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Output-feedback proportional-integral-derivative-type control with multiple saturating structure for the global stabilization of robot manipulators with bounded inputs

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Abstract

An output-feedback proportional integral derivative-type control scheme for the global regulation of robot manipulators with constrained inputs is proposed. It guarantees the global stabilization objective—avoiding input saturation—releasing the feedback not only from the exact knowledge of the system structure and parameter values but also from velocity measurements. With respect to previous approaches of the kind, the proposed controller is expressed in a generalized form whence multiple saturating structures may be adopted, thus enlarging the degree of design flexibility. Furthermore, experimental tests on a two-degree-of-freedom direct-drive manipulator corroborate the efficiency of the proposed scheme.

Keywords

Output feedback, PID control, global stabilization, robot manipulators, bounded inputs, saturation

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Introduction

The classical proportional-integral-derivative (PID) control law has been frequently implemented in industrial manipulators.[1–3] This is mainly due to the practical certainty on the achievement of the regulation goal experienced through its simple linear structure.[4] A simple structure that avoids involving the system model and exact knowledge of the system parameters.[2] Nevertheless, through the classical PID linear structure, it has not been possible to derive a global proof of the closed-loop stability properties experimentally observed. This is why alternative nonlinear versions of the PID controller, mainly oriented to guarantee global regulation, have been developed for instance in Arimoto,[5] Kelly[6] and Santibáñez and Kelly.[7] However, these algorithms implicitly assume that actuators can furnish any required torque value. Unfortunately, this is impossible in practice in view of the saturation nonlinearity that generally relates the controller outputs to the plant inputs in actual feedback systems. Furthermore, disregarding such natural constraints may lead

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to unexpected behaviors and/or degraded closed-loop performances.[8,9] For this reason, bounded PID-type approaches have been further developed. For instance, semiglobal regulators with different saturating PID-type structures have been proposed in a frictionless setting by Alvarez-Ramrez J and colleagues.[10,11] The closed-loop analysis in these works was carried out using singular perturbation methodology. Through such a methodology, the authors show the existence of some suitable tuning, mainly characterized by the requirement of small enough integral action gains and sufficiently high proportional and derivative ones. Furthermore, the first globally stabilizing bounded PID-type control scheme, that the authors are aware of, was previously achieved by Gorez.[12] The control algorithm developed therein was carried out through the explicit consideration of friction forces on the system open-loop dynamics. The resulting algorithm gives the alternative to include or disregard velocities in the feedback. Nevertheless, the approach presented by Gorez[12] is quite complex. This inspired other researchers to find alternative bounded PID-type structures. Such efforts gave rise, for instance, to the Saturating-Proportional Saturating-Integral Saturating-Derivative (SP-SI-SD) type algorithm. This was developed by Meza et al.[13] via passivity theory and later on by Su et al.[14] through Lyapunov stability analysis. A Saturating-Proportional-Derivative Saturating-Integral (SPD-SI) type controller was further proposed by Santibáñez et al.[15] More recently, a state-feedback PID-type scheme with a generalized saturating structure, that includes both the SP-SI-SD and SPD-SI as particular cases, was presented by Mendoza et al.[16] In particular, Su et al.'s work[14] includes a velocity-free version of the presented controller by involving the dirty derivative. Further concerns on the bounded input problem have led to the additional consideration of the saturation effects of the electronic control devices of practical PID regulators.[17,18] Exponential and/or global asymptotic stabilization conditions were obtained under such natural restrictions for several implementation structures that are common in industrial robots.

The above-cited bounded PID-type approaches give a solution to the formulated problem under input constraints and restricted data. In this direction, output-feedback schemes, like the velocity-free extensions of the algorithms presented by Gorez[12] and Su et al.,[14] are particularly important. This is so since they achieve regulation not only without the need for the exact knowledge of the system structure and parameter values but also through the exclusive feedback of the position variables. This proves to be particularly useful when velocity measurements are unavailable, which seems a common practical situation. However, how dirty-derivative-based output-feedback PID-type bounded schemes with alternative saturating structures (different to the SP-SI-SD one) can be designed and analytically supported is not yet clear. Although this has already been treated in the state-feedback context,[16] it remains an unsolved problem within the dirty-derivative-based output-feedback framework, where the analytical complication is considerably higher. A solution to such an open problem is not only motivated by the implicated analytical challenge but also by the nice performance expectations generated by analog saturating structures in *gravity-compensation*-type state-feedback contexts.[19] These arguments actually constitute the main motivation of the present work, which aims at giving a formal solution to the referred unsolved problem. As a result, an output-feedback PID-type control scheme with generalized saturating structure for the global stabilization of robot manipulators with bounded inputs is contributed here. With respect to previous approaches of the kind, it increases the degree of design flexibility through its generalized form that permits the implementation of multiple saturating structures. The proposed scheme finds potential applications in numerous types of autonomous robot systems, saving these from undesirable behaviors due to actuator saturation, releasing them from the need for speed sensors, and opening new control design possibilities to improve their closed-loop behavior. The result is developed through formal analysis based on Lyapunov stability theory. Furthermore, experimental tests on a two-degree-of-freedom (DOF) direct-drive manipulator support the analytical developments.

Preliminaries

Let $X \in \mathbb{R}^{m\times n}$ and $y \in \mathbb{R}^n$. Throughout this article, X_{ij} stands for the element of X at its i th row and j th column and y_i represents the i th element of y. 0_n denotes the origin of $\mathbb{R}^n$ and I_n the $n \times n$ identity matrix. $\|\cdot\|$ represents the standard Euclidean norm for vectors, that is, $\|y\| = \sqrt{\sum_{i=1}^n y_i^2}$, and induced norm for matrices, that is, $\|X\| = \sqrt{\lambda_{\max}\{X^{\mathrm{T}}X\}}$ where $\lambda_{\max}\{X^{\mathrm{T}}X\}$ is the maximum eigenvalue of $X^{\mathrm{T}}X$. For a continuous scalar function $\psi : \mathbb{R} \to \mathbb{R}$, ψ' represents its derivative, when differentiable, $D^+\psi$ its upper right-hand (Dini) derivative, that is, $D^+\psi(\varsigma) = \lim\sup_{h\to 0^+} \frac{\psi(\varsigma+h)-\psi(\varsigma)}{h}$, with $D^+\psi = \psi'$ at points of differentiability (Appendix C.2 in Khalil[20]), and ψ^{-1} its inverse, when invertible. Consider the n-DOF serial rigid robot manipulator dynamics with viscous friction [21]

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau \tag{1}$$

where q, $\dot{q}$, $\ddot{q} \in \mathbb{R}^n$ are, respectively, the position (generalized coordinates), velocity and acceleration vectors. $H(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, which is a continuously differentiable symmetric matrix function being positive definite and bounded on the whole configuration space, that is

$$\mu_m I_n \leq H(q) \leq \mu_M I_n \tag{2}$$

$\forall q \in \mathbb{R}^n$, for some constants $0 < \mu_m \leq \mu_M$. $C(q,\dot{q})\dot{q}$ is the Coriolis and centrifugal (generalized) force vector, with $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ (the Coriolis matrix) satisfying

$$\|C(q,\dot{q})\| \leq k_C\|\dot{q}\| \tag{3}$$

$\forall (q,\dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$, for some constant $k_C \geq 0$,

$$\dot{q}^{\mathrm{T}}\left[\frac{1}{2}\dot{H}(q,\dot{q}) - C(q,\dot{q})\right]\dot{q} = 0 \tag{4}$$

$\forall(q,\dot q)\in\mathbb{R}^n\times\mathbb{R}^n$, where $\dot H$ denotes the rate of change of H, that is, $\dot H:\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^{n\times n}$ with $\dot H_{ij}(q,\dot q)=\frac{\partial H_{ij}}{\partial q}(q)\dot q$, $i,j=1,\ldots,n$, and actually

$$\dot H(q,\dot q)=C(q,\dot q)+C^{\mathrm{T}}(q,\dot q) \tag{5}$$

$\forall(q,\dot q)\in\mathbb{R}^n\times\mathbb{R}^n$. $F\dot q$ is the viscous friction force vector, with $F\in\mathbb{R}^{n\times n}$ being a positive definite constant diagonal matrix whose entries $f_i>0$, $i=1,\ldots,n$, are the viscous friction coefficients, such that

$$f_m\|\dot q\|^2\le\dot q^{\mathrm{T}}F\dot q\le f_M\|\dot q\|^2 \tag{6}$$

$\forall\dot q\in\mathbb{R}^n$, where $0<f_m\triangleq\min_i\{f_i\}\le\max_i\{f_i\}\triangleq f_M$. $g(q)=\nabla U(q)$ is the gravity force vector, with $U(q)$ being the gravitational potential energy, or equivalently

$$\mathcal{U}(q)=\mathcal{U}(q_0)+\int_{q_0}^{q}g^{\mathrm{T}}(r)dr \tag{7a}$$

$$\begin{aligned}\int_{q_0}^{q}g^{\mathrm{T}}(r)dr=&\int_{q_{01}}^{q_1}g_1(r_1,q_{02},\ldots,q_{0n})dr_1\\&+\int_{q_{02}}^{q_2}g_2(q_1,r_2,q_{03},\ldots,q_{0n})dr_2\\&+\cdots+\int_{q_{0n}}^{q_n}g_n(q_1,\ldots,q_{n-1},r_n)dr_n\end{aligned} \tag{7b}$$

for any $q,q_0\in\mathbb{R}^n$. From the conservative character of $g(q)=\nabla\mathcal{U}(q)$, for any $q,q_0\in\mathbb{R}^n$, the inverse relation $\mathcal{U}(q)=\mathcal{U}(q_0)+\int_{q_0}^{q}g^{\mathrm{T}}(r)dr$ is independent of the integration path (Khalil et al., p. 120).[20] Equation (7b) considers integration along the axes. This way, on every axis (that is at every integral in the right-hand side of (7b)), the corresponding coordinate varies (according to the specified integral limits) while the rest of the coordinates remain constant.

This work is addressed to robots whose gravity force term $g(q)$ is a continuously differentiable bounded vector function with bounded Jacobian matrix $\frac{\partial g}{\partial q}$. Equivalently, manipulators whose gravity force vector components, $g_i(q)$, $i=1,\ldots,n$, satisfy

$$|g_i(q)|\le B_{gi} \tag{8}$$

$\forall q\in\mathbb{R}^n$, for some positive constant B_{gi}, and $\frac{\partial g_i}{\partial q_j}$, $j=1,\ldots,n$, exist and are continuous and such that

$$\left|\frac{\partial g_i}{\partial q_j}(q)\right|\le\left\|\frac{\partial g}{\partial q}(q)\right\|\le k_g \tag{9}$$

$\forall q\in\mathbb{R}^n$, for some positive constant k_g, and consequently $|g_i(x)-g_i(y)|\le\|g(x)-g(y)\|\le k_g\|x-y\|$, $\forall x,y\in\mathbb{R}^n$. This is satisfied, for instance, by robot manipulators having only revolute joints (Kelly et al.,[21] § 4.3).

Finally, $\tau\in\mathbb{R}^n$ is the external input force vector, whose elements τ_i, $i=1,\ldots,n$, are assumed in this work to be constrained by a given saturation bound $T_i>0$, that is, $|\tau_i|\le T_i$, $i=1,\ldots,n$. More precisely, letting u_i represent the control variable (controller output) relative to the i th DOF, we have that

$$\tau_i=T_i\,\mathrm{sat}(u_i/T_i) \tag{10}$$

where $\mathrm{sat}(\cdot)$ is the standard saturation function, that is, $\mathrm{sat}(\varsigma)=\mathrm{sign}(\varsigma)\min\{|\varsigma|,1\}$. From equations (1) and (10), one sees that $T_i\ge B_{gi}$ (see (8)), $\forall i\in\{1,\ldots,n\}$, is a necessary condition for the robot manipulator to be stabilizable at any desired equilibrium configuration $q_d\in\mathbb{R}^n$. Thus, the following assumption turns out to be important within the analytical setting considered here.

Assumption 1. *$T_i>\alpha B_{gi}$, $\forall i\in\{1,\ldots,n\}$, for some scalar $\alpha\ge1$.*

The control scheme proposed in this work involves functions fulfilling the following definition.

Definition 1. Given a positive constant M, a nondecreasing Lipschitz-continuous function $\sigma:\mathbb{R}\to\mathbb{R}$ is said to be a generalized saturation with bound M if

(a) $\varsigma\sigma(\varsigma)>0$, $\forall\varsigma\ne0$
(b) $|\sigma(\varsigma)|\le M$, $\forall\varsigma\in\mathbb{R}$

If in addition

(c) $\sigma(\varsigma)=\varsigma$ when $|\varsigma|\le L$

for some positive constant $L\le M$, σ is said to be a *linear saturation* for (L,M).

Lemma 1. *Let $\sigma:\mathbb{R}\to\mathbb{R}$ be a generalized saturation with bound M and let k be a positive constant. Then,*

1. $\lim_{|\varsigma|\to\infty}D^+\sigma(\varsigma)=0$;
2. $\exists\sigma'_M\in(0,\infty)$ such that $0\le D^+\sigma(\varsigma)\le\sigma'_M$, $\forall\varsigma\in\mathbb{R}$;
3. $\varsigma[\sigma(\varsigma+\eta)-\sigma(\eta)]\ge0$, $\forall\varsigma,\eta\in\mathbb{R}$;
4. $[\sigma(k\varsigma+\eta)-\sigma(\eta)]^2\le\sigma'_Mk\varsigma[\sigma(k\varsigma+\eta)-\sigma(\eta)]\le(\sigma'_Mk\varsigma)^2$, $\forall\varsigma,\eta\in\mathbb{R}$;
5. $\frac{\sigma^2(k\varsigma)}{2k\sigma'_M}\le\int_0^{\varsigma}\sigma(kr)dr\le\frac{k\sigma'_M\varsigma^2}{2}$, $\forall\varsigma\in\mathbb{R}$;
6. $\int_0^{\varsigma}\sigma(kr)dr>0$, $\forall\varsigma\ne0$;
7. $\int_0^{\varsigma}\sigma(kr)dr\to\infty$ as $|\varsigma|\to\infty$;
8. if σ is strictly increasing then, for any constant $a\in\mathbb{R}$, $\bar\sigma(\varsigma)=\sigma(\varsigma+a)-\sigma(a)$ is a strictly increasing generalized saturation function with bound $\bar M=M+|\sigma(a)|$.

Proof. The proof of items 1, 2, 5–8 is found in López-Araujo et al.,[22] while items 3 and 4 are proven in Zavala-Rió et al.[23] □

The proposed control scheme

Consider the following *generalized* output-feedback bounded PID-type control law

$$u(q,\vartheta,\phi)=-s_P(K_P\bar q)+s_I(K_I\phi)-s_d(\bar q,\vartheta,\phi) \tag{11}$$

where $\bar{q} = q - q_d$, for any constant desired equilibrium position vector $q_d \in \mathbb{R}^n$. $\phi, \vartheta \in \mathbb{R}^n$ are the output vector variables of the integral-action dynamics, defined as

$$\dot{\phi}_c = -\varepsilon K_P^{-1} s_P(K_P \bar{q}), \quad \phi = -\bar{q} + \phi_c \tag{12}$$

and the velocity estimation auxiliary subsystem, defined as

$$\dot{\vartheta}_c = -A[\vartheta_c + B\bar{q}], \quad \vartheta = \vartheta_c + B\bar{q} \tag{13}$$

Under time parametrization of the system trajectories, the integral-action dynamics in equations (12) adopts the (equivalent) integral-equation form $\phi(t) = \phi(0) + \bar{q}(0) - \bar{q}(t) - \int_0^t \varepsilon K_P^{-1} s_P(K_P \bar{q}(\varsigma)) d\varsigma$, for any initial vector values $\phi(0), \bar{q}(0) \in \mathbb{R}^n$. For any $x \in \mathbb{R}^n$, $s_P(x) = (\sigma_{P1}(x_1), \ldots, \sigma_{Pn}(x_n))^{\mathrm{T}}$ and $s_I(x) = (\sigma_{I1}(x_1), \ldots, \sigma_{In}(x_n))^{\mathrm{T}}$, with $\sigma_{Pi}(\cdot)$, $i = 1, \ldots, n$, being *linear saturation functions* for (L_{Pi}, M_{Pi}) and $\sigma_{Ii}(\cdot)$, $i = 1, \ldots, n$, being *strictly increasing generalized saturation functions* with bounds M_{Ii}, such that

$$L_{Pi} > 2B_{gi} \quad , \quad M_{Ii} > B_{gi} \tag{14}$$

$i = 1, \ldots, n$. $s_d : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuous vector function satisfying

$$\|s_d(\bar{q}, \vartheta, \phi)\|^2 \leq \kappa \vartheta^{\mathrm{T}} s_d(\bar{q}, \vartheta, \phi) \leq \kappa^2 \|\vartheta\|^2 \tag{15a}$$

$\forall (\bar{q}, \vartheta, \phi) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$, for some positive constant κ, and

$$|u_i(q, \vartheta, \phi)| = |-\sigma_{Pi}(k_{Pi}\bar{q}_i) + \sigma_{Ii}(k_{Ii}\phi_i) - s_{di}(\bar{q}, \vartheta, \phi)| < T_i \tag{15b}$$

$i = 1, \ldots, n$, $\forall (\bar{q}, \vartheta, \phi) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$, for suitable bounds M_{Pi} and M_{Ii} of $\sigma_{Pi}(\cdot)$ and $\sigma_{Ii}(\cdot)$. $K_P, K_I, A, B \in \mathbb{R}^{n \times n}$ are positive definite diagonal matrices—that is, $K_P = \mathrm{diag}[k_{P1}, \ldots, k_{Pn}]$, $K_I = \mathrm{diag}[k_{I1}, \ldots, k_{In}]$, $A = \mathrm{diag}[a_1, \ldots, a_n]$ and $B = \mathrm{diag}[b_1, \ldots, b_n]$ with $k_{Pi} > 0$, $k_{Ii} > 0$, $a_i > 0$, $b_i > 0$, $\forall i = 1, \ldots, n$—such that

$$k_{Pm} \triangleq \min_i \{k_{Pi}\} > k_g \tag{16a}$$

$$\beta_d \triangleq \min_i \left\{ \frac{a_i}{b_i} \right\} > \frac{\kappa}{2f_m} \tag{16b}$$

with κ as defined through (15a). Finally, ε (in equations (12)) is a positive constant satisfying

$$\varepsilon < \varepsilon_M \triangleq \min\{\varepsilon_1, \varepsilon_2, \varepsilon_3\} \tag{17}$$

$$\varepsilon_1 \triangleq \sqrt{\frac{\beta_0 \beta_P \mu_m}{\mu_M^2}} \quad , \quad \varepsilon_2 \triangleq \frac{\beta_0 \beta_d k_{Pm}}{\kappa} \quad ,$$

$$\varepsilon_3 \triangleq \frac{f_m - \frac{\kappa}{2\beta_d}}{\beta_M + \frac{f_M^2}{\beta_0 k_{Pm}}} < \frac{f_m - \frac{\kappa}{2\beta_d}}{\beta_M} \triangleq \varepsilon_4$$

(observe that by inequality (16b): $f_m - \frac{\kappa}{2\beta_d} > 0$), with $\beta_0 \triangleq 1 - \max\left\{ \frac{k_g}{k_{Pm}}, \max_i\left\{ \frac{2B_{gi}}{L_{Pi}} \right\} \right\}$, $\beta_P \triangleq \min_i\left\{ \frac{k_{Pi}}{\sigma'_{PiM}} \right\}$, $\beta_M \triangleq k_C B_P + \mu_M \sigma'_{PM}$, $B_P \triangleq \sqrt{\sum_{i=1}^n \left(\frac{M_{Pi}}{k_{Pi}} \right)^2}$ and $\sigma'_{PM} \triangleq \max_i\{\sigma'_{PiM}\}$ (observe that by inequalities (16a) and (14): $0 < \beta_0 < 1$), σ'_{PiM} being the positive bound of $D^+\sigma_{Pi}(\cdot)$, in accordance with item 2 of Lemma 1, and μ_m, μ_M, k_C, f_m, f_M, B_{gi} and k_g as defined through the system properties expressed by inequalities (2), (3), (6), (8) and (9).

Remark 1. Note that $\dot{q}$ is not involved in any of the expressions in equations (11)–(13). In fact, $\dot{q}$ is estimated online through the auxiliary subsystem in equations (13), driven by $\bar{q}$ as input variable. Its output variable ϑ gives the estimated vector value of $\dot{q}$. As a matter of fact, the auxiliary subsystem in equations (13) gives rise to the so-called dirty derivative of $\bar{q}$. This is the derivative of $\bar{q}$ (or the velocity vector $\dot{q}$) with each of its components going through a first-order low pass filter. This is commonly done in practice to bound the high-frequency gains, giving rise to a causal (approximated) derivative operator.

Remark 2. In order to preserve the main feature of PID-type controllers, the vector function s_d in equation (11) shall not involve any term of the open-loop system dynamics (whether as online or desired compensation) or the exact value of any of its parameters. In general, s_d will include a computed-derivative-action term (acting on the estimated velocity vector) and may involve some form of the proportional and/or the integral ones, as illustrated in Appendix 1.

Remark 3. It is important to note that, depending on the specific choice of the vector function s_d, Assumption 1 may be required to be satisfied with some α strictly greater than unity. This arises as a requirement to guarantee the feasibility of the simultaneous fulfillment of (15b) and inequalities (14). For instance, in the particular control structure cases presented in Appendix 1, such a feasibility is achieved by requiring $\alpha = 3$, as pointed out in Remark 5. A similar condition on the control input bounds has been required by other approaches where input constraints have been considered.[24] In some saturating PID-type schemes from previous references, a similar or analog condition on the control input bounds remains implicit, by requiring corresponding parameters to be high enough to satisfy conditions coming from the stability analysis and simultaneously low enough to fulfill the input-saturation-avoidance inequalities.

Closed-loop analysis

Consider system (1),(10) taking $u = u(q, \vartheta, \phi)$ as defined through equations (11)–(13). Let us define the variable transformation

$$\begin{pmatrix} \bar{q} \\ \vartheta \\ \bar{\phi} \end{pmatrix} = \begin{pmatrix} q - q_d \\ \vartheta_c + B(q - q_d) \\ \phi - \phi^* \end{pmatrix} \tag{18}$$

with $\phi^* = (\phi_1^*, \ldots, \phi_n^*)^{\mathrm{T}}$ such that $s_I(K_I \phi^*) = g(q_d)$, or equivalently $\phi_i^* = \sigma_{Ii}^{-1}(g_i(q_d))/k_{Ii}$, $i = 1, \ldots, n$ (notice that their strictly increasing character renders σ_{Ii} invertible). Observe that the satisfaction of (15b), under the consideration of (10), shows that

$$T_i > |u_i(\bar{q}+q_d, \vartheta, \bar{\phi}+\phi^*)| = |u_i| = |\tau_i| \quad i=1,\ldots,n \quad \forall(\bar{q},\vartheta,\bar{\phi}) \in \mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n \tag{19}$$

Hence, under the consideration of the variable transformation (18), the closed-loop dynamics adopts the (equivalent) form

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = -s_d(\bar{q},\vartheta,\phi) - s_P(K_P\bar{q}) + \bar{s}_I(\bar{\phi}) + g(q_d) \tag{20a}$$

$$\dot{\vartheta} = -A\vartheta + B\dot{q} \tag{20b}$$

$$\dot{\bar{\phi}} = -\dot{q} - \varepsilon K_P^{-1} s_P(K_P\bar{q}) \tag{20c}$$

where $\bar{s}_I(\bar{\phi}) = s_I(K_I\bar{\phi} + K_I\phi^*) - s_I(K_I\phi^*)$. Observe that, by item 1 of Lemma 1, the elements of $\bar{s}_I(\bar{\phi})$, that is, $\bar{\sigma}_{Ii}(\bar{\phi}_i) = \sigma_{Ii}(k_{Ii}\bar{\phi}_i + k_{Ii}\phi_i^*) - \sigma_{Ii}(k_{Ii}\phi_i^*)$, $i=1,\ldots,n$, turn out to be strictly increasing generalized saturation functions.

Proposition 1. *Consider the closed-loop system in equations* (20)*, under the satisfaction of inequalities* (14)*, the conditions on the vector function s_d stated through expressions* (15)*, and Assumption 1 with suitable value of α. Thus, for any positive definite diagonal matrices A, B, K_I and K_P such that inequalities* (16) *are fulfilled, and any ε satisfying inequality* (17)*, global asymptotic stability of the closed-loop trivial solution $(\bar{q},\vartheta,\bar{\phi})(t) \equiv (0_n, 0_n, 0_n)$ is guaranteed with $|\tau_i(t)| = |u_i(t)| < T_i$, $i=1,\ldots,n$, $\forall t \geq 0$.*

Proof. By (19), one sees that, along the system trajectories, $|\tau_i(t)| = |u_i(t)| < T_i$, $\forall t \geq 0$. This proves that, under the proposed scheme, the input saturation values, T_i, are never attained. Now, in order to carry out the stability analysis, the following scalar function is defined

$$V(\bar{q},\dot{q},\vartheta,\bar{\phi}) = \frac{1}{2}\dot{q}^{\mathrm{T}}H(q)\dot{q} + \varepsilon s_P^{\mathrm{T}}(K_P\bar{q})K_P^{-1}H(q)\dot{q} + \mathcal{U}(q) - \mathcal{U}(q_d) - g^{\mathrm{T}}(q_d)\bar{q} + \int_{0_n}^{\bar{q}} s_P^{\mathrm{T}}(K_P r)dr + \int_{0_n}^{\bar{\phi}} \bar{s}_I^{\mathrm{T}}(r)dr + \frac{\kappa}{2}\vartheta^{\mathrm{T}}B^{-1}\vartheta$$

where $\int_{0_n}^{\bar{q}} s_P^{\mathrm{T}}(K_P r)dr = \sum_{i=1}^n \int_0^{\bar{q}_i} \sigma_{Pi}(k_{Pi}r_i)dr_i$, $\int_{0_n}^{\bar{\phi}} \bar{s}_I^{\mathrm{T}}(r)dr = \sum_{i=1}^n \int_0^{\bar{\phi}_i} \bar{\sigma}_{Ii}(r_i)dr_i$ and recall that $\mathcal{U}$ represents the gravitational potential energy. Note, by recalling equations (2), that the defined scalar function can be rewritten as

$$V(\bar{q},\dot{q},\vartheta,\bar{\phi}) = \frac{1}{2}\dot{q}^{\mathrm{T}}H(q)\dot{q} + \varepsilon s_P^{\mathrm{T}}(K_P\bar{q})K_P^{-1}H(q)\dot{q} + \gamma_0\int_{0_n}^{\bar{q}} s_P^{\mathrm{T}}(K_P r)dr + \mathcal{U}_{\gamma_0}^c(\bar{q}) + \int_{0_n}^{\bar{\phi}} \bar{s}_I^{\mathrm{T}}(r)dr + \frac{\kappa}{2}\vartheta^{\mathrm{T}}B^{-1}\vartheta$$

$$\mathcal{U}_{\gamma_0}^c(\bar{q}) = \int_{0_n}^{\bar{q}} [g(r+q_d) - g(q_d) + (1-\gamma_0)s_P(K_P r)]^{\mathrm{T}} dr = \sum_{i=1}^n \int_0^{\bar{q}_i} [\bar{g}_i(r_i) - g_i(q_d) + (1-\gamma_0)\sigma_{Pi}(k_{Pi}r_i)]dr_i$$

$$\begin{aligned}\bar{g}_1(r_1) &= g_1(r_1+q_{d1}, q_{d2}, \ldots, q_{dn})\\ \bar{g}_2(r_2) &= g_2(q_1, r_2+q_{d2}, q_{d3}, \ldots, q_{dn})\\ &\vdots\\ \bar{g}_n(r_n) &= g_n(q_1, q_2, \ldots, q_{n-1}, r_n+q_{dn})\end{aligned}$$

and γ_0 is a constant satisfying

$$\beta_0\frac{\varepsilon^2}{\varepsilon_1^2} < \gamma_0 < \beta_0 \tag{21}$$

(observe, from inequality (17) and the definition of β_0, that $0 < \beta_0\varepsilon^2/\varepsilon_1^2 < \beta_0 < 1$). Under this consideration, $\mathcal{U}_{\gamma_0}^c(\bar{q})$ turns out to be lower-bounded by

$$W_{10}(\bar{q}) = \sum_{i=1}^n w_i^{10}(\bar{q}_i) \tag{22a}$$

$$w_i^{10}(\bar{q}_i) \triangleq \begin{cases} \frac{k_{li}}{2}\bar{q}_i^2 & \text{if } |\bar{q}_i| \leq \bar{q}_i^* \\ k_{li}\bar{q}_i^*\left(|\bar{q}_i| - \frac{\bar{q}_i^*}{2}\right) & \text{if } |\bar{q}_i| > \bar{q}_i^* \end{cases} \tag{22b}$$

with $0 < k_{li} \leq (1-\gamma_0)k_{Pi} - k_g$ and $\bar{q}_i^* = [L_{Pi} - 2B_{gi}/(1-\gamma_0)]/k_{Pi}$ (note that by inequality (21) and the definition of β_0: $0 < (1-\gamma_0)k_{Pi} - k_g$ and $\bar{q}_i^* > 0$); this is proven in Appendix 2 of Mendoza et al.[16] From this, inequality (2) and item 5 of Lemma 1, we have

$$\begin{aligned}V(\bar{q},\dot{q},\vartheta,\bar{\phi}) \geq{}& \frac{\mu_m}{2}\|\dot{q}\|^2 - \varepsilon\mu_M\|K_P^{-1}s_P(K_P\bar{q})\|\|\dot{q}\| + \gamma_0\sum_{i=1}^n \frac{\sigma_{Pi}^2(k_{Pi}\bar{q}_i)}{2k_{Pi}\sigma_{PiM'}} + W_{10}(\bar{q}) + \int_{0_n}^{\bar{\phi}} \bar{s}_I^{\mathrm{T}}(r)dr \\ &+ \frac{\kappa}{2}\vartheta^{\mathrm{T}}B^{-1}\vartheta \geq W_{11}(\bar{q},\dot{q}) + W_{10}(\bar{q}) + \int_{0_n}^{\bar{\phi}} \bar{s}_I^{\mathrm{T}}(r)dr + \frac{\kappa}{2}\vartheta^{\mathrm{T}}B^{-1}\vartheta\end{aligned} \tag{23}$$

$$\begin{aligned}W_{11}(\bar{q},\dot{q}) &= \frac{\mu_m}{2}\|\dot{q}\|^2 - \varepsilon\mu_M\|K_P^{-1}s_P(K_P\bar{q})\|\|\dot{q}\| + \frac{\gamma_0\beta_P}{2}\|K_P^{-1}s_P(K_P\bar{q})\|^2 \\ &= \frac{1}{2}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix}^{\mathrm{T}} Q_{11}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\| \\ \|\dot{q}\|\end{pmatrix}\end{aligned}$$

$$Q_{11} = \begin{pmatrix}\gamma_0\beta_P & -\varepsilon\mu_M \\ -\varepsilon\mu_M & \mu_m\end{pmatrix} = \begin{pmatrix}\gamma_0\beta_P & -\frac{\varepsilon}{\varepsilon_1}\sqrt{\beta_0\beta_P\mu_m} \\ -\frac{\varepsilon}{\varepsilon_1}\sqrt{\beta_0\beta_P\mu_m} & \mu_m\end{pmatrix}$$

By inequality (21), $W_{11}(\bar{q}, \dot{q})$ is positive definite (since with $\varepsilon < \varepsilon_M \leq \varepsilon_1$, in accordance to inequality (17), any γ_0 satisfying (21) renders Q_{11} positive definite) and observe that $W_{11}(0_n, \dot{q}) \to \infty$ as $\|\dot{q}\| \to \infty$. Further, from equations (22), items 6 and 7 of Lemma 1 and the positive-definite and diagonal characters of B, it is clear that the three last terms in the right-hand side of (23) are radially unbounded positive definite functions of $\bar{q}$, $\bar{\phi}$ and ϑ respectively. Thus, $V(\bar{q}, \dot{q}, \vartheta, \bar{\phi})$ is concluded to be positive definite and radially unbounded. Its upper right-hand derivative along the system trajectories, $\dot{V} = D^+V$ (see section 6.1A of Michel et al.[25]), is given by

$$\begin{aligned}\dot{V}(\bar{q}, \dot{q}, \vartheta, \bar{\phi}) &= \dot{q}^{\mathrm{T}}H(q)\ddot{q} + \frac{1}{2}\dot{q}^{\mathrm{T}}\dot{H}(q,\dot{q})\dot{q}\\ &\quad + \varepsilon s_P^{\mathrm{T}}(K_P\bar{q})K_P^{-1}H(q)\ddot{q} + \varepsilon s_P^{\mathrm{T}}(K_P\bar{q})K_P^{-1}\dot{H}(q,\dot{q})\dot{q}\\ &\quad + \varepsilon\dot{q}^{\mathrm{T}}s_P'(K_P\bar{q})H(q)\dot{q} + g^{\mathrm{T}}(q)\dot{q} - g^{\mathrm{T}}(q_d)\dot{q}\\ &\quad + s_P^{\mathrm{T}}(K_P\bar{q})\dot{q} + \bar{s}_I^{\mathrm{T}}(\bar{\phi})\dot{\bar{\phi}} + \kappa\vartheta^{\mathrm{T}}B^{-1}\dot{\vartheta}\\ &= -\dot{q}^{\mathrm{T}}F\dot{q} - \dot{q}^{\mathrm{T}}s_d(\bar{q},\vartheta,\phi) - \varepsilon s_P^{\mathrm{T}}(K_P\bar{q})K_P^{-1}F\dot{q}\\ &\quad - \varepsilon s_P^{\mathrm{T}}(K_P\bar{q})K_P^{-1}s_d(\bar{q},\vartheta,\phi)\\ &\quad - \varepsilon s_P^{\mathrm{T}}(K_P\bar{q})K_P^{-1}[g(q) + s_P(K_P\bar{q}) - g(q_d)]\\ &\quad + \varepsilon\dot{q}^{\mathrm{T}}C(q,\dot{q})K_P^{-1}s_P(K_P\bar{q}) + \varepsilon\dot{q}^{\mathrm{T}}s_P'(K_P\bar{q})H(q)\dot{q}\\ &\quad - \kappa\vartheta^{\mathrm{T}}B^{-1}A\vartheta + \kappa\vartheta^{\mathrm{T}}\dot{q}\end{aligned}$$

where $H(q)\ddot{q}, \dot{\vartheta}$ and $\dot{\bar{\phi}}$ have been replaced by their equivalent expressions from the closed-loop dynamics in equations (20), equations (4)–(5) have been used and $s_P'(K_P\bar{q}) \triangleq \mathrm{diag}[D^+\sigma_{P1}(k_{P1}\bar{q}_1), \ldots, D^+\sigma_{Pn}(k_{Pn}\bar{q}_n)]$. The resulting expression can be rewritten as

$$\begin{aligned}\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) &= -\dot{q}^{\mathrm{T}}F\dot{q} + \dot{q}^{\mathrm{T}}[\kappa\vartheta - s_d(\bar{q},\vartheta,\phi)]\\ &\quad - \varepsilon s_P^{\mathrm{T}}(K_P\bar{q})K_P^{-1}F\dot{q} - \varepsilon s_P^{\mathrm{T}}(K_P\bar{q})K_P^{-1}s_d(\bar{q},\vartheta,\phi)\\ &\quad - \varepsilon\gamma_1 s_P^{\mathrm{T}}(K_P\bar{q})K_P^{-1}K_PK_P^{-1}s_P(K_P\bar{q}) - \varepsilon\mathcal{W}_{\gamma_1}(\bar{q})\\ &\quad + \varepsilon\dot{q}^{\mathrm{T}}C(q,\dot{q})K_P^{-1}s_P(K_P\bar{q}) + \varepsilon\dot{q}^{\mathrm{T}}s_P'(K_P\bar{q})H(q)\dot{q}\\ &\quad - \kappa\vartheta^{\mathrm{T}}B^{-1}A\vartheta\\ \mathcal{W}_{\gamma_1}(\bar{q}) &= s_P^{\mathrm{T}}(K_P\bar{q})K_P^{-1}[(1-\gamma_1)s_P(K_P\bar{q}) + g(q) - g(q_d)]\\ &= \sum_{i=1}^{n}\left[\frac{(1-\gamma_1)}{k_{Pi}}\sigma_{Pi}^2(k_{Pi}\bar{q}_i) + \frac{\sigma_{Pi}(k_{Pi}\bar{q}_i)}{k_{Pi}}[g_i(q) - g_i(q_d)]\right]\end{aligned}$$

with γ_1 being a constant that satisfies

$$\beta_0\left[\max\left\{\frac{\varepsilon}{\varepsilon_2}, \frac{\varepsilon}{\varepsilon_3}\left(\frac{\varepsilon_4 - \varepsilon_3}{\varepsilon_4 - \varepsilon}\right)\right\}\right] < \gamma_1 < \beta_0 \qquad (24)$$

(from inequality (17) and the definition of β_0, one verifies, after simple developments, that $0 < \beta_0[\max\{\varepsilon/\varepsilon_2, \varepsilon(\varepsilon_4 - \varepsilon_3)/[\varepsilon_3(\varepsilon_4 - \varepsilon)]\}] < \beta_0 < 1$; in particular, $\varepsilon\varepsilon_3/\varepsilon_4 < \varepsilon < \varepsilon_3 \Leftrightarrow \varepsilon\varepsilon_3 < \varepsilon\varepsilon_4 < \varepsilon_3\ \varepsilon_4 \Leftrightarrow 0 < \varepsilon(\varepsilon_4 - \varepsilon_3) < \varepsilon_3(\varepsilon_4 - \varepsilon) \Leftrightarrow 0 < \varepsilon(\varepsilon_4 - \varepsilon_3)/[\varepsilon_3(\varepsilon_4 - \varepsilon)] < 1$). Under this consideration, $\mathcal{W}_{\gamma_1}(\bar{q})$ turns out to be lower-bounded by

$$W_{20}(\bar{q}) = \sum_{i=1}^{n} w_i^{20}(\bar{q}_i) \qquad (25a)$$

$$w_i^{20}(\bar{q}_i) = \begin{cases} c_i\bar{q}_i^2 & \text{if } |\bar{q}_i| \leq L_{Pi}/k_{Pi}\\ \dfrac{d_i}{k_{Pi}}(|\sigma_{Pi}(k_{Pi}\bar{q}_i)| - L_{Pi}) + c_i\left(\dfrac{L_{Pi}}{k_{Pi}}\right)^2 & \text{if } |\bar{q}_i| > L_{Pi}/k_{Pi}\end{cases} \qquad (25b)$$

with $d_i = (1-\gamma_1)L_{Pi} - 2B_{gi}$, $c_i = \min\left\{h, \frac{d_ik_{Pi}}{L_{Pi}}\right\}$ and $h = (1-\gamma_1)k_{Pm} - k_g$ (notice, from inequality (24) and the definition of β_0, that $d_i > 0$ and $h > 0$, hence $c_i > 0$). This is proven in Appendix 3 of Mendoza et al.[16]

From this, inequalities (2), (3) and (6), items 2 of Lemma 1 and (b) of Definition 1, and the positive definite character of K_P, we have that

$$\begin{aligned}\dot{V}(\bar{q}, \dot{q}, \vartheta, \bar{\phi}) &\leq -f_m\|\dot{q}\|^2 + \|\dot{q}\|\|\kappa\vartheta - s_d(\bar{q}, \vartheta, \phi)\|\\ &\quad + \varepsilon f_M\|K_P^{-1}s_P(K_P\bar{q})\|\|\dot{q}\|\\ &\quad + \varepsilon\|K_P^{-1}s_P(K_P\bar{q})\|\|s_d(\bar{q}, \vartheta, \phi)\|\\ &\quad - \varepsilon\gamma_1k_{Pm}\|K_P^{-1}s_P(K_P\bar{q})\|^2 - \varepsilon W_{20}(\bar{q})\\ &\quad + \varepsilon k_CB_P\|\dot{q}\|^2 + \varepsilon\mu_M\sigma_{PM}'\|\dot{q}\|^2 - \kappa\beta_d\|\vartheta\|^2\end{aligned}$$

Let us further note that by (16), we have that $\|\kappa\vartheta - s_d(\bar{q}, \vartheta, \phi)\|^2 = [\kappa\vartheta - s_d(\bar{q}, \vartheta, \phi)]^{\mathrm{T}}[\kappa\vartheta - s_d(\bar{q}, \vartheta, \phi)] = \kappa^2\vartheta^{\mathrm{T}}\vartheta - 2\kappa\vartheta^{\mathrm{T}}s_d(\bar{q}, \vartheta, \phi) + s_d^{\mathrm{T}}(\bar{q}, \vartheta, \phi)s_d(\bar{q}, \vartheta, \phi) \leq \kappa^2\|\vartheta\|^2 - \|s_d(\bar{q}, \vartheta, \phi)\|^2 \leq \kappa^2\|\vartheta\|^2$, that is $\|\kappa\vartheta - s_d(\bar{q}, \vartheta, \phi)\| \leq \kappa\|\vartheta\|, \forall(\bar{q}, \vartheta, \phi) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$. From this and inequality (16), we get

$$\begin{aligned}\dot{V}(\bar{q}, \dot{q}, \vartheta, \bar{\phi}) &\leq -f_m\|\dot{q}\|^2 + \kappa\|\dot{q}\|\|\vartheta\|\\ &\quad + \varepsilon f_M\|K_P^{-1}s_P(K_P\bar{q})\|\|\dot{q}\| + \varepsilon\kappa\|K_P^{-1}s_P(K_P\bar{q})\|\|\vartheta\|\\ &\quad - \varepsilon\gamma_1k_{Pm}\|K_P^{-1}s_P(K_P\bar{q})\|^2 + \varepsilon k_CB_P\|\dot{q}\|^2\\ &\quad + \varepsilon\mu_M\sigma_{PM}'\|\dot{q}\|^2 - \kappa\beta_d\|\vartheta\|^2 - \varepsilon W_{20}(\bar{q})\\ &\leq -\varepsilon W_{21}(\bar{q}, \vartheta) - W_{22}(\bar{q}, \dot{q}, \vartheta) - \varepsilon W_{20}(\bar{q})\end{aligned}$$

$$\begin{aligned}W_{21}(\bar{q}, \vartheta) &= \frac{\gamma_1k_{Pm}}{2}\|K_P^{-1}s_P(K_P\bar{q})\|^2 - \kappa\|K_P^{-1}s_P(K_P\bar{q})\|\|\vartheta\|\\ &\quad + \frac{\kappa\beta_d}{2\varepsilon}\|\vartheta\|^2\\ &= \frac{1}{2}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\|\\ \|\vartheta\|\end{pmatrix}^{\mathrm{T}} Q_{21}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\|\\ \|\vartheta\|\end{pmatrix}\end{aligned}$$

$$Q_{21} = \begin{pmatrix}\gamma_1k_{Pm} & -\kappa\\ -\kappa & \dfrac{\kappa\beta_d}{\varepsilon}\end{pmatrix} = \begin{pmatrix}\gamma_1k_{Pm} & -\kappa\\ -\kappa & \dfrac{\kappa^2\varepsilon_2}{\beta_0k_{Pm}\varepsilon}\end{pmatrix}$$

$$\begin{aligned}W_{22}(\bar{q}, \dot{q}, \vartheta) &= \frac{\varepsilon\gamma_1k_{Pm}}{2}\|K_P^{-1}s_P(K_P\bar{q})\|^2\\ &\quad - \varepsilon f_M\|K_P^{-1}s_P(K_P\bar{q})\|\|\dot{q}\| + (f_m - \varepsilon\beta_M)\|\dot{q}\|^2\\ &\quad - \kappa\|\dot{q}\|\|\vartheta\| + \frac{\kappa\beta_d}{2}\|\vartheta\|^2\\ &= \frac{1}{2}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\|\\ \|\dot{q}\|\\ \|\vartheta\|\end{pmatrix}^{\mathrm{T}} Q_{22}\begin{pmatrix}\|K_P^{-1}s_P(K_P\bar{q})\|\\ \|\dot{q}\|\\ \|\vartheta\|\end{pmatrix}\end{aligned}$$

$$
Q_{22} = \begin{pmatrix} \varepsilon\gamma_1 k_{Pm} & -\varepsilon f_M & 0 \\ -\varepsilon f_M & 2(f_m - \varepsilon\beta_M) & -\kappa \\ 0 & -\kappa & \kappa\beta_d \end{pmatrix}
$$

$$
= \begin{pmatrix} \varepsilon\gamma_1 k_{Pm} & -\varepsilon\sqrt{2\beta_0\beta_M k_{Pm}\left(\frac{\varepsilon_4-\varepsilon_3}{\varepsilon_3}\right)} & 0 \\ -\varepsilon\sqrt{2k_{Pm}\beta_M\beta_0\left(\frac{\varepsilon_4-\varepsilon_3}{\varepsilon_3}\right)} & 2\beta_M(\varepsilon_4-\varepsilon)+\frac{\kappa}{\beta_d} & -\kappa \\ 0 & -\kappa & \kappa\beta_d \end{pmatrix}
$$

By inequality (24), $W_{21}(\bar{q},\vartheta)$ and $W_{22}(\bar{q},\dot{q},\vartheta)$ are positive definite (since with $\varepsilon < \varepsilon_M \leq \min\{\varepsilon_2,\varepsilon_3\} < \varepsilon_4$, in accordance to inequality (24), any γ_1 satisfying (30) renders Q_{21} and Q_{22} positive definite). Further, from equations (25), it is clear that W_{20} is a positive definite function of $\bar{q}$. Hence, $\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) \leq 0$ with $\dot{V}(\bar{q},\dot{q},\vartheta,\bar{\phi}) = 0 \Leftrightarrow (\bar{q},\dot{q},\vartheta) = (0_n,0_n,0_n)$. Furthermore, from the closed-loop dynamics in equations (20), we see that $\bar{q}(t)\equiv\dot{q}(t)\equiv\vartheta(t)\equiv 0_n \Leftrightarrow \ddot{q}(t)\equiv 0_n \Leftrightarrow \bar{s}_I(\bar{\phi}(t))\equiv 0_n \Leftrightarrow \bar{\phi}(t)\equiv 0_n$ (at any $(\bar{q},\dot{q},\vartheta,\bar{\phi})$ on $Z = \{(w,x,y,z)\in\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n : w = x = y = 0_n\}$ with $\bar{\phi}\neq 0_n$, the resulting unbalanced force term $\bar{s}_I(\bar{\phi})$ acts on the closed-loop dynamics forcing the system trajectories to leave Z). Therefore, by the invariance theory (Michel et al.[25], §7.2)—more precisely, by Corollary 7.2.1 of Michel et al.[25]—, the closed-loop trivial solution $(\bar{q},\vartheta,\bar{\phi})(t)\equiv(0_n,0_n,0_n)$ is concluded to be globally asymptotically stable, which completes the proof. □

Remark 4. Let us note that the fulfillment of inequalities (14), (16)–(17) is not necessary but only sufficient for the closed-loop analysis to hold. This permits a breach tolerance margin without destabilizing the closed loop.

Experimental results

In order to corroborate the efficiency of the proposed scheme, several real-time control tests were implemented on a two-DOF robot manipulator. The experimental setup, shown in Figure 1, is a two-revolute-joint mechanical arm (on a vertical plane) located at the *Instituto Tecnológico de la Laguna*, Mexico. The robot actuators are direct-drive brushless servomotors operated in torque mode: that is, they act as torque sources and receive an analog voltage as a torque reference signal. Joint positions are obtained using incremental encoders on the motors. In order to get the encoder data and generate reference voltages, the robot includes a motion control board based on a DSP 32-bit floating point microprocessor. The control algorithm is executed at a 2.5 millisecond sampling period on a PC-host computer. Further technical information on this robot, as well as its model and parameter values, can be found in Reyes and Kelly.[26]

For the experimental manipulator, inequalities (2), (3), (6), (8) and (9) are satisfied with $\mu_m = 0.088$ kg m^2, $\mu_M = 2.533$ kg m^2, $k_C = 0.1455$ kg m ^{2}, $f_m = 0.175$ kg m^2/s, $f_M = 2.288$ kg m^2/s, $B_{g1} = 40.29$ N m, $B_{g2} = 1.825$ N m and $k_g = 40.373$ N m/rad. The maximum allowed torques (input saturation bounds) are $T_1 = 150$ N m and $T_2 = 15$ N m for the first and second links, respectively. From these data, one easily corroborates that Assumption 1 is fulfilled with $\alpha = 3$.

Figure 1. Experimental setup: two-DOF robot manipulator.

The proposed scheme in equations (11)–(13) was tested in every one of the forms presented in Appendix 1. That is, in the SP-SI-SD form—with $s_d(\bar{q},\vartheta,\phi) = s_D(K_D\vartheta)$—that is

$$u(q,\vartheta,\phi) = -s_P(K_P\bar{q}) - s_D(K_D\vartheta) + s_I(K_I\phi)$$

with the saturation functions involved in s_D—that is $\sigma_{Di}(\cdot)$, $i = 1,2$—being generalized saturations with bounds M_{Di}, and the involved bound values satisfying

$$M_{Pi} + M_{Di} + M_{Ii} < T_i \tag{26}$$

$i = 1,2$. The SPD-SI form—with $s_d(\bar{q},\vartheta,\phi) = s_P(K_P\bar{q} + K_D\vartheta) - s_P(K_P\bar{q})$—that is

$$u(q,\vartheta,\phi) = -s_P(K_P\bar{q} + K_D\vartheta) + s_I(K_I\phi)$$

with bound values fulfilling

$$M_{Pi} + M_{Ii} < T_i \tag{27}$$

$i = 1,2$. The Saturating-Proportional-Integral-Derivative like (SPID-like) form—with $s_d(\bar{q},\vartheta,\phi) = s_I(K_I\phi) - s_P(K_P\bar{q}) - s_0(s_I(K_I\phi) - s_P(K_P\bar{q}) - K_D\vartheta)$—that is

$$u(q,\vartheta,\phi) = s_0(-s_P(K_P\bar{q}) - K_D\vartheta + s_I(K_I\phi))$$

with the saturation functions involved in s_0—that is $\sigma_{0i}(\cdot)$, $i = 1,2$—being linear saturation functions for (L_{0i}, M_{0i}), and the involved linear/generalized saturation function parameters satisfying

$$M_{Pi} + M_{Ii} < L_{0i} \leq M_{0i} < T_i \tag{28}$$

$i = 1,2$. And the SP-SID form—with $s_d(\bar{q},\vartheta,\phi) = s_I(K_I\phi) - s_I(K_I\phi - K_D\vartheta)$—that is

$$u(q,\vartheta,\phi) = -s_P(K_P\bar{q}) + s_I(-K_D\vartheta + K_I\phi)$$

with bound values fulfilling

$$M_{Pi} + M_{Ii} < T_i \tag{29}$$

$i = 1, 2$. Letting $\sigma_h(\varsigma; M) = M\,\mathrm{sat}(\varsigma/M)$ (observe that this is a linear saturation with $L = M$) and

$$\sigma_s(\varsigma; L, M) = \begin{cases} \varsigma & \text{if } |\varsigma| \le L \\ \mathrm{sign}(\varsigma)L + (M - L)\left(\dfrac{\varsigma - \mathrm{sign}(\varsigma)L}{M - L}\right) & \text{if } |\varsigma| > L \end{cases}$$

with $0 < L < M$, the saturation functions used for the implementation were defined as: $\sigma_{Pi}(\varsigma) = \sigma_h(\varsigma; M_{Pi})$, $\sigma_{Di}(\varsigma) = \sigma_h(\varsigma; M_{Di})$, $\sigma_{Ii}(\varsigma) = \sigma_s(\varsigma; L_{Ii}, M_{Ii})$, $i = 1, 2$, in the SP-SI-SD case; $\sigma_{Pi}(\varsigma) = \sigma_h(\varsigma; L_{Pi}, M_{Pi})$, $\sigma_{Ii}(\varsigma) = \sigma_s(\varsigma; L_{Ii}, M_{Ii})$, $i = 1, 2$, in the SPD-SI case; $\sigma_{0i}(\varsigma) = \sigma_h(\varsigma; M_{0i})$, $\sigma_{Pi}(\varsigma) = \sigma_h(\varsigma; M_{Pi})$, $\sigma_{Ii}(\varsigma) = \sigma_s(\varsigma; L_{Ii}, M_{Ii})$, $i = 1, 2$, in the SPID-like cases; and $\sigma_{Pi}(\varsigma) = \sigma_h(\varsigma; M_{Pi})$, $\sigma_{Ii}(\varsigma) = \sigma_s(\varsigma; L_{Ii}, M_{Ii})$, $i = 1, 2$, in the SP-SID case. Let us note that with these saturation functions, we have $\sigma'_{PiM} = \sigma_{IiM}' = \sigma_{DiM}' = \sigma_{0iM}' = 1$, $\forall i \in \{1, 2\}$. As a consequence, for all the four controllers, inequalities (15a) and (16b) are satisfied with $\kappa = \max_i\{k_{Di}\}$ (see equations (39)).

For comparison purposes, additional experimental tests were implemented using the output-feedback version of the bounded PID-type controller presented in Su et al.[14] The choice was made taking into account the analog nature of the compared algorithms: globally stabilizing via output feedback developed in a bounded-input context, and the recent appearance of Su et al.[14] That is

$$u = -K_P\,\mathrm{Tanh}(\bar{q}) - K_D\,\mathrm{Tanh}(\vartheta) - K_I\,\mathrm{Tanh}(\phi) \tag{30a}$$

$$\dot{\vartheta}_c = -A[\vartheta_c + Bq], \quad \vartheta = \vartheta_c + Bq \tag{30b}$$

$$\dot{\phi}_c = \mathrm{Tanh}(\bar{q}), \quad \phi = \eta^2\bar{q} + \eta\phi_c \tag{30c}$$

with η being a (sufficiently large) positive constant and $\mathrm{Tanh}(x) = (\tanh x_1, \ldots, \tanh x_n)^{\mathrm{T}}$ for any $x \in \mathbb{R}^n$. In place of equations (30c), the work of Su et al.[14] defines $\phi(t) = \eta^2\bar{q}(t) + \eta\int_0^t \mathrm{Tanh}(\bar{q}(\varsigma))d\varsigma$, which imposes the auxiliary variable initial condition $\phi(0) = \eta^2\bar{q}(0)$ (or, equivalently, $\phi_c(0) = 0_n$ in the context of equations (30c)). Instead, equations (30c)—or their (equivalent) time representation $\phi(t) = \phi(0) + \eta^2[\bar{q}(t) - \bar{q}(0)] + \eta\int_0^t \mathrm{Tanh}(\bar{q}(\varsigma))d\varsigma$— keeps the required auxiliary dynamics while permitting any initial condition for ϕ (or, equivalently, for ϕ_c in the context of equations (30c)). This proves to be more appropriate in the global stabilization framework considered in Su et al.[14] (and what is generally expected from an approach developed within such a framework). For the sake of simplicity, this algorithm is subsequently referred to as the S10 controller.

At all the experiments, the desired joint positions were fixed at $q_d = (\pi/4, \pi/4)^{\mathrm{T}}$ (rad), that is $q_{d1} = \pi/4$ rad for the shoulder and $q_{d2} = \pi/4$ rad for the elbow. The initial conditions were $q(0) = 0_2$ (the *home* position), $\dot{q}(0) = 0_2$ and, for the algorithms obtained through the proposed scheme, $\phi_c(0)$ was taken so as to have $\phi(0) = 0_2$, while $\phi_c(0) = 0_2$ was taken for the S10 controller in view of the way it is presented by Su et al.[14]

The control and saturation function parameter values were set so as to achieve pre-specified performance requirements. Two such performance requirements were a priori considered. The first one consisted in getting closed-loop responses with small transient peak values (whether as overshoot or undershoot) within a tolerance margin level of 40% of the desired position value at every link. The second aimed at achieving closed-loop responses with stabilization times shorter than five seconds. Both such requirements were achieved through a single test by each one of the algorithms obtained from the proposed scheme. On the contrary, the tuning procedure presented by Su et al.[14] permitted the S10 controller to achieve the first requirement only through long stabilization times, and the second only with high transient peak values, but not both requirements simultaneously. The resulting control and saturation function parameter values are presented in Table 1. One can corroborate that inequalities (16)–(17) are fulfilled by all the controllers obtained through the proposed scheme, as well as the corresponding saturation-avoidance inequalities (26)–(29) (through which (15b) is guaranteed).The considered performance requirements were achieved under an additional control-parameter adjustment procedure that does not only take into account the conditions obtained through the closed-loop analysis (in the eponymous section) but also adopts the spirit of *performance-oriented* tuning methods.[27] Guidelines are given in Appendix 2. The tunings for the S10 controller are labeled as S10a and S10b for the peak and stabilization-time requirements respectively. This labeling is subsequently used to differentiate from the tests under tunings S10a and S10b.

Figures 2 and 3 show the experimental results. One sees from the graphs that in all the experiments the control objective is achieved avoiding input saturation. In order to establish a comparison criterion, a performance index was evaluated for every controller: the integral of the square of the position error (ISE), that is $\int_0^{t_f}[\sum_{i=1}^2 \bar{q}_i^2(t)]dt$ (with t_f the final time of the experiment). We further show evaluations of the stabilization time, taken as $t_s = \inf\{t_e \ge 0 : \|\bar{q}(t)\| \le 0.02\|q_d\|, \forall t \ge t_e\}$, and the largest transient peak (LTP) at every link, measured as a percentage of the corresponding desired position. For each one of the considered quantifications (ISE, t_s, LTP), the lowest estimated value indicates the best evaluated performance. Table 2 shows the resulting evaluations.

One sees from the obtained values that the controller with the lowest ISE index evaluation was the SPD-SI algorithm (indicated by a check mark). On the other hand, the algorithm with the highest ISE index value is the S10

Table 1. Control parameter values.

Parameter	SP-SI-SD	SPD-SI	SPID-like	SP-SID	S10a	S10b	units
k_{P1}	6000	6000	7000	6000			N m/rad
					74	108	N m
k_{P2}	500	500	350	500			N m/rad
					8.5	11.5	N m
k_{I1}	900	900	900	175			N m/rad
					40.5	40.5	N m
k_{I2}	1500	1500	700	1			N m/rad
					1.9	1.9	Nm
k_{D1}	2	2	2	2			N m s/rad
					10.5	0.5	N m
k_{D2}	2	2	2	2			N m s/rad
					4.5	0.1	N m
a_1	60	60	60	60	10	60	s^{-1}
a_2	60	60	60	60	210	40	s^{-1}
b_1	5	5	5	5	15	70	s^{-1}
b_2	5	5	5	5	210	20	s^{-1}
ε	0.024	0.024	0.021	0.024			s^{-1}
η					30	170	s/rad
M_{P1}	81	81	81	81			N m
M_{P2}	7	7	7	7			N m
M_{I1}	41	48	41	41			N m
M_{I2}	2	5	2	4			N m
$L_{I1/2}$	$0.9M_{I1/2}$	$0.9M_{I1/2}$	$0.9M_{I1/2}$	$0.9M_{I1/2}$			N m
M_{D1}	22						
M_{01}			125				N m
M_{D2}	5						
M_{02}			14				N m

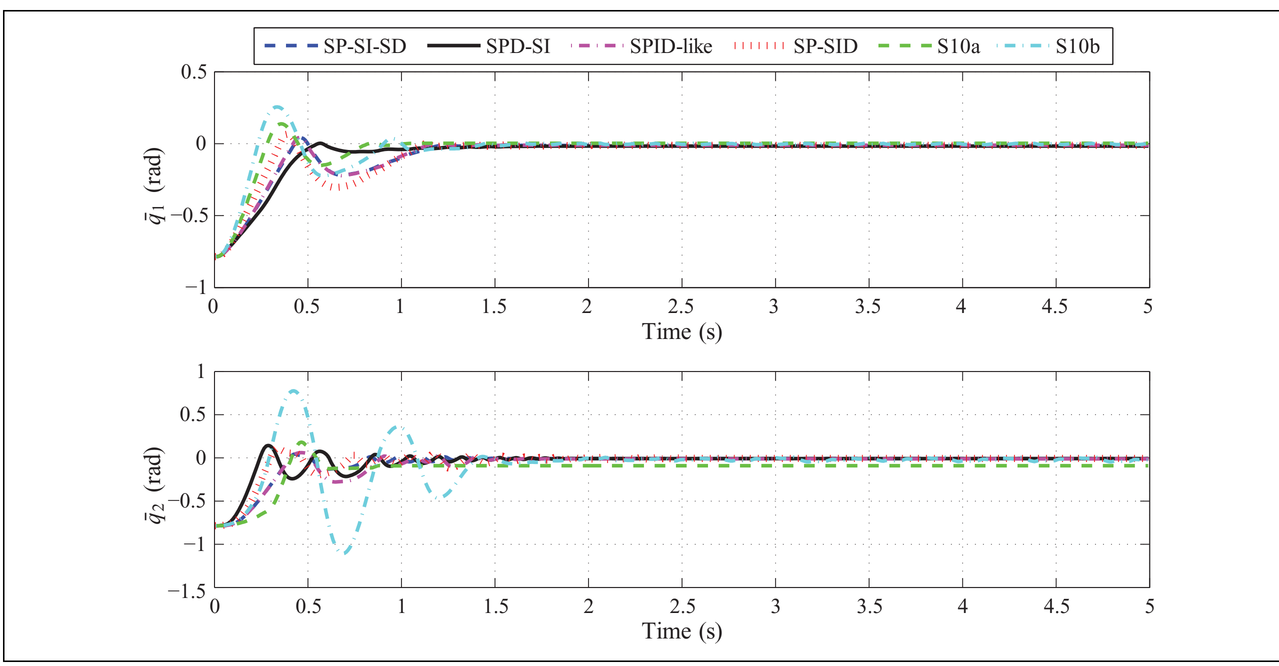

Figure 2. Position errors.

controller (indicating through asterisks the two higher ones, with double asterisk for the highest). The rest of the evaluations give an analog idea on the system performance at every implementation. They confirm that the algorithms obtained from the proposed methodology were able to meet both pre-specified performance requirements through a single test. On the other hand, the S10 controller is the one with the highest number of largest index evaluations.

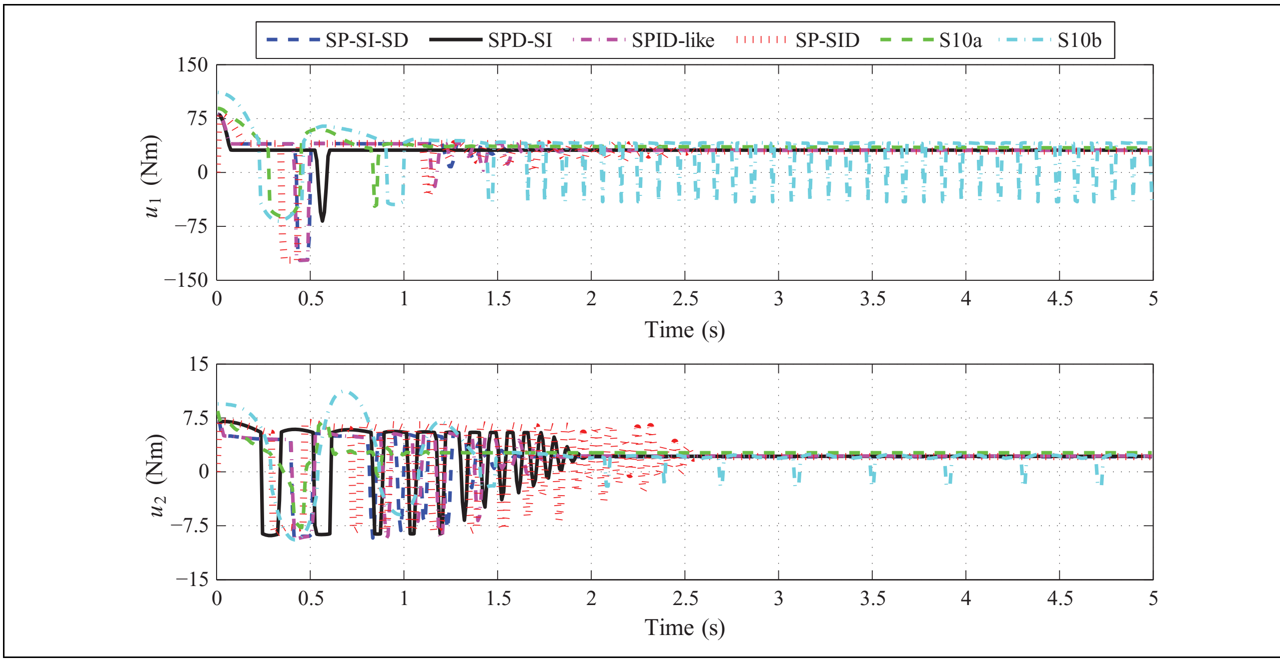

Figure 3. Control signals.

Table 2. Performance index evaluations.

Perf. index		SP-SI-SD	SPD-SI	SPID-like	SP-SID	S10a	S10b
ISE		0.2633	0.2242 ✓	0.2731	0.2454	0.3023*	0.5327**
t_s		1.3227 ✓	1.6623	1.4945	1.8879	>5**	4.9134*
LTP	link 1	27.9%	7.3% ✓	28.1%	38.8%**	19.3%	32.6%*
	link 2	16.6% ✓	30.9%	35.5%*	24.8%	23.3%	141.3%**

Conclusions

Up to the submission of the present article, a methodology for the design of output-feedback bounded PID-type controllers for robot manipulators with constrained inputs, leading to multiple saturating structures, was lacking in the literature. For instance, it was not clear how to get a velocity-free version of the SPD-SI state-feedback structure of Santibáñez et al.[15] Such an open problem was tackled in this work, leading to a generalized design method and the corresponding closed-loop analysis, developed with the required rigorous formality. The proposed scheme gives rise to bounded PID-type controllers with multiple saturating structures, extending the degree of design flexibility when velocity measurements are not available. For instance, it does not only extend the SPD-SI approach to such a velocity-free context and includes the SP-SI-SD as a particular case, but it also offers the possibility to generate innovative saturating structures as thoroughly shown. In addition, the design and analysis were further addressed so as to include not only smooth, but also nonsmooth (Lipschitz-continuous), saturation functions in the control structure. Further efforts made possible the corroboration of the analytical developments through experimental tests on a two-DOF manipulator, which showed the efficiency of the proposed controller. The contributed approach is thus concluded to find potential applications in numerous types of autonomous robot systems, saving these from undesirable behaviors due the actuator saturations, releasing them from the need for speed sensors, and opening new control design possibilities to improve their closed-loop behavior.

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Appendix I

On the basis of bounded algorithms from previous references, several particular control structures arise through the proposed generalized scheme. For instance, let $K_D \in \mathbb{R}^n$ be a positive definite diagonal matrix. An *SP-SI-SD* type algorithm[13,28] is obtained by defining

$$s_d(\bar{q}, \vartheta, \phi) = s_D(K_D \vartheta) \tag{31}$$

giving rise to

$$u(q, \vartheta, \phi) = -s_P(K_P \bar{q}) - s_D(K_D \vartheta) + s_I(K_I \phi)$$

where, for any $x \in \mathbb{R}^n$, $s_D(x) = (\sigma_{D_1}(x_1), \ldots, \sigma_{D_n}(x_n))^{\mathrm{T}}$, with $\sigma_{Di}(\cdot)$, $i = 1, \ldots, n$, being *generalized saturation functions* with bounds M_{Di}, and the involved bound values satisfying

$$M_{Pi} + M_{Di} + M_{Ii} < T_i \tag{32}$$

An *SPD-SI* type scheme[15,19] is obtained by defining

$$s_d(\bar{q}, \vartheta, \phi) = s_P(K_P\bar{q} + K_D\vartheta) - s_P(K_P\bar{q}) \tag{33}$$

resulting in

$$u(q, \vartheta, \phi) = -s_P(K_P\bar{q} + K_D\vartheta) + s_I(K_I\phi)$$

with bound values fulfilling

$$M_{Pi} + M_{Ii} < T_i \tag{34}$$

An *SPID-like* scheme[18,19] is obtained by defining

$$s_d(\bar{q}, \vartheta, \phi) = s_0(s_I(K_I\phi) - s_P(K_P\bar{q})) - s_0(s_I(K_I\phi) - s_P(K_P\bar{q}) - K_D\vartheta) \tag{35}$$

where, for any $x \in \mathbb{R}^n$, $s_0(x) = (\sigma_{01}(x_1), \ldots, \sigma_{0n}(x_n))^{\mathrm{T}}$, with $\sigma_{0i}(\cdot)$, $i = 1, \ldots, n$, being *linear saturation functions* for (L_{0i}, M_{0i}), and the involved linear/generalized saturation function parameters satisfying

$$M_{Pi} + M_{Ii} < L_{0i} \leq M_{0i} < T_i \tag{36}$$

whence, by virtue of item (c) of Definition 1, we have that $s_0(s_I(K_I\bar{\phi}) - s_P(K_P\bar{\phi})) \equiv s_I(K_I\bar{\phi}) - s_P(K_P\bar{\phi})$, giving rise to

$$u(q, \vartheta, \phi) = s_0(-s_P(K_P\bar{q}) - K_D\vartheta + s_I(K_I\phi))$$

Furthermore, the general character of the proposed scheme permits the generation of control laws with innovative saturating structure. For instance, an *SP-SID* type controller can be obtained by defining

$$s_d(\bar{q}, \vartheta, \phi) = s_I(K_I\phi) - s_I(K_I\phi - K_D\vartheta) \tag{37}$$

resulting in

$$u(q, \vartheta, \phi) = -s_P(K_P\bar{q}) + s_I(-K_D\vartheta + K_I\phi)$$

with bound values fulfilling

$$M_{Pi} + M_{Ii} < T_i \tag{38}$$

One can verify that in all the above cases the expressions (15a) and (15b) are satisfied. In particular, the input-saturation-avoidance requirement stated through (15b) is accomplished through the fulfillment of inequalities (32), (34), (36) and (38). Furthermore, from item 4 of Lemma 1, one sees that $s_d(\bar{q}, \vartheta, \phi)$ in every one of the above cases in (31), (33), (35) and (37) satisfies inequality (15a) with

$$\kappa = \max_i\{\sigma'_{iM} k_{Di}\} \tag{39a}$$

$$\sigma'_{iM} = \begin{cases} \sigma'_{DiM} & \text{in the SP} - \text{SI} - \text{SD case} \\ \sigma'_{PiM} & \text{in the SPD} - \text{SI case} \\ \sigma'_{0iM} & \text{in the SPID} - \text{like case} \\ \sigma'_{IiM} & \text{in the SP} - \text{SID case} \end{cases} \tag{39b}$$

σ'_{DiM}, σ'_{PiM}, σ'_{0iM} and σ'_{IiM} respectively being the positive bounds of $D^+\sigma_{Di}(\cdot)$, $D^+\sigma_{Pi}(\cdot)$, $D^+\sigma_{0i}(\cdot)$ and $D^+\sigma_{Ii}(\cdot)$, in accordance with item 2 of Lemma 1.

Remark 5. Observe that the input-saturation-avoidance conditions for the particular control structures presented in this appendix, that is inequalities (32), (34), (36) and (38), imply that $M_{Pi} + M_{Ii} < T_i$. On the other hand, the satisfaction of inequalities (14) implies that $M_{Pi} + M_{Ii} > 3B_{gi}$. Hence, for the specific choices of s_d presented in equations (31), (33), (35) and (37), the feasibility of the simultaneous fulfillment of inequalities (14) and the corresponding input-saturation-avoidance condition—(32), (34), (36) or (38), respectively—is ensured by requiring the satisfaction of Assumption 1 with $\alpha = 3$. Other particular choices of s_d in the generalized scheme (11) could require different values of $\alpha \geq 1$.

Appendix 2

The performance-oriented tuning procedure used to obtain the experimental results shown in the eponymous section is sketched as follows:

1. Set the saturation function parameters (M_{Pi}, M_{Di}, M_{0i}, M_{Ii}, L_{Pi}) so as to guarantee the satisfaction of inequalities (14) and (26)–(29).
2. Set low control gains, under the consideration of (16a).
3. Adjust the velocity-estimation-subsystem parameters (a_i, b_i) under the consideration of (16b), fixing a_i such that $1/a_i$ be six to 10 times the sampling period of the controller, and $b_i < a_i$ to reduce inertial effects (inherent to the velocity estimation dynamics), such as oscillations.
4. Run simulations/experiments with coefficient ε adhering to (17), if possible, or as small as the closed-loop stability permits it. If the resulting closed-loop response is satisfactory then stop, otherwise perform the following steps.
5. Increase integral gains, k_{Ii}, so as to strengthen the elimination of position errors, aiming at reducing stabilization times.
6. Increase proportional gains, k_{Pi}, in order to reduce the rise time (speed up the closed-loop response).
7. Increase derivative gains, k_{Di}, in order to reduce inertial effects (particularly added by the integral actions), such as the overshoot.
8. Repeat steps 3–4.