A Study of Solitary Plasma Rings in Axisymmetric Plasma Configurations

by

Tenzin Rabga

Submitted to the Department of Physics in partial fulfillment of the requirements for the degree of

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Abstract

In this thesis, we search for the plasma and field configurations that can exist under stationary conditions around a collapsed object such as a black hole. Regimes where the iso rotational condition corresponding to negligible magnetic field diffusion have been considered. Under the basic assumptions made in this analysis, we find axisymmetric radially localized solitary plasma configurations. We identify the constraint that restricts separability of solutions in the radial and vertical directions. Taking different limits of the ratio Δ_r^2/Δ_z^2 we find plasma configurations with a solitary or a pair of rings. Considering the restrictions imposed by the constraint equation and the basic assumptions we suggest problems for further investigation.

Thesis Supervisor: Bruno Coppi
Title: Professor of Physics

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Finally, I thank my family and friends who have constantly encouraged me to taking up the challenge of pursuing an undergraduate education in physics. Their moral support has been immense throughout the four years here at MIT.

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Chapter 1

Introduction

For a currentness plasma disk surrounding a compact object, a spectrum of characteristic modes has been identified (Pringe, 1981), in which both gravitation and toroidal rotation play a key role. The modes that can be excited (Coppi, 2009) are standing in the vertical direction and are found to be axisymmetric as well as the spiral type. The relevant driving factors are the plasma differential rotation and the vertical temperature gradient along the density gradient. These "thermo-rotational" modes satisfy the 'frozen-in-law' $\vec{E} + \vec{V} \times \vec{B}/c = 0$, that is they cannot be excited in the resistive plasmas disks where the magnetic field diffusion is significant.

Therefore we can argue that simple currentless plasma disks when immersed in a vertical seed magnetic field will evolve toward different stationary plasma and field configurations. This justifies the efforts made in the papers (Coppi and Rousseau,2006; Coppi, 2011) to identify these configurations. For simplicity we deal with an axisymmetric configurations.

In this thesis, we show that under certain limitations solitary plasma rings can form, and the vertical temperature profile and the gradient have a significant influence on their properties. Dealing with simple axisymmetric currentless plasma disks we determine the conditions necessary for obtaining solutions that are separable in the radial and the vertical directions. The conditions are indeed non-trivial and have significant effects on the plasma configurations that we obtain. In Chapter 2, we outline the relevant form of the equations that govern the evolution of these plasma configurations. We start with a discussion of the basic assumptions made in this simplistic analysis. Then we derive the **Master Equation** (Coppi, 2011), that relates the plasma density profile to the magnetic surface function. In Chapter 3 we use the vertical equilibrium equation to obtain the plasma pressure and temperature profiles. This allows us to check that the solutions of the **Master Equation** is consistent with the realistic pressure and temperature profiles. In Chapter 4 we outline the constraints on separability of solutions in the z and $r - R$ variables. We identify three different conditions that are later used in analyzing solutions of the **Master Equation** at different limits. In Chapter 5 we obtain plasma configurations in two important limits: $\Delta_r^2/\Delta_z^2 \ll 1$ and $\Delta_r^2/\Delta_z^2 < 1$. Moreover we note the collapse of ring pairs as this ratio is increased above the threshold value of 2/3. Chapter 6 summarizes and discusses the analysis of these solitary plasma configurations suggesting problems for further investigation.

Chapter 2

The Master Equation

We begin our study of the magnetic surface and plasma density profiles by obtaining a convenient form of the total momentum conservation equation. As we will see later in this chapter, this allows us to relate the plasma density ρ to the magnetic surface ψ. However before we get to that we would like to clearly state a set of basic assumptions that we make throughout this discussion.

2.1 Basic Assumptions:

Since we deal with the simplest of plasma and field configurations that can, at least theoretically, exist around around compact collapsed objects with strong gravitational fields, such as black holes, we limit our analysis to axisymmetric geometries and make the following assumptions [3]. Given the axisymmetric geometry it is suggestive to use cylindrical coordinates (r, ϕ, z).

(a) We consider perfectly conducting plasma conditions. This consequently gives

$$\vec{V} = \alpha_V \vec{B} + \Omega(\psi) r \, \vec{e}_\phi \tag{2.1}$$

where $\vec{V}$ is the plasma flow velocity and $\psi = \psi(r, z)$ is the magnetic surface function.

(b) We take $\alpha_V \simeq 0$ as no appreciable poloidal flow velocity is included in the theory, and

$$\vec{V} \simeq \Omega(\psi) r\, \vec{e}_\phi \tag{2.2}$$

(c) The relevant particle distributions in the phase space are predominantly Maxwellian and for the purposes of this case we refer to a scalar pressure $\mathbf{P} = p\mathbf{I}$.

(d) The relevant Lorentz force does not have any toroidal component and the corresponding magnetic field configurations are given as a function of the magnetic surface function by

$$\vec{B} = \frac{1}{r}[\vec{\nabla}\psi \times \vec{e}_\phi + I(\psi)\vec{e}_\phi] \tag{2.3}$$

where $I(\psi)$ is the toroidal plasma current. In this case, the Lorentz force $\vec{F}_L$ is simply given by

$$\vec{F}_L = \frac{1}{4\pi r^2}\left(\Delta_*\psi + I\frac{dI}{d\psi}\right)\vec{\nabla}\psi \tag{2.4}$$

where

$$\Delta_*\psi \equiv \frac{\partial^2\psi}{\partial z^2} + r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\psi\right) \tag{2.5}$$

(e) For simplicity we include a Newtonian potential Φ_G. In particular, for the relatively thin plasma structures that we analyze we have

$$\vec{\nabla}\Phi_G \simeq -\frac{V_K^2}{r}\left(\vec{e}_r + \frac{z}{r}\vec{e}_z\right) \tag{2.6}$$

where

$$V_K^2 \equiv \frac{GM}{r} \equiv \Omega_K^2 r^2 \tag{2.7}$$

and Ω_K is the Keplerian angular frequency.

2.2 Deriving the Master Equation

We begin by analyzing the total momentum conservation equation

$$-\rho(\vec{\nabla}\Phi_G + \Omega^2 r\vec{e}_r) = -\vec{\nabla}p + \frac{1}{c}\vec{J}\times\vec{B} \tag{2.8}$$

where ρ is the plasma density. Taking the curl of both sides we observe that

$$\vec{\nabla} \times (\rho\vec{\nabla}\Phi_G + \rho\Omega^2 r\vec{e}_r) = \vec{\nabla} \times \left[\left(\frac{\partial\Phi_G}{\partial r} + \Omega^2 r\right)\vec{e}_r + \left(\frac{\partial\Phi_G}{\partial z}\right)\vec{e}_z\right]$$

$$= \left\{\frac{\partial}{\partial z}\left(\frac{\partial\Phi_G}{\partial r} + \Omega^2 r\right) - \frac{\partial}{\partial r}\left(\frac{\partial\Phi_G}{\partial z}\right)\right\}\vec{e}_\phi \tag{2.9}$$

$$= \left\{\left(\frac{\partial\Phi_G}{\partial r} + \Omega^2 r\right)\frac{\partial\rho}{\partial z} + \rho r 2\Omega\frac{\partial\Omega}{\partial z} - \frac{\partial\rho}{\partial r}\frac{\partial\Phi_G}{\partial z}\right\}\vec{e}_\phi$$

and

$$-\vec{\nabla} \times \left(-\vec{\nabla}p + \frac{1}{c}\vec{J} \times \vec{B}\right) = -\vec{\nabla} \times \left(\frac{1}{c}\vec{J} \times \vec{B}\right)$$

$$-\vec{\nabla} \times \left(\frac{1}{c}\vec{J} \times \vec{B}\right) = -\vec{\nabla} \times \left(\frac{1}{4\pi}(\vec{\nabla} \times \vec{B}) \times \vec{B}\right) \tag{2.10}$$

$$= \frac{1}{4\pi}\vec{\nabla} \times (\vec{B} \cdot \vec{\nabla}B)$$

where we use $\vec{J} = (\vec{\nabla} \times \vec{B})c/4\pi$. This can be further reduced by expressing the magnetic field in terms of the magnetic surface function using equation (2.3). We get

$$-\vec{\nabla} \times \left(\frac{1}{c}\vec{J} \times \vec{B}\right) = \frac{1}{4\pi r^2}\left[-\frac{2}{r}\left(\Delta_*\psi + I\frac{dI}{d\psi}\right)\vec{e}_r + \vec{\nabla}(\Delta_*\psi)\right] \times \vec{\nabla}\psi$$

$$= \frac{1}{4\pi r^2}\left[\left\{-\frac{2}{r}\left(\Delta_*\psi + I\frac{dI}{d\psi}\right) - \frac{\partial}{\partial r}(\Delta_*\psi)\right\}\frac{\partial\psi}{\partial z} + \left\{\frac{\partial}{\partial z}(\Delta_*\psi)\right\}\frac{\partial\psi}{\partial r}\right]\vec{e}_\phi \tag{2.11}$$

We can further simplify equation (2.9) by expanding Φ_G in z^2/R^2 and $\Omega \simeq \Omega_K + \delta\Omega$. This allows us to write the **Master Equation** that relates the magnetic surface function ψ to the plasma density ρ.

$$2\Omega_K r\frac{\partial}{\partial z}(\rho\delta\Omega) + z\Omega_K^2\left(\frac{\partial\rho}{\partial r} + \frac{3}{2}\frac{z}{r}\frac{\partial\rho}{\partial z}\right) \simeq \frac{1}{4\pi r^2}$$

$$\left[\left\{-\frac{2}{r}\left(\Delta_*\psi + I\frac{dI}{d\psi}\right) - \frac{\partial}{\partial r}(\Delta_*\psi)\right\}\frac{\partial\psi}{\partial z} + \left\{\frac{\partial}{\partial z}(\Delta_*\psi)\right\}\frac{\partial\psi}{\partial r}\right] \tag{2.12}$$

We now consider a localized plasma in an interval $|r - R| < R$ around $r - R$ and we introduce characteristic scale distances over which $r - R$ and z vary by Δ_r and Δ_z respectively. This reduces the **Master Equation** to

$$2\Omega_K R\frac{\partial}{\partial z}(\rho\delta\Omega) + z\Omega_K^2\frac{\partial\rho}{\partial r} \simeq \frac{1}{4\pi R^2}\left\{\left[\left(\frac{\partial^3}{\partial z^3}\psi\right)\frac{\partial\psi}{\partial r} - \left(\frac{\partial^3}{\partial r^3}\psi\right)\frac{\partial\psi}{\partial z}\right]\right\}$$

$$+\frac{1}{4\pi R^2}\left\{\left[\left(\frac{\partial}{\partial z}\frac{\partial^2}{\partial r^2}\psi\right)\frac{\partial\psi}{\partial r} + \left(\frac{\partial}{\partial z}\frac{\partial^2}{\partial z^2}\psi\right)\frac{\partial\psi}{\partial r}\right]\right\} \tag{2.13}$$

We have $\rho = \rho(r_*, \bar{z}^2)$, $\psi = \psi(r_*, \bar{z}^2)$ for $r_* \equiv (r - R)/\Delta_r$, and $\bar{z} \equiv z/\Delta_z$ and $\Delta_r^2 < \Delta_z^2 << R^2$. In particular, we consider ρ to be a positive even function of both r_* and $\bar{z}$. This implies that $\delta\Omega$ and ψ are odd functions of r_* and even functions of $\bar{z}$. For $\psi \simeq \psi_0 + \psi_1$ where $\psi_0 \equiv B_0 R r$, $|\psi_1| < \psi_0$ and B_0 is the seed magnetic field. This gives

$$\delta\Omega = \left(\frac{d\Omega_K}{dr}/\frac{d\psi_0}{dr}\right)\psi_1 \tag{2.14}$$

Furthermore if we consider the asymptotic limits for which $R\Delta_r > \Delta_z^2 > \Delta_r^2$, the **Master Equation** reduces to

$$-\Omega_D^2 R\frac{\partial}{\partial z}\left(\rho\frac{\psi_1}{\psi_0}\right) \simeq \frac{1}{4\pi R^2}\left\{\left[\left(\frac{\partial^3}{\partial z^3}\psi_1\right)\frac{\partial\psi_1}{\partial r} - \left(\frac{\partial^3}{\partial r^3}\psi_1\right)\frac{\partial\psi_1}{\partial z}\right]\right\}$$

$$+\frac{1}{4\pi R^2}\left\{\left[\left(\frac{\partial}{\partial z}\frac{\partial^2}{\partial r^2}\psi_1\right)\frac{\partial\psi}{\partial r} + \left(\frac{\partial}{\partial z}\frac{\partial^2}{\partial z^2}\psi_1\right)\frac{\partial\psi_1}{\partial r}\right]\right\} \tag{2.15}$$

where

$$\Omega_D^2 = -R\frac{d}{dr}\Omega_k^2 \tag{2.16}$$

is the differential rotation. In fact, $\rho\Omega_D^2$ is the driving factor for the field configurations given by ψ_1. Another important feature being, the scale distance Δ_z does not affect equation (2.15) in the limit $\Delta_r^2/\Delta_z^2 \ll 1$. (Coppi and Rousseau, 2006).

Chapter 3

Plasma Pressure and Temperature Profiles

We begin by extracting the vertical equilibrium equation from the total momentum conservation equation (2.8). As we will see in this chapter, this allows us to determine the expressions for the plasma temperature and pressure profiles. It is an important exercise to analyze the behavior of these profiles and check that they are realistic.

3.1 Gravitational Pressure p_G and Temperature T_G

The vertical equilibrium equation that involves the plasma pressure p confined both by the gravity and the plasma currents can be written as

$$\frac{\partial p}{\partial z} = -\Omega_K^2 z \rho + F_{Lz} \tag{3.1}$$

where $\vec{F}_L$ is the Lorentz force. Now the corresponding plasma temperature can be related to the plasma pressure and density. The plasma temperature is given by

$$T = \frac{p}{\rho}\frac{m_p}{2} \tag{3.2}$$

where m_p is the average mass of the particles which constitute the plasma. The electron and ion temperatures are considered equal. Then p can be separated into $p = p_G + p_L$. This allows us to write

$$2\frac{\partial p}{\partial z^2} = -\Omega_K^2 z\rho \tag{3.3}$$

This is the gravitational confinement of the relevant plasma structure. Then we write

$$p_G = 2\frac{T_G}{m_p}\rho \tag{3.4}$$

For

$$\Delta_z^2 \equiv \frac{2T_G^0}{m_p\Omega_K^2}, \quad \frac{T_G(\bar{z}^2, r_*)}{T_G^0} \equiv \frac{1}{C_0(\bar{z}^2, r_*)} \equiv \hat{T} \tag{3.5}$$

and

$$p_G = p_{G*}(r_*) \exp\left[-\frac{1}{2\Delta_z^2}\int_0^{\bar{z}^2} C_0(r_*, \bar{z'}^2)\, d\bar{z'}^2\right] \tag{3.6}$$

we obtain

$$\rho = \rho_*(r_*)C_0(\bar{z}^2, r_*) \exp\left[-\frac{1}{2\Delta_z^2}\int_0^{\bar{z}^2} C_0(r_*, \bar{z'}^2)\, d\bar{z'}^2\right] \tag{3.7}$$

where T_G^0 is reference value for the T_G (gravitational) contribution to the temperature. Simply from the above expressions we notice that p_G and ρ are separable in r_* and $\bar{z}^2$ only if $C_0(r_*, z'^2)$ is independent of r_*. For simplicity we assume that $(\partial T_G/\partial r_* = 0)$. However we have to verify that this is compatible with the **Master Equation** on imposing the co-rotation condition

$$\frac{\delta\Omega}{\Omega_K} \propto \psi \tag{3.8}$$

This allows us to write the magnetic surface function as

$$\psi = \psi_*(r_*) f_0\left(\frac{dC_0}{d\bar{z}^2}\right) \exp\left[-\frac{1}{2}\int_0^{\bar{z}} C_0(\bar{z'}^2)\, d\bar{z'}^2\right] \tag{3.9}$$

where $f_0(dC_0/d\bar{z}^2 = 0) = 1$ allows for separable magnetic surface functions. We will further elaborate on the separability of solutions in the next chapter. In fact we shall see that relatively simple solutions maintaining the separability of the dependence of ψ on r_* and $\bar{z}^2$ for inhomogenous T_G temperature profiles, $(dT_G/dz^2 \neq 0)$ can not be easily found.

3.2 Lorentz Pressure p_L and Temperature T_L

Now we focus on the other half of the plasma confinement pressure; the contribution from the Lorentz force. We note that

$$\frac{\partial p_L}{\partial z} = \frac{1}{c}\left(J_r B_\phi - J_\phi B_r\right)$$

$$= -\frac{1}{4\pi r^2}\frac{\partial \psi}{\partial z}\left(\Delta_* \psi + I\frac{dI}{d\psi}\right) \tag{3.10}$$

$$\simeq -\frac{1}{8\pi R^2}\left\{\frac{\partial}{\partial \bar{z}}\left[I^2 + \frac{1}{\Delta_z^2}\left(\frac{\partial \psi}{\partial \bar{z}}\right)^2\right] + \frac{1}{\Delta_r^2}\frac{\partial \psi}{\partial \bar{z}}\frac{\partial^2 \psi}{\partial r_*^2}\right\}$$

This can be integrated to get

$$p_L \simeq -\frac{1}{8\pi R^2}\left[I^2 + \frac{1}{\Delta_z^2}\left(\frac{\partial \psi}{\partial \bar{z}}\right)^2 + \frac{1}{\Delta_r^2}\psi\frac{\partial^2 \psi}{\partial r_*^2}\right] + p_L^0(r_*) \tag{3.11}$$

Now an important criterion for selecting acceptable solutions of the **Master Equation** would be the positive and finite temperature condition. We require that the total temperature $T = T_G + T_L = m_p(p_G + p_L)(2\rho)$ to be both positive and finite.

Chapter 4

Separable Solutions of the Master Equation

As mentioned in the previous chapter, separation of variables in the expressions for ρ and ψ is not always possible. In fact as we shall see in this chapter, there is a specific constraint condition that has to be met for there to be such separable solutions of the **Master Equation**.

If we adopt the expressions (3.7) and (3.9) for ρ and ψ, respectively, and define

$$\psi_* \equiv \psi_N \bar{\psi}_*(r_*)$$

$$\rho_* \equiv \rho_N \bar{\rho}_*(r_*) \tag{4.1}$$

$$\Delta_r \equiv \left(\frac{\psi_N \psi_0}{8\pi \rho_N R^6 \Omega_D^2} \right)^{1/3}$$

where $\bar{\psi}_*$ and $\bar{\rho}_*$ are dimensionless and $\bar{\psi}_* \sim \bar{\rho}_* \sim 1$.

Plugging these expressions into the **Master Equation** gives

$$(\bar{\rho}_*\bar{\psi}_*)\left(f_0' - C_0 f_0 + \frac{f_0}{C_0}C_0'\right) \simeq \left[\frac{d^3\bar{\psi}_*}{dr_*^3}\bar{\psi}_* - \frac{d^2\bar{\psi}_*}{dr_*^2}\frac{d\bar{\psi}_*}{dr_*} + \frac{\Delta_r^2}{\Delta_z^2}\left(\frac{d}{dr_*}\bar{\psi}_*^2\right)\right]$$
$$[f_0(2f_0' - C_0 f_0)] + \frac{\Delta_r^2}{\Delta_z^2}\left(\frac{d}{dr_*}\bar{\psi}_*^2\right)Q(f_0, C_0) \tag{4.2}$$

where the primes denote partial derivative with respect to $\bar{z}^2$, $f' \equiv \partial f/\partial \bar{z}^2$ and

$$Q \equiv 2f_0' + (f_0^2)'(2C_0 - 1) - 6f_0 f_0'' + 3f_0 C_0'$$
$$+ 2\bar{z}^2\left[(f_0'^2)' + 2C_0(f_0 f_0'' - f_0'^2) + (f_0^2)'C_0'' - \frac{1}{2}f_0^2(C_0^2)' - 2f_0''' f_0 + C_0'' f_0^2\right] \tag{4.3}$$

All the z dependence are carried by the functions f_0 and C_0 and their derivatives. Clearly, $Q = 0$ for $f_0' = 0$ and $C_0' = 0$. We then observe that a separable solution ψ of the form as represented in (3.9) is compatible with the **Master Equation** under the following circumstances:

(a) if $dC_0/d\bar{z}^2 = 0$ and $f_0 = 1$, corresponding to $T_G =$ constant.
The **Master Equation** in this case reduces to

$$\bar{\rho}_*\bar{\psi}_* \simeq \frac{d}{dr_*}\left[\frac{d^2\bar{\psi}_*}{dr_*^2}\bar{\psi}_* - \left(\frac{d\bar{\psi}_*}{dr_*}\right)^2 + \frac{\Delta_r^2}{\Delta_z^2}\bar{\psi}_*^2\right] \tag{4.4}$$

(b) for $f_0(\bar{z}^2) = C_0(\bar{z}^2)$ and assuming $C_0(\bar{z}^2)$ profiles that are solutions of $Q = 0$, besides $C_0' = 0$, can be found. In this case we can use the same expression for the **Master Equation** as determined in (a).

(c) for all temperature T_G profiles ($dC_0/d\bar{z}^2 \neq 0$) in the limit where $\Delta_r^2/\Delta_z^2 \ll 1$ and can be neglected, provided that $f_0 = C_0$.

The **Master Equation** in this case reduces to

$$\bar{\rho}_* \bar{\psi}_* \simeq \frac{d}{dr_*} \left[\frac{d^2 \bar{\psi}_*}{dr_*^2} \bar{\psi}_* - \left(\frac{d\bar{\psi}_*}{dr_*} \right)^2 \right] \tag{4.5}$$

Chapter 5

Solitary Ring Solutions

We now identify solutions to the relevant form of the **Master Equation** as solitary plasma rings. We attempt to find separable solutions keeping in mind the constraints outlined in the previous chapter. We eventually find solutions that correspond to solitary rings and determine the respective density and magnetic surface function profiles.

5.1 The $\Delta_r^2/\Delta_z^2 \ll 1$ Limit:

Considering the difficulty of finding separable solutions when Δ_r^2/Δ_z^2 becomes significant and when T_G is in homogenous over Δ_z, we begin by focussing on the case where Δ_r^2/Δ_z^2 is very small.

In this limit we have

$$\rho = \rho_N \bar{\rho} C_0 \exp\left(-\frac{1}{2}\int_0^{\bar{z}^2} C_0' d\bar{z}'^2\right) \tag{5.1}$$

while

$$\psi_1 = \psi_N \frac{r_*}{\sqrt{1+r_*^2}} C_0 \exp\left(-\frac{1}{2}\int_0^{\bar{z}^2} C_0' d\bar{z}'^2\right) \tag{5.2}$$

with $\bar{\psi}_* = r_*/(1+r_*^2)^{1/2}$, we compute the following derivatives

$$\frac{d\bar{\psi}_*}{dr_*} = \frac{1}{(1+r_*^2)^{3/2}}$$

$$\frac{d^2\bar{\psi}_*}{dr_*^2} = -\frac{3r_*}{(1+r_*^2)^{5/2}} \tag{5.3}$$

$$\frac{d^3\bar{\psi}_*}{dr_*^3} = \frac{12r_*^2 - 3}{(1+r_*^2)^{7/2}}$$

Plugging these into the relevant form of the **Master Equation**, we get

$$\bar{\rho}_*\bar{\psi}_* = \frac{d}{dr_*}\left[\frac{d^2\bar{\psi}_*}{dr_*^2}\bar{\psi}_* - \left(\frac{d\bar{\psi}_*}{dr_*}\right)^2\right]$$

$$= \frac{12r_*^3}{(1+r_*^2)^4} \tag{5.4}$$

$$\bar{\rho}_* = \frac{12r_*^2}{(1+r_*^2)^{7/2}}$$

Hence we obtain a pair of radially localized rings. This is as shown in figure 5-1.

5.2 $\Delta_r^2/\Delta_z^2 < 1$:

We now focus on the effect of finite ratios $\Delta_r^2/\Delta_z^2 < 1$ taking into account that separable solutions can be found for $T_G =$ constant. We begin with an expression of ψ_1 that has characteristics in common with the expression in equation (5.2), given by

$$\bar{\psi}_* = r_* e^{-\frac{1}{2}r_*^2} \tag{5.5}$$

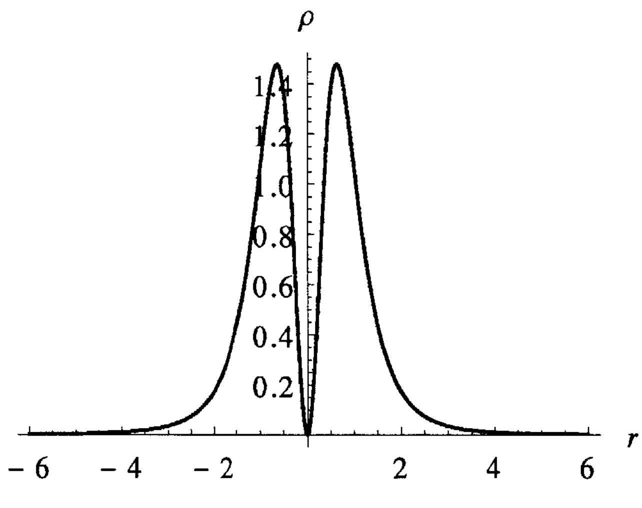

Figure 5-1: Plot of $\bar{\rho}_*$ as a function of r_*. We observe 2 radially localized rings

In order to include the effects of finite Δ_r^2/Δ_z^2, we take $C_0 = 1$. Solving the relevant form of the **Master Equation**

$$\bar{\rho}_* \bar{\psi}_* \simeq \frac{d}{dr_*}\left[\frac{d^2\bar{\psi}_*}{dr_*^2}\bar{\psi}_* - \left(\frac{d\bar{\psi}_*}{dr_*}\right)^2 + \frac{\Delta_r^2}{\Delta_z^2}\bar{\psi}_*^2\right] \tag{5.6}$$

we obtain

$$\bar{\rho}_* = 2\left[\frac{\Delta_r^2}{\Delta_z^2} + r_*^2\left(1 - \frac{\Delta_r^2}{\Delta_z^2}\right)\right] e^{-\frac{1}{2}r_*^2} \tag{5.7}$$

requiring that $\Delta_r^2/\Delta_z^2 \leq 1$. We then determine the total magnetic surface function to be

$$\psi = \psi_1(r_*, \bar{z}) + r_*\frac{\Delta_r}{R}\psi_0 = \psi_N\left[\bar{\psi}_* e^{-\frac{1}{2}\bar{z}^2} + r_*\frac{\Delta_r}{R}\frac{\psi_0}{\psi_N}\right] \tag{5.8}$$

considering $\psi_N/\Delta_r > \psi_0/R$, we can represent the relevant magnetic surfaces by

$$r_* e^{-\frac{1}{2}(r_*^2+\bar{z}^2)} + \epsilon r_* = \text{constant} \tag{5.9}$$

where $\epsilon = (\psi_0\Delta_r/\psi_N R) < 1$.

The relevant magnetic surface and the density profile are as shown in figure 5-2.

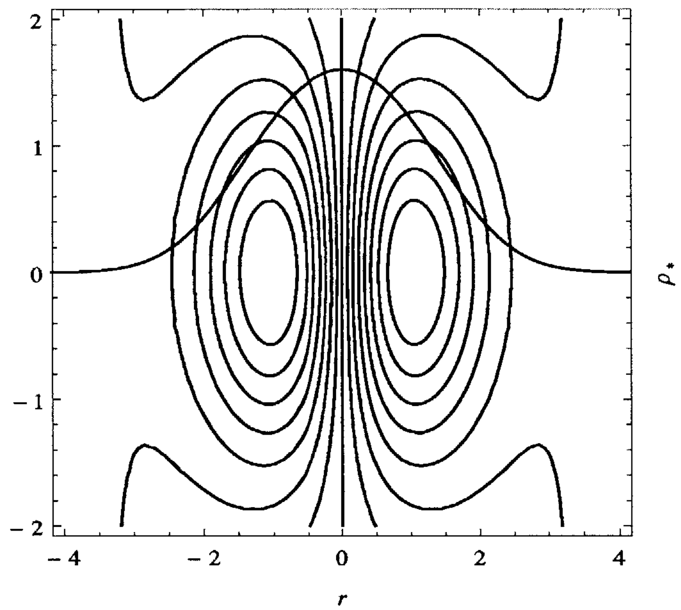

Figure 5-2: The contour plot shows the magnetic surface function and the single radial ring is the density profile as a function of the radius for $\Delta_r^2/\Delta_z^2 = 4/5$ and $\epsilon = 1/20$.

We note that density profile in (5.7) corresponds to a single ring when

$$\frac{2}{3} < \frac{\Delta_r^2}{\Delta_z^2} \leq 1 \tag{5.10}$$

We observe a pair of rings in the $\Delta_r^2/\Delta_z^2 \ll 1$ limit as shown in figure 5-1. Thus we may also argue that the pair of rings collapse into one ring as the ratio Δ_r^2/Δ_z^2 is increased. Figure 5-3 shows the collapse of the pair of rings as the ratio Δ_r^2/Δ_z^2 is raised above the threshold 2/3.

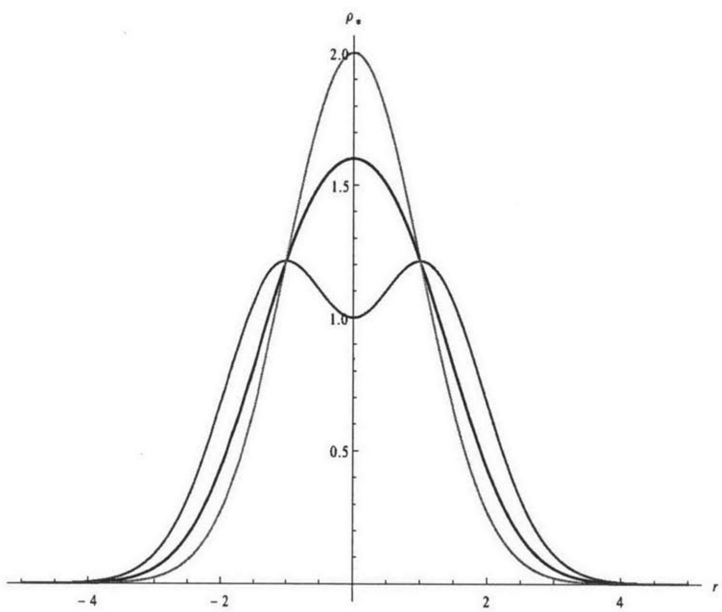

Figure 5-3: Plot of density profiles for different Δ_r^2/Δ_z^2 ratios. $\bar{\rho}_*(r_* = 0)$ values of 2, 1.6 and 1 correspond to Δ_r^2/Δ_z^2 ratios of 1, 4/5 and 1/2, respectively.

Chapter 6

Conclusions

In this thesis we discuss and analyze the plasma configurations surrounding a compact object. Working under the basic assumptions outlined in Chapter 1 we find the relevant form of the **Master Equation**. We also find the restriction on finding separable solutions that limit our ability to analytically solve for the plasma configurations. We note the possibility of solitary plasma configurations that correspond to plasma ring(s). This is nonetheless a very simplistic treatment of the plasma configurations surrounding a compact object and the results obtained provide ample avenue for further work. The following highlights possible questions for further investigation.

One problem for further consideration concerns the stability of these plasma configurations and their evolutions into different axisymmetric or tridimensional configurations. In fact, the experimental observation of the so called Quasi Periodic Oscillators of X-ray emission, in one of the radiation emission regimes associated with galactic black holes (Remillard and McClintock, 2006), indicates the need to include non-axisymmetric configurations into the evolution of the relevant plasma structures (Coppi and Rebusco, 2008).

Considering the axisymmetric configurations, we note certain restrictions on the separability of solutions in the $r - R$ and z variables, as shown in Chapter 4. This suggests the need for computational efforts in studying these non-separable solutions (Regev and Umurhan, 2008).

And finally, one of the key assumptions made in this analysis is the Maxwellian distribution of the phase space. This is a serious limitation considering the fact that there are important radiation regimes associated with compact objects such as galactic black holes, have been experimentally observed to be non-thermal (Remillard and McClintock, 2003). This suggests the need to investigate non-thermal phase space distributions.

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