path in the coupling medium (water) is subtracted from the overall time of flight along the path in the sample at a given incident angle, we get time that the ultrasonic wave takes to pass through the material. This time difference  $\Delta t$  is then used in equation 3 to calculate the phase velocity  $V_p$ . Rokhlin et al. [4] explained the mathematical process to obtain equation 3 in function of  $\Delta t$ , the reference velocity in the water  $V_0$ , the incident angle  $\theta_i$ , the thickness of the sample  $h$  and the velocity in the immersion fluid  $V_0$ .

$$V_p(\theta_r) = \left( \frac{1}{V_0^2} - \frac{\Delta t \cos \theta_i}{hV_0} + \frac{(\Delta t)^2}{4h^2} \right)^{-1/2} \quad (3)$$

Then the propagation angle can be calculated with the Snell's law [4,5] showed in equation 4 where  $\theta_i$  = incidence angle (radians)  $\theta_r$  = refracted angle (radians),  $V_0$  = velocity in water (m/s) and  $V_p$  = velocity in the specimen (m/s).

$$\theta_r = \sin^{-1} \left( \frac{V_p(\theta_r) \sin \theta_i}{V_0} \right) \quad (4)$$

The phase velocity and its propagation angle can be used in the Christoffel equation (equation 1) to determinate the elastic constants of the material. To solve equation 1, one can use the nonlinear least-square optimization technique. This technique minimizes the deviations between the measured and calculated ultrasonic velocities at different angles of propagation by determining the set of elastic constants. The resolution process is valid for materials of arbitrary anisotropy and for arbitrary directions.

### 3. Specimen and experimental setup

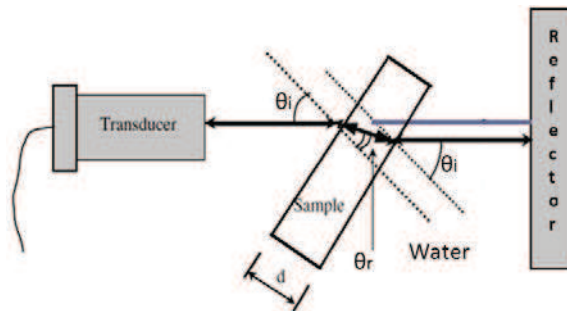
#### 3.1. Specimen

A carbon fiber reinforced composite is used to calculate its elastic constants from the phase velocity. The sample is made of 12 unidirectional plies of prepreg. Sample fabrication is made using the manual lay up technique. Then the sample is cured at 125°C for 90 minutes and a pressure of 2 Bars. The sample dimensions are 150 x 100 x 1.8 mm<sup>3</sup>. The density of this composite is given by the material manufacturer [Hexcel Corporation<sup>®</sup>]: 1570 kg/m<sup>3</sup>. This sample is representative of an orthotropic composite. Even if the plies are placed unidirectionally, orthotropy may appear as it was described in section 1.

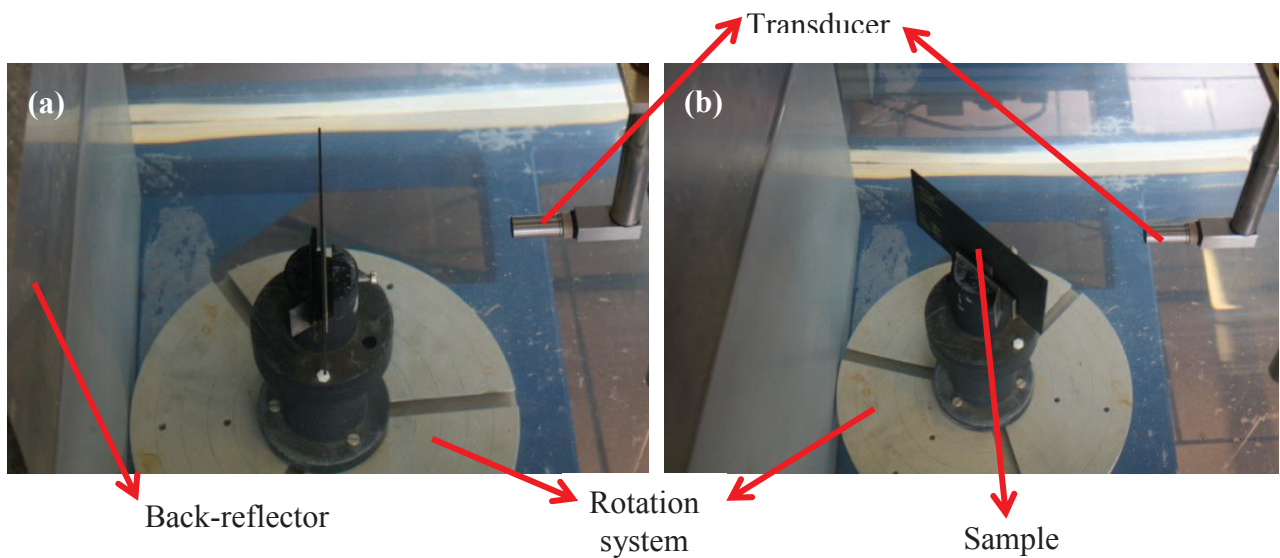
#### 3.2. Experimental setup

In the present work, the ultrasonic velocities are measured for different orientations of wave propagation in the sample. The immersion ultrasonic technique is used. For this technique, water acts as a couplant to transfer the mechanical energy from the transducer to the sample under inspection. The transducer is not in direct contact with the sample and hence consistent coupling is ensured. In this way, it is possible to measure the velocities at different angles of propagation either by adjusting transducer orientation or by rotating the sample with respect to the transducers [10,16]. In this specific work, the setup was made in such a way that the sample is held and rotated using a rotation system to provide different angles of propagation (Fig. 1 and 2). As ultrasonic generator, an Omniscan 32: 128 PR is used with a mono-element transducer connected to it. The transducer is both ultrasonic source and receiver at the same time (reflection mode). The frequency of it is 5 MHz. In the present work, back-reflection technique explained in section 1 is used (Fig. 1). Figure 2 shows the experimental setup for the back-reflection immersion technique. Transducer, back-reflector and specimen were checked for alignment at normal incidence.

**Figure 1.** Schematic of back-reflection technique [5].  $\theta_i$  is the incident angle and  $\theta_r$  is the refraction angle

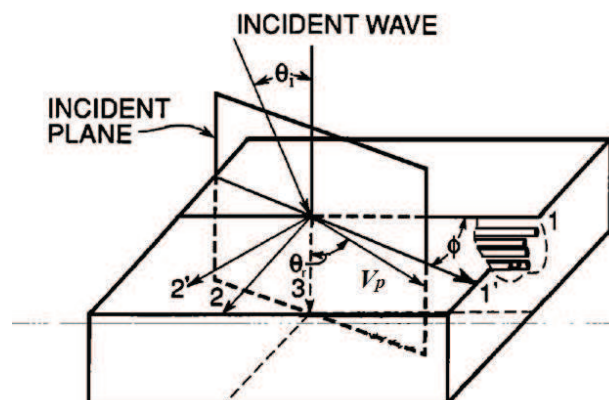


**Figure 2.** Back-reflection immersion setup. (a) Incident angle =  $0^\circ$ . (b) Incident angle  $\neq 0^\circ$



The measurements have been performed for three different angles  $\Phi$  between fiber and incident plane direction (figure 3)  $0^\circ$ ,  $45^\circ$  and  $90^\circ$ . For a selected angle  $\Phi$ , the sample is rotated around the axis perpendicular to the incidence plane by an angle  $\theta_i$ . At each set of angles the time of flight is measured.

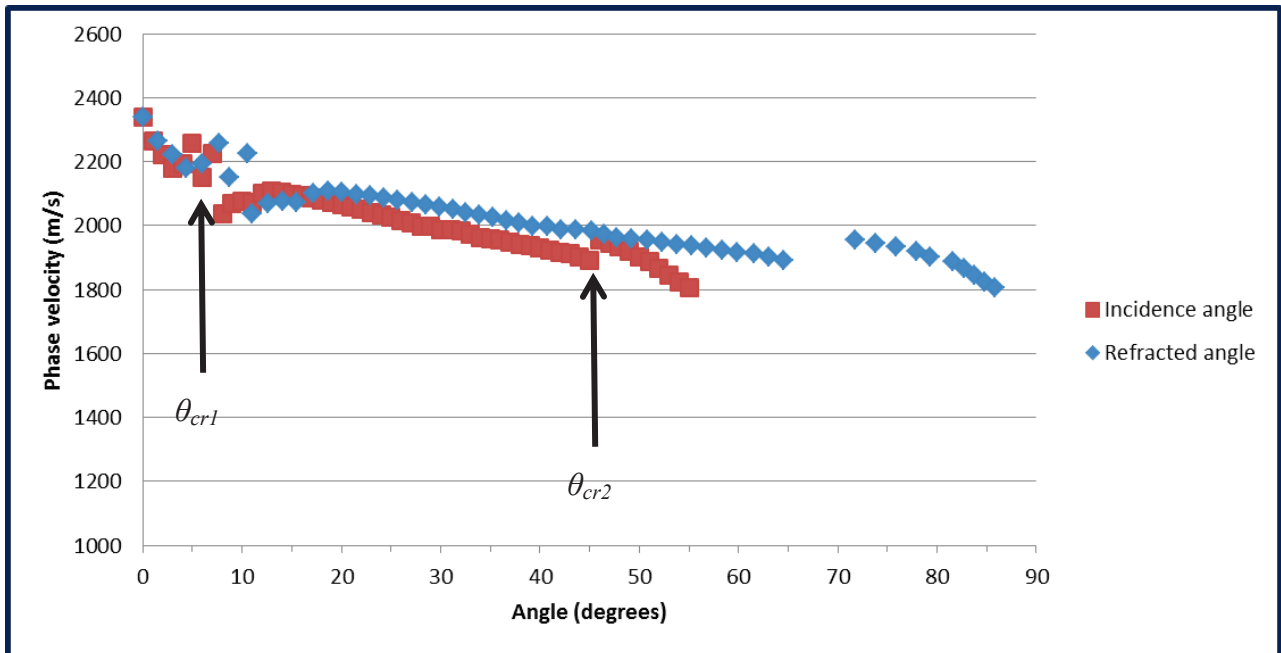
**Figure 3.** Incident angle and refracted angle.



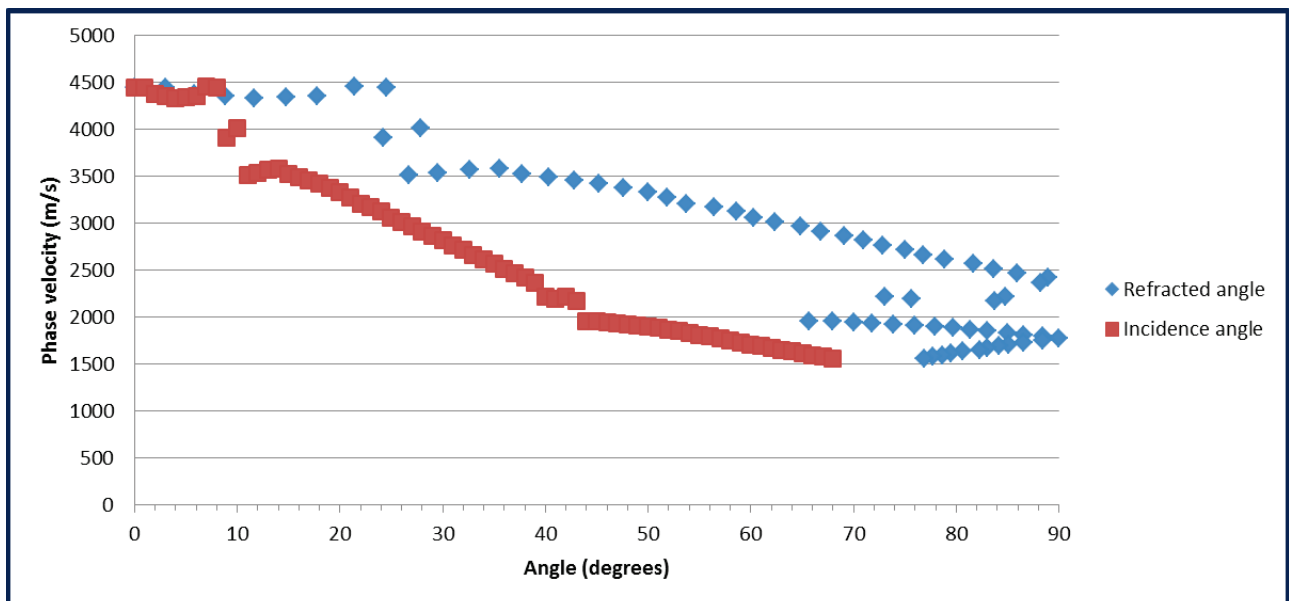
#### 4. Results

Figures 4, 5 and 6 show the phase velocity for an ultrasonic wave as function of the propagation angle for  $\Phi = 0^\circ$ ,  $45^\circ$  and  $90^\circ$  respectively.

**Figure 4.** Phase velocity vs. refracted and incident angle for  $\Phi = 0^\circ$ . (1-3) plane

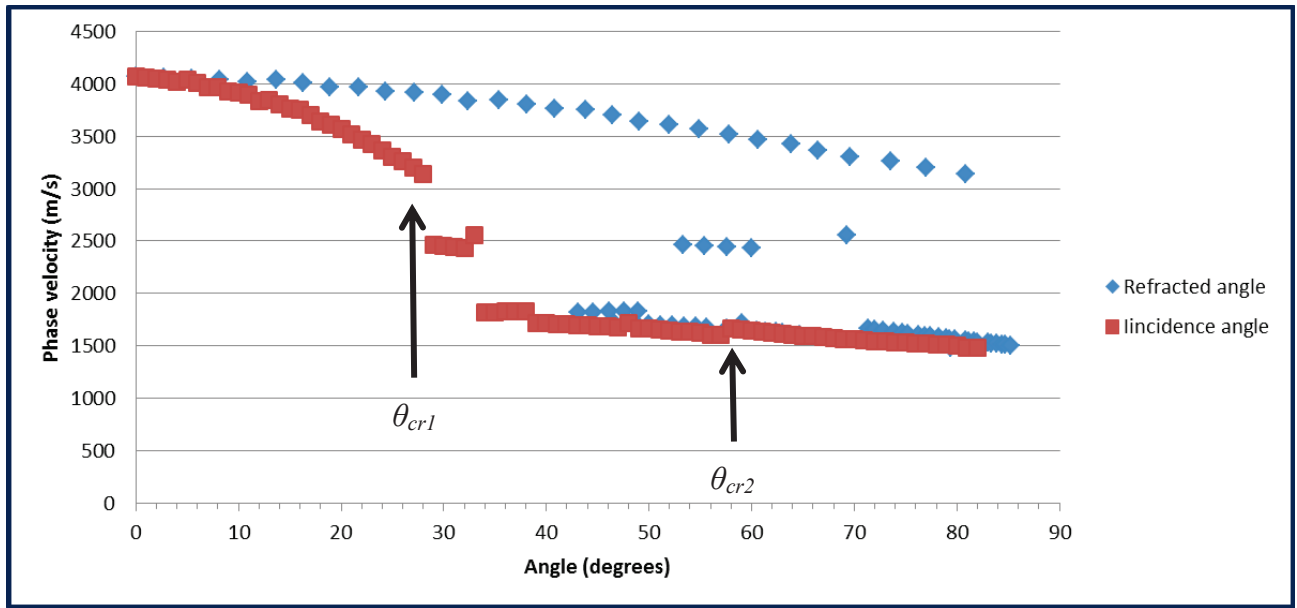


**Figure 5.** Phase velocity vs. refracted and incident angle for  $\Phi = 45^\circ$ . (3-45°) plane





**Figure 6.** Phase velocity vs. refracted and incident angle for  $\Phi = 90^\circ$ . (2-3) plane



The longitudinal wave propagating in 2–3 plane (Fig. 3) shows anisotropy of the material since the velocity is decreasing with the incident angle (Fig. 6). The shear velocity in (2–3) plane is almost constant for all angles of propagation between first ( $\theta_{cr1}$ ) and second critical ( $\theta_{cr2}$ ) angles exhibiting transversely isotropy of the material (Fig. 6). Because of the anisotropy of the medium in (1–3) plane, both longitudinal and shear velocities are changing with the angle. The longitudinal wave velocity in (1–3) plane is increasing with the increase in incident angle whereas the shear wave velocity is decreasing with angle (Fig. 4).

It is interesting to note that, at  $\Phi = 90^\circ$  (Fig. 6), except for some scatter the measured phase velocities are almost independent of the refraction angle, indicating that this particular composite sample is in fact almost nearly transversely isotropic in the plane normal to the fibers. When the incident plane is not a plane of symmetry, such as  $\Phi = 45^\circ$ , all the elastic waves should be excited in the solid at oblique incidence.

These three measurements ( $\Phi = 0^\circ, 45^\circ$  and  $90^\circ$ ) are enough to calculate the elastic constants by mathematical methods as explained in sections 1 and 2. The calcul of the nine elastic constants for this material is in progress.

## 5. Conclusions

Measurements of time delay of waves passing through a sample with parallel surfaces and arbitrary anisotropy can be used for phase velocity determination.

The through-transmission method used in previous works requires precise adjustment of the position of the receiving transducer, which is necessary for an anisotropic plate scanning in two dimensions. This significantly complicates the measurement process and reduces precision. The back-



reflection technique is free from this disadvantage, since the back-reflected ultrasonic wave goes through the path inverse to which it came and is received by the single transmitting/receiving transducer working in pulse-echo mode.

Finally, reconstruction of the elastic constants from ultrasonic data can be done by non-linear least-square optimization. The advantage of using data from non-symmetry planes is that the whole set of elastic constants for an orthotropic material can be obtained using the back-reflection method described in this work, without additional data in planes parallel to the sample surface (such as data from the critical angle method).

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