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MINIMUM-VARIANCE CONTROL OF NONLINEAR SYSTEMS

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A minimum-variance self-tuning controller with a nonlinear difference equation structure is described with the adaptiveness being provided by an extended least-squares estimation algorithm. Performance analysis is discussed in terms of a cumulative loss function and high order correlation functions that have been derived specifically for use in conjunction with nonlinear ARMAX models. Simulation results using the model of a practical process are described.

1. Introduction

In recent years Billings and Leontaritis [8] have descibed a new class of nonlinear difference equation model, the NARMAX model, that has a more general appeal for adaptive control algorithms than previously reported methods. Research work so far reported has concentrated on the Hammerstein model [2,6,7] and the bilinear model [5,10]. Although they are both subsets of the NARMAX model, the failings of these models lie in their specific nature and that they avoid the issue of multiplicative nonlinear noise terms. The problem of having to estimate several hundred parameters effectively precludes functional series models from being a viable alternative, especially as this is compounded with the fact that such models are difficult to interpret in terms of parametric models and prior knowledge is not easily incorporated into this description.

2. A control algorithm for the NARMAX model

The NARMAX model in difference equation form is defined by,

$$z(t) = F^{l}[z(t-1), ..., z(t-n_z); u(t-k), ..., u(t-k-n_u+1); e(t-1), ..., e(t-n_e)] + e(t) \quad (1)$$

$$= F^{l}[\bullet, t]$$

where z(t) is the system ouput, u(t) the controllable input, e(t) a zero-mean Gaussian noise sequence, l the degree of nonlinearity and n_z, n_u and n_e the orders of z(t), u(t) and e(t) respectively

The purpose of the minimum-variance strategy [1,6] is to minimise the k-step ahead loss function,

$$J_2(t+k) = E[(z(t+k) - w(t))^2] \quad (2)$$

where w(t) is the demand input and $E[\cdot]$ the expectation operator.

Substituting the k-step ahead form of (1) into (2) gives

$$J_2(t+k) = E[(F^{l}[\bullet, t+k] - w(t))^2] \quad (3)$$

Minimising (3) with respect to u(t) to find the new control action

$$\frac{dJ_2(t+k)}{du(t)} = E[(F^{l}[\bullet, t+k] - w(t))\ 2\ \frac{dF^{l}[\bullet, t+k]}{du(t)}] = 0 \quad (4)$$

and the control law becomes

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$$E[z(t+k-1),\ldots,z(t+k-n_z);u(t),\ldots,u(t-n_u+1);e(t+k-1),\ldots,e(t+k-n_e)] - w(t)] = 0 \tag{5}$$

3. Self-tuning strategy

The algorithm may be made self-tuning by employing an extended RLS estimator to directly estimate the controller parameters [4]. Equation (5) may be written as

$$E[\chi^T(t)\ \hat{\theta}(t)] = 0 \tag{6}$$

where $\chi^T(t)$ is the data vector and $\hat{\theta}(t)$ the parameter estimate vector at time t.

$$\begin{aligned}\chi^T(t) = [&z(t-1),\ldots,z(t-n_z);u(t-k),\ldots,u(t-k-n_u+1);e(t-1),\ldots,e(t-n_e),w(t)\\ &z^2(t-1),\ldots\quad z(t-1)u(t-k),\ldots\quad z(t-1)e(t-1),\ldots\\ &\ldots\ldots\text{ higher terms}]\end{aligned}$$

$$\begin{aligned}\hat{\theta}(t) = [&\hat{\theta}_1,\ldots\quad \hat{\theta}_{n_z},\quad \hat{\theta}_{n_z+1},\ldots\quad \hat{\theta}_{n_z+n_u},\quad \hat{\theta}_{n_z+n_u+1},\ldots\hat{\theta}_{n_z+n_u+n_e},-1.0\\ &\hat{\theta}_{11},\ldots\quad \hat{\theta}_{1n_z+1},\ldots\quad \hat{\theta}_{1n_z+n_u+1},\ldots\\ &\ldots\ldots\text{ higher terms}]\end{aligned}$$

In practice the parameter for w(t) is preset to -1.0, (this is consistent with linear systems) and not estimated to ensure uniqueness.

4. Calculation of the noise estimates

The current noise terms may be estimated at each iteration by considering (1) and (6)

$$\hat{e}(t) = z(t) - \chi^T(t-k)\hat{\theta}(t-k) \tag{7}$$

Note that this is actually the system residual.

If $k > 1$ then future noise terms exist in the control law. However, e(t) is a random sequence and hence, is unpredictable. These values are set to zero; their conditional mean, in the control law. Thus, the control law becomes

$$F'[z(t+k-1),\ldots,z(t+k-n_z);u(t),\ldots,u(t-n_u+1);\hat{e}(t),\ldots,\hat{e}(t-n_e)] - w(t) = 0 \tag{8}$$

5. Prediction of the future output terms

The control law above contains future unknown but predictable output terms, z(t+k-d), d = 1, 2,.., k-1. These terms may be proxied by their predictions. From (7)

$$z_p(t+d) = \chi_p^T(t-k+d)\hat{\theta}(t-k) \tag{9}$$

since $\hat{e}(t+d)$ is unpredictable.

$$\begin{aligned}\chi_p^T(t-k+1) = [&z(t),z(t-1),\ldots,z(t-n_z+1);u(t-k+1),\ldots,u(t-k-n_u+2);\hat{e}(t),\ldots,\hat{e}(t-n_e+1),w(t)\\ &\ldots\ldots\text{ higher terms}]\end{aligned}$$

$$\begin{aligned}\chi_p^T(t-k+2) = [&z_p(t+1),z(t),\ldots,z(t-n_z+2);u(t-k+2),\ldots,u(t-k-n_u+3);\hat{e}(t),\ldots,\hat{e}(t-n_e+2),w(t)\\ &\ldots\ldots\text{ higher terms}]\end{aligned}$$

6. Performance analysis

Performance analysis is concerned with monitoring the operation of the controller and estimator. One common and simple test is the cumulative loss function which can be calculated as the sum of the squares of the errors between the demand and the system output. It should converge to the sum of the squares of the unknown noise terms. This test gives little insight into the cause of failure and hence high-order correlation functions have been derived. In model validation, Billings and Voon [3] have shown that the conventional linear correlation functions do not necessarily detect all possible nonlinear terms and have defined a set of functions suitable for use with the NARMAX model. These have been re-interpreted for use in closed-loop control and the system can be said to be performing satisfactorily if and only if [9]

$$\left.\begin{array}{ll} \phi_{\hat{e}\hat{e}}(\tau) & = \delta(t) \\ \phi_{u\hat{e}}(\tau),\quad \phi_{\hat{e}(\hat{e}u)}(\tau),\quad \phi_{u^{2'}\hat{e}}(\tau),\quad \phi_{u^{2'}\hat{e}^2}(\tau), & = 0 \quad \forall\ \tau \geq k \\ \phi_{z\hat{e}}(\tau),\quad \phi_{\hat{e}(\hat{e}z)}(\tau),\quad \phi_{z^{2'}\hat{e}}(\tau),\quad \phi_{z^{2'}\hat{e}^2}(\tau), & = 0 \quad \forall\ \tau \geq k \\ \phi_{w\hat{e}}(\tau) & = 0 \quad \forall\ \tau \end{array}\right\} \qquad (10)$$

7. Simulation results

The model used in this experiment was identified from data collected whilst a patient was being anaesthetised. The ouput is a "calculated score" based on pulse rate, blood arterial pressure, respiratory rate, tidal volume, etc. and the controllable input is the drug flow rate. The model is given as

$$\begin{aligned} z(t) = {} & 1.376z(t-1) - 0.00866z(t-2) - 0.4703z(t-3) + 0.3315z(t-4) - 0.1442z(t-5) \\ & + 0.00856u(t-3) + 0.00485u(t-4) - 0.3504z^2(t-1) + 0.1067z(t-1)z(t-4) \\ & - 0.8175z(t-1)z(t-5) - 0.1949z(t-1)u(t-3) - 0.02395z(t-1)u(t-4) - 0.3903z^2(t-2) \\ & + e(t) - 0.6322e(t-1) + 0.2928e(t-3) \end{aligned}$$

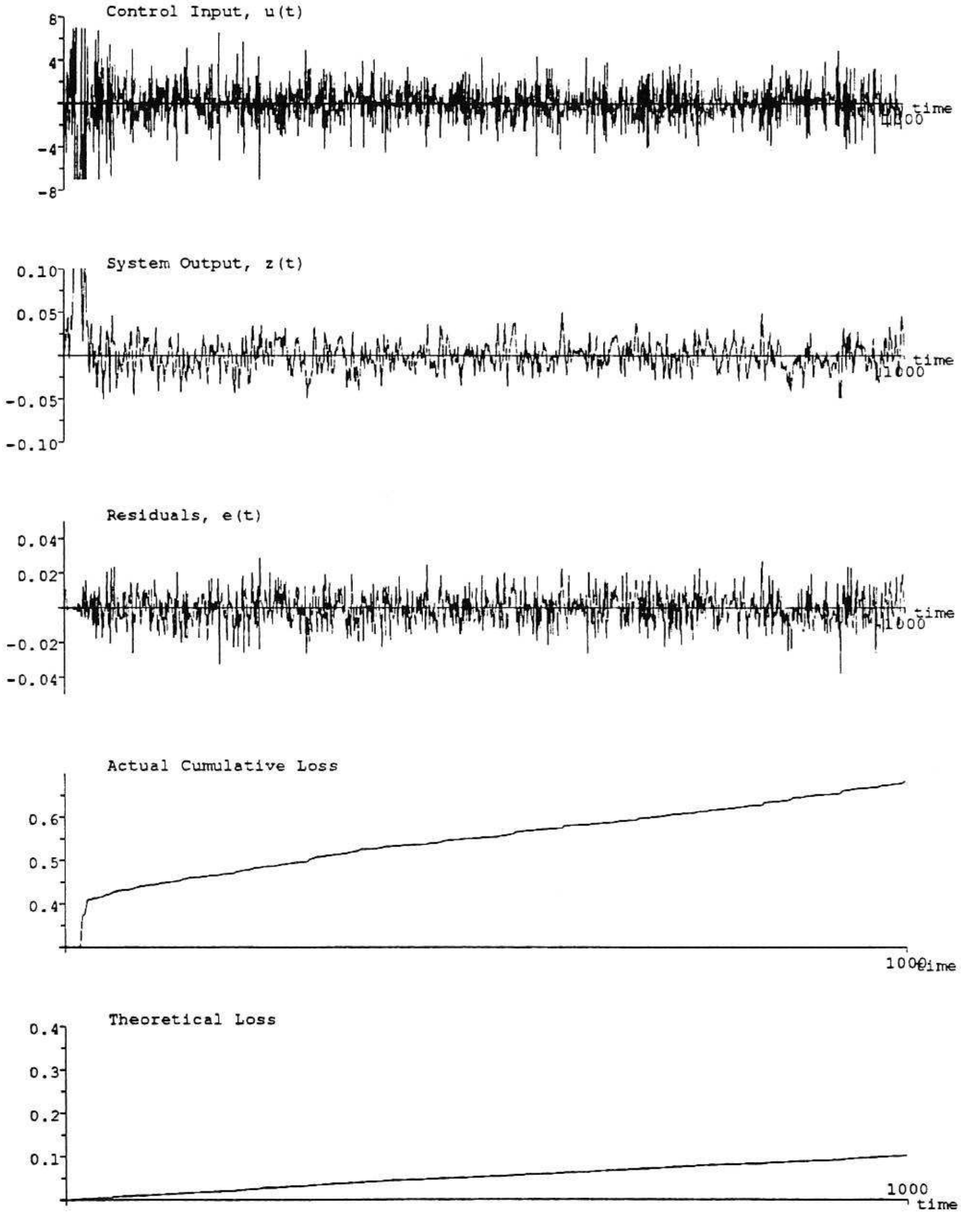
Using the derivation of §§2-5, the control law is

$$\begin{aligned} & 1.376z_p(t+2) - 0.00866z_p(t+1) - 0.4703z(t) + 0.3315z(t-1) - 0.1442z(t-2) \\ & + 0.00856u(t) + 0.00485u(t-1) - 0.3504z^2{}_p(t+2) + 0.1067z_p(t+2)z(t-1) \\ & - 0.8175z_p(t+2)z(t-2) - 0.1949z_p(t+2)u(t) - 0.02395z_p(t+2)u(t-1) - 0.3903z^2{}_p(t+1) \\ & + 0.2928\hat{e}(t) - w(t) = 0 \end{aligned}$$

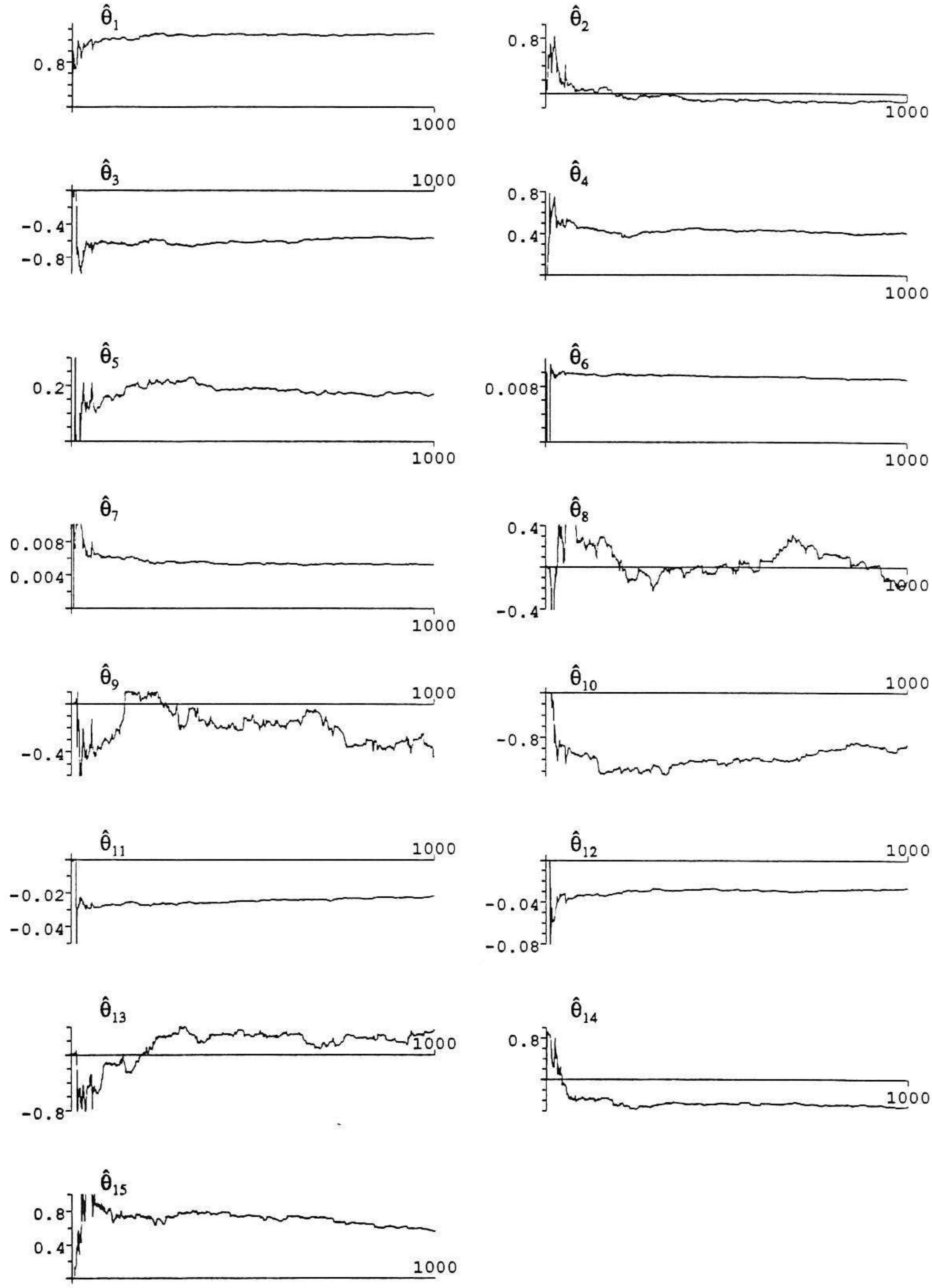
Initially the regulation is poor, the control input (fig 1.2) is limited to ±7.0 and until the parameter estimates (figs 2.1-2.15) start to converge, after about 50 samples, the action desired by the controller is not implementable. Since the control law is unable to take account of future noise terms and includes future output terms, it is inevitable that the actual loss function (fig 1.4) rises more steeply than the theoretical value (fig 1.5). A more realistic test is to consider the residuals (fig 1.3) and model validity tests (figs 3.1-3.9). The latter have been calculated over the final 500 samples in order to allow the system to tune. Only $\phi_{u^{2'}\hat{e}}(\tau)$ at lags 10 and 12 and $\phi_{z^{2'}\hat{e}}(\tau)$ at lag 8 are outside the 95% confidence bound, which suggests that non-convergence of noise model parameters is being detected. These failings are of only minor concern since the discrepancies are small and are at high lags in a very severe set of tests. These points could reasonably be expected to come within the confidence bounds over a longer run. The parameter estimates themselves have, in general, tuned well - the exceptions being those that are associated with future output terms and the noise model, which for the latter is consistent with linear systems.

An attempt to control the model using a "best-fit" linear controller resulted in both the control and estimation blowing-up, even if the parameter estimates were preset to their desired values, and no control being attainable.

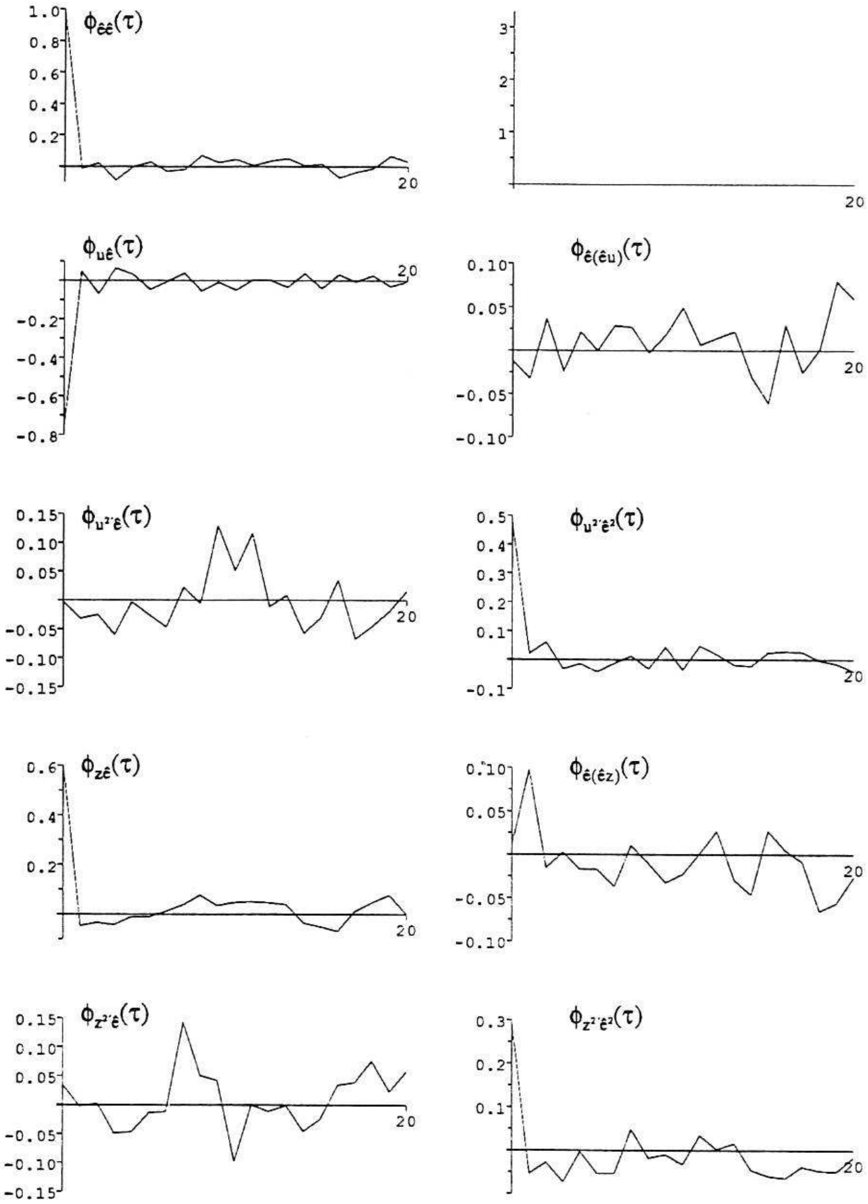
Graphical results



Figs 1.1-1.5 Input-Output data



Figs 2.1-2.15 Parameter Estimates



Figs 3.1-3.9 Model Validity Tests

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