Robust Reconnaissance Asset Planning Under Uncertainty

by

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Abstract

This thesis considers the tactical reconnaissance asset allocation problem in military operations. Specifically this thesis presents methods to optimize, under uncertain conditions, tactical reconnaissance asset allocation in order to maximize, within acceptable levels of asset risk exposure, the expected total information collection value. We propose a deterministic integer optimization formulation and two robust mixed-integer optimization extensions to address this problem. Robustness is applied to our model using both polyhedral and ellipsoidal uncertainty sets resulting in tractable mixed integer linear and second order cone problems. We show through experimentation that robust optimization leads to overall improvements in solution quality compared to non-robust and typical human generated plans. Additionally we show that by using our robust models, military planners can ensure better solution feasibility compared to non-robust planning methods even if they seriously misjudge their knowledge of the enemy and the battlefield. We also compare the trade-offs of using polyhedral and ellipsoidal uncertainty sets. In our tests our model using ellipsoidal uncertainty sets provided better quality solutions at a cost of longer average solution times to that of the polyhedral uncertainty set model. Lastly we outline a special case of our models that allows us to improve solution time at the cost of some solution quality.

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David M. Culver, Major, U.S. Army
May 10, 2013

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Chapter 1

Introduction

In this thesis we provide an analytic approach to increase the planning effectiveness for tactical surveillance and reconnaissance. This chapter outlines our motivation for this research and introduces our general approach.

1.1 Research Motivation

According to Army planners the United States military faces a future of persistent conflict characterized by complex environments and an adaptive and creative enemy [19]. This projection is based on the US counter-insurgency experience in Iraq and Afghanistan as well as recent conflicts in places such as Lebanon, among others. As US doctrine has evolved as a result of these conflicts, the importance of accurate and timely intelligence, defined as analyzed information regarding the enemy, has become clear. Intelligence drives operations, which results in new intelligence, which leads to further operations. This intelligence/operations cycle is critical to the success of any current or future military operation.

Due to the importance of information and intelligence in military operations it is vital for Army planners to use their limited Intelligence, Surveillance, and Reconnaissance (ISR) collection assets wisely. ISR in this context refers to the Army's systems used to collect and process information needed by commanders and key decision makers. Current doctrine emphasizes that the Army must focus its ISR operations for maximum collection by a limited number of assets and resources to produce the best intelligence possible [22]. Unfortunately, based on the author's first hand experience in Iraq, military planners are frequently overwhelmed with potential reconnaissance and surveillance targets. Additionally, due to time constraints and competing priorities planners are often unable to conduct a thorough analysis of how to optimally assign collection assets to these targets. Instead planners either default to an assignment plan that focuses all of their assets on collecting the information that they value the highest or one that provides the longest coverage possible of the most areas.

This thesis focuses on one component of ISR operations, tactical air-ground reconnaissance, and provides an optimization approach to improve planning synchronization and integration in this area. ISR synchronization ensures that the most appropriate assets, both internal and external to the organization, collect information. ISR integration ensures the efficient tasking of these assets to collect on the information requirements that will return the most value [21]. We term this problem the Tactical Reconnaissance Asset Allocation Problem (TRAAP). Tactical air-ground reconnaissance is a mission, at the brigade level or below, to obtain information useful to the commander using both aerial and ground collection assets. The thesis addresses the TRAAP by developing a method to optimize, under uncertain conditions, reconnaissance asset allocation in order to maximize, within acceptable levels of asset risk exposure, the expected total information value collected. The in-corporation of uncertainty into the model allows planners to develop mathematically robust solutions, i.e., solutions that remain optimal or near optimal, when executed, under a range of conditions.

Specifically, this thesis presents deterministic and robust optimization formulations to determine the optimal allocation of reconnaissance assets to targets. We apply robust optimization methods to our deterministic formulations in order to develop realistic, useful, and flexible asset allocation models. Robust optimization is particularly useful in this case as it addresses a number of operational and computational considerations of the TRAAP. The tactical "fog of war," defined by military analyst Carl von Clausewitz as the uncertainty in situational awareness in military operations, forces military planners to rely on inexact data. Robust optimization allows planners to account for this uncertainty and balance, based on the preferences of the commander, the feasibility and optimality of the solution within some known probabilistic bounds. Even more important in the time constrained environment of combat operations, the robust optimization techniques used in this thesis often generate computationally tractable problems that are able to be solved in a relatively short period of time. The methods proposed in this thesis result in linear or second-order cone mixed-integer optimization problems that are computationally tractable using commercially available optimization solvers.

This research is broadly applicable to both military and civilian security scenarios. In a military context our models can provide combat battalion and brigade planners assistance in generating their reconnaissance plans. It can also assist planners in identifying collection gaps in their reconnaissance operations. Similarly this method can be used by civilian border security personnel, for example along the US/Mexico border, to improve the effectiveness of their border interdiction efforts.

1.2 Thesis Organization

Chapter 2 provides background on the problem and a literature review. We introduce relevant background on reconnaissance operations and ISR synchronization and integration. Additionally we present an overview of robust optimization and asset allocation problem literature. In Chapter 3 we introduce a deterministic formulation of the TRAAP that incorporates reconnaissance asset risk considerations, multiple asset types, missions, and mission configurations. The formulation successfully optimizes the expected total information value collected of a given scenario. In Chapter 4 we present two robust extensions to the deterministic model. The robust extensions successfully account for uncertainty in both asset risk exposure and target information value. Each extension uses a different method to model uncertainty, allowing us to compare the advantages and disadvantages of each method. Chapter 5 outlines a useful special case of both the deterministic and robust formulations. This special case provides drastic improvements in solution time at the cost of some solution quality. In Chapter 6 we compare the solution quality of our models using simulation. Chapter 7 provides concluding remarks and areas of further research on this topic.

Chapter 2

Background

Famed American General George S. Patton once asserted that "You can never have too much reconnaissance." [16] He was referring to the tremendous advantage an army has when it can effectively collect information on battlefield conditions and the dispositions and activities of its adversaries. This chapter provides background on military reconnaissance and surveillance operations and planning doctrine. We also present background on optimization approaches previously applied to reconnaissance and surveillance operations and the robust optimization techniques used in this thesis.

2.1 Reconnaissance and Surveillance Operations

The goal of this research is to assist planners in developing effective reconnaissance and surveillance plans. Reconnaissance, as defined by the Army, "is a mission to obtain, by visual observation or other detection methods, information about the activities and resources of an enemy or adversary, or to secure data concerning the meteorological, hydrographic, or geographic characteristics of a particular area." [24] Surveillance operations are often executed as a part of reconnaissance operations. Both surveillance and reconnaissance involve observation and reporting. The two are differentiated by the nature of how information is collected. Surveillance is typically a passive observation of an area or areas and can be continuous. Reconnaissance is generally a more active and short term collection of information and can involve fighting for information [21]. In this thesis we do not attempt to dictate how information is collected. Because surveillance is often a task completed as part of reconnaissance, for the remainder of this thesis we will use the term reconnaissance as an overarching description of both traditional reconnaissance and surveillance missions.

2.1.1 Commander's Inputs into the Reconnaissance Plan

Effective reconnaissance plans allow leaders to more efficiently apply their available combat power, leading to fewer casualties and greater chance for mission success. To illustrate this idea we present the following example.

Consider a light infantry battalion conducting security operations in Afghanistan near the Pakistani border. The border creates a region in Pakistan for Taliban insurgents to organize, train, and equip themselves free from attacks from US and coalition forces. Due to the length of the border, over 1500 miles (2430km) in total, US forces have difficulty providing persistent and effective overwatch of all potential crossing points. As a result many commanders are tempted, or pressured, into devoting too many of their resources towards denying the border to the enemy, at the expense of other critical missions such as engaging with and building trust with the local population, a proven counter-insurgency tactic. An efficient use of reconnaissance assets in this case would allow the battalion commander to provide effective border overwatch in the most critical areas, while keeping free the bulk of his forces to execute more population-centric missions.

Effective reconnaissance plans are focused collection efforts. Commanders ensure a focused collection effort by orienting their reconnaissance on a reconnaissance objective. The reconnaissance objective "is a terrain feature, geographic area, enemy force, or specific civil considerations about which the commander wants to obtain additional information." [21] In our example the battalion commander's reconnaissance objective could be the Taliban insurgents in Pakistan or possibly the border crossing points he suspects the insurgents use most frequently. In either case by selecting a reconnaissance objective the battalion's planners and intelligence personnel can begin to determine where and when to conduct reconnaissance. The commander's reconnaissance objective also allows planners to begin to set reconnaissance priorities.

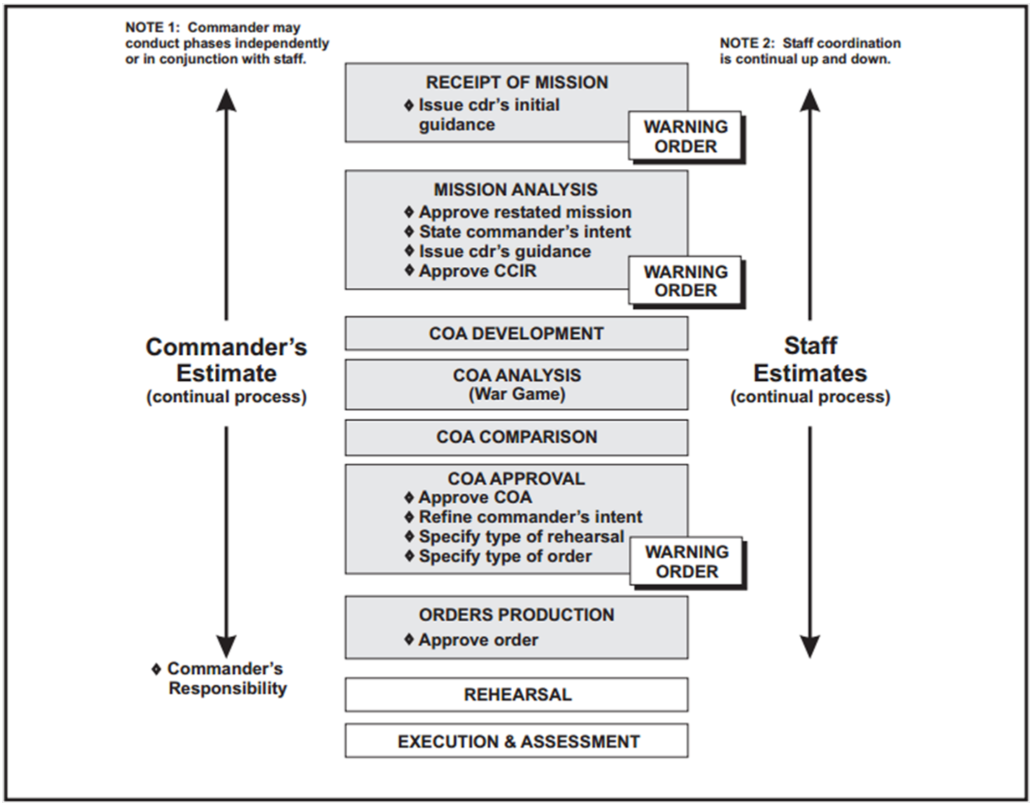
In addition to determining the reconnaissance objective the commander is responsible for approving the Commander's Critical Information Requirements (CCIR). The CCIR is a list of information requirements that facilitate the commander's decision making. Within the CCIR is a list of Priority Intelligence Requirements (PIR). PIR ask a specific question that, if answered, provides intelligence to support a single decision [21]. In our example scenario the commander may designate, "Does the enemy infiltrate our area along Route Cubs?" as a PIR, where Route Cubs is a path across the border from Pakistan. The CCIR and PIR allow planners to focus reconnaissance assets on collecting information that will allow the commander to make better decisions.

2.1.2 Reconnaissance Planning and the Military Decision Making Process

The commander's reconnaissance objective, included in the commander's guidance, and CCIR/PIR are inputs into the Military Decision Making Process (MDMP). The MDMP is a seven step analytical process that assists commanders and their staffs in reaching logical decisions [23]. The steps are, in order of execution, receipt of mission, mission analysis, course of action development, course of action analysis, course of action comparison, course of action approval, and orders production. Reconnaissance generally takes place as soon as possible in this process and is executed continuously throughout. An outline of the MDMP is shown in Figure 2-1.

Reconnaissance planning is based on the commander's guidance and staff outputs of the mission analysis step of the MDMP. During mission analysis the staff will conduct an intelligence analysis of the situation. Based on this analysis and the CCIR, staff planners will develop an initial reconnaissance plan, also known as an ISR or collection synchronization matrix. When developing a reconnaissance plan military planners must allocate their available assets in such a manner that they optimize the expected total information value of the plan. At the lowest level they must assign each available reconnaissance asset to a set of missions. A mission is defined as a time window and location where reconnaissance can be conducted. Mission locations are referred to as Named Areas of Interest (NAIs), that are generally associated with one or more PIR. When considering mission assignments planners must take into consideration, among other things, the amount of risk exposure each asset will experience. For example, a commander may not wish to assign a highly valued asset to a mission where enemy contact is likely. A commander's tolerance for risk will be reflected in his commander's guidance. Once the plan is complete

Figure 2-1: The Military Decision Making Process



and approved by the commander a collection asset will then be officially tasked to execute reconnaissance in one or more NAIs with the mission of answering the NAI's associated PIR.

In an extended security operation like our Afghanistan example the reconnaissance planning process can be a daily event. As battlefield conditions change the commander's PIR, reconnaissance objective, and risk tolerance will adjust accordingly. Additionally, reconnaissance asset availability will differ on a daily basis based on weather conditions, other operations, and numerous other variables that the battalion cannot control. This requires the planning staff to continuously reassess their reconnaissance plans and adapt it to the dynamic conditions of the battlefield.

2.1.3 Types and Capabilities of Reconnaissance Assets

We now present a brief description of common reconnaissance assets used by the United States Army and an overview of their capabilities and drawbacks. This is by no means an exhaustive list but is meant to provide an example of the various types of reconnaissance assets in use.

The majority of tactical reconnaissance operations are conducted by scout platoons. Soldiers in scout platoons are specifically trained in the execution of reconnaissance and surveillance operations. A typical light infantry battalion, such as the one in our example, will have one scout platoon consisting of 19 soldiers divided into three sections. A light infantry scout platoon is not equipped with vehicles, although scout platoons in other types of combat units possess HMMWVs, Stryker Reconnaissance Vehicles, and Bradley Reconnaissance Vehicles. Relying on ground transportation limits the mobility of scout platoons, especially over difficult terrain. The platoon is designed so that each scout section can independently conduct a separate reconnaissance mission, although this rarely happens in practice. Typically the platoon works as a single unit to conduct reconnaissance on a single location. With external support a scout platoon is capable of conducting continuous reconnaissance for several days. However, scout platoons are generally employed to short duration (less than 12 hours) reconnaissance missions and then given time to rest and refit before their next mission. In addition to scout platoons, other types of ground units such as infantry platoons and sniper teams also conduct reconnaissance.

Unmanned Aerial Vehicles (UAVs) are also typically employed in reconnaissance operations, usually in a surveillance role. A light infantry battalion is generally equipped with at least three RQ-11B Raven UAVs, see Figure 2-2. The Raven is a hand-launched man-portable tactical UAV designed to conduct reconnaissance and surveillance. It operates at an altitude of approximately 500ft above ground level at speeds of 28 to 60mph. It has a maximum operational radius of 6 miles with an endurance limit of 1 to 1.5 hours. The Raven is equipped with IR thermal cameras making it capable of operating at night [2].

Figure 2-2: Raven RQ-11B UAV

Combat battalions also have the capability to request the support of more capable UAVs from higher headquarters. The most common of these UAVs is the RQ-7 Shadow, see Figure 2-3. The Shadow uses a vehicle towed catapult system for launch and a ground arresting system, similar to those used on an aircraft carrier, for recovery. It has a wingspan of 14ft and can operate at altitudes as high as 15,000ft at speeds of 81 to 127mph. It has an operational radius of 68 miles with an endurance limit of 6 to 9 hours. The Shadow is also equipped with IR cameras for night operations [1].

UAVs are highly mobile, have a long flight endurance, and possess excellent optics. These features make them excellent at providing persistent surveillance of a small area or numerous small areas over time. However, UAVs do suffer from certain drawbacks making them not always the ideal reconnaissance asset for a mission. Their primary drawback is their limited observation area. Some have referred to UAV

Figure 2-3: Shadow RQ-7 UAV

reconnaissance as conducting reconnaissance by looking through a straw. Their limited observation area make UAVs most useful for conducting reconnaissance in small or well defined areas such as mountain passes or roads. Some UAVs, such as both the Raven and Shadow, suffer from a large audio signature. Due to their relatively low service ceilings and loud engines it is often easy to know if a UAV is operating in the area. This characteristic can prevent the UAV from collecting information on the enemy. UAVs are also limited by weather conditions. Poor weather can force a UAV to land or diminish the quality of its optics to make them nearly useless. These drawbacks make UAVs useful for only certain types of reconnaissance missions.

The last type of reconnaissance asset we will present are air reconnaissance assets. Air assets include both fixed and rotary winged manned aircraft. Most Army combat units do not have air assets organic to their organization and must request the support of air assets from higher headquarters. Although fixed wing aircraft, such as F/A-18s and F-16s, do occasionally provide reconnaissance support to Army units, the majority of air asset reconnaissance is conducted by rotary wing aircraft. The OH-58D Kiowa Warrior helicopter, see Figure 2-4, was specifically designed to conduct battlefield reconnaissance. The OH-58D is a single engine, single rotor helicopter with a crew of two. It has a maximum speed of 149mph, range of 345 miles, and a service ceiling of 15,000ft. A typical Kiowa mission will last no longer than two hours. The OH-58D is also equipped with a Mast Mounted Sight (MMS) above the rotor that provides thermal imaging, range finding, and target designating capability. The Kiowa Warrior is armed with machine guns, rockets, and air to ground missiles [15].

Figure 2-4: OH-58D Kiowa Warrior Scout Helicopter

Rotary winged reconnaissance assets like the OH-58D are excellent at providing short duration reconnaissance of large areas. Their optics allow them to observe locations from a distance and from behind cover. Their armament also allows them to fight for information if necessary. Unfortunately Kiowas are extremely vulnerable to ground fire, adding a significant amount of risk to their employment. Air assets, just like UAVs, are also subject to weather restrictions. Their relatively short flight endurance also limits the utility of employing Kiowas in certain situations.

Reconnaissance planners must take the advantages and disadvantages of each type of asset into consideration when formulating their reconnaissance plan. Based on the weather, terrain, enemy situation, and a number of other variables, each NAI will have an appropriate asset type or group of asset types that will provide the best reconnaissance in the area. For example, if we wish to conduct reconnaissance in a high mountain pass that requires persistent observation for an extended period of time, a Kiowa Warrior may not be the ideal asset to employ. The Kiowa's limited endurance would leave critical gaps in observation. A UAV would be a better choice as it can loiter in the area and focus on a small area due to the restricted terrain. However, if poor weather is expected during the collection window a scout platoon may be the best choice. A scout platoon is capable of conducting extended observation of the area even in poor weather conditions.

2.2 Robust Optimization

In Section 2.1.2 we provided an overview of the process military planners use to develop reconnaissance plans. The key problem military planners face when developing a reconnaissance plan is how to best allocate their available assets in an uncertain environment so that they optimize the expected information value of their plan, within certain physical and commander dictated constraints. This problem lends itself to an optimization based approach.

The TRAAP falls into the category of optimization problems known as resource allocation problems. More specifically, the TRAAP can be considered a general knapsack problem. Knapsack problems consist of a set of items, each with a specified value and weight, and a "knapsack" with a defined capacity. The goal is to place items in the knapsack in such a manner as to maximize the total value of the items in the knapsack without exceeding the weight capacity of the knapsack. Knapsack problems have numerous applications and have been extensively studied. In the TRAAP our "items" are mission assignments that have an associated expected information value and risk exposure. Our knapsack consists of the commander's risk tolerance of our assets.

Traditionally, optimization problems such as the knapsack problem assume we have perfect knowledge of the problem parameters. In the case of the knapsack problem we generally assume that we know with certainty the values, weights, and capacity of the items and knapsack. In real world applications this assumption is usually incorrect and can lead to highly unstable solutions. In fact, solutions "can exhibit remarkable sensitivity to perturbations in the parameters of the problem, thus often rendering a computed solution highly infeasible, suboptimal, or both (in short, potentially worthless)" [7]. Robust optimization seeks to address this problem.

Robust optimization attempts to find solutions that remain optimal or near optimal under a range of conditions. It does this by modeling uncertainty in a deterministic, set-based manner. By using convex, closed sets to model uncertainty robust optimization allows us to derive a solution that gives up some expected solution value but will remain optimal for all data realizations within the uncertainty set. This approach has numerous advantages. First, unlike stochastic optimization, robust optimization has been shown to remain tractable for large problems. Second, we can vary the trade off of robustness and optimality in our solution by changing the size or shape of the uncertainty set. Third, robust optimization requires no prior knowledge of the nature or distribution of the data uncertainty. Fourth, robust optimization allows us to derive probabilistic bounds on constraint violation.

2.3 Literature Review

2.3.1 Robust Optimization Literature

The first robust model approach was proposed by Soyster in 1973 [18]. Soyster proposed a linear optimization model that would remain feasible for all realizations of data within a convex set. He showed that his approach was the equivalent of solving the model after setting all unknown parameters to their worst case values. The drawback of Soyster's method is that it results in extremely conservative solutions. Furthermore, Soyster's method can only account for column-wise uncertainty.

In the late 1990's Ben-Tal and Nemirovski [4, 5, 6] and El Ghaoui et al. [13] independently addressed the problem of over-conservatism in Soyster's method by proposing the use of ellipsoidal uncertainty sets. Ellipsoidal uncertainty sets are appealing for many reasons, the foremost being that they closely resemble typical measurement errors. Using the Ben-Tal and Nemirovski/El Ghaoui et al. approach one must derive and solve a robust counterpart of the linear optimization problem that results in a solution that is robust and much less conservative than the solution produced using Soyster's method. However, tractability suffers using this approach as the robust counterparts are non-linear conic quadratic problems.

In 2003 Bertsimas and Sim [9] proposed an approach that avoids both the over-conservatism of Soyster's method and the tractability concerns of the method's proposed by Ben-Tal and Nemirovski/El Ghaoui et al. The Bertsimas/Sim method seeks to protect against a pre-specified number, Γ, of uncertain parameters assuming their worst case values. They show that the method ensures constraint feasibility if the number of uncertain coefficients assuming their worst case values is less than Γ. Furthermore, they provide probabilistic guarantees that even if more than Γ

coefficients change, the robust solution will be feasible with high probability. The Bertsimas/Sim approach also maintains the tractability of the model. For example, the robust counterparts of linear problems, remain linear. Therefore this approach is suitable for applications involving a large number of variables and constraints. We further discuss and apply the Bertsimas/Sim approach in Chapter 4.

2.3.2 Robust Reconnaissance Asset Allocation Literature

In 2004, Bertucceli et al. [10, 11] applied robust optimization using a modified Soyster's method to the UAV task assignment problem. In this approach attack UAVs are assigned to targets with the goal of maximizing the total reward of targets that are attacked. The reward for striking each target is subject to uncertainty. In their model the uncertainty associated with each target can be reduced if a reconnaissance UAV is assigned to the target prior to an attack UAV.

Bertucceli et al. apply robustness to their formulation by using a modified Soyster's method. The reward for attacking target i at time k, $\bar{c}_{ki}$, is uncertain with a known standard deviation, σ_{ki}. A scalar, μ, allows the planner to adjust the amount of robustness in the solution from no protection against uncertainty to complete protection against uncertainty. Complete protection against uncertainty in their formulation is equivalent to Soyster's method. This modified Soyster's formulation is outlined below

$$\max \quad \sum_{i=1}^{|N_T|} \bar{c}_{ki} x_{ki} - \mu \sigma_{ki} x_{ki} + \mu \sigma y_{ki}$$

$$\text{s.t.} \quad \sum_{i=1}^{|N_T|} x_{ki} = |N_{VS}|$$

$$\sum_{i=1}^{|N_T|} y_{ki} = |N_{VR}|$$

$$x_{ki}, y_{ki} \in \{0, 1\}$$

Where $|N_T|$ is the number of targets, $|N_{VS}|$ is the number of strike UAVs, and $|N_{VR}|$ is the number of reconnaissance UAVs. x_{ki} is 1 if a strike UAV is assigned to target i at time k and zero otherwise. y_{ki} is 1 if a reconnaissance UAV is assigned to target i at time k and zero otherwise.

In 2006, Bryant [12] and Sakamoto [17] applied the robust optimization framework presented in Bertsimas and Sim [9] to address UAV task assignment and routing problems. Bryant proposed a UAV assignment formulation based on the military's Effects Based Operations (EBO) framework. Tasks are valued based on their ability to achieve desired effects. UAV assignments to tasks are then made to maximize the total desired effects of the plan. Bryant then applied robustness to his planning formulation using both the Bertsimas and Sim method with polyhedral uncertainty sets and using chance constrained programming.

Sakamoto approached the UAV assignment problem as a vehicle routing problem. In his formulation a UAV is presented with a set of tasks, with associated reward values, located throughout a geographic area. His formulation seeks to select the group of tasks and order of execution so as to maximize the total reward of the UAV mission. He then applies robustness using the Bertsimas and Sim method using polyhedral uncertainty sets.

This thesis seeks to build on this work. Whereas previous optimization approaches to reconnaissance asset allocation problems have focused exclusively on

UAV task assignment, this thesis extends this approach to include UAV, air, and ground reconnaissance assets. By doing so, this thesis also allows us to model the benefits of *mixing*, using assets of different types to simultaneously collect information on the same target, and *redundancy*, using more than one asset of the same type to simultaneously collect information on a single target, which have been neglected in previous work [24]. Lastly, this thesis uses more advanced methods to model uncertainty, i.e., ellipsoidal and central-limit theorem based uncertainty sets, than previous work on reconnaissance asset allocation problems. To the best of our knowledge this is the first application of these specific modeling methods in a military reconnaissance context.

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Chapter 3

Deterministic Approach

We first consider a deterministic optimization model approach to the TRAAP. Before we do this, we first describe the available data and key decisions facing tactical reconnaissance planners. Our deterministic model successfully generates a reconnaissance plan that maximizes the expected information value returned of the plan. We also present two algorithms that approximate a human planner's approach to the TRAAP. The plans generated by these algorithms will be used to help evaluate the performance of our optimization models.

3.1 Data and Decisions

Reconnaissance planning is based on the commander's guidance and staff outputs of the mission analysis step of the Military Decision Making Process (MDMP), see Section 2.1.2 for further discussion of reconnaissance planning and the MDMP. This information comprises the data used in our optimization model. In particular, reconnaissance planners use the staff estimates for the number and type of available reconnaissance assets, the capabilities of each of these assets, the locations where information needs to be collected, known as Named Areas of Interest (NAIs), the collection time windows associated with each NAI, the collection priority for each NAI, and the commander's risk assessment to develop their plans.

Using this information military planners must allocate their available reconnaissance assets, generally in what is known as an ISR synchronization matrix or reconnaissance plan, in such a manner that they optimize the expected total information collection value of the plan. At the lowest level they must assign each available reconnaissance asset to a set of missions. A mission is defined as a time window and location, i.e., NAI, where reconnaissance can be conducted. These assignment instructions constitute the reconnaissance plan.

When developing their reconnaissance plan, planners must also consider the impact of mixing collection assets and of redundancy. *Mixing* is using two or more different types of assets to simultaneously collect against a single NAI, while *redundancy* is using more than one of the same type of asset to simultaneously collect on a single NAI [24]. Typically using mixing or redundancy increases the collected information value of a particular reconnaissance mission. Therefore, planners must decide not only where and when each asset should be assigned, but also the configuration of assets that are assigned to each NAI. A configuration is defined as a collection of one or more assets, not necessarily of the same type, that are assigned simultaneously to the same NAI. An example of a configuration could be one platoon and one Unmanned Aerial Vehicle (UAV), while another example of a valid configuration could be simply a single UAV.

Planners must also incorporate operational risk into their reconnaissance plans. Risk, in this context, refers to hazards that exist on the battlefield, such as enemy forces, weather conditions, or dangerous terrain, that can result in mission degradation or mission failure [20]. Planners identify potential risk during mission analysis and use this information when developing their plans. In practice, the commander will identify what he feels is an acceptable level of risk exposure for his assets. Planners will then use the information they have regarding potential operational risk and allocate assets in such a way as to not expose any asset to risk levels above the prescribed limit.

Another consideration when forming a reconnaissance plan is the effect of unit movements during collection time windows. Each NAI has an associated collection window or windows. A collection window is a block of time when the planner believes he has the greatest chance of gathering useful information in an NAI. Sometimes it is useful for some or all of the assets that are conducting reconnaissance in an NAI to be reassigned before the end of a collection window. A planner may consider this action if, during a portion of the collection window, the expected information value in a different NAI is significantly greater than the expected information value of the original NAI. Mid-collection window asset transitions of this type allow a greater utilization of available assets and increase the expected total information value of the reconnaissance plan.

Although asset transitions typically generate plans with higher expected information value, asset transitions also have associated penalties. Generally ground reconnaissance units wish to minimize their total number of movements on the battlefield. A reconnaissance unit's greatest asset is its ability to remain unobserved by the enemy. When a ground unit is told to move from one location to another it must expose itself, to some degree, to enemy observation. Often, once a reconnaissance unit is exposed, the enemy will adjust their activity in the area for a duration of time in order to deny the friendly unit of the information it is trying to collect, thus lowering the expected information value of further reconnaissance in this area.

Therefore, in order to accurately model asset transitions our formulation must only allow a transition when the expected increase in total information value more than offsets the associated reduction in information value of further reconnaissance in the original NAI. It is also important to note that not all asset types are subject to transition penalties. For example, many UAVs and manned reconnaissance aircraft fly at altitudes where they do not increase their chances of enemy observation as they move around the battlefield.

Although aerial reconnaissance platforms generally do not suffer from transition penalties they are restricted by endurance limitations. UAVs and manned aircraft are subject to mission duration limitations based on fuel capacity, weather conditions, and other environmental factors. As a result, each system can only conduct reconnaissance for a specified period of time before being forced to return to an airfield to refuel and conduct maintenance. By incorporating asset endurance limitations into our model we can generate more realistic plans and gain insight into the best times to conduct resupply and maintenance for aerial assets.

3.2 Deterministic Model Formulation

With these considerations in mind we now model the TRAAP as an integer optimization problem. Using the outputs of mission analysis we identify a set J of available reconnaissance assets and I of NAIs where we wish to conduct reconnaissance. Assets are divided into K types, e.g., platoons, UAVs, sniper teams, etc., where the set of assets of type $k \in K$, A_k, is a subset of J. All assets of the same type are assumed to have equivalent capability. In order to account for mixing and redundancy each mission can be serviced by multiple configurations of assets. As mentioned earlier, an example of a configuration could be one platoon and one UAV. This configuration would be deemed feasible if at least one mission calls for at a minimum one platoon and one UAV to conduct reconnaissance. The feasible configurations for all potential missions are compiled into a set C. The quantity of asset type $k \in K$ in configuration $c \in C$ is defined by the parameter a_{kc}.

We also have the sets T_i, defined as the set of time intervals when we wish to conduct reconnaissance in NAI i, to reflect collection time windows. We then have the set T, of which $T_i \subset T$, as the complete set of discrete time intervals in our planning time horizon. Note that this formulation gives us the flexibility to schedule multiple, non-consecutive collection windows in each NAI during our planning time horizon. We then define the parameter $e^k_{i_1 i_2}$ as the number of time intervals required for assets of type k to transition from NAI i_1 to NAI i_2.

The remaining model parameters are also developed from the mission analysis outputs. The expected information value f_{ic} of executing reconnaissance in NAI i in configuration c is based on a number of variables, including the commander's collection priorities, the number and capabilities of the reconnaissance assets in configuration c, expected weather conditions, target characteristics, among others. The anticipated risk r_{jic} associated with asset j conducting reconnaissance in NAI i in configuration c is, much like the information value, a function of multiple variables. These variables include the enemy situation, the distance of the NAI from medical facilities, the number and capabilities of nearby friendly units, etc. and can be derived from the commander's risk assessment. The risk assessment, along with the commander's intent, also provides the model with the acceptable level of accumulated risk m_j that each asset can be exposed to, we refer to this as an asset's risk budget. In our model we assume that both the information value, f_{ic}, and the risk value, r_{jic}, per time period remain constant in each NAI throughout the planning time horizon. The extension to time varying parameters is straightforward.

Next, we define the set $B \subset K$ of asset types that are subject to asset transition penalties. Asset transition penalties are defined by the constants α and β as follows. If a combination (j, i, c) is engaged at time $t-1$ for some j, but not engaged at time t, then for the next β time periods the value $f_{i,c}$ we obtain for conducting reconnaissance at (i, c) is reduced to $f_{i,c}(1-\alpha)$. That is, the expected information value is reduced by $\alpha\%$ during these periods. The values of the α and β parameters are adjustable and based on the commander's preferences and staff intelligence estimates from the MDMP.

We then define the set $L \subset K$ of asset types that have endurance limitations. For each element k in L we have a new parameter, s_k, that we define as the maximum consecutive time periods assets of type k can conduct reconnaissance before needing to refuel or conduct maintenance. In our model we assume that travel to and from, and execution of refueling and maintenance activities can be conducted in one time period. This assumption can easily be adjusted in our model to reflect different operational conditions.

Our model has four types of binary decision variables. The first set of variables, y_{ict}, represent whether reconnaissance is conducted in NAI i in configuration c during time period t. The second type of decision variable, x_{jict}, corresponds to whether asset j conducts reconnaissance in NAI i in configuration c during time interval t. We also define auxiliary binary decision variables, w_{it} and p_{ict} to account for asset transitions. The logic for w_{it} is as follows. If $x_{jic,t-1} = 1$ and $x_{jict} = 0$, then $w_{it} = ... = w_{i,t+\beta-1} = 1$. Moreover, if $y_{ict} = 1$ and $w_{it} = 1$, then $p_{ict} = 1$, and in this case the reward f_{ic} we receive will be $f_{ic}(1-\alpha)$ due to the asset transition penalty.

Using this information we have the following formulation.

$$\max \quad \sum_{i\in I,c\in C,t\in T} f_{ic}\left(y_{ict} - \alpha p_{ict}\right)$$

$$\text{s.t.} \quad \sum_{c\in C} y_{ict} \leq 1 \qquad \forall i \in I, \forall t \in T_i \tag{3.1}$$

$$y_{ict} = 0 \qquad \forall i \in I, \forall t \notin T_i \tag{3.2}$$

$$x_{jict} \leq y_{ict} \qquad \forall j \in J, \forall i \in I, \forall t \in T \tag{3.3}$$

$$\sum_{j\in A_k} x_{jict} = a_{kc} y_{ict} \qquad \forall k \in K, \forall i \in I, \forall c \in C, \forall t \in T \tag{3.4}$$

$$1 - \sum_{c\in C} x_{ji_1ct} \geq \sum_{c\in C} x_{ji_2ct'} \qquad \forall j \in A_k, \forall k \in K, \forall i_1, i_2 \in I : i_1 \neq i_2, \tag{3.5}$$

$$\forall t \in T : t \leq t' \leq t + e^k_{i_1 i_2}$$

$$\sum_{i\in I,c\in C,t\in T_i} r_{jic} x_{jict} \leq m_j \qquad \forall j \in J \tag{3.6}$$

$$\sum_{t'=t-s_k}^{t} \sum_{i\in I,c\in C} x_{jict'} \leq s_k \qquad \forall j \in A_k, \forall k \in L, \forall t \in T : t > s_k \tag{3.7}$$

$$\beta \sum_{c\in C} \left(x_{jict-1} - x_{jict}\right) \leq \sum_{t'=t}^{t+\beta-1} w_{it'} \qquad \forall j \in A_k, \forall k \in B, \forall i \in I, \forall t \in T : t \geq 2 \tag{3.8}$$

$$y_{ict} + w_{it} \leq p_{ict} + 1 \qquad \forall i \in I, \forall c \in C, \forall t \in T \tag{3.9}$$

$$x_{jict}, y_{ict}, w_{it}, p_{ict} \in \{0, 1\} \qquad \forall j \in J, \forall i \in I, \forall c \in C, \forall t \in T$$

Constraints (3.1) allow missions to be executed in only one feasible configuration. Constraints (3.2) ensure that no assets can be allocated to NAIs during time intervals that are not part of the set T_i, that is time intervals not in NAI i's collection windows. Constraints (3.3) ensure assets can only be assigned to missions and configurations that are executed. Constraints (3.4) ensure that the correct number of each asset type are assigned to executed configurations and missions. Constraints (3.5) ensure that assets are not assigned to a new NAI until after the appropriate number of transition time intervals. Constraints (3.6) are risk constraints. They restrict assets from being exposed to more risk than their maximum allowable risk level. Constraints (3.7) enforce endurance limitations on aerial assets. Lastly, constraints (3.8) and (3.9) determine when an asset transition penalty occurs.

3.3 Human Planning Approximation Algorithms

In order to gain a better understanding of the quality of our models we compare our optimization based reconnaissance plans to estimates of reconnaissance plans developed by human planners. As discussed in Section 1.1, due to time constraints and competing priorities military planners generally default to reconnaissance plans that they feel will return the information that they value the highest or one that provides coverage of the most areas. Thus, based on the authors first hand observations, human plans typically have all of their collection assets focused on the highest priority areas or have their assets spread thin over the entire battlefield. In either case, human planners nearly always disregard the level of uncertainty associated with each mission when developing their plans. With these observations in mind we present two planning algorithms, one for each type of typical human reconnaissance plan, for comparison with our models.

3.3.1 The High Priority Mission Algorithm

We now consider the first type of planner, one who focuses his assets exclusively on the highest priority missions. A high priority mission, in this context, is a mission with a high expected information value. Thus, a greedy algorithm, where mission/configuration assignments are made solely on their expected information value, provides a reasonable approximation of how a human planner might act. In this algorithm the planner prioritizes his collection efforts on the NAIs and configurations that produce the highest expected information value. Beginning with the highest priority NAI/configuration pair, the algorithm assigns assets in such a manner as to maximize the total number of time periods that the NAI/configuration pair can be executed without violating an asset's risk budget. Once no more assets can be assigned to an NAI/configuration pair the algorithm steps to the next highest priority NAI/configuration pair. The algorithm continues until no more assets can be assigned to any NAI/configuration pairs without violating their risk budget.

The High Priority algorithm is outlined in detail in the steps below.

1. Create a list of all feasible NAI/configuration pairs.

2. IF list of feasible NAI/configuration pairs is empty, exit algorithm.

3. ELSE select NAI/configuration pair, (i, c), from list with highest expected information value.

4. Create a list of feasible time periods, from the list of time periods that reconnaissance can be conducted in NAI i, where assets are available to execute configuration c.

5. Create a list of all possible asset groups that fulfill configuration c.

6. FOR all groups in the asset group list:

    (a) Create a list of asset group available time periods, from the list of feasible time periods, that the asset group has all assets available. Record the total number of time periods that the asset group is available, t_a.

    (b) Determine the number of time periods, t_r, that the asset group can conduct reconnaissance in NAI i without any member of the group violating its risk budget.

7. Assign all asset groups an asset group score. The asset group score equals the minimum of t_a and t_r.

8. Determine the highest asset group score.

9. IF the highest asset group score is zero, skip to step 15.

10. IF multiple asset groups have the highest asset group score, select the asset group, A, with the highest sum of remaining asset risk budgets.

11. ELSE select the asset group, A, with the highest asset group score.

12. Assign each asset in A to NAI i to the first t_r time periods in the list of asset group available time periods determined in step 6(a). Call this the set of executed time periods.

13. Reduce the risk budget of each asset in A by the accumulated risk of this assignment.

14. Mark all assets in A as unavailable during all executed time periods plus respective transition times.

15. Remove (i, c) from list of feasible NAI/configuration pairs and return to step 2.

3.3.2 The Maximum NAI Coverage Algorithm

Next, we present a planning algorithm to model the behavior of a planner that is primarily concerned with conducting reconnaissance in many different NAIs. This algorithm is also a greedy algorithm, in that asset assignments to NAIs are prioritized by their expected information value, but this algorithm also attempts to avoid, as much as possible, assigning more than one asset to an NAI. This algorithm considers each asset separately, beginning with the asset that has the highest remaining risk budget. It will then attempt to assign each asset in such a manner as to avoid NAIs with assets already assigned to them during any portion of the NAI's collection windows. If this is not possible the algorithm will then attempt to assign the asset to any NAI/time period with no other assets assigned. Only then will the algorithm consider pairing the asset with a previously assigned asset. The algorithm stops once all assets are unable to be assigned without violating their risk budgets.

The NAI Coverage algorithm is outlined in detail below.

1. IF all assets have been considered for assignment, select asset, j, with highest remaining risk budget

2. ELSE select asset type, k, with highest expected information value for a single asset configuration with at least one asset that has not been considered for assignment.

    (a) Select asset, j, of type k, that has not been considered for assignment with the highest remaining risk budget.

3. Create a list of NAIs that request an asset of the same type as asset j, and have no assets currently assigned to them.

4. IF this unassigned NAI list is not empty, select NAI i with highest expected information value for a configuration with only asset j.

   (a) Calculate the number of time periods, t, asset j can be assigned to NAI i without violating its risk budget and transition constraints.

   (b) IF $t > 0$, assign asset j to NAI i for up to t time periods it is available or all feasible time periods in NAI i, whichever is smaller.

      i. Reduce asset js risk budget by the amount of accumulated risk from this assignment.

      ii. Return to step 1

   (c) ELSE, remove NAI i from the unassigned NAI list created in step 3 and return to step 4.

5. ELSE, create a new list of NAIs that request an asset of the same type as j and have at least one asset currently assigned to them.

6. IF this assigned NAI list is not empty, select NAI, i, with the highest expected information value for a configuration, c, that includes asset j and all previously assigned assets to NAI i.

   (a) Calculate the number of time periods, t, asset j can be assigned to NAI i in configuration c without violating its risk budget and transition constraints.

   (b) IF $t > 0$, assign asset j to NAI i for up to t time periods when j is available and when its assignment will result in configuration c.

i. Reduce asset js risk budget by the amount of accumulated risk from this assignment.

ii. Increase asset budgets of all other assets assigned to NAI i in configuration c by the difference between the accumulated risk of the asset in configuration c and the accumulated risk of the previous configuration.

iii. Return to step 1

(c) ELSE, remove NAI i from the assigned NAI list created in step 5 and return to step 6.

7. ELSE IF unable to assign any asset, exit algorithm.

8. ELSE select asset, j, with next highest remaining risk budget and return to step 3.

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Chapter 4

Robust Approach

In this chapter we apply robust optimization to our TRAAP formulation to account for uncertainty in our model parameters. We consider modeling uncertainty using both polyhedral and ellipsoidal uncertainty sets.

4.1 Modeling Uncertainty Using Uncertainty Sets

Our deterministic formulation assumes that we know explicitly the values for our expected information value, f_{ic}, and our expected risk accumulation, r_{jic} per time period. Clearly under real world circumstances it is impossible to estimate the exact realized values for these parameters. It is, however, possible to accurately estimate a range in which we expect, with high confidence, the true values for these parameters to take. Using our information value parameter, f_{ic}, as an example we redefine our parameters as follows, see Bertsimas and Sim [9].

$$\tilde{f}_{ic} = \left[\bar{f}_{ic} - \hat{f}_{ic}, \bar{f}_{ic} + \hat{f}_{ic}\right]$$

In this notation $\bar{f}_{ic}$ represents the nominal, or expected, value of the uncertain information value parameter $\tilde{f}_{ic}$. $\hat{f}_{ic}$ represents the half-length of the range in which we expect the realized information value, f_{ic}, to fall in. We then expect the following to be true for all i and c.

$$\left|\frac{f_{ic} - \bar{f}_{ic}}{\hat{f}_{ic}}\right| \leq 1$$

Motivated by the central limit theorem, see Bandi and Bertsimas [3], as a means to efficiently aggregate the uncertainty of these parameters, we propose the following polyhedral uncertainty sets.

$$\mathcal{U}_{f_{ic}} = \left\{ f_{ic} \middle| \left|\frac{f_{ic} - \bar{f}_{ic}}{\hat{f}_{ic}}\right| \leq 1, \forall i \in I, \forall c \in C, \sum_{i \in I, c \in C} \left|\frac{f_{ic} - \bar{f}_{ic}}{\hat{f}_{ic}}\right| \leq \Gamma\sqrt{D} \right\}$$

$$\mathcal{U}_{r_{jic}} = \left\{ r_{jic} \middle| \left|\frac{r_{jic} - \bar{r}_{jic}}{\hat{r}_{jic}}\right| \leq 1, \forall j \in J, i \in I, c \in C, \sum_{i \in I, c \in C} \left|\frac{r_{jic} - \bar{r}_{jic}}{\hat{r}_{jic}}\right| \leq \Phi_j\sqrt{D}, \forall j \in J \right\}$$

where $D = |I| \cdot |C|$

The new parameters Γ and Φ_j allow us to adjust the level of robustness we wish to include in our model. When Γ and Φ_j equal zero we do not protect against any uncertainty and our formulation is equivalent to the non-robust formulation we outlined in Chapter 3. As we increase the values of these parameters we progressively add robustness into our model at the expense of possibly more conservative solutions. The ability to adjust the robustness of our model is especially useful as it allows planners to tailor the model to the risk preferences of the commander and easily generate multiple reconnaissance plans for consideration.

4.2 The Robust Formulation Under Polyhedral Uncertainty

We will now derive the robust counterpart to our deterministic TRAAP model using polyhedral uncertainty sets. For ease of notation we define

$$Z_{ic} = Z_{ic}^{+} - Z_{ic}^{-} = \frac{f_{ic} - \bar{f}_{ic}}{\hat{f}_{ic}}$$

We can then redefine our uncertainty set for the information value parameter, f_{ic}, as shown below.

$$\mathcal{Z} = \left\{ Z \,\middle|\, \left(Z_{ic}^{+} + Z_{ic}^{-}\right) \leq 1, \forall i \in I, \forall c \in C, \sum_{i \in I, c \in C} \left(Z_{ic}^{+} + Z_{ic}^{-}\right) \leq \Gamma\sqrt{D} \right\}$$

Using these robust parameters our model's objective function now becomes:

$$\max_{x,y,w,p} \sum_{i \in I, c \in C, t \in T} \bar{f}_{ic} y_{ict} + \left(\min_{\mathcal{Z}} \sum_{i \in I, c \in C, t \in T} \hat{f}_{ic} \left(Z_{ic}^{+} - Z_{ic}^{-}\right) y_{ict} \right) - \alpha \sum_{i \in I, c \in C, t \in T} p_{ict}$$

The interpretation of this new objective function is that we are trying to maximize the expected total information value of the solution while also trying to minimize the amount of uncertainty in our solution. As shown in Bertsimas and Sim [9] we can restructure the inner minimization problem into an equivalent linear optimization problem using the properties of linear optimization duality. After applying this method we have the following robust objective function and additional constraints.

$$\max_{x,y,w,p,\mu,\nu,\gamma} \sum_{i \in I, c \in C, t \in T} \bar{f}_{ic} y_{ict} + \Gamma\sqrt{D}\mu + \sum_{i \in I, c \in C} \nu_{ic} - \sum_{i \in I, c \in C} \gamma_{ic} - \alpha \sum_{i \in I, c \in C, t \in T} p_{ict}$$

$$
\begin{aligned}
\text{s.t.} \quad & \mu + \nu_{ic} + \gamma_{ic} \leq \hat{f}_{ic} \sum_{t \in T} y_{ict} && \forall i \in I, \forall c \in C \\
& \mu - \nu_{ic} - \gamma_{ic} \leq -\hat{f}_{ic} \sum_{t \in T} y_{ict} && \forall i \in I, \forall c \in C \\
& \mu \leq 0 && \\
& \nu_{ic} \leq 0 && \forall i \in I, \forall c \in C \\
& \gamma_{ic} \geq 0 && \forall i \in I, \forall c \in C
\end{aligned}
$$

Applying the same process described above to our risk constraints we have the complete robust TRAAP model formulation using polyhedral uncertainty sets shown here.

$$
\max \quad \sum_{i \in I, c \in C, t \in T} \bar{f}_{ic} \left(y_{ict} - \alpha p_{ict} \right) + \Gamma \sqrt{D} \mu + \sum_{i \in I, c \in C} \nu_{ic} - \sum_{i \in I, c \in C} \gamma_{ic}
$$

$$
\begin{aligned}
\text{s.t.} \quad \mu + \nu_{ic} + \gamma_{ic} \leq \hat{f}_{ic} \sum_{t \in T} y_{ict} \quad & \forall i \in I, \forall c \in C && (4.1) \\
\mu - \nu_{ic} - \gamma_{ic} \leq -\hat{f}_{ic} \sum_{t \in T} y_{ict} \quad & \forall i \in I, \forall c \in C && (4.2) \\
\sum_{c \in C} y_{ict} \leq 1 \quad & \forall i \in I, \forall t \in T_i && (4.3) \\
y_{ict} = 0 \quad & \forall i \in I, \forall t \notin T_i && (4.4) \\
x_{jict} \leq y_{ict} \quad & \forall j \in J, \forall i \in I, && (4.5) \\
& \forall t \in T && \\
\sum_{j \in A_k} x_{jict} = a_{kc} y_{ict} \quad & \forall k \in K, \forall i \in I, && (4.6)
\end{aligned}
$$

$$\forall c \in C, \forall t \in T$$

$$1 - \sum_{c \in C} x_{ji_1ct} \geq \sum_{c \in C} x_{ji_2ct'} \quad \forall j \in A_k, \forall k \in K, \tag{4.7}$$

$$\forall i_1, i_2 \in I : i_1 \neq i_2,$$

$$\forall t \in T : t \leq t' \leq t + e^k_{i_1i_2}$$

$$\sum_{i \in I, c \in C, t \in T_i} \bar{r}_{jic} x_{jict} + \Phi_j \sqrt{D} \rho_j + \sum_{i \in I, c \in C} \phi_{jic} - \sum_{i \in I, c \in C} \lambda_{jic} \leq m_j \quad \forall j \in J \tag{4.8}$$

$$\rho_j + \phi_{jic} + \lambda_{jic} \geq \hat{r}_{jic} \sum_{t \in T_i} x_{jict} \quad \forall j \in J, \forall i \in I, \tag{4.9}$$

$$\forall c \in C$$

$$\rho_j - \phi_{jic} - \lambda_{jic} \geq -\hat{r}_{jic} \sum_{t \in T_i} x_{jict} \quad \forall j \in J, \forall i \in I, \tag{4.10}$$

$$\forall c \in C$$

$$\sum_{t'=t-s_k}^{t} \sum_{i \in I, c \in C} x_{jict'} \leq s_k \quad \forall j \in A_k, \forall k \in L, \tag{4.11}$$

$$\forall t \in T : t > s_k$$

$$\beta \sum_{c \in C} (x_{jict-1} - x_{jict}) \leq \sum_{t'=t}^{t+\beta-1} w_{it'} \quad \forall j \in A_k, \forall k \in B, \tag{4.12}$$

$$\forall i \in I, \forall t \in T : t \geq 2$$

$$y_{ict} + w_{it} \leq p_{ict} + 1 \quad \forall i \in I, \forall c \in C, \tag{4.13}$$

$$\forall t \in T$$

$$x_{jict}, y_{ict}, w_{it}, p_{ict} \in \{0, 1\} \quad \forall j \in J, \forall i \in I, \forall c \in C,$$

$$\forall t \in T$$

$$\gamma_{ic}, \rho_j, \phi_{jic} \geq 0 \quad \forall j \in J, \forall i \in I, \forall c \in C$$

$$\mu, \nu_{ic}, \lambda_{jic} \leq 0 \quad \forall j \in J, \forall i \in I, \forall c \in C$$

This new model is equivalent to our non-robust, deterministic model proposed in Section 3.2 with the exception of the new robust constraints and objective function. Constraints (4.1) and (4.2), along with the additions to the objective function, add robustness against uncertainty in the information value parameter, f_{ic}. Constraints (4.8), (4.9), and (4.10) replace the risk constraints (3.6) in the deterministic model and add robustness against uncertainty in the risk parameter, r_{jic}. The remaining constraints are identical, and serve the same purpose, as those proposed in our deterministic model.

The robust model has $2\mathcal{I}\mathcal{C}(\mathcal{J}+1)$ additional constraints than our non-robust model, where $\mathcal{I}$, $\mathcal{C}$, and $\mathcal{J}$ represent the number of non-zero elements in the sets I, C, and J respectively. The model also added complexity by the addition of six types of new variables. Despite the addition of new constraints and variables we have maintained the linear structure, and therefore the tractability, of our original deterministic model. As a result this model can still be efficiently solved using readily available commercial solvers.

4.3 The Robust Formulation Under Ellipsoidal Uncertainty

The ability to model uncertainty in different ways is one appealing aspect of robust optimization. Motivated by Ben-Tal and Nemirovski [4, 5] we now consider modeling uncertainty using ellipsoidal uncertainty sets.

In this model we still expect the realized value of our uncertain parameters to fall within some symmetrical range around a nominal value. Instead of aggregating the uncertainty using the central limit theorem, as we did when constructing our polyhedral uncertainty sets, we now restrict the total "distance" that we expect the aggregate realized values of our uncertain parameters to fall away from their nominal values. This is accomplished using the Euclidean norm. This approach yields the following ellipsoidal uncertainty sets.

$$\mathcal{U}_{f_{ic}} = \left\{ f_{ic} \middle| \left| \frac{f_{ic} - \bar{f}_{ic}}{\hat{f}_{ic}} \right| \leq 1, \forall i \in I, c \in C, \sum_{i \in I, c \in C} \left(\frac{f_{ic} - \bar{f}_{ic}}{\hat{f}_{ic}} \right)^2 \leq \Theta^2 \right\}$$

$$\mathcal{U}_{r_{jic}} = \left\{ r_{jic} \middle| \left| \frac{r_{jic} - \bar{r}_{jic}}{\hat{r}_{jic}} \right| \leq 1, \forall j \in J, \forall i \in I, c \in C, \sum_{i \in I, c \in C} \left(\frac{r_{jic} - \bar{r}_{jic}}{\hat{r}_{jic}} \right)^2 \leq \Psi_j^2, \forall j \in J \right\}$$

In this case the parameters Θ and Ψ_j represent the level of robustness in the model. When these parameters are zero, just as before, the model does not protect against any deviations in our uncertain parameters from their nominal values. As we increase the values of Θ and Ψ_j we increase the level of robustness of our solution.

Using these new uncertainty sets we apply robustness to our model in the same manner as before. This results in the following robust formulation, which is a second order cone problem with binary variables.

$$\max \quad \sum_{i \in I, c \in C, t \in T} \bar{f}_{ic} \left(y_{ict} - \alpha p_{ict} \right) - \Theta \left\| \hat{f}_{ic} \sum_{t \in T} y_{ict} \right\|_2$$

$$\text{s.t.} \quad \sum_{c \in C} y_{ict} \leq 1 \qquad \forall i \in I, \forall t \in T_i \tag{4.14}$$

$$y_{ict} = 0 \qquad \forall i \in I, \forall t \notin T_i \tag{4.15}$$

$$x_{jict} \leq y_{ict} \qquad \forall j \in J, \forall i \in I, \forall t \in T \tag{4.16}$$

$$\sum_{j \in A_k} x_{jict} = a_{kc} y_{ict} \qquad \forall k \in K, \forall i \in I, \forall c \in C, \forall t \in T \tag{4.17}$$

$$1 - \sum_{c \in C} x_{ji_1ct} \geq \sum_{c \in C} x_{ji_2ct'} \qquad \forall j \in A_k, \forall k \in K, \forall i_1, i_2 \in I : i_1 \neq i_2, \forall t \in T : t \leq t' \leq t + e^k_{i_1 i_2} \tag{4.18}$$

$$\sum_{i \in I, c \in C, t \in T_i} \bar{r}_{jic} x_{jict} + \Psi_j \left\| \hat{r}_{jic} \sum_{t \in T_i} x_{jict} \right\|_2 \leq m_j \qquad \forall j \in J \tag{4.19}$$

$$\sum_{t'=t-s_k}^{t} \sum_{i \in I, c \in C} x_{jict'} \leq s_k \qquad \forall j \in A_k, \forall k \in L, \forall t \in T : t > s_k \tag{4.20}$$

$$\beta \sum_{c \in C} (x_{jict-1} - x_{jict}) \leq \sum_{t'=t}^{t+\beta-1} w_{it'} \qquad \forall j \in A_k, \forall k \in B, \forall i \in I, \forall t \in T : t \geq 2 \tag{4.21}$$

$$y_{ict} + w_{it} \leq p_{ict} + 1 \qquad \forall i \in I, \forall c \in C, \forall t \in T \tag{4.22}$$

$$x_{jict}, y_{ict}, w_{it}, p_{ict} \in \{0, 1\} \qquad \forall j \in J, \forall i \in I, \forall c \in C, \forall t \in T$$

Just as with our robust model using polyhedral uncertainty sets, our robust model using ellipsoidal uncertainty sets is equivalent to our deterministic model with the exception of the new robust constraints and objective function. This model protects against uncertainty in the information value parameter, f_{ic}, by adding a non-linear term to the objective function. Robustness against uncertainty in the risk parameter, r_{jic}, is added by replacing constraints (3.6) in the deterministic model with the robust, non-linear constraints (4.19). The remaining constraints are identical, and serve the same purpose, as those proposed in our deterministic model.

Note that when using ellipsoidal uncertainty sets we do not add additional constraints and variables. Instead, complexity is added to our model through the nonlinearity of both our objective function and our risk constraints. However, due to the structure of our uncertainty sets our robust model is now a second-order cone optimization problem. Efficient solution methods for second-order cone problems with binary variables exist and have been implemented in commercial solvers such as CPLEX and Gurobi. Thus, tractability of our robust model is maintained.

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Chapter 5

Fixed Allocation Approach

We now propose special cases of our deterministic and robust formulations that prohibit assets from conducting transitions during NAI collection windows. By fixing the allocation of assets during each collection window we reduce the complexity of our models and achieve significant computational benefits at the expense of some optimality and robustness. We provide these models for operational considerations. Potentially a planner may prefer a sub-optimal plan immediately instead of a higher quality plan later. These models give the military planner that option. Note that unless explicitly discussed in this section all parameters from our earlier models retain their previous definitions.

5.1 Deterministic Formulation of the Fixed Allocation Approach

In order to reduce the complexity of our models outlined in Chapters 3 and 4 we propose a few modifications and additions to their parameters. Previously we defined the sets I, to reflect the locations or NAIs where we wish to conduct reconnaissance, and T_i, to describe what time periods we wish to conduct reconnaissance in NAI i. By combining these temporal and spatial attributes we generate a new set, N, of missions. A mission, just as before, is defined as a time window and location where reconnaissance can be conducted. Whereas before we could define multiple collection windows for each NAI, in this new formulation each new collection window in an NAI is considered a distinct mission. Just as before each mission can be serviced by multiple configurations of assets. Therefore the expected information value parameter and asset risk parameters are now defined as f_{nc} and r_{jnc} respectively.

We then state that if an asset is assigned to a particular mission, its allocation is fixed and it will be considered unavailable to other missions from the mission start time to the mission end time plus some transition time. This information is captured in a mission compatibility matrix, g^k, for each asset type. Two missions are considered not compatible when their collection windows occur simultaneously or when an asset cannot transition from one NAI to the other before the collection window for the second mission begins. The values in each compatibility matrix are represented as $g^k_{n_1 n_2}$, that is one, if mission n_1 is compatible with mission n_2 for assets of type k and zero, otherwise.

By restricting asset assignments to the entire collection window of a mission, as opposed to individual time periods in the collection window, we eliminate the possibility of mid-collection window asset transitions and their associated penalties. It also forces us to reassess how asset endurance limitations are modeled. It is possible, using this formulation, to assign an asset to a mission with a collection window longer than the asset's endurance limit. We therefore assume that lower level planners will decide what periods, within the mission collection window, the asset will conduct reconnaissance and when it will return to base for maintenance and refueling. Despite the periodic gaps in reconnaissance by these assets the effects of the asset on the mission, i.e., information value added, risk reduction from mixing and redundancy, and risk accumulation, are assumed to be equally distributed throughout the mission collection window.

Because we no longer have asset transition penalties our model has only two types of binary decision variables. The first set of variables, y_{nc}, takes the value one, if mission n is executed in configuration c and is zero, otherwise. The second set of variables, x_{jnc}, takes the value one, if asset j is assigned to mission n in configuration c and is zero, otherwise.

Applying these changes we have the following deterministic formulation.

$$\max \quad \sum_{n \in N, c \in C} f_{nc} y_{nc}$$

$$\text{s.t.} \quad \sum_{c \in C} (x_{jn_1c} + x_{jn_2c}) \leq 1 \qquad \forall n_1 \in N, \forall n_2 \in N, \forall j \in A_k, \quad \forall k \in K : g^k_{n_1 n_2} = 0 \tag{5.1}$$

$$\sum_{c \in C} y_{nc} \leq 1 \qquad \forall n \in N \tag{5.2}$$

$$x_{jnc} \leq y_{nc} \qquad \forall j \in J, \forall n \in N \tag{5.3}$$

$$\sum_{j \in A_k} x_{jnc} = a_{kc} y_{nc} \qquad \forall k \in K, \forall n \in N, \forall c \in C \tag{5.4}$$

$$\sum_{n \in N, c \in C} r_{jnc} x_{jnc} \leq m_j \qquad \forall j \in J \tag{5.5}$$

$$y_{nc}, x_{jnc} \in \{0, 1\} \qquad \forall j \in J, \forall n \in N, \forall c \in C$$

Constraints (5.2), (5.3), (5.4), and (5.5) have the same functions as constraints (3.1), (3.3), (3.4), and (3.6) in our previous deterministic model outlined in Chapter 3. Constraints (5.1) enforce the mission compatibility requirements, namely they ensure that an asset cannot be assigned to two incompatible missions.

5.2 Robust Formulations of the Fixed Allocation Approach

We now propose robust formulations of our special case model using both polyhedral and ellipsoidal uncertainty sets. We begin by modeling uncertainty using a polyhedral uncertainty set motivated by the central limit theorem.

Applying robustness using the same method described in Section 4.2 we have the following formulation.

$$\max \quad \sum_{n\in N, c\in C} \bar{f}_{nc} y_{nc} + \Gamma\sqrt{D}\mu + \sum_{n\in N, c\in C} \nu_{nc} - \sum_{n\in N, c\in C} \gamma_{nc}$$

$$\begin{aligned}
\text{s.t.} \quad \mu + \nu_{nc} + \gamma_{nc} &\leq \hat{f}_{nc} y_{nc} && \forall n \in N, \forall c \in C && (5.6)\\
\mu - \nu_{nc} - \gamma_{nc} &\leq -\hat{f}_{nc} y_{nc} && \forall n \in N, \forall c \in C && (5.7)\\
\sum_{c\in C} (x_{jn_1c} + x_{jn_2c}) &\leq 1 && \forall n_1 \in N, \forall n_2 \in N, && (5.8)\\
& && \forall j \in A_k, &&\\
& && \forall k \in K : g^k_{n_1n_2} = 0 &&\\
\sum_{c\in C} y_{nc} &\leq 1 && \forall n \in N && (5.9)
\end{aligned}$$

$$
\begin{aligned}
x_{jnc} &\le y_{nc} && \forall j \in J, \forall n \in N && (5.10)\\
\sum_{j \in A_k} x_{jnc} &= a_{kc} y_{nc} && \forall k \in K, \forall n \in N, && (5.11)\\
& && \forall c \in C &&\\
\sum_{n \in N, c \in C} \bar{r}_{jnc} x_{jnc} + \Phi_j \sqrt{D} \rho_j + \sum_{n \in N, c \in C} \phi_{jnc} - \sum_{n \in N, c \in C} \lambda_{jnc} &\le m_j && \forall j \in J && (5.12)\\
\rho_j + \phi_{jnc} + \lambda_{jnc} &\ge \hat{r}_{jnc} x_{jnc} && \forall j \in J, \forall n \in N, && (5.13)\\
& && \forall c \in C &&\\
\rho_j - \phi_{jnc} - \lambda_{jnc} &\ge -\hat{r}_{jnc} x_{jnc} && \forall j \in J, \forall n \in N, && (5.14)\\
& && \forall c \in C &&\\
y_{nc}, x_{jnc} &\in \{0, 1\} && \forall j \in J, \forall n \in N, \forall c \in C &&\\
\gamma_{nc}, \rho_j, \phi_{jnc} &\ge 0 && \forall j \in J, \forall n \in N, \forall c \in C &&\\
\mu, \nu_{nc}, \lambda_{jnc} &\le 0 && \forall j \in J, \forall n \in N, \forall c \in C &&\\
\text{where } D &= |N| \cdot |C| &&&&
\end{aligned}
$$

This new model is equivalent to our deterministic model proposed in Section 5.1 with the exception of the new robust constraints and objective function. Constraints (5.6) and (5.7), along with the new terms in the objective function, add robustness against uncertainty in the information value parameter, f_{nc}. Constraints (5.12), (5.13), and (5.14) replace the risk constraints (5.5) in the deterministic model and add robustness against uncertainty in the risk parameter, r_{jnc}. The remaining constraints are identical, and serve the same purpose, as those proposed in our deterministic model.

The robust model has $2\mathcal{N}\mathcal{C}(\mathcal{J}+1)$ additional constraints than our non-robust model, where $\mathcal{N}$, $\mathcal{C}$, and $\mathcal{J}$ represent the number of non-zero elements in the sets N, C, and J respectively.

We next present a robust formulation of the Fixed Allocation model using ellipsoidal uncertainty sets based on the Euclidean norm. Derivation of this model follows the same procedure outlined in Section 4.3.

$$\max \quad \sum_{n\in N, c\in C} \bar{f}_{nc} y_{nc} - \Theta \left\| \hat{f}_{nc} y_{nc} \right\|_2$$

$$\text{s.t.} \quad \sum_{c\in C} (x_{jn_1c} + x_{jn_2c}) \le 1 \qquad \forall n_1 \in N, \forall n_2 \in N, \forall j \in A_k, \tag{5.15}$$

$$\forall k \in K : g^k_{n_1n_2} = 0$$

$$\sum_{c\in C} y_{nc} \le 1 \qquad \forall n \in N \tag{5.16}$$

$$x_{jnc} \le y_{nc} \qquad \forall j \in J, \forall n \in N \tag{5.17}$$

$$\sum_{j\in A_k} x_{jnc} = a_{kc} y_{nc} \qquad \forall k \in K, \forall n \in N, \forall c \in C \tag{5.18}$$

$$\sum_{n\in N, c\in C} \bar{r}_{jnc} x_{jnc} + \Psi_j \left\| \hat{r}_{jnc} x_{jnc} \right\|_2 \le m_j \qquad \forall j \in J \tag{5.19}$$

$$y_{nc}, x_{jnc} \in \{0,1\} \qquad \forall j \in J, \forall n \in N, \forall c \in C$$

Just as before, our robust model using ellipsoidal uncertainty sets is equivalent to our deterministic model with the exception of the new robust constraints and objective function. This model protects against uncertainty in the information value parameter, f_{nc}, by adding a non-linear term to the objective function. Robustness against uncertainty in the risk parameter, r_{jnc}, is added by replacing constraints (5.5) in the deterministic model with the robust, non-linear constraints (5.19). The remaining constraints are identical, and serve the same purpose, as those proposed in our deterministic model.

By using an uncertainty set based on the Euclidean norm our robust counterpart is a second-order cone problem, and therefore remains tractable using commercially available solvers.

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Chapter 6

Computational Results and Analysis

To test and evaluate our models we desire test scenarios similar to current and anticipated operational problems facing military reconnaissance forces. Keeping this in mind, and using the author's first hand operational experience, we present a representative scenario that a battalion-level reconnaissance planner may face in a conflict similar to those in Afghanistan or Iraq. We then use this scenario to conduct analysis of the solution quality, using simulation, of each of our models. In particular, we focus on the effects of adding robustness into our models on solution quality. Additionally we present further analysis on the computational demands of our models.

6.1 The Operational Scenario

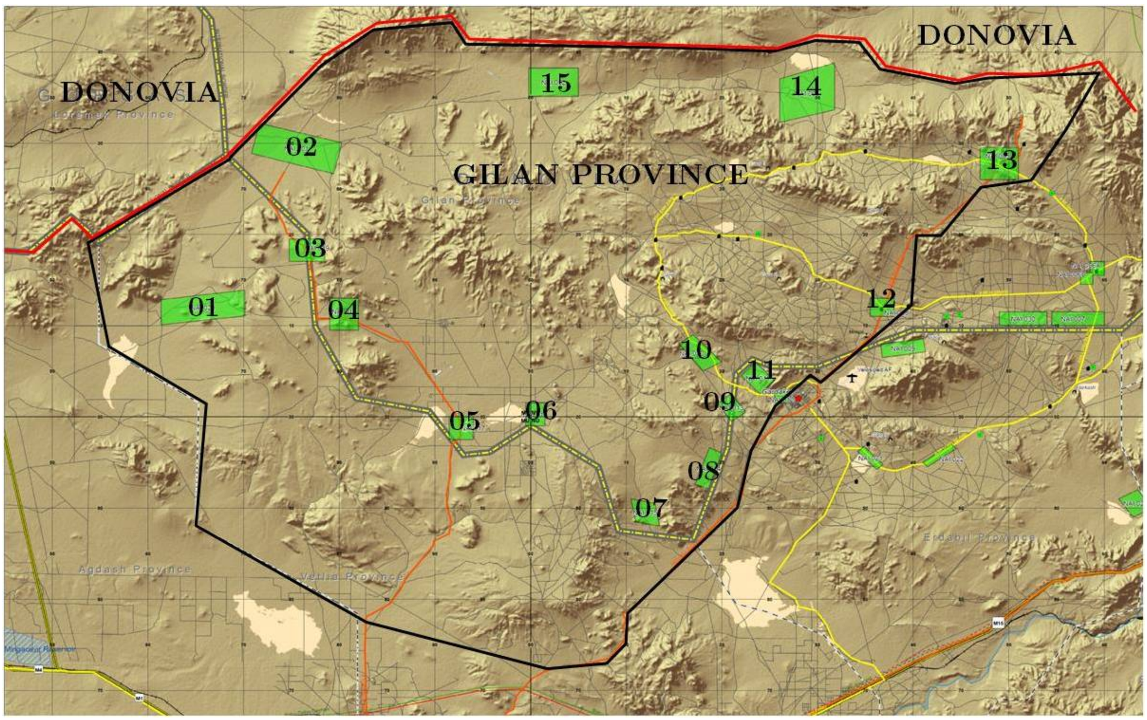
Our test scenario is designed to reflect a typical operational problem facing battalion-level reconnaissance planners in an environment similar to that found in Iraq or Afghanistan during the height of US military involvement in these conflicts.

The test scenario we propose here is based on a fictional scenario used at the National Training Center (NTC) to train US Army and Marine Corps units preparing to conduct security and counterinsurgency operations in Afghanistan. The NTC is located at Ft. Irwin, CA and serves as a world class military training facility focused on preparing units to fight in the contemporary operating environment and future battlefields [14]. The training staff at the NTC is responsible for developing complex and difficult training scenarios designed to tax all elements and systems in a brigade sized unit. Using the NTC scenario as our guide we scaled the problem to a battalion sized operation and generated representative scenario data to feed into our TRAAP formulations and algorithms. By doing so we hope to illustrate the effectiveness of our models on a difficult problem used in real world training.

6.1.1 Test Scenario Background

In our test scenario a battalion task force is conducting security operations in the Gilan Province of the fictional country of Atropia. Figure 6-1 depicts a map of the battalion's area of operations (AO) and NAIs. Gilan Province consists of mountainous terrain in the north, transitioning to rolling plains in the south. The battalion AO has no major population centers but does contain numerous small villages connected by a sparse, mostly unimproved road network. In total, the battalion's assigned section of Gilan Province has a population of around 50,000. The battalion's area of operations also borders the country of Donovia. Although Donovia is considered an ally, due to sparse and ineffective Donovian military operations near the border, the enemy uses the mountainous Donovian borderlands as a safe haven and support zone. Most of the battalion's reconnaissance efforts focus on suspected enemy infiltration routes from Donovia.

Figure 6-1: Test Scenario Area of Operations



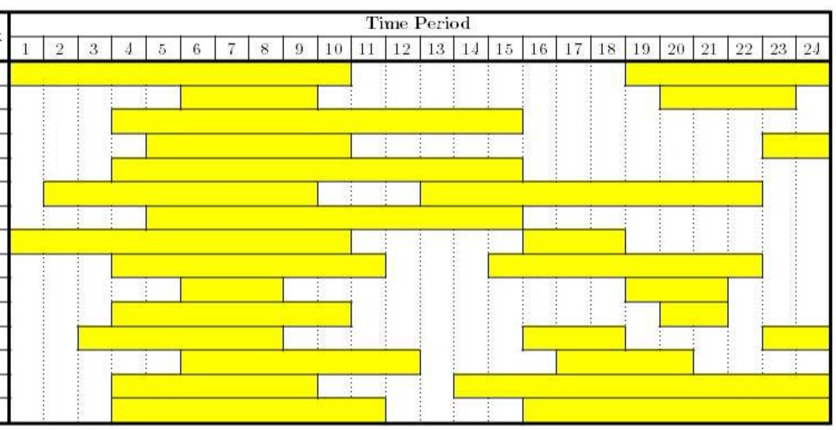
6.1.2 Test Scenario Data

Before generating the test scenario data we established certain scenario parameters, such as the number and type of assets, number of NAIs, number of time periods in the planning horizon, etc., in order to appropriately scale the scenario to a battalion sized problem. The remaining scenario data was generated randomly and then manually modified, where necessary, to better reflect the conditions of the NTC scenario. We chose to create our scenario data in this manner in order to ensure some randomness in the data while still approximating a realistic battlefield situation. In this section we will provide an overview of the scenario metrics and discuss in more detail how our scenario data was created.

The test scenario consists of twenty-eight possible reconnaissance missions spread across fifteen NAI's over a twenty-four period (hour) planning horizon. Figure 6-2 depicts the scenario asset requests and collection windows for each NAI. The NAI asset requests in Figure 6-2 are broken down by asset type; platoon's, UAV's, and scout weapons teams (SWT). The asset requests for each NAI were randomly generated and then manually adjusted to better reflect the NTC scenario's conditions. Asset requests were capped for each NAI at a maximum of two platoons, one UAV, and one SWT; resulting in the number of feasible asset configurations in the scenario being 12 (including a configuration with no assets).

Figure 6-2: Test Scenario NAI Asset Requests and Collection Windows

NAI	PLT	UAV	SWT	Max InV	Risk
01	0	1	1	0.37	L
02	1	0	1	0.54	L
03	2	0	0	1.00	M
04	2	1	1	1.32	M
05	0	1	0	0.13	H
06	1	1	1	0.79	H
07	2	0	1	0.89	H
08	2	0	1	1.24	L
09	0	0	1	0.19	H
10	1	0	0	0.40	M
11	0	1	0	0.17	L
12	2	0	0	1.08	L
13	1	1	1	0.74	L
14	1	0	0	0.41	L
15	0	1	0	0.13	M



The NAI collection windows in the scenario are correlated to reflect times of high expected enemy activity, implying a high demand in reconnaissance assets, and low activity in the area. In order to generate pseudo-random collection windows and still ensure a period of high asset demand we randomly selected a collection window start time for each NAI between time periods 1 and 6. This ensured that there would be significant overlap in the collection windows for all NAIs. After establishing the first collection window for each NAI, further collection windows were created by randomly selecting a start time at least three time periods after the previous collection window ended. The length for all collection windows was randomly chosen between 2 and 12 time periods.

Figure 6-2 also shows the maximum possible expected information value per time period for conducting reconnaissance in an NAI. These values, along with the information values for other asset configurations, were generated using the number and type of assets in each configuration and a randomly assigned NAI priority. All NAIs were first randomly ranked from highest to lowest priority, where high priority in this context means that, based on the scenario, we would expect a relatively higher information value for reconnaissance conducted in the NAI. A few manual adjustments to the priorities were then made to ensure consistency with the NTC scenario. Nominal information values, $\bar{f}_{ic}$, were then assigned to each NAI/configuration pair based on the NAI's priority and the number and type of assets in the configuration. The number and type of assets requested in each NAI was used to develop this data due to the assumption that NAIs with larger asset requests will typically return more valuable information. In reality this is not always the case, and we use the NAI priority parameter to reflect this characteristic of real world scenarios. By doing so, we ensured that some NAIs with relatively few assets requested would have higher expected information values than other NAIs with more assets requested. The values for the $\bar{f}_{nc}$ parameters were calculated by summing the values of $\bar{f}_{ic}$ for the number of time periods in mission n.

Figure 6-2 also provides a general description (High, Medium, and Low) of the level of risk when conducting reconnaissance in each NAI. Risk data was generated by randomly assigning each NAI an overall risk value between 0.0 and 1.0. Again, a small number of manual adjustments were made to these assignments to ensure scenario consistency. We categorized each NAI in Figure 6-2 based on these assignment. For example, a high risk NAI has an overall risk value in the interval 0.667 to 1.0 and a low risk NAI has an overall risk value in the interval 0.0 to .333. Risk values for each asset type were then derived using this overall NAI risk value and the number and type of assets in each configuration. Asset risk values were assigned so that the level of risk exposure per time period is decreased for each additional asset in the configuration.

We used two methods to establish the amount of uncertainty in our uncertain parameters, $\bar{f}_{ic}$ and $\bar{r}_{jic}$. In the scenario we assumed that all parameter uncertainty was normally distributed around the nominal values. We also felt, based on the NTC scenario information on hand, that the reconnaissance planners had a better estimate of the amount of uncertainty in the information value parameters compared to the risk value parameters. Thus, we randomly assigned each NAI a level of information value uncertainty, $\hat{f}_{ic}$, between 10% and 90% of the nominal information value for each configuration. The varied levels of uncertainty reflects the battalion's belief that they having a reasonable idea of the level of information value uncertainty in each NAI. In contrast, the level of uncertainty for the risk parameters, $\hat{r}_{jic}$, was set equal to 50% of the nominal risk value for all NAIs and configurations. This implies that the battalion planners do not have a good estimate of which NAIs have more or less risk uncertainty relative to the risk parameter nominal values. The values for the parameters $\hat{f}_{ic}$ and $\hat{f}_{ic}$ represent two standard deviations of the normally distributed uncertain parameters. This ensures, under the assumption that the parameters accurately represent the real world, that the realized values of the uncertain parameters will fall with high probability, roughly 95%, within the interval $[\bar{f}_{ic} - \hat{f}_{ic}, \bar{f}_{ic} + \hat{f}_{ic}]$.

In the scenario the planner has approximately one third (four platoons) of the battalion's ground forces available to devote to reconnaissance tasks. This assumes that the remaining battalion forces are devoted to other vital missions such as force protection, offensive operations, civil-affairs, etc. The battalion is being supported by two RQ-7B Shadow UAV's and a dedicated scout weapons team (SWT) of OH-58D Kiowa helicopters. Transition times between NAIs vary by asset type between one and four time periods and only platoons are subject to transition penalties in this scenario. The Shadow UAVs are subject to a nine hour endurance constraint while the SWT has an endurance limit of two hours. Furthermore, the commander has set the asset type risk budgets, meaning the amount of risk each type of asset can assume, to protect platoons the most, followed by the SWT and then UAVs. All of these parameters were selected manually to best reflect the conditions in the NTC scenario. Table 6.1 provides a summary of these scenario metrics.

Table 6.1: Test Scenario Asset Summary

Asset Type	**Quantity**	**Trans. Times (Per)**	**Trans. Penalties**	**Endurance Limit (Per)**	**Risk Budget**
Platoon	4	1 to 4	Yes	None	Low
UAV	2	1	No	9	High
SWT	1	1	No	2	Medium

6.2 Model Simulation

In order to evaluate the solutions produced by our TRAAP formulation and algorithms it was necessary to use simulation. There are numerous approaches to simulation testing and, ideally, one would use a closed-loop method. A closed-loop simulation for our scenario would generate realized information value and risk data for each time period, starting at the beginning of our planning time horizon. The simulation would implement each TRAAP solution time period by time period until an asset violates its risk budget. At that time the simulation would input the remaining risk levels of all assets, and other pertinent data, back into our TRAAP formulation and generate a new solution for the remainder of the planning time horizon. The simulation would resume and progress in this manner until all assets have violated their risk budgets or the simulation reaches the end of the planning time horizon. A closed-loop simulation such as this is very computationally expensive and difficult to implement.

We selected to use a different, less complicated, form of simulation known as Monte Carlo simulation. In a Monte Carlo simulation realized values for all of our unknown data are generated. We then apply our solutions using this realized data. After applying the data we can determine if an asset or assets have violated their risk budgets and calculate the total realized information value of the solution. By repeating this process multiple times we can generate statistics on how each solution performs and use this information to compare solutions against one another.

In our testing we generated 5,000 separate realizations of our unknown risk and information value parameters. All of our solutions were tested using this set of data realizations in order to fairly evaluate their performance against one another. In each simulation the unknown risk and information value parameters for each time period were sampled from a normal distribution with a mean equal to $\bar{f}_{ic}$ and $\bar{r}_{jic}$ and standard deviation equal to $\frac{\hat{f}_{ic}}{2}$ and $\frac{\hat{r}_{jic}}{2}$. If a time period was subject to an asset transition penalty the simulation randomly determined whether the asset transition was observed or not. We assumed that all asset transitions had an 80% chance of being observed by the enemy. If the asset transition was observed the corresponding realized information values were sampled from a distribution with mean equal to $\frac{\bar{f}_{ic}}{4}$ and standard deviation equal to $\frac{\hat{f}_{ic}}{4}$. If the asset transition was not observed by the enemy the realized information value was sampled from the original distribution. To avoid negative information and risk values, if the realized parameter value was less than zero it was reset to equal zero. The parameters $\hat{f}_{nc}$ and $\hat{r}_{jnc}$ were calculated by summing the realized parameter values for the time periods and NAI corresponding to mission n.

6.3 Model Performance and Comparison

After simulation we compared our model solutions using three primary performance measures; the mean of the realized information value collected over all simulations, the standard deviation of the collected information value over all simulations, and model feasibility. Model feasibility is a measure of constraint violation frequency. A model was deemed feasible for a simulated scenario only if no assets exceeded their maximum risk allowance. These three performance measures dictate the solution quality of the model solution.

We selected these performance measures because they directly translate to characteristics of desirable reconnaissance plans. The mean realized information value is the expected intelligence benefit received from executing the plan. Clearly maximizing this value is the primary goal of any reconnaissance plan. The standard deviation of the collected information value is a measure of how certain the plan is of achieving the expected outcome. By minimizing this value it is less likely that our reconnaissance assets will return to base without useful information. The final performance measure, model feasibility, directly translates into saving soldiers' lives. A risk constraint violation implies that a unit is exposed to an unacceptable level of risk. Solutions that minimize the likelihood of constraint violation reduce the chances of soldiers being injured. Therefore a model solution with high solution quality will have a large mean realized information value, small standard deviation, and high probability of model feasibility.

6.3.1 Human Approximation Algorithm and Deterministic Model Performance

Table 6.2 displays the solution performance of the deterministic and human approximation algorithms after five thousand simulated realizations of the test scenario.

Table 6.2: Human Approximation Algorithm and Deterministic Model Solution Performance

Model	**Mean**	**Std Dev**	**% Feasible**
High Priority Algorithm	28.5	0.989	60.2%
NAI Coverage Algorithm	22.3	0.476	1.8%
Deterministic Model	38.3	0.800	27.1%

In total, our optimization based deterministic model outperformed both human planning approximation algorithms. The High Priority and NAI Coverage algorithm solutions each bested the deterministic model solution in a single performance category; the High Priority solution improved solution feasibility by over 33% and the NAI Coverage solution reduced the standard deviation of the total information value by over 40% from the deterministic model. However, these performance gains came at a very steep cost to mean information value collected, 26% for the High Priority algorithm and 42% for the NAI Coverage algorithm. In addition to the cost in mean information value both algorithm solutions performed worse than the deterministic model solution in the other performance categories. Based on this evidence, particularly the prohibitively high cost in mean information value, we can conclude that our deterministic model provides an improvement in reconnaissance planning performance from typical human planning methods.

Further analysis of our solutions provides some insight into why our human approximation algorithms performed poorly compared to our deterministic model. Figures 6-3, 6-4, and 6-5 provide an overview of the solutions derived using our deterministic model, High Priority algorithm, and NAI Coverage algorithm, respectively. The green sections represent the time periods when reconnaissance is conducted in an NAI. The labels on, or adjacent to, the green sections state the number of platoons, UAVs, and SWTs assigned to conduct reconnaissance in the NAI during that time interval.

Figure 6-3: Test Scenario Deterministic Model Solution

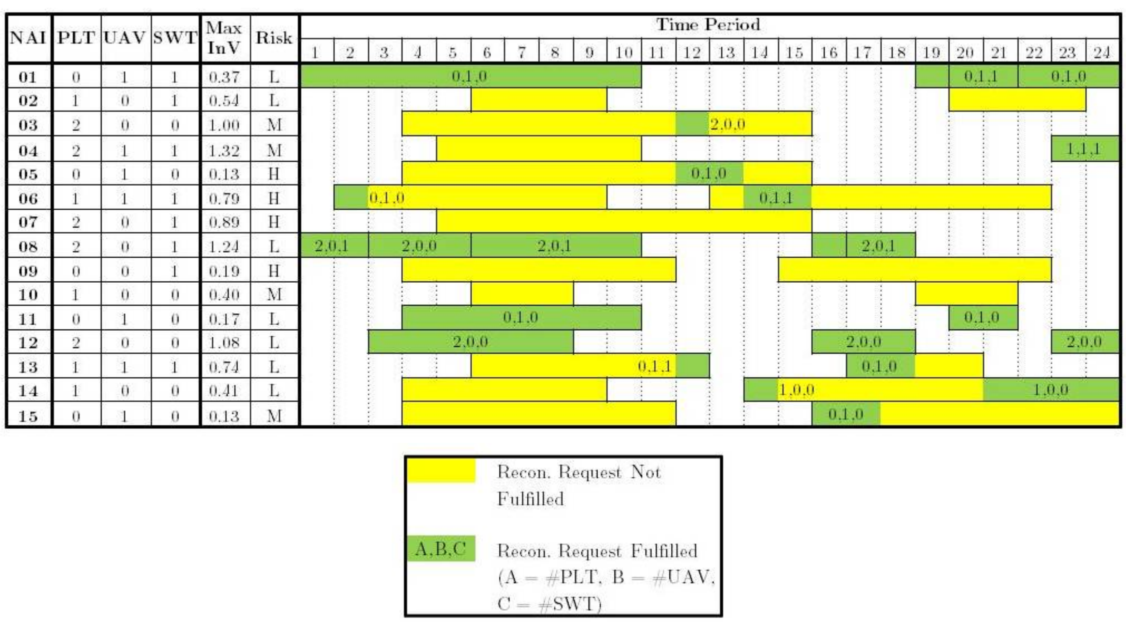
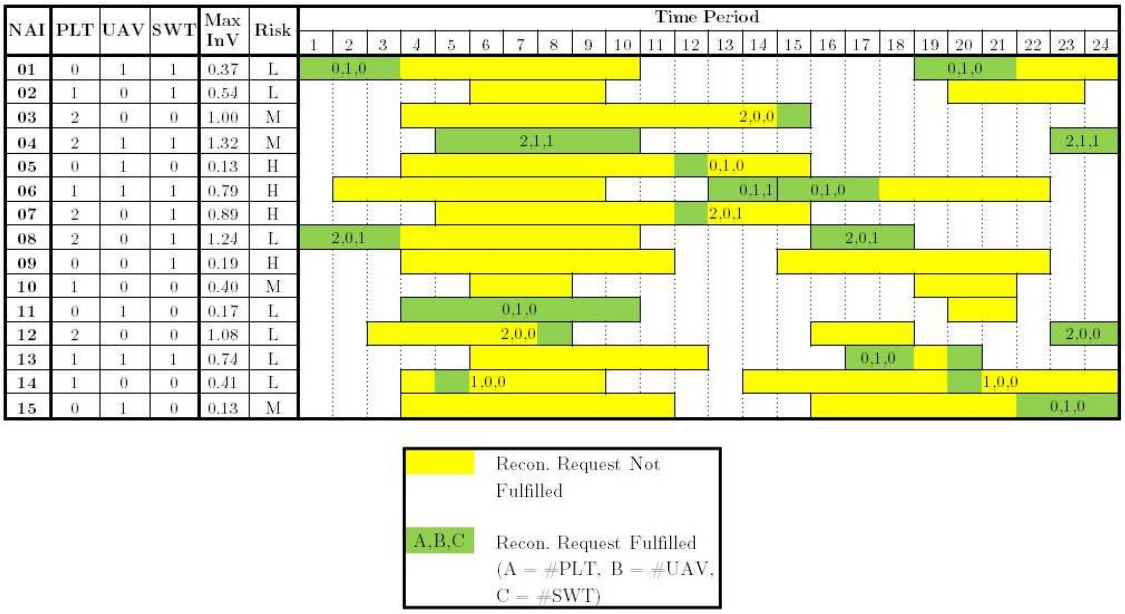


Figure 6-4: Test Scenario High Priority Algorithm Solution



NAI	PLT	UAV	SWT	Max InV	Risk
01	0	1	1	0.37	L
02	1	0	1	0.54	L
03	2	0	0	1.00	M
04	2	1	1	1.32	M
05	0	1	0	0.13	H
06	1	1	1	0.79	H
07	2	0	1	0.89	H
08	2	0	1	1.24	L
09	0	0	1	0.19	H
10	1	0	0	0.40	M
11	0	1	0	0.17	L
12	2	0	0	1.08	L
13	1	1	1	0.74	L
14	1	0	0	0.41	L
15	0	1	0	0.13	M

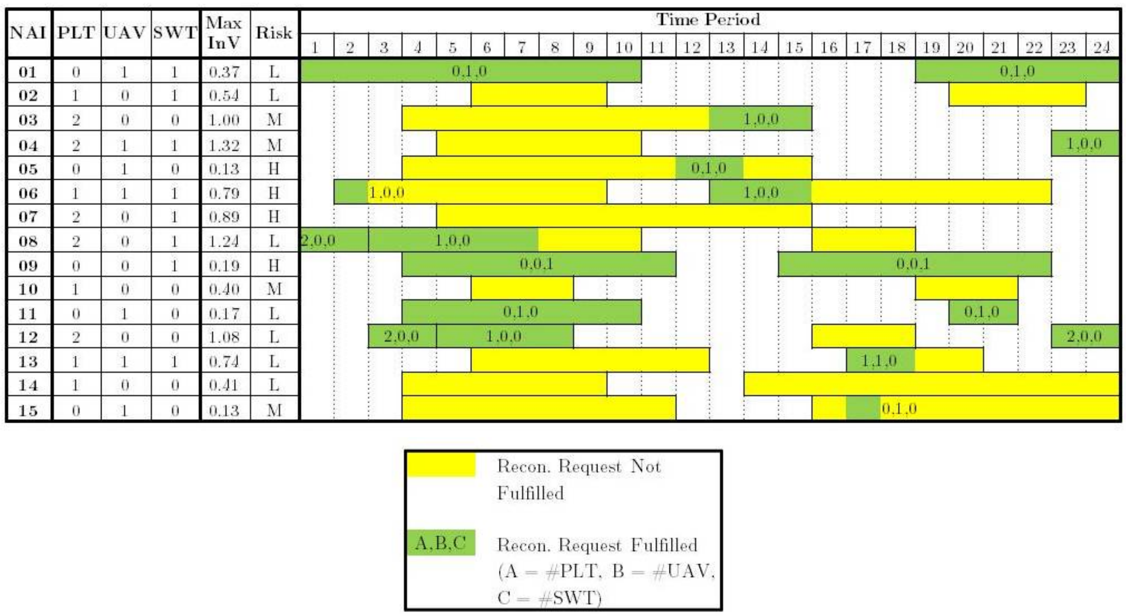
After studying these results we conclude that the High Priority algorithm performs poorly due to its failure to fully account for the level of risk of conducting reconnaissance in each NAI. The algorithm selects locations to conduct reconnaissance solely on the expected information value returned. It only accounts for the risk level of an NAI when determining how many time periods it can assign assets to conduct reconnaissance in it. By using risk in a secondary manner such as this it leaves the algorithm susceptible to assigning assets to a relatively high risk mission where the expected information value return may not necessarily be worth the risk invested. This appears to be the case in our test scenario. In Figure 6-4 we see that the algorithm chose to maximize reconnaissance in NAI 04. This intuitively makes sense as NAI 04 provides the highest expected information value return per time period. However, NAI 04 is a medium risk NAI and therefore conducting reconnaissance in NAI 04 comes with a significant risk investment, even with the risk reducing effects of mixing and redundancy. As a result over half of our available assets (2 PLT's, 1 UAV, and 1 SWT) devote a substantial part of their risk budgets to conducting reconnaissance in NAI 04. This limits the availability of these assets for additional missions and reduces the overall information value of the solution.

The deterministic model avoids this problem. By design the model weighs the costs and benefits of conducting reconnaissance in each NAI. Thus, the deterministic model tends to select reconnaissance missions with the most efficient reward to risk ratio. For example, in Figure 6-3 we see that the deterministic model prioritized platoon reconnaissance in NAI 08 and NAI 12. Both of these NAIs are low risk and have high, although not the highest, expected information value per time period. This high information value to risk ratio makes these NAIs an efficient use of our low risk budget assets.

Being efficient in allocating assets does not mean that our deterministic model ignores high or medium risk NAIs. In fact, the deterministic model assigns assets to nearly all high and medium risk NAIs for at least a portion of their requested collection windows. However, when doing so, the model generally assigns assets with higher risk budgets, UAVs and SWTs, to these high and medium risk NAIs. By completely considering the cost/benefit trade-offs of asset assignments the deterministic model is able to maximize the expected information value of the solution.

The High Priority algorithm's higher solution standard deviation compared to the deterministic model is also likely due to the algorithm's emphasis on executing missions with the highest expected information value. In general, NAI's with higher expected information value tended to have more uncertainty in the information value parameter. This reflects the idea that if we highly value a certain piece of information, it is likely that our enemy will go to greater lengths to deny us this information. The enemy's counter-reconnaissance efforts in high priority NAIs present an additional variable that leads to greater uncertainty in the quality of information we obtain. Because the High Priority algorithm stresses execution of missions with high expected information value we would expect that the variability of our realized information value would also be greater.

Figure 6-5: Test Scenario NAI Coverage Algorithm Solution



When considering the NAI Coverage algorithm we conclude that its performance suffers, primarily, due to its failure to exploit the benefits of mixing and redundancy. Figure 6-5 depicts the solution derived using the NAI Coverage algorithm. As the name suggests, we would expect that the NAI Coverage algorithm would assign assets to all, or nearly all, NAIs for at least a portion of their collection windows. In fact, the solution produced by this algorithm provides less total NAI coverage than both the deterministic model and High Priority solutions. This is a result of the algorithm's emphasis on minimizing the number of assets assigned to conduct reconnaissance in each NAI, in other words, its avoidance of asset mixing and redundancy. Mixing and redundancy provide risk reducing benefits, many units working in coordination are all generally exposed to less risk, but come at a cost of devoting a large portion of the available assets to a single NAI, as was seen in our High Priority model. In the case of our test scenario, by avoiding the benefits of mixing and redundancy our assets, especially the low risk budget assets, accumulated risk at a much higher rate than in our other approaches. This resulted in an overall reduction in NAI coverage and a steep loss of mean information value collected.

The deterministic model addresses this issue by selectively using mixing and redundancy to minimize the risk accumulation of the low risk budget assets. In Figure 6-3 we see that in nearly all platoon assignments the platoon is supported by at least one other asset. The only instance where this does not occur is in NAI 14, where only a single platoon is requested. This strategy of supporting low risk budget assets also reduces the risk exposure to the supporting assets, allowing them to conduct further reconnaissance in other NAIs. By inherently protecting low risk budget assets the deterministic model avoids the performance obstacles observed in the NAI Coverage algorithm and maintains a significantly higher mean information value.

6.3.2 Robust Model Performance

Table 6.3 outlines the performance of solutions to the robust model using ellipsoidal uncertainty sets after five thousand simulated realizations of the test scenario. This table displays results when robustness is added to both the information value (Θ) and risk constraint (Ψ) parameters. For comparison we also include the solution performance of our deterministic model and human planning approximation algorithms.

Table 6.3: Robust Model Solution Performance

Model	**Mean**	**Std Dev**	**% Feasible**	Θ	Ψ
High Priority Algorithm	28.5	0.989	60.2%	N/A	N/A
NAI Coverage Algorithm	22.3	0.476	1.8%	N/A	N/A
Deterministic	38.3	0.800	27.1%	N/A	N/A
Robust Model - Ellipsoidal Uncertainty	37.9	0.782	38.0%	0.1	0.1
	37.8	0.782	61.2%	0.2	0.2
	37.5	0.772	89.2%	0.4	0.4
	37.3	0.771	97.8%	0.6	0.6
	37.0	**0.767**	**100.0%**	**1.0**	**1.0**
	34.9	0.741	100.0%	2.0	2.0

The simulation results show that the robust model formulation consistently produces higher quality solutions, in terms of solution feasibility and standard deviation, for a relatively minor cost to expected information value collected when compared to the deterministic model. For example at robustness level of $\Theta/\Psi = 1.0$ we can achieve a 4% reduction in solution standard deviation for a cost in mean total information value of around 3% from the deterministic model. We also improve solution feasibility by a dramatic 73% when applying robustness at this level. Although gains in solution variability are modest in this case the most significant improvements in solution performance occur in, arguably, the most important performance measure, solution feasibility.

Our robust model also decisively outperforms both of our human planning approximation algorithms. For example, the robust model can achieve similar solution feasibility performance to the High Priority model (around 60%) but at a fraction of the cost in expected information value while also still maintaining a significantly lower standard deviation. Although, in the cases we tested, we were unable to match the NAI Coverage solution's standard deviation using our robust models, it is unlikely that an operational commander would be willing to give up over 42% of his expected information value and assume a significant risk of reconnaissance plan infeasibility to achieve these improvements; especially when one considers that solution standard deviation is a less significant, in an operational context, performance measure.

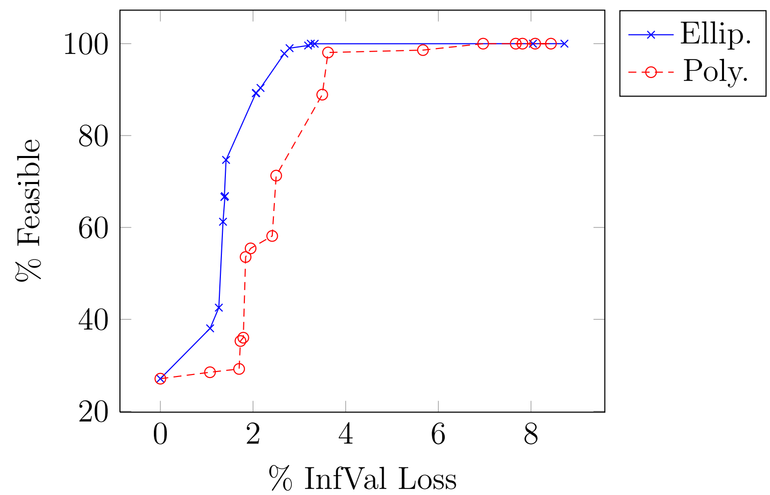
These results show that by using our robust model during planning a commander will considerably improve his reconnaissance plan solution quality compared to typical human generated plans. Additionally, our results imply that by using our robust models a commander can trade a small reduction in expected intelligence information to dramatically reduce the risk exposure of his soldiers compared to our deterministic model.

6.3.3 Ellipsoidal vs. Polyhedral Uncertainty

It is evident that our robust model using ellipsoidal uncertainty sets provides improved solutions over our deterministic model. We now compare the solution performance of our robust model using ellipsoidal uncertainty sets with the solution performance of our robust model using polyhedral uncertainty sets. Figures 6-6, 6-7, and 6-8 depict the performance measures of both robust models as a function of the relative cost of robustness, in terms of average percent total information value loss, compared to the non-robust model. All figures depict solution performance when robustness is added to both the information value and risk parameters.

Figure 6-6 depicts the solution feasibility performance of both robust models. In our scenario the model using ellipsoidal uncertainty sets strictly outperforms the model using polyhedral uncertainty sets in this measure. For instance, the ellipsoidal uncertainty model achieves near 100% solution feasibility (greater than 97.5%) at the cost of around 2.7% in expected total information value while the polyhedral uncertainty model achieves this mark at the cost of nearly 3.6% in expected total

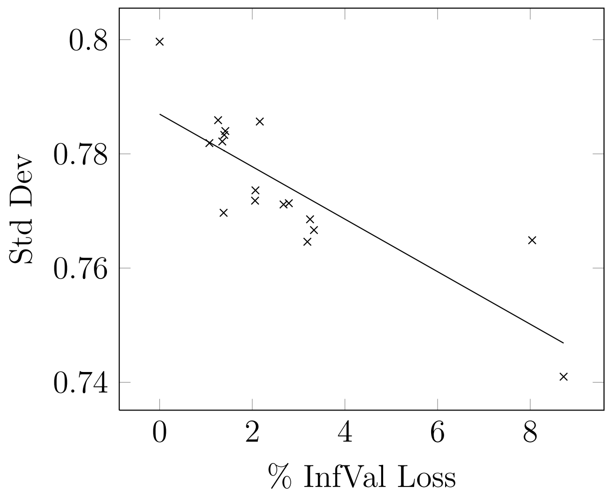
Figure 6-6: Robust Model Solution Feasibility



information value. The ellipsoidal uncertainty model outperforms the polyhedral uncertainty model in this manner at all levels of solution feasibility performance.

Figure 6-7 and Figure 6-8 show the solution standard deviation as a function of expected total information value lost from the non-robust model. We can see that both models reduce solution standard deviation as robustness is added into the model. The trend, depicted by a linear trend line, for both models clearly shows that there is a reduction in solution standard deviation at higher levels of robustness, but predicting how much decrease one will see from one solution to the next is difficult. Regardless, by adding robustness into the model, solution quality in terms of variability will remain similar or improved. The bottom line is that robustness will not hurt solution variability. The inconsistency in solution variability improvement of both models makes it impossible to decisively state which model outperforms the other in this performance measure. Despite this we can state that, in general, both models have an increased mission success rate, meaning a higher rate of reconnaissance assets returning from a mission with useful information, as

Figure 6-7: Robust Solution Std. Deviation - Ellipsoidal Uncertainty



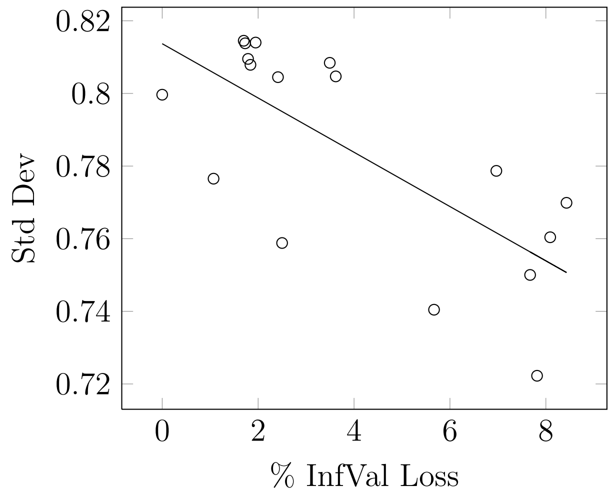
robustness is added into the model.

Based on this analysis we conclude that our model using ellipsoidal uncertainty sets outperforms our model using polyhedral uncertainty sets in terms of solution quality. The solutions produced by the model using ellipsoidal uncertainty sets consistently achieve higher feasibility rates with less cost than the solutions produced by the model using polyhedral uncertainty sets. Both models also reduced solution variability as robustness was added into the model, but in an inconsistent manner. In operational terms these results suggest that reconnaissance plans developed by our model using ellipsoidal uncertainty sets will have greater risk reductions to assets and more consistently successful missions than plans generated by our robust model using polyhedral uncertainty sets, our deterministic model, or human planners.

6.3.4 Solution Quality Under Varying Levels of Uncertainty

When developing our models we made certain assumptions about the amount of uncertainty in the risk and information value parameters. We chose values for the

Figure 6-8: Robust Solution Std. Deviation - Polyhedral Uncertainty



$\hat{f}_{ict}$, $\hat{f}_{nc}$, $\hat{r}_{jic}$, and $\hat{r}_{jnc}$ parameters that we felt represented the half-length of the range in which we expected, with high confidence, that the true values of these parameters would take. Specifically, the $\hat{f}_{ict}$, $\hat{f}_{nc}$, $\hat{r}_{jic}$, and $\hat{r}_{jnc}$ parameters encompass a range that is two standard deviations from the nominal value. Meaning, if we assume a normal distribution for each parameter, we estimate that roughly 95% of the time the true, realized value of our parameters would fall within our chosen intervals. In our simulations we assumed that the values we chose for these parameters were a reasonable representation of reality. However, in a real world operational situation it is possible that these assumptions are inaccurate. For this reason we now present an analysis on the stability of our robust and non-robust solutions as simulated parameter uncertainty is varied.

In this test we first solved our models under a certain assumption of the level of uncertainty in the parameters, we term this “ideal” uncertainty. We then varied the value of uncertainty in our simulation to understand how solution performance was effected. In order to compare our robust model based on polyhedral uncertainty sets and our robust model using ellipsoidal uncertainty sets we tested solutions from each model that had similar performance in simulation, in terms of mean total information value, information value standard deviation, and solution feasibility, under "ideal" uncertainty; meaning the realizations of our uncertain parameters were likely to fall in the range we predicted. We also include the solution performance of our High Priority algorithm, deterministic model, and of our model using ellipsoidal uncertainty sets at robustness level of $\Theta/\Psi = 1.0$ that was highlighted in Section 6.3.2 for comparison. The solution performance for these points under "ideal" conditions are depicted in Table 6.4.

Table 6.4: Performance of Comparable Solutions of Both Robust Models Under "Ideal" Uncertainty

Model	**Mean**	**Std. Dev.**	**% Feasible**	Γ/Θ	Φ/Ψ
Deterministic	38.3	0.800	27.1%	N/A	N/A
High Priority	28.5	0.989	60.2%	N/A	N/A
Robust - Ellip.	37.0	0.767	100.0%	1.0	1.0
Robust - Ellip.	37.9	0.782	38.0%	0.1	0.1
Robust - Poly.	37.6	0.810	36.0%	0.06	0.06
Robust - Ellip.	37.8	0.782	61.2%	0.2	0.2
Robust - Poly.	37.4	0.804	58.1%	0.09	0.09

Figure 6-9 shows the feasibility performance for each of the solutions in Table 6.4 as the level of uncertainty in our parameters is varied during simulation. The x-axis in Figure 6-9 represents the multiplicative factor used in simulation of the uncertain parameter standard deviation we assumed when generating our solutions. Therefore 1 depicts solution performance when the level of uncertainty in our parameters during simulation is "ideal", meaning it equals what we assumed when developing our solutions. The value 2, for example, represents a parameter distribution in simulation with twice the standard deviation than we assumed, and values less than 1 represent less uncertainty in simulation than we originally assumed.

Figure 6-9: Solution Feasibility Under Varying Levels of Uncertainty

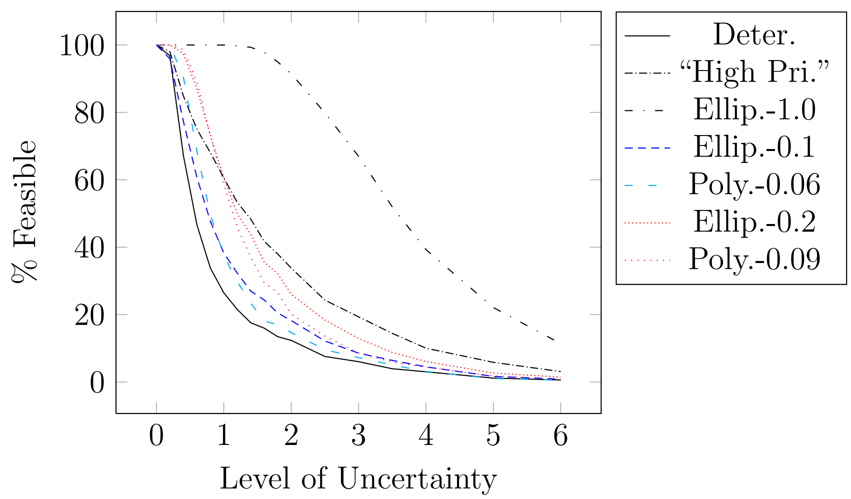


Figure 6-9 clearly shows that our robust solutions provide higher quality solutions than our deterministic model when underestimating the level of uncertainty in our parameters. The results for robustness level of $\Theta/\Psi = 1.0$ are particularly encouraging. For a cost of around 3% in mean information value we can underestimate the uncertainty of our parameters by a factor of three and still achieve a greater than 50% feasibility rate. Our non-robust model does not achieve this level of feasibility even when we know the exact amount of uncertainty in our parameters. When we compare the stability of our two robust model solutions we see that in both cases we tested, the solution derived using ellipsoidal uncertainty maintained a higher level of performance as parameter uncertainty was increased than its counterpart derived using polyhedral uncertainty. Similar, although less striking, performance is seen in solution variability as uncertainty is increased in our simulation.

Also of interest is the performance of the High Priority algorithm solution compared to our deterministic and robust solution performance. The High Priority solution performs slightly better in this metric when compared to our deterministic solution and robust solutions with similar feasibility rates under "ideal" conditions (Ellip.-0.2 and Poly.0.09). However, just as before, when we consider the cost in expected information value to achieve this slight performance advantage we still must conclude that our deterministic and robust models provide higher quality solutions overall. Furthermore, our robust solution with robustness level of $\Theta/\Psi = 1.0$ provides significantly better solution stability than the High Priority algorithm solution at dramatically less cost in information value. This example implies that solutions derived from our models will nearly always outperform human generated solutions.

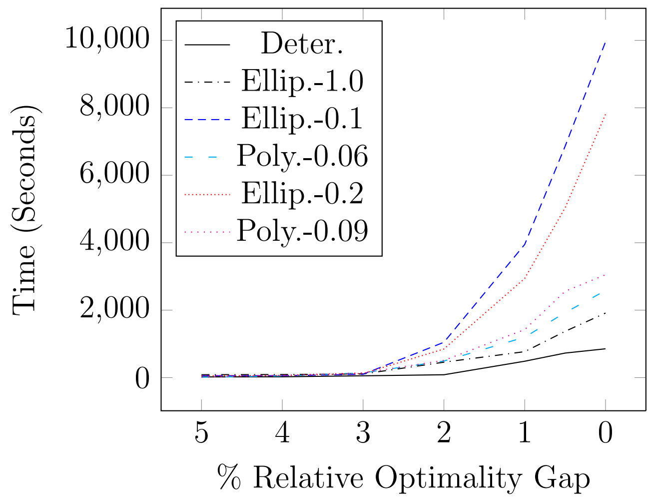
Overall, these results mean that by using our robust models to develop reconnaissance plans military planners can ensure better mission results compared to non-robust and typical human planning methods, even if they seriously misjudge their knowledge of the enemy and the battlefield.

6.3.5 Solution Time Performance

One of the appealing attributes of the robust optimization techniques used in this paper is that they preserve the tractability of our model. This is important as reconnaissance planning is generally conducted in a time constrained environment. If our robust models cannot be solved in a reasonable amount of time they will not be useful in application despite their significant improvement in solution quality. Therefore a comparison of the computational demands of our models is necessary.

Figure 6-10 displays a comparison of solution times for our deterministic and robust models. The data points used for comparison in Section 6.3.4 are used in Figure 6-10 to show the differences in solution progress over time, in seconds, between solutions from our model using ellipsoidal uncertainty sets and from our model using polyhedral uncertainty sets with similar performance. The results depicted in Figure 6-10 were derived by solving our models using Gurobi 5.0.2 on a computer with 8xIntel Core i7-860(2.8GHz) processors and 16GB of memory.

Figure 6-10: Solution Time Comparison



Understandably the deterministic model solves considerably faster than the more complex robust models, especially when solving to proof of optimality. However, when one considers the 3% relative optimality gap solution times we see that the robust models do produce high quality solutions in a reasonable amount of time. In fact, in all cases we tested the solver reached solutions within 5% of optimality in under 300 seconds and in most instances, our robust models reached solutions within 3% of optimality in under 300 seconds. Even better performance, with respect to finding the optimal solution, was achieved when the solver was set to prioritize finding feasible solutions over proving optimality. Between the two robust models the polyhedral model generally reached the optimal solution and proved optimality faster than the ellipsoidal model. However, this advantage was too inconsistent between tests, as can be seen by the robustness level $\Theta/\Psi = 1.0$ results, to provide a definitive explanation of the extent of the polyhedral model's dominance in this area. Despite this, our results show that both the polyhedral or ellipsoidal robust models can generate high quality solutions in a time constrained, operational environment.

We can reduce solution times further by using our fixed allocation model. Table 6.5 illustrates the trade-offs of using the robust fixed allocation model as opposed to our standard robust models. The table shows a selection of robust fixed allocation and robust model solutions that have similar performance in solution variability and feasibility. The column "Solution Time" in the table depicts the time, in seconds, for the solver to prove optimality of the solution. The columns "5% Time" and "3% Time" refer to the seconds required for the solver to achieve a 5% and 3% relative optimality gap, respectively.

Table 6.5: Fixed Allocation Model Solution Time Comparison

Model	**Mean**	**Std. Dev.**	**% Feasible**	**5% Time**	**3% Time**	**Solution Time**
FA Deterministic	34.1	0.762	64.5%	0.2	0.2	0.2
FA Robust - Ellip.	34.1	0.762	91.4%	2	2	2.8
FA Robust - Poly.	34.1	0.762	91.4%	0.2	0.2	0.2
Robust - Ellip.	37.5	0.786	90.4%	101	431	4536
Robust - Poly.	36.9	0.808	88.9%	29	356	2712

In this case both of the robust fixed allocation models reached identical solutions in under three seconds. It is worthwhile to note that solution times to optimality remained under eight seconds for all fixed allocation models tested. The standard robust models took around 45 minutes and over an hour to prove optimality of solutions with similar standard deviations and feasibility to the robust fixed allocation solutions. This impressive reduction in solution time to optimality comes at a cost of expected information collected, as can be seen by the lower mean total information values of the fixed allocation models.

These results do not discount the usefulness of our standard robust models. As can be seen in Figure 6-10 and Table 6.5 high quality solutions (within 5% of optimality) can be achieved within a few minutes using our standard models. We propose both our standard and fixed allocation models in order to provide military planners more flexibility when conducting operations. In an environment where higher quality solutions are more important than producing an immediate plan, a planner can chose to use our standard robust models. If the planner is subject to severe time constraints and optimality is less important, he can still generate multiple high quality plans for consideration using our fixed allocation robust models.

Chapter 7

Conclusions

We have shown that robust optimization using the techniques described in Bertsimas and Sim [9] provides an effective, flexible, and tractable method to model the TRAAP with uncertainty. Most importantly we have demonstrated the ability of our robust models to reduce the risk exposure of soldiers at a minimal cost to expected information value. Although this research is only a small contribution in the area of reconnaissance planning, we feel that it presents a solid case for using robust optimization in future work in this area.

7.1 Summary of Results and Contributions

An overview of our contributions and findings is outlined below.

- We propose a deterministic integer optimization formulation of the Tactical Reconnaissance Asset Allocation Problem (TRAAP). The inputs to the deterministic model can be derived from the outputs of the Military Decision Making Process (MDMP) allowing simpler implementation into the current

Army planning methodology.

- We introduce two algorithms that produce reconnaissance plans representative of a typical plan developed by a human planner under time constraints. The algorithms model the two typical approaches of reconnaissance planners. The first algorithm attempts to focus as many assets as possible on collecting on the NAIs with highest expected information value per time period. The second algorithm seeks to conduct reconnaissance in as many NAIs as possible by minimizing the total number of assets assigned to collect in an NAI.

- We propose two robust extensions to our deterministic model. The first extension models uncertainty using a polyhedral uncertainty set motivated by the central-limit theorem. The second extension models uncertainty using an ellipsoidal uncertainty set based on the Euclidean Norm. The resulting robust formulations are mixed integer linear and second order cone problems, respectively.

- We propose special cases of our deterministic and robust formulations that prohibit assets from conducting transitions during NAI collection windows. We then show that our Fixed Allocation models drastically improve solution times while still producing quality solutions.

- We show that both solutions derived using our deterministic and robust models significantly outperform solutions generated using our human planner approximation algorithms. The human approximation algorithms provide marginal improvement in certain performance measures, but at a prohibitively larger cost in expected information value collected.

- We show that our robust models consistently produce higher quality solutions than our deterministic model. Our results imply that by using our robust models a commander can trade a small reduction in expected intelligence information to dramatically reduce the risk exposure of his soldiers compared to our deterministic and human approximation models. This is true even if planners seriously underestimate the level of uncertainty of their parameters.

- We show that our robust model using ellipsoidal uncertainty outperforms our model using polyhedral uncertainty in terms of solution quality. However, this improved performance comes at a cost of generally longer solution times.

7.2 Future Work

Here we present some recommendations for future work in the area of robust reconnaissance planning.

- *Validate models using data from training centers such as the NTC or from operations in Iraq and Afghanistan.* The US military has developed and executed innumerable reconnaissance plans in both combat situations and training. In many circumstances, especially at training centers such as the NTC, the military has kept detailed records of the effectiveness of these plans after implementation. This information could help validate our models by confirming assumptions and highlighting areas for model improvement. In this thesis our testing was based on a scenario using contrived data. Although we took measures to make this scenario as realistic as possible, only data from real world military operations can truly confirm the utility of our models.

- *Develop methods to more accurately quantify uncertain parameters.* A significant hurdle to implementing our robust models is the difficulty in estimating values for our uncertain parameters. Although we showed that our models still provide quality solutions even if we drastically misjudge our uncertain parameters, solution quality in this case still suffers. Further research using operational data from Iraq and Afghanistan could be done on developing formulas to better guide planners on estimating uncertain parameters in our models.

- *Extend testing to different types of operational scenarios.* Our testing was completed using an operational scenario representing a battalion level security and counterinsurgency operation in an area similar to Afghanistan. Further testing could be done on model performance when using scenarios with varying conditions. For example, how does increased asset transition times in a very large area of operations effect solution quality? Other scenario factors such as number of NAIs, number and type of assets, and length of planning horizon, among others, could all be varied to examine their impact on overall solution performance.

- *Evaluate solution quality when adding and changing scenario uncertainty.* In our scenario we made certain assumptions regarding the type of uncertainty in our models and scenario. Within the scenario, for example, we assumed that all uncertain parameters were normally distributed. Further testing using other symmetric distributions would show the versatility of our robust models. Additionally, we only assumed uncertainty in our information value and risk parameters. Extensions to our robust models incorporating uncertainty in asset availability, weather effects, and unit transition times could enhance the applicability and usefulness of our models.

- *Compare our robust TRAAP models to TRAAP models developed using different robust optimization approaches.* In this thesis we only considered the Bertsimas and Sim approach to robust optimization. This is not the only method to account for uncertainty and a comparison with other approaches, such as those discussed in Section 2.3, could illustrate certain conditions when other methods provide higher quality solutions.

- *Apply a heuristic approach to developing solutions to the TRAAP.* Although heuristics do not account for uncertainty, it still may be possible to develop a heuristic that generates high quality reconnaissance plans with some robust characteristics. A heuristic approach could significantly improve solution time and avoid the use of an optimization solver, making operational implementation much easier.

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Appendix A

Abbreviations and Acronyms

Abbreviation/ Acronym	**Term**
AO	Area of Operations
CCIR	Commander's Critical Information Requirements
CDR	Commander
COA	Course of Action
GB	Gigabyte
GHz	Gigahertz
HMMWV	High Mobility Multipurpose Wheeled Vehicle
IEEE	Institute of Electrical and Electronics Engineers
ISR	Intelligence, Surveillance, and Reconnaissance
LP	Linear Program/Linear Programming
MDMP	Military Decision Making Process
MILP	Mixed Integer Linear Program
MMS	Mast Mounted Sight
NAI	Named Area of Interest
NTC	National Training Center
Per	Time Periods
PLT	Platoon
PIR	Priority Intelligence Requirements
SOCP	Second Order Cone Problem
SWT	Scout Weapons Team

Abbreviation/ Acronym	**Term**
TRAAP	Tactical Reconnaissance Asset Allocation Problem
TRADOC	Training and Doctrine Command
UAV	Unmanned Aerial Vehicle
US	United States

Appendix B

Sets, Parameters, and Variables

Type	Symbol	Description	Unit	Min	Max
Sets	J	set of all assets	-	-	-
	K	set of all asset types	-	-	-
	$B \subset K$	set of asset types subject to asset transition penalties	-	-	-
	$L \subset K$	set of asset types subject to endurance limits	-	-	-
	$A_k \subset J$	set of assets of type k	-	-	-
	I	set of all NAIs	-	-	-
	C	set of all asset configurations	-	-	-
	T	set of discrete time intervals	-	-	-
	$T_i \subset T$	set of time intervals when missions in NAI i can be executed	-	-	-
	N	set of all missions	-	-	-
Parameters	a_{kic}	quantity of asset type k in NAI i configuration c	Units	0	2
	f_{ic}	point forecast of the information value per time period when executing mission in NAI i in configuration c	-	0.0	∞

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Type	Symbol	Description	Unit	Min	Max
Parameters	$\hat{f}_{ic}$	half-length of the information value range forecast centered at $\bar{f}_{ic}$	-	0.0	∞
	m_j	risk budget for asset j	-	0.0	∞
	$\bar{r}_{jic}$	point forecast of the accumulated risk per time period of asset j when executing mission in NAI i in configuration c	-	0.0	∞
	$\hat{r}_{jic}$	half-length of the accumulated risk range forecast centered at $\bar{r}_{jic}$	-	0.0	∞
	$e^k_{i_1 i_2}$	time periods required for assets of type k to transition from NAI i_1 to NAI i_2	Hours	1	4
	s_k	maximum consecutive time periods assets of type k can conduct reconnaissance	Hours	2	9
	α	information value asset transition penalty constant	-	0.0	1.0
	β	time periods asset transition penalty is in effect following an asset transition	Hours	0	4
	Γ	budget of uncertainty for information value (Polyhedral Uncertainty)	-	0.0	∞
	Φ_j	budget of uncertainty for risk for asset j (Polyhedral Uncertainty)	-	0.0	∞
	Θ	budget of uncertainty for information value (Ellipsoidal Uncertainty)	-	0.0	∞
	Ψ_j	budget of uncertainty for risk for asset j (Ellipsoidal Uncertainty)	-	0.0	∞

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Type	**Symbol**	**Description**	**Unit**	**Min**	**Max**
Variables	x_{jict}	Binary: asset j assigned to mission in NAI i in configuration c at time t	-	0	1
	y_{ict}	Binary: mission in NAI i executed in configuration c at time t	-	0	1
	w_{it}	Binary: a mission in NAI i is subject to an asset transition penalty during time t if executed	-	0	1
	p_{ict}	Binary: mission in NAI i executed in configuration c is subject to an asset transition penalty during time t	-	0	1
	ρ_j	dual variable (information value)	-	0	∞
	ϕ_{jic}	dual variable (information value)	-	0	∞
	λ_{jic}	dual variable (information value)	-	$-\infty$	0
	μ	dual variable (risk)	-	$-\infty$	0
	ν_{ic}	dual variable (risk)	-	$-\infty$	0
	γ_{ic}	dual variable (risk)	-	0	∞

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Bibliography

[1] AAI Corporation. RQ-7 Datasheet. 2010. http://www.aaicorp.com/pdfs/Shadow20200.pdf.

[2] Aerovironment. RQ-11B Datasheet. 2010. http://www.avinc.com/downloads/AV_RAVEN-DOM_V10109.pdf.

[3] Bandi, C., Bertsimas, D. Tractable Stochastic Analysis in High Dimensions via Robust Optimization, *Mathematical Programming*. Vol 134, 1, 23-70, 2012.

[4] Ben-Tal, A., Nemirovski, A. Robust Convex Optimization, *Mathematics of Operations Research*. Vol 23, 4, 769-805, 1998.

[5] Ben-Tal, A., Nemirovski, A. Robust Solutions of Uncertain Linear Programs, *Operations Research Letters*. Vol 25, 1, 1-13, 1999.

[6] Ben-Tal, A., Nemirovski, A. Robust solutions of linear programming problems contaminated with uncertain data, *Mathematical Programming*. Vol 88, 411-424, 2000.

[7] Bertsimas, D., Brown, D., Caramanis, C. Theory and applications of Robust Optimization, *Society for Industrial and Applied Mathematics Review*. Vol 53, 3, 464-501, 2011.

[8] Bertsimas, D., Sim, M. Robust Discrete Optimization and Network Flows, *Mathematical Programming, Series B.* Vol 98, 49-71, 2003.

[9] Bertsimas, D., Sim, M. The Price of Robustness, *Operations Research.* Vol 52, 35-53, 2004.

[10] Bertuccelli, L. *Robust planning for heterogeneous UAVs in uncertain environments*, SM Thesis. Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA, 2004.

[11] Bertuccelli, L., Alighabari, M., How, JP. Robust planning for coupled cooperative UAV missions, Proceedings of the IEEE Conference on Decision and Control. Paradise Island, Bahamas, 2917-2922, 2004.

[12] Bryant, C. *Robust Planning for Effects Based Operations*, SM Thesis. Sloan School of Management, Massachusetts Institute of Technology, Cambridge, MA, 2006.

[13] El Ghaoui L., Oustry F., Lebret H. Robust solutions to uncertain semidefinite programs, *Society for Industrial and Applied Mathematics Journal on Optimization.* Vol 9, 33-52, 1998.

[14] Headquarters, National Training Center. NTC Mission and Vision, *www.irwin.army.mil.* 2013. `http://www.irwin.army.mil/CommandGroupUnits/NTCcommand/Pages/NTCmissionvision.aspx`.

[15] Jackson, P., Peacock L., Munson K., Taylor J. *Jane's All the World's Aircraft, 199697.* Coulsdon, UK: Jane's Information Group, 1996.

[16] Patton, G. *War As I Knew It.* Houghton Mifflin Co., Boston, MA, 1947.

[17] Sakamoto, P. *UAV Mission Planning Under Uncertainty*, MS Thesis. Sloan School of Management, Massachusetts Institute of Technology, Cambridge, MA, 2006.

[18] Soyster, A. Convex programming with set-inclusive constraints and applications to inexact linear programming. *Operations Research*. Vol 21, 1154-1157, 1973.

[19] United States. The Army Capstone Concept: TRADOC Pam 525-3-0. Ft. Monroe, VA: Headquarters, Training and Doctrine Command, Department of the Army, 2009.

[20] United States. Composite Risk Management: Field Manual 5-19. Washington, DC: Headquarters, Department of the Army, 2006.

[21] United States. Information Collection: Field Manual 3-55. Washington, DC: Headquarters, Department of the Army, 2012.

[22] United States. ISR Synchronization: Field Manual 2-01. Washington, DC: Headquarters, Department of the Army, 2009.

[23] United States. The Operations Process: Army Doctrine Reference Publication 5-0. Washington, DC: Headquarters, Department of the Army, 2012.

[24] United States. Tactics: Field Manual 3-90. Washington, DC: Headquarters, Department of the Army, 2001.