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ALFVÉN WAVE HEATING AND DISSIPATIVE INSTABILITIES OF ASTROPHYSICAL PLASMAS

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1. Introduction

It is well known that matter in the universe consists predominantly of plasma. However, the fundamental role which this fact plays for the description of nature has only recently been realised. In particular, the search for the generation of energy from controlled thermonuclear reactions has vastly increased our knowledge of the interaction of magnetic fields with hot ionised gases (plasmas). In turn, this research has stimulated the modelling of astrophysical plasmas, where magnetic fields turn out to play an increasingly more important role. A prominent example is the magnetohydrodynamics of the sun, where virtually all phenomena that are important in tokamaks (presently the most promising nuclear fusion experiment) return in a modified form. The appropriate tool for the description of plasmas in magnetic fields is the theory of magnetohydrodynamics (MHD). Consequently, research of magnetohydrodynamics from the two points of view of nuclear fusion and astrophysics offers a promising perspective. The Twinning Project on *'Alfvén wave heating and dissipative instabilities of astrophysical plasmas'* aims at a contribution to the solution of the problems of heating and stability of solar plasmas by bringing together experts from the two mentioned fields.

The Twinning Project is concerned with two major astrophysical problems, viz. the question of why the sun and other stars have hot, X-ray emitting, coronae and the opposite question about the stability and fate of cold plasma structures (prominences) on the sun. Since both problems involve the simultaneous consideration of the global geometry of the magnetic configurations involved (closed magnetic flux loops emanating from the solar surface) and several dissipation mechanisms (ranging from collective effects of Alfvén continuum modes to resistivity and thermal transport) a large-scale numerical approach exploiting supercomputers was considered necessary.

In the Project three small scientific groups collaborate, viz. scientists from the FOM-Instituut voor Plasmafysica 'Rijnhuizen' (Nieuwegein, the Netherlands) under the guidance of J.P. Goedbloed, scientists from the Astronomisch Instituut of the Katholieke Universiteit Leuven (Heverlee, Belgium) under the guidance of M. Goossens, and scientists from the Max-Planck-Institut für Plasmaphysik (Garching, Germany) under the guidance of W. Kerner (presently at JET, Culham, U.K.). This collaboration has established a group of critical size in terms of manpower to ensure the effective use of the numerical programs that are developed. In addition, it has proved to be an extremely fertile environment for the training of young physicists working on their Ph.D. theses. During the twinning project many visits of the scientists to the collaborating institutes were made (see section 7), resulting in publications which are beginning to make a real impact in the field of solar physics (section 6), whereas some spin-off for the field of thermonuclear plasmas also is reported (section 5). The main contributions to astrophysical plasmas are reported in section 3 on Alfvén wave heating and in section 4 on thermal instabilities. Obviously, success in these fields hinges on the development of a set of well structured numerical codes requiring a non-trivial effort in maintenance, which is described in section 2. The report would be incomplete if it would not mention the educational activity involved in attracting new Ph.D. students and training them in the field of numerical solar physics. The twinning collaboration has turned out to be extremely effective in motivating young people to give their best in this exciting new field of research. During the twinning project two Ph.D. theses were successfully completed, viz. by R.A.M. Van der Linden on the subject *"The Thermal Instability in the Solar Corona: A Mechanism for the Formation of Cool Condensations"* and by G.T.A. Huysmans on the subject *"External Resistive Modes in Tokamaks"*.

2. Numerical codes

Purposely, this technical section on the different numerical tools, codes, and modular structuring has been put at the beginning of our report to stress the fact that this activity should be considered as basic to the whole effort of the twinning collaboration.

In the initiating meeting of the new twinning collaboration at Garching in December 1989, two lines of research were defined, embodied in two major 2D codes called CASTOR (**C**omplex **A**lfvén **S**pectra for **TOR**oidal plasmas) and POLLUX (**P**rogram **O**n **L**ine-tied **L**oops **U**nder e**X**citation). These programs solve for the complex spectrum of MHD waves and instabilities of tokamak and solar coronal plasmas, respectively, exploiting the same numerical techniques. Both codes are derived from the 1D spectral code developed in the previous Leuven–Garching twinning project, which has been baptised LEDA (**L**arge-scale **E**igenvalue solver for the **D**issipative **A**lfvén spectrum) for this purpose.

As a working principle, new physical effects are first introduced in the 1D (cylindrical) program LEDA in order to avoid, in first instance, the mathematical complication of the poloidal mode coupling (for tokamaks) or of the longitudinal mode coupling (for coronal loops). When the new effect is mathematically under control in 1D, it is generalised to 2D. This working method has already proved to be very valuable upon the introduction of the plasma resistivity.

Apart from the spectral codes mentioned above, other numerical programs have been developed during the twinning collaboration, some of which are auxiliary programs: HELENA, CSCAS, and CSPOL (see section 2d), and some of which were derived from the spectral codes: stationary state codes and temporal-evolution codes (see section 2c).

2a. REVISE: a portable facility for structured program development

J.P. Goedbloed, S. Poedts, E. Schwarz

In an international group of scientists collaborating on related subjects and using the same numerical codes an immediate need arises with respect to communication on the development of these codes: How to control and to exchange the progression of changes, avoiding the introduction of mistakes, informing fellow collaborators of yet another version of their daily working tool, while stimulating growth of these tools? REVISE is a package of portable FORTRAN codes developed precisely for this purpose. Its use requires the source codes to be brought in a special precompile format so that changes can be detected, documented, and communicated easily. The five functions of creating a new source list, precompiling a FORTRAN program from the source, comparing a new source with a previous list, creating a modification set, and extracting a new source code from a REVISE list are cast in five auxiliary programs called NEW, PRE, COM, MOD, and EXT. These programs and their companion system procedures have been perfected and installed at the computers of the collaborating institutes (IPP Garching, KUL Leuven, FOM Nieuwegein, i.e. SARA Amsterdam, and JET, Culham). The agreement on this way of communication has turned out to be invaluable for the coherence of the numerical work of the twinning group.

2b. PPPLIB: Plasma Physics Plotting Library

J.P. Goedbloed, G.T.A. Huysmans, S. Poedts (in collaboration with E. Westerhof)

In a similar vein, the portable package of FORTRAN plotting routines PPPLIB has been installed on the above mentioned computers of the collaborating groups. This has turned out to be another means of safeguarding the coherence of the numerical programs since this avoids the need of constant rewriting diagnostic parts of the codes calling local plotting facilities. A modernised version of this package has been developed which may deliver CALCOMP as well as Postscript files to be directly processed by a laser printer.

2c. Development of the programs LEDA, CASTOR, and POLLUX

E. Schwarz, S. Poedts, J.P. Goedbloed, R.A.M. Van der Linden, G. Halberstadt, and G.T.A. Huysmans

The spectral codes LEDA, CASTOR and POLLUX basically solve the full (unreduced) set of dissipative linearised MHD equations. The aim is to compute the ideal and dissipative MHD spectra of axisymmetric cylindrical, both periodic (LEDA) and line-tied (POLLUX), and toroidal (CASTOR) plasmas and, in particular, to determine the ideal and dissipative MHD stability of such systems. A combination of cubic Hermite and quadratic finite elements is employed for the discretization in the radial direction and a Fourier expansion in the other two co-ordinates resulting in the generalised eigenvalue problem $\mathbf{A}\ \mathbf{x} = \lambda\ \mathbf{B}\ \mathbf{x}$, with a non-Hermitian matrix $\mathbf{A}$ and a symmetric matrix $\mathbf{B}$, typically with dimensions in the range $1000 < d < 60000$. These large dimensions arise from the need to represent the fine-scale motion of the plasma in the two directions of inhomogeneity. The use of finite elements results in matrices with a block-tridiagonal band structure. Very large systems are handled by storing data out of core.

Additional codes, to be considered as offspring of LEDA (although developed by S. Poedts before the name LEDA was introduced), concern the calculation of the stationary state (SS) and the temporal evolution (TE) of externally driven cylindrical plasmas described by the same equations that constitute the eigenvalue problem solved by LEDA so that the matrices $\mathbf{A}$ and $\mathbf{B}$ may be exploited again. Denoting the spectral codes by SP, it is clear that the triplet SP/SS/TE represents a significant proliferation of all codes mentioned. This enables the investigation of different aspects of a specific magnetohydrodynamic phenomenon by different approaches which lead to a deeper insight of the studied physical phenomenon. So far, only LEDA has been developed in all three modes, the SP version of POLLUX has been supplemented by an SS version, whereas CASTOR only exists in the SP variant.

During the twinning project, the normal mode code CASTOR has been created and further developed in order to include new physics and to improve the performance and possibilities. Also LEDA and POLLUX have been further developed. A profound change of the lay-out has enhanced the readability of the codes. In addition, a description of the input variables is provided in the latest versions of the programs. Improvements of the eigenvalue solvers, built-in test cases, and the new physics added to the codes are discussed in the technical appendices 1–6 (Section 8).

2d. Other numerical programs

In addition to the above-mentioned creation and extensions of the spectral codes, new codes were developed, viz. HELENA, CSCAS, and CSPOL, and significant extensions were made of the existing cylindrical stationary state and temporal evolution codes that were derived from LEDA. In this section, the new codes that were developed are shortly discussed.

HELENA

G.T.A. Huysmans, W. Kerner and J.P. Goedbloed

An important subject in nuclear fusion research is the magnetohydrodynamic stability of hot plasmas contained in a tokamak. For the numerical calculation of the linear stability of ideal and resistive modes, the equilibrium quantities of the plasma have to be known very accurately. This has been obtained by describing the solution of the equilibrium equation with the use of isoparametric bicubic Hermite finite elements. These two-dimensional higher order elements were shown to yield very accurate solutions with good convergence properties. The equilibrium quantities needed in ideal and resistive MHD stability codes like CASTOR are usually represented in a co-ordinate system with ψ as a radial co-ordinate. The angular co-ordinate χ is then chosen such that the magnetic field lines become straight in the projection. The evaluation of the equilibrium quantities consists of line integrals over surfaces of constant ψ with $\nabla\psi$ in the integrand. Bicubic Hermite elements allow for a representation of $\psi(x,y)$ such that both ψ and $\nabla\psi$ are continuous across element boundaries.

CSCAS

S. Poedts and E. Schwarz

Based on the CASTOR matrices, the auxiliary one-dimensional code CSCAS (**C**ontinuous **S**pectrum from **CAS**TOR) was developed. This numerical code computes the range and the internal structure of the continuous ideal MHD spectrum of axisymmetric toroidal plasmas by solving a reduced eigenvalue problem on a specific flux surface. This code turned out to be indispensable for the study of the gap modes (see next section) and will evidently be of similar importance for the study of resonant absorption in 2D fusion plasmas, given the vital role that the continuous spectrum plays for this heating mechanism.

When the equilibrium quantities vary only in one spatial direction, the continuous parts of the spectrum can be determined easily as they correspond to the singularities of the coefficients of a second-order differential equation. In equilibria with inhomogeneity in two directions, however, the loss of symmetry and the corresponding poloidal mode coupling make the determination of the continuous spectrum considerably more complicated. Consequently, CASTOR is not efficient for the determination of the continuous spectrum. The Galerkin procedure leads to a general matrix eigenvalue problem of the form $\mathbf{A} \cdot \mathbf{a} = \lambda\, \mathbf{B} \cdot \mathbf{a}$, with $\mathbf{a}$ a vector of $16 \times N_\psi \times N_m$ expansion coefficients, where N_ψ is the number of radial mesh points and N_m the number of poloidal Fourier components ($\sim e^{im\theta}$). Owing to the finite spatial resolution, the continua show up as closely spaced discrete eigenvalues. For a good approximation of the (singular) continuum modes (*i.e.* one that enables one to distinguish them from real discrete modes), N_ψ has to be large which leads to intolerable memory and CPU-time requirements, especially when the mode coupling is strong since N_m has to be large too in this case. Therefore, CSCAS was developed. This code determines the internal structure of the continuous spectrum of axisymmetric, toroidal plasmas in a very efficient way: instead of solving the full eigenvalue problem, a reduced (non-singular) eigenvalue problem is solved on each flux surface. This yields a discrete set of eigenvalues on each flux surface. However, each eigenvalue of this discrete set maps out a continuous spectrum when the magnetic surface is varied. We discovered a very convenient way to obtain the equations of the reduced eigenvalue problem that determines the continuous spectrum from the full set of ideal MHD equations that is solved by CASTOR. This method consists of focusing on one magnetic flux surface and prescribing the radial dependence — which is known to be logarithmic in nature — on that particular surface. The matrices of the reduced eigenvalue problem have dimensions $(d \times d)$, with $d = 8 \times N_m$ and, hence, the memory and CPU-time problems are eliminated.

CSPOL

G. Halberstadt, S. Poedts, E. Schwarz, R.A.M. Van der Linden

In the same vein as the development of the continuum code CSCAS, a code CSPOL (Continuous Spectrum from **POLLUX**) was constructed in order to be able to calculate the 2D coronal loop continuous spectrum. The main difference between CSCAS and CSPOL is the implementation of the line-tying boundary conditions in the latter code. In the original POLLUX eigenvalue code, the line-tying is implemented by eliminating the surface terms from the equations in favour of the line-tying constraint. However, as a result of this procedure, the **B**-matrix is no longer positive definite and therefore the eigenvalue problem cannot be solved by the QR-algorithm. Therefore, in order to calculate the continuous spectrum, in CSPOL the QZ-algorithm is exploited or the line-tying constraint is imposed by using a projection matrix **P**. In the latter case the resulting eigenvalue problem reads $\mathbf{B}^{-1}\mathbf{APx} = \lambda\mathbf{x}$, where **P** accounts for the line-tying constraint. This eigenvalue problem can be solved by the QR-algorithm. CSPOL does not only serve to calculate line-tied continuous spectra, but will also reveal interesting new physics if the recently implemented 2D density profiles (see appendix 6) are used, either with or without combination with the line-tying constraint.

3. Resonant Absorption and phase-mixing

During the twinning collaboration, the properties of resonant absorption and phase-mixing have been studied extensively in various physical configurations where these mechanisms can operate. The ideal quasi-modes that play an essential role for resonant absorption were analysed and the efficiency and time-scales of plasma heating by resonant absorption have been studied in laboratory configurations as well as solar coronal loops. The efficiency of resonant absorption of pressure driven modes in sunspots was also investigated, both analytically and by means of numerical investigations. Parallel to these investigations, the possibilities of heating line-tied solar coronal loops by the resonant absorption of shear Alfvén waves and heating by phase-mixing due to footpoint oscillations were studied.

3a. Ideal quasi-modes reviewed in resistive MHD

S. Poedts

In agreement with the philosophy outlined in Section 2, the cylindrical configuration considered in the 1D spectral code LEDA was enlarged with a vacuum region between the plasma (with radius r_p) and a perfectly conducting wall (at radius r_w). This extension of the code made it possible to study free-boundary modes in cylindrical, axisymmetric configurations in the framework of linearised resistive MHD. With this extended 1D code ideal quasi-modes were reviewed in resistive MHD and it was shown that these modes correspond to eigenmodes of the resistive MHD differential operator. The damping of these modes becomes independent of the plasma resistivity in the limit of vanishing η. Hence, for the first time resistive eigenmodes have been found in the range of the ideal Alfvén continuum which converge to their ideal MHD counterparts when $\eta \to 0$. This result has been published in Physical Review Letters[19].

During the second year of the twinning project, the study of ideal quasi-modes in resistive MHD was continued. The path in the complex plane followed by the external kink frequency as the conducting wall is shifted towards the plasma has been studied extensively. First, the system parameters were chosen such that the external kink mode is unstable ($Re(\lambda) > 0$). As the wall is shifted towards the plasma ($r_w \longrightarrow r_p$), the kink mode is stabilised, its frequency gets an oscillatory part ($Im(\lambda) \neq 0$) and $Re(\lambda)$ becomes negative (damping). As the wall is shifted closer and closer to the plasma surface, $Im(\lambda)$ crosses first the slow magnetosonic continuum and later the Alfvén continuum. When located in a continuous spectrum the mode couples to continuum modes and the frequency becomes independent of the plasma resistivity in the limit of vanishing η. The coupling to the continuum modes is weak in the slow magnetosonic continuum and strong in the Alfvén continuum due to, respectively, nearly perpendicular and parallel polarisations. Outside the continua, only the oscillatory part of the frequency becomes independent of η in the limit $\eta \longrightarrow 0$, while the damping part ($| Re(\lambda)|$) scales as $\eta^{1/2}$. It has been suggested in the literature that the quasi-mode joins and completes the discrete fast magnetosonic spectrum as the wall approaches the plasma. However, this suggestion was based on ideal MHD calculations. In resistive MHD, this conclusion is invalidated because the external kink frequency behaves differently above the Alfvén continuum than the fast modes: its frequency does not join the lower end of the fast spectrum and scales differently with resistivity.

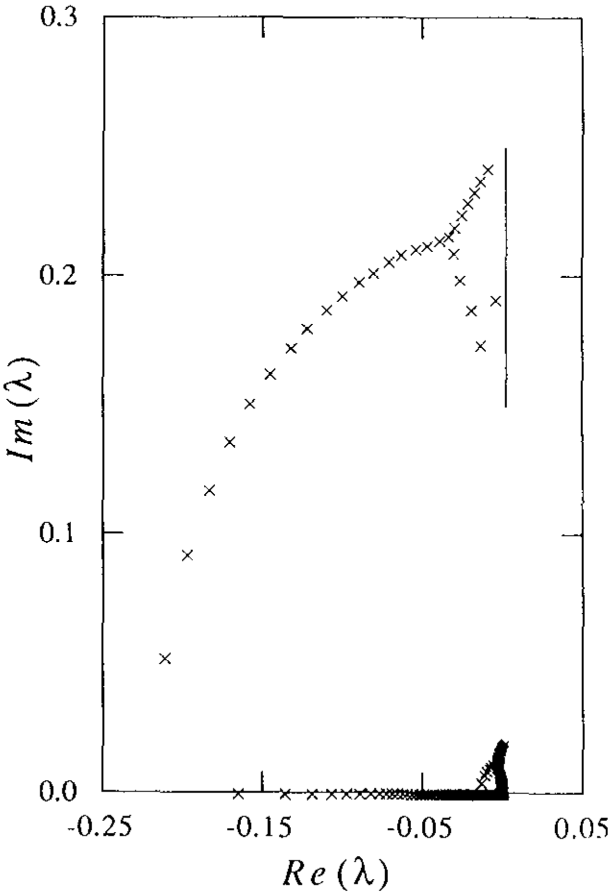

Figure 1: Typical Alfvénic and slow magnetosonic part of the resistive-MHD spectrum for $\eta = 5 \times 10^{-5}$, as obtained with the QR-algorithm. The ideal Alfvén continuum, ranging from $\lambda = (0, 0.15)$ to $\lambda = (0, 0.25)$, is also indicated. The weakly damped eigenmode with frequency situated in the triangle formed by the ideal Alfvén continuum and the curves with resistive (Alfvén) modes in the complex plane, corresponds to the 'ideal quasi-mode'.

3b. Efficiency and time-scales of resonant absorption

S. Poedts and W. Kerner

The time-scales and the efficiency of plasma heating by resonant absorption of Alfvén waves have been investigated in the framework of linearised compressible and resistive MHD by means of numerical simulations with the stationary state and temporal evolution codes based on the spectral code LEDA. The considered configuration is the same as in LEDA (*i.e.* a cylindrical plasma-vacuum-wall configuration) and the plasma is excited by an external 'antenna' situated in the vacuum region. We considered different ways of driving, such as periodic driving, multi-periodic driving, and random (stochastic) driving, but most attention was given to periodic external sources.

The variation of the energetics in periodically driven resistive systems has been analysed in detail for three different choices of the driving frequency, *viz.* an arbitrary continuum frequency, the frequency of an ideal quasi-mode, and a discrete Alfvén frequency. We showed that the so-called ideal quasi-modes manifest themselves as the natural plasma oscillations and play a fundamental role in resonant absorption

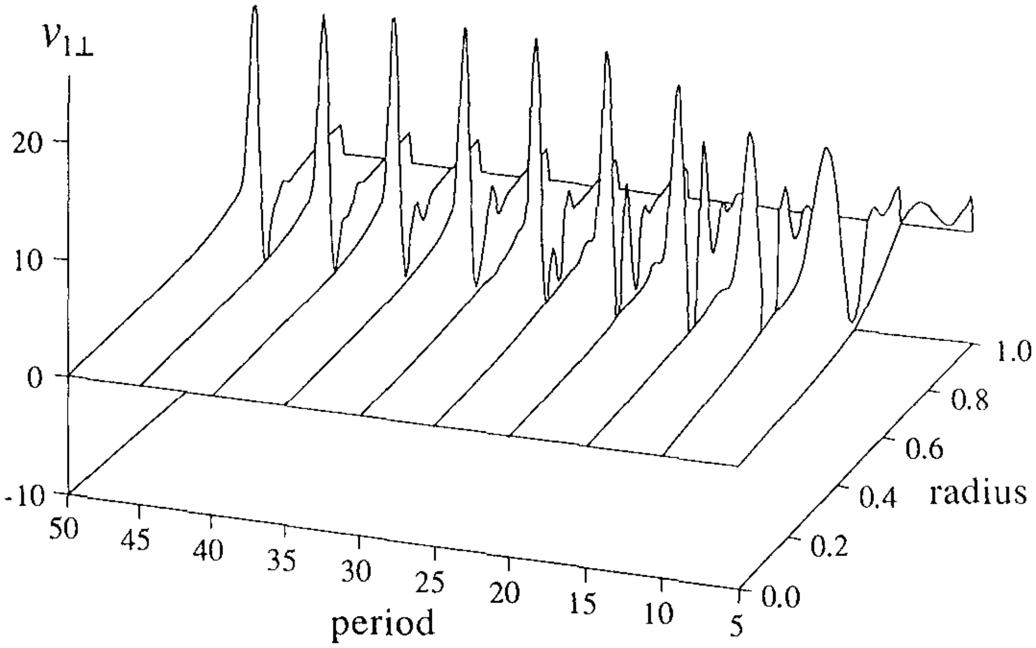

Figure 2: Snapshots of $v_{1\perp}(r)$ for a driving frequency of 0.205 and $\eta = 10^{-6}$ (other parameters as in Figure 1). In an initial phase, the energy gradually accumulates in a resistive plasma layer around the ideally singular layer at $r = 0.67$. The driven dissipative system evolves to a stationary state.

in the sense that they affect both the time-scales and the efficiency of this heating mechanism considerably. When a plasma is driven at an arbitrary frequency within the range of the ideal Alfvén continuum, this quasi-mode is also excited in the initial phase, giving rise to the production of beats. When the driving frequency is close enough to the oscillatory part of the frequency of the ideal quasi-mode, this leads to a higher Ohmic dissipation rate than in the steady state. Excitation of a plasma at the frequency of such a weakly-damped quasi-mode yields 100% absorption, which means that all the energy that is supplied by the external source is absorbed by the plasma and, in the stationary state, converted into heat by Ohmic dissipation. So, the coupling of the external driver to the plasma is perfect in this case. Driving at an arbitrary frequency in the range of the ideal Alfvén continuum yields a less efficient plasma-driver coupling. However, the ratio of the absorbed power to the total power emitted by the external source can still be very high, depending on how close the driving frequency is to the frequency of the ideal quasi-mode. We found that the basic time-scale of resonant absorption is proportional to $\eta^{-\nu}$ with $1/5 \leq \nu \leq 1/3$, depending on the proximity of the driving frequency to the quasi-mode frequency: for driving at the quasi-mode frequency the time τ_{SS} needed to reach a stationary state, is proportional to $\eta^{-1/5}$, while for driving frequencies in the range of the ideal continuum, but not close to the quasi-mode frequency, $\tau_{SS} \sim \eta^{-1/3}$.

For typical small tokamak parameter values, the time-scale needed to reach the steady state is very short: for $\eta = 10^{-8}$ the steady state is reached after about 150×10^{-6} sec, typically. For realistic solar coronal loop parameter values, on the other hand, the frequencies involved are much lower and the time-scale needed to reach the steady state typically varies from a few minutes to a few hours and, hence, the basic time-scale of resonant absorption is much smaller than the typical life time of the magnetic loops (1 day). Driving at the frequency of discrete Alfvén waves also yields 100% absorption. However, the time-scales to reach the steady state are extremely long in this case compared to the time-scales for driving at continuum frequencies. The energy supplied by the external source produces a change of the kinetic and potential plasma energy and the Ohmic dissipation rate remains modest over very long time-scales. These results have been submitted for publication to the Journal of Plasma Physics[21]. An application to solar coronal loop heating was presented at the Heidelberg conference on 'Mechanisms of Chromospheric and

Coronal Heating'[14].

3c. P-mode absorption by sunspots

S. Poedts, M. Goossens, H. Stenuit (in collaboration with J.V. Hollweg and T. Sakurai)

Observations of acoustic oscillations around sunspots have shown that sunspots absorb a large fraction of the acoustic energy flux that is incident upon them. Analysis of incoming and outgoing acoustic waves outside sunspots that were carried out by D.C. Braun and collaborators indicate a loss of up to 70% in the outgoing acoustic power. This fascinating property of sunspots has provided an indirect possibility for probing the subphotospheric structure of sunspots. Resonant absorption of acoustic oscillations was investigated in order to explain the observed loss of acoustic power in sunspots.

First, analytical expressions for the jump conditions (or connection formulae) across the dissipative resonant layer were derived and conserved quantities were obtained for 1D cylindrical magnetic flux tubes that are stratified only in the radial

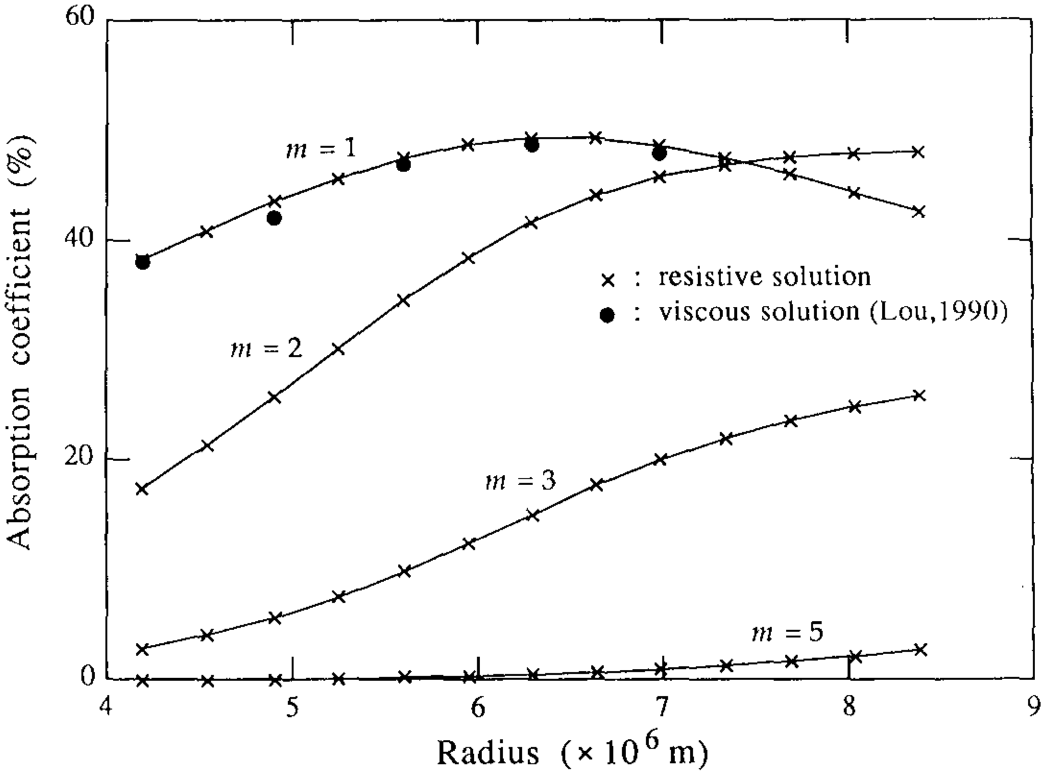

Figure 3: Absorption coefficient versus the radius of the sunspot for typical sunspot parameter values. *cross* refers to a solution obtained in resistive MHD; *filled circle* refers to a solution obtained by Y.Q. Lou in viscous MHD.

direction. Once these jump conditions are known, it suffices to solve the *ideal* MHD equations on either side of the ideal singularity and to connect these solutions across the singularity by applying the connection formulae, instead of solving the full set of resistive MHD equations. This approach was followed in modelling the resonant absorption of sound waves in a simple sunspot equilibrium model. The equilibrium is cylindrically symmetric with a straight magnetic field $\mathbf{B}_0 = (0, 0, B_{0z}(r))$ and consists of three parts. The inner part (1) is a uniform plasma with a straight magnetic field. The outer part (3) consists of a uniform non-magnetised plasma, whereas the physical variables vary from their values in (1) to their values in (3) in an intermediate layer (2). Linear expressions were used for the square of the Alfvén speed $V_A^2(r)$, for the square of the sound speed $c_s^2(r)$, and for the density $\rho_0(r)$. For this equilibrium model the obtained absorption coefficients are comparable to the observed values. An important shortcoming of this model is the assumption of a straight magnetic field.

Subsequently a numerical study of the efficiency of resonant absorption of acoustic oscillations in sunspots was carried out. The sunspot was idealised as a cylindrical axisymmetric flux tube stratified only in the radial direction and surrounded by a uniform unmagnetised plasma. The resonant absorption of acoustic oscillations that are incident was studied by means of numerical simulations in the framework of linear resistive MHD. The existing code (Section 3b) for calculation of coronal loop heating in resistive MHD was adapted. The surrounding vacuum region was substituted by a uniform non-magnetic plasma and the antenna was replaced by a mixture of incoming and outgoing sound waves. The resulting code was first used to calculate resonant absorption of sound waves in the equilibrium models considered by Y.Q. Lou (*Astrophys. J.*, **350**, (1990) 452). In Lou's calculations of resonant absorption viscosity was chosen as the dissipation mechanism whereas, here, dissipation by resistivity was considered. Lou's results were recovered, thereby providing the first numerical confirmation that resonant absorption is independent of the dissipative mechanism considered. A parametric study revealed that the efficiency of resonant absorption depends significantly on both the equilibrium model and the characteristics of the p-modes themselves. The overall picture resulting from our numerical survey of the relevant parameter domain is that the resonant absorption of p-modes is more efficient in larger sunspots with twisted magnetic fields. This is particularly true for p-modes with higher azimuthal wave numbers. The observed absorption coefficients of 50% to 60% can easily be explained in this model. Even the recently observed higher absorption coefficients in large sunspots can be explained. As a matter of fact, we showed that it is possible to have 100% absorption, i.e. that the incident p-mode is completely absorbed by the sunspot. Such a high efficiency is possible in large sunspots with a sufficiently twisted magnetic field. The possibility of total absorption is a consequence of the appearance of ideal quasi-modes in the range of the continuous spectrum of such sunspot plasmas. However, absorption coefficients of 100% have not yet been observed.

The dependency of the fractional absorption on the driving frequency and, in particular, the conditions for maximal and, if possible, total absorption were studied analytically for a 1D cylindrical plasma. It was found that the eigenvalue problem of MHD radiating (or leaky) eigenmodes is the key element for understanding the dependency of resonant absorption efficiency on the driving frequency. The eigenfrequencies of these MHD radiating eigenmodes are complex. Their real part differs only slightly from the real part of the classic non-radiating eigenmodes (unless there is very strong outgoing MHD radiation). Because of the outgoing radiation the eigenmodes are damped and the eigenvalues have an imaginary part. A resonance in ideal MHD can cause additional damping. Driving at frequencies that are equal or close to the real part of these MHD radiating eigenmodes produces substantial absorption. The driving frequencies producing the maximal fractional absorption are not exactly equal to but differ (very) slightly from the real parts of the eigenvalues of the MHD radiating eigenmodes. Total absorption occurs if in the radiating eigenvalue problem the damping rate due to the resonance equals the damping rate due to the outgoing radiation. If this condition is satisfied 100% absorption occurs even for thin non-uniform layers. Analytical expressions for the solutions in the neighbourhood of the resonant layer have enabled us to understand the typical change in the spatial behaviour of the solutions around a maximal fractional absorption. The behaviour of the spatial wave solutions is determined by the variation of the real and imaginary parts of the dispersion relation of the MHD radiating eigenmodes for real frequencies. For a driving frequency far enough away from the oscillation frequency of an MHD radiating eigenmode both the real part (d_v) and the imaginary part (d_i) differ from zero and $Re\xi_r$ and $Im\xi_r$ (ξ_r is the radial component of the Lagrangian displacement) have $\ell n|s|$ and $H(s)$ contributions. Since, in general, d_r is in absolute value much larger than $d_i, Re\xi_r$ is characterized by a dominant $\ell n|s|$ singularity and $Im\xi_r$ by a dominant $H(s)$ contribution. When the frequency of the incoming wave equals the oscillation frequency of an MHD radiating eigenmode d_r vanishes and $Re\xi_r$ has only a $H(s)$ contribution and $Im\xi_r$ a $\ell n|s|$ singularity. Because of continuity there is an interval around this oscillation frequency in which the spatial wave solutions undergo a rapid change as a function of the driving frequency. This method based on jump conditions and integration of the ideal MHD equations is a powerful means for understanding and interpreting results of large scale simulations in non-ideal MHD. Furthermore this method has now been used to recover the results by Goossens and Poedts for sunspots with straight magnetic fields.

3d. Coronal heating by resonant absorption in line-tied coronal loops

G. Halberstadt, J.P. Goedbloed, S. Poedts, R.A.M. Van der Linden

The solar corona consists of tenuous plasma that is highly structured by the solar magnetic field. One generally distinguishes between open magnetic field regions and magnetically closed coronal regions, that consist of magnetic loops with a length that is of the order of the solar radius. The magnetic field lines of the tenuous loops enter the much denser photosphere, which therefore constrains the motion of the field lines at the footpoints of the loop, i.e., coronal loops are effectively *line-tied* to the photosphere.

One of the most striking features of the corona is its extraordinarily high temperature of a few million degrees, which is several orders of magnitude higher than the photospheric temperature. After this was discovered in 1946, it has become clear more recently that virtually all stars have a hot X-ray emitting corona. During the past thirty years extensive research has been carried out to find the physical mechanisms that may transport energy into a corona, and finally heat it by dissipation. Although several viable heating mechanisms have been proposed, most of them are ill-understood and the fundamental question why the corona is so hot still lacks a conclusive answer and constitutes one of the most challenging problems in present day solar physics.

The magnetic pressure in the corona is much higher than the gas pressure, so that most physical processes are dominated by the magnetic field. Hence, one expects the coronal heating mechanism to be magnetic in nature. However, for typical solar coronal loop parameters (radius $a \sim 10^4$ km, Alfvén speed $v_A \sim 6 \times 10^3$ km/s and resistivity $\eta \sim 10^{-6}\Omega \cdot \mathrm{m}$) the magnetic Reynolds number $R_m := \mu_0 v_A a/\eta$, which reflects the magnitude of the ideal terms compared to the dissipative terms in the MHD equations, is very high: about 10^{12}. Consequently, the dissipation in the coronal plasma due to processes that take place on the indicated length scale is by far insufficient to explain the coronal temperature. Theoretical remedies that considerably decrease the magnetic Reynolds number are provided by theories in which the resistivity is enhanced by anomalous effects, by considering processes that take place on much smaller length scales. The current coronal heating research concentrates on two viable magnetic heating mechanisms: dissipation of static magnetic energy by current sheet formation (i.e., small length scales) and subsequent heating during reconnection, and magnetic wave heating by resonant absorption.

We investigate coronal heating by resonant absorption. This process relies on the existence of Alfvén continuum modes in ideal MHD. The importance of these modes for coronal heating stems from the fact that they behave singular on a single flux surface and yield finite dissipation that becomes independent of the plasma resistivity η, as $\eta \rightarrow 0$. Hence, even for the very low plasma resistivity encountered in coronae, resonant absorption yields a finite dissipation, and relatively ill-understood effects that might enhance η can, in principle, be left out of the consideration.

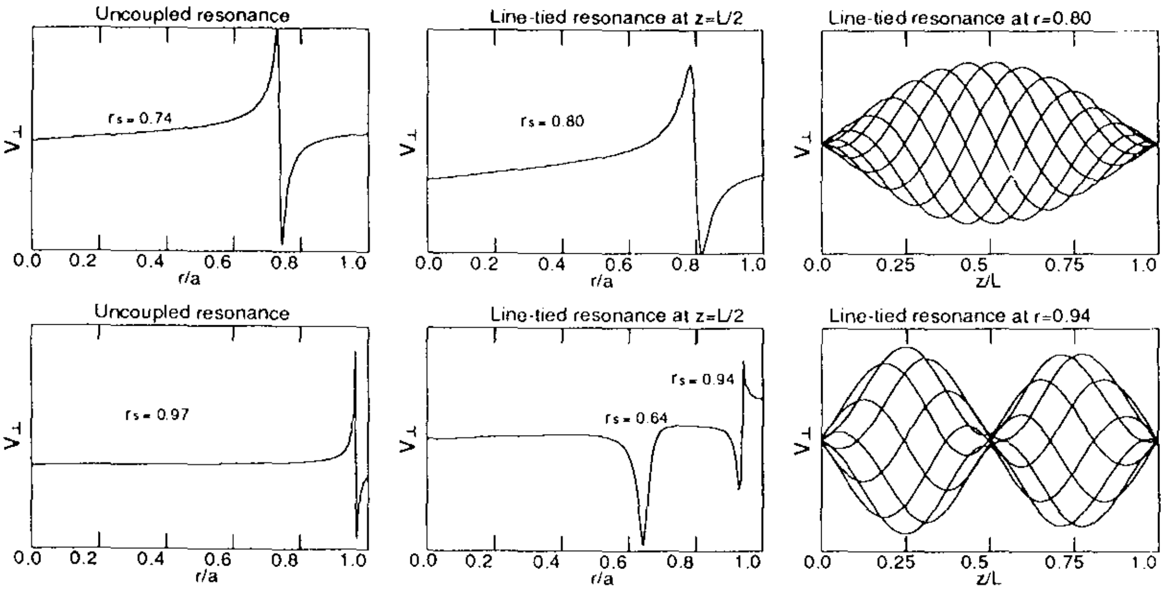

Figure 4: Perturbations $v_{1\perp}$ for two typical driving frequencies exciting an $n = 1$ (upper half) and an $n = 2$ (lower half) line-tied continuum mode in a solar coronal loop.

A physical picture of resonant loop heating that one could have in mind is that the convective motion of the atmosphere excites Alfvén continuum waves at the footpoints of coronal loops. These waves are then resonantly absorbed and effectively dissipated in very thin layers in the loop and thereby heat the corona. So far, most research on resonant heating has been restricted to one dimensional models of infinite plasma cylinders. However, solar coronal loops are constrained by the photosphere and in order to determine the importance of resonant absorption for coronal heating, it seems inevitable to take the photospheric physics into account: the energy source for coronal heating resides in the photosphere and the waves that are excited have to satisfy the appropriate line-tying boundary conditions at the photosphere. Therefore, we work on a model in which the constraints imposed by the photosphere are taken into account. Our numerical code POLLUX models a coronal loop as a straight plasma cylinder *with* inclusion of the end effects, which makes the resulting problem intrinsically two dimensional.

It is obvious that the axisymmetric Alfvén continuum modes that occur in unconstrained cylinders, cannot remain unchanged if a line-tying boundary constraint is imposed, and one might even wonder whether the one dimensional Alfvén continuum survives the line-tying boundary conditions at all. Alfvén continuum modes are restricted to a single flux surface and with this basic ingredient we first carried out a one dimensional analysis, based on the ideal MHD variational principle, in order to determine the Alfvén continuum frequencies in a line-tied cylinder. In this analysis, the MHD energy of a line-tied plasma cylinder was evaluated on each flux surface separately. The Alfvén continuum frequencies were then found by solving the resulting eigenvalue problem. This analysis showed that the Alfvén continuum as it exists in an unconstrained cylinder no longer exists in a line-tied cylinder. Instead, an entirely new continuum arises, the line-tied continuum, which has the surprising feature of being independent of the poloidal magnetic field strength. In relation to this, the line-tied continuum waves have a global ballooning character that is absent in unconstrained cylinders.

The next step was to carry out a two dimensional calculation of resonant absorption with POLLUX, which obviously requires a model for the external excitation source. In a first approach we considered excitation of Alfvén continuum waves in the loop by waves that impinge on the surface of the loop, whereas the photospheric velocity field was constrained to zero, in order to account for the photospheric line-tying effect. This approach not only provides a robust means of mapping the internal absorption spectrum, but theories in which this kind of excitation is considered actually exist. Originally, POLLUX has been developed as an eigenvalue code which solves the generalised eigenvalue problem $\mathbf{Ax} = \lambda\mathbf{Bx}$. The explicit form of the matrices $\mathbf{A}$ and $\mathbf{B}$ depends on the equilibrium configuration and $\mathbf{x}$ is an eigenmode with eigenvalue λ. In the present problem, the excitation source enters as a forcing term and the resulting problem that is solved by POLLUX reads $\mathbf{Ax} - i\omega_e\mathbf{Bx} = \mathbf{f}$, where $\mathbf{f}$ represents the excitation source and ω_e is the excitation frequency. These calculations can be viewed as the direct 2D generalisation of the extensive 1D calculations of Poedts et al. 1990 (Poedts, S., Goossens, M., Kerner, W. 1990, Comp. Phys. Comm. **59**, 75–84.). Our results fully recovered the results of the single flux surface analysis discussed above: the two dimensional POLLUX calculations show that the Alfvén continuum of a line-tied coronal loop is independent of the poloidal mode number and the poloidal magnetic field strength, and the plasma response clearly exhibits the ballooning character of the waves. This ballooning character of the modes led us to use a mode representation that involves a ballooning factor that is determined by the poloidal magnetic field. Indeed, this turned out to be a much more favourable representation than the ordinary Fourier representation, which considerably decreases the amount of required computer memory.

In a second approach, we modelled the excitation source by imposing the velocity field at the footpoints of the loop. In our calculations these velocity fields are arbitrary, but we expect that in the near future both the excitation spectrum and the photospheric velocity fields will be provided by observation. Our calculations with footpoint excitation revealed the importance of the line-tied continuum for resonant absorption as a coronal heating mechanism: footpoint excitation yields resonances which correspond with the line-tied spectrum, as originally provided by our one dimensional analysis, and the corresponding modes have the salient ballooning character. Moreover, as long as the photospheric velocity field is regular, the boundary constraint only affects the regular part of the solution and leaves the singular part unchanged, so that this result applies to *any* regular velocity field. Hence, resonant heating of footpoint excited loops is due to the line-tied Alfvén continuum and the associated global ballooning modes. These results, including a precise calculation of the absorption rates of the excited waves are being prepared for publication. So far, the features of the Alfvén waves that occur in footpoint excited loops have been established and the absorption spectrum can be calculated. Our future work will concentrate on a more sophisticated model of the photosphere, so that in addition to the absorption spectrum, the effectiveness of the coupling between the excitation source and the loop can be calculated. This will both show to what extent resonant absorption contributes to coronal heating and lead to a better understanding of resonant absorption in 2D systems in which the co-ordinate along the field lines is not ignorable.

3e. Generation of MHD waves by footpoint motion

R. Scheurwater, M. Goossens

The wave equations of ideal MHD are solved for a uniform slab of magnetised plasma, which is perturbed by footpoint motion at the boundary of the slab. Solutions are obtained for a semi-infinite slab, occupying the half-space $z \geq 0$ and driven at the boundary $z = 0$, as well as a finite slab, occupying the region $0 \leq z \leq H$ and driven at the boundaries $z = 0$ and $z = H$. The efficiencies for generation of Alfvén waves and of fast and slow magnetosonic waves are calculated as a function of the prescribed boundary values at the footpoints and of the plasma parameter β (the ratio of gas pressure to magnetic pressure). The effects of viscosity and resistivity on the ideal wave solutions are investigated. The wave solutions are not only damped but also modified by the contribution of evanescent modes so as to form a dissipative boundary layer at the footpoints. The thickness of the boundary layer is just the Alfvén wave length divided by the characteristic Hartmann number. As long as the Hartmann number is much larger than $1/\beta^{1/2}$, the efficiencies obtained from the ideal wave solutions are not modified by dissipative effects.

The boundary value approach is generalised to investigate phase-mixing and resonant absorption of Alfvén waves in a non-uniform slab of magnetised plasma, which is perturbed by footpoint motions. In an attempt to make the mathematical treatment as simple as possible a linear variation of the square of the Alfvén velocity in the direction normal to the magnetic surfaces (i.e. along the x-as, with the magnetic field along the z-as) is considered. The class of perturbations is limited to perturbations that are incompressible and invariant along the y-axis. For these perturbations the visco-resistive MHD equations can be reduced to two uncoupled differential equations governing resonant absorption and phase mixing respectively. Special attention has been given to phase mixing. The dependency on height z of the Ohmic heating by phase mixing is given by a product of two factors. The first factor is a quadratic and increasing function of z, while the second factor is an exponential function with negative argument. Hence there is always a well defined height z_0 where the Ohmic heating is maximal. This critical height depends on the characteristics of the equilibrium and of the footpoint motions and on the value of resistivity and viscosity.

4. Ideal and dissipative instabilities in astrophysical plasmas

The ideal and dissipative instabilities of astrophysical plasmas were extensively investigated. Special attention was given to thermal instabilities. One line of research is devoted to a better understanding of the initial, linear phase of thermal instabilities. In particular, we want to study how such a thermal instability can form prominences in the solar corona. Prominences are large, long-lived phenomena in the atmosphere of the sun. They are characterised by a high density and a low temperature as compared to the solar corona. Prominences are supported against gravity and fixed in space by a strong magnetic field. This magnetic field also serves to shield the prominences against thermal influx from the surrounding corona. The formation of prominences is often attributed to an instability in the thermal equilibrium of the coronal plasma, the 'thermal instability'. This mechanism is also thought to be the cause of the formation of similar condensations in tokamak edge plasma ('marfes', see section 5c). We have therefore conducted a detailed study of the linear phase of this thermal instability, using a normal mode approach. The results of these investigations are reported in sections 4a–4e. We also investigated the effect of line-tying on the stability of solar coronal loops (section 4f) and the effect of equilibrium flows on waves and instabilities (section 4g).

4a. The thermal continuum

R.A.M. Van der Linden, M. Goossens

The normal mode spectrum of a system may contain both discrete solutions and continuous subspectra. As is well-known, in ideal MHD there exist two continuous subspectra (slow and Alfvén), linked to a mobile regular singularity in the linearised differential equations. We have shown that when thermal conduction, plasma heating and optically thin plasma radiation are included, a new continuum arises when conduction across the field lines is neglected. This new continuum (called the thermal continuum) is linked to the slow continuum (which becomes complex rather than purely oscillatory). We have shown that because of the existence of this thermal continuum, instability criteria can be derived for general one-dimensional equilibria.

When perpendicular conduction across the field lines is included, the thermal continuum is replaced by a closely spaced set of discrete solutions, but since the coefficient of thermal conduction across the field lines is very small, the dissipative broadening of the singularity is very small. The eigenmodes remain nearly singular and are therefore called quasi-continuum modes, as the continuum characteristics are still present. The broadening of the singular layer results in strongly localised condensations, with a width proportional to the fourth root of the perpendicular conduction coefficient. This is of obvious relevance for the fine-structure of prominences (see below).

4b. Prominence fine-structure

R.A.M. Van der Linden, M. Goossens

Observations show that virtually all prominences are characterised by a very pronounced fine-structure in the form of many (apparently vertical) 'threads'. Typical sizes of these threads are below 1000 km, while latest observations suggest sizes even below 100 km.

There still is no definite explanation for the formation of such fine-structure. We have however already shown previously that the interplay of magnetic fields and thermal conduction automatically leads to fine-structure along the field lines in the thermal instability eigenfunctions. This is a consequence of the fact that thermal conduction perpendicular to the magnetic field lines is strongly reduced in a highly magnetised plasma as the corona. Condensations formed due to the thermal instability mechanism in a strongly magnetised plasma must therefore show similar fine-structure. The idea that perpendicular conduction could create prominence fine-structure has always been rejected on the basis of an oversimplified order-of-magnitude estimate of the length scales involved. This estimate suggests that the length scale of the fine-structure scales like the square root of the coefficient of perpendicular conduction. We have shown however that the actual scale in *the most unstable mode* is proportional to the *fourth root* of the conduction coefficient. This implies that for typically coronal plasmas, the resulting length scale is actually pretty close to the observed values.

4c. Interaction of magnetic and thermal instabilities

R.A.M. Van der Linden, M. Goossens (in collaboration with A.W. Hood)

In many studies of the thermal instability, the magnetic field is considered to be of secondary importance only. However, we have found that magnetic and thermal effects can become strongly interrelated. We have therefore studied the coupling of magnetic and thermal instabilities in more detail. For simplicity, perpendicular thermal conduction has so far been neglected in this study.

Mathematically, some strong mutual influence was to be expected whenever the time scales of both types of instabilities are of the same order. In that case, the discrete magnetic instabilities may be situated in the region of the thermal continuum. The singular nature of this continuum then leads to strong interaction. We have studied a few typical cases and found that the mutual influence is indeed very drastic. Varying the wave numbers makes the magnetic and thermal instability growth rates change (also relative to each other), and allowed us to study the interaction in more detail. We found that at specific critical wave numbers, the thermal and magnetic instabilities of the same radial order (i.e. with the same mode number) coalesce and form a complex conjugate pair of *overstably oscillating modes.* This emphasises the necessity to consider both types of instabilities simultaneously.

These results have been submitted for publication.

4d. Magnetothermal stability of line-tied coronal loops

R.A.M. Van der Linden, M. Goossens

For a plasma with typically coronal properties, the time scale for magnetic instabilities (essentially the Alfvén transit time) is in general much shorter than the time scale for thermal instabilities. This implies that prominences can only be formed by condensation in a coronal equilibrium if this equilibrium is magnetically stable. Otherwise, the magnetic structure would be disrupted long before the condensation has had time to develop. We are thus led to a 'stability paradox' because the coronal plasma needs to be thermally unstable and yet magnetically stable. We have shown that this paradox can be resolved by including the anchoring of the magnetic footpoints to the solar photosphere (line-tying). It has been shown by several authors that line-tying exerts a strong stabilising influence on coronal magnetic structures. In particular, magnetic instabilities can be completely eliminated if the magnetic field lines are sufficiently short.

Of course, also the thermal instability is influenced by line-tying. But we have demonstrated for a typical coronal loop model that the stabilising influence of line-tying on the thermal instability is much less pronounced than on the magnetic instability. Complete elimination of the thermal instability requires much shorter field lines than is needed for magnetic stability. This finding supports a simple heuristic model for prominence formation and eruption due to a thermal and a magnetic instability respectively. In this model, the evolution starts when a bipolar magnetic region has just emerged. The field lines are considered straight and sufficiently short to give complete magnetothermal stability. Further expansion of the magnetic field lines combined with random footpoint motions stretch the field lines until the thermal marginal stability point is crossed. A thermal instability sets in, and a condensation

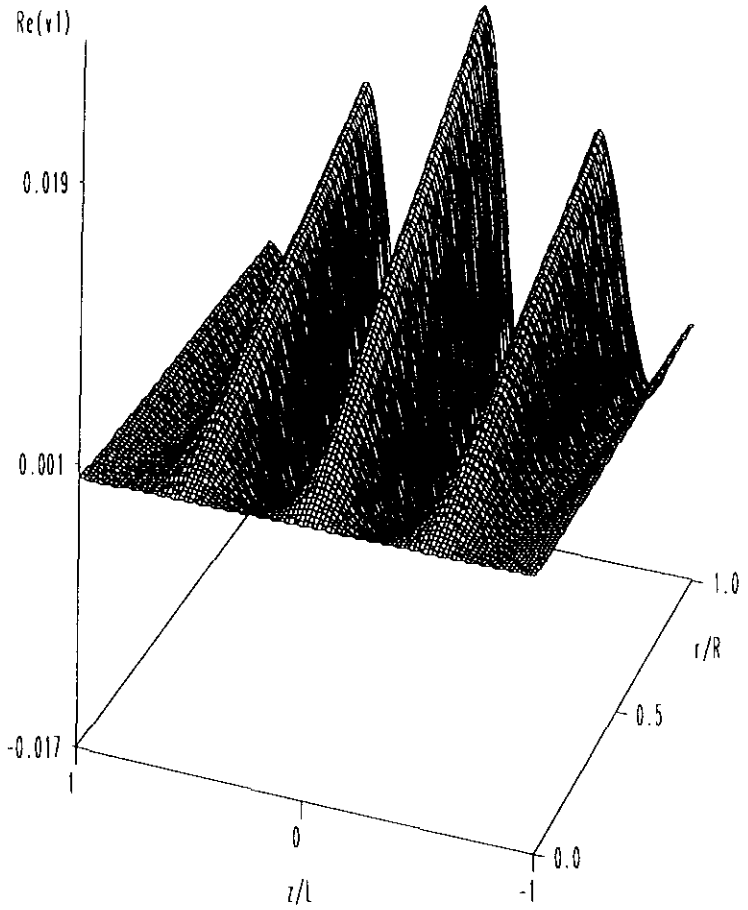

Figure 5: Three-dimensional view of a global $m = 1$ line-tied kink mode in a solar coronal loop.

forms over the magnetic neutral line provided the magnetic structure can support it. During and after prominence formation, line-tying still dominates in the magnetic stability, which explains the long lifetimes of prominences. Further footpoint motions, and field line stretching due to the (observed) ascending motion of the prominence, ultimately render the magnetic structure unstable, and a fastly growing magnetic disruption destroys the prominence.

4e. Non-adiabatic discrete Alfvén waves and thermal instabilities

R. Keppens, M. Goossens, R.A.M. Van der Linden

Goedbloed has shown that in ideal MHD the Alfvénic sub-spectrum can contain discrete modes with regular eigenfunctions in addition to the well-known Alfvén continuum of singular eigenmodes if the equilibrium magnetic field has twist. These discrete modes have eigenvalues below and/or above the Alfvén continuum and a WKB-analysis can be used to obtain the dispersion relation for discrete Alfvén waves of high radial order and frequencies close to an internal extremum of the Alfvén continuum. These discrete Alfvén waves can become damped or overstable when non-ideal effects are included, as shown by a WKB-analysis by Van der Linden and Goossens. Their possible overstability makes these modes relevant for explaining oscillations in prominences and the onset of magnetically driven disruptions. A WKB-analysis only applies to modes of high radial order and does not give any information on the modes of low radial order. A numerical study of discrete non-adiabatic Alfvén waves was carried out for 1D cylindrical equilibrium states with twisted magnetic fields. The numerical results for modes of high radial order were in good agreement with the analytical predictions on the basis of the WKB-analysis. But, more importantly, the numerical results showed that the low order modes were the most overstable and, consequently physically the most important ones. Moreover, overstable modes of low order were found when the WKB-analysis correctly predicted damping for the modes of high radial order. These results show that discrete Alfvén waves can be important for solar applications and also that the applicability of the WKB-analysis is limited in this context.

A numerical study of the thermal sub-spectrum was carried out. This thermal sub-spectrum has a continuous branch if perpendicular thermal conduction is neglected (see Van der Linden, Goossens and Goedbloed). When perpendicular thermal conduction is included this continuum is replaced by many discrete modes which still show strong traces of the continuum modes. For this reason these discrete modes are called quasi-continuum modes. The aim of the numerical study was to extend the work by Van der Linden and Goossens and to see if there exist quasi-continuum modes with centrally concentrated eigenfunctions for realistic equilibrium models of coronal loops. Although incomplete, the numerical survey was successful in the sense that such an equilibrium state was indeed found. These results add further support to the suggestion by Van der Linden and Goossens for explaining the fine-scale structure in prominences.

4f. On the photospheric boundary conditions of solar coronal loops

J.P. Goedbloed, G. Halberstadt, R.A.M. Van der Linden

Twisted coronal magnetic flux loops emanating from the solar photosphere are manifestly more stable than toroidal flux loops in the laboratory (e.g. tokamaks). This is usually attributed to the large density increase at the photosphere which would give rise to the so-called line-tying boundary conditions. However, it is easy to see that the stability of coronal loops is strictly unaffected by the density of the photosphere: the energy principle does not involve the plasma density. The usual line-tying boundary condition of vanishing plasma displacement at the photosphere is in fact very stabilising, but it is not equivalent to having a large photospheric density. The two viewpoints may be reconciled by exploiting a σ-stability criterion in which stability is redefined as absence of instabilities exponentiating faster than $e^{\sigma t}$. The appropriate energy principle is then given by the so-called modified energy principle in which the photospheric density does appear. The resulting stability picture becomes more involved though than the usual one as has been demonstrated by the results of a recently developed 2D eigenvalue code computing the MHD modes of a flux loop with density variations along the loop.

4g. Waves and instabilities in equilibrium states with flows

M. Goossens (in collaboration with J.V. Hollweg and T. Sakurai)

The resonances that appear in the linear compressible MHD formulation of waves are studied for equilibrium states with flow. The conservation laws and the jump conditions at the Alfvén and the slow resonance points obtained by Sakurai, Goossens and Hollweg (1991) for static equilibrium states have been generalised to include an equilibrium flow. For equilibrium states with straight magnetic field lines and equilibrium flow along the magnetic field lines the fundamental conserved quantity is the Eulerian perturbation of the total pressure. Curvature of the magnetic field lines and/or velocity field lines leads to more complicated conservation laws. Rewritten in terms of the displacement components in the magnetic surfaces parallel and perpendicular to the magnetic field lines, the conservation laws simply state that the waves are dominated by the parallel motions for the modified slow resonance and by the perpendicular motions for the modified Alfvén resonance.

The conservation laws and the jump conditions are first used for studying surface waves in cylindrical plasmas. These surface waves are characterised by resonances and have complex eigenfrequencies when the classic true discontinuity is replaced by a non-uniform layer. A thin non-uniform layer is considered in an attempt to obtain analytical results. An important result related to earlier work by Hollweg et al. (1990) for incompressible planar plasmas is found for equilibrium states with straight magnetic field lines and straight velocity field lines. For these equilibrium states the incompressible and compressible surface waves have the same eigenvalues, at least in the long wave length (or thin flux tube) approximation, and there is an exact correspondence with the planar case. As a consequence, the conclusions formulated by Hollweg et al. on resonant overstability also hold for the straight cylindrical case. For equilibrium states with curved magnetic field lines and/or curved velocity field lines the dispersion relation becomes rather complicated and numerical computation for specific equilibrium profiles is required in order to formulate statements on the eigenvalues and possible resonant overstability.

The conserved quantities and jump conditions are then used for studying the effects of a mass flow on the resonant absorption of MHD waves on magnetic flux tubes. In a first attempt the attention was focused on an equilibrium that is uniform apart from a "thin" non-uniform layer. In this "thin" layer the equilibrium quantities vary from their constant values inside the flux tube to their constant values outside the flux tube. The variation of the fractional absorption is studied as a function of the frequency of the incoming wave and its relation to the eigenvalue problem of the

MHD radiating eigenmodes of a stationary non-uniform flux tube is established. The optimal frequencies producing maximal absorption are determined and identified as frequencies that slightly differ from the oscillation frequencies of the MHD radiating eigenmodes. The condition for total absorption is obtained and this condition defines an impedance matching which is fulfilled for an equilibrium that is fine tuned with respect to the incoming wave. The numerical results show that a modest equilibrium flow with a velocity of only 10% of the Alfvén velocity can substantially change fractional absorption for frequencies around the optimal frequencies.

An equilibrium flow can not only change the absorption of MHD waves but can also cause over reflection where energy from the equilibrium flow is fed into the reflected wave.

5. Ideal and dissipative spectrum of tokamak plasmas

The investigations on Alfvén wave heating and dissipative instabilities of astrophysical plasmas carried out in the twinning project yielded some spin-off for the field of thermonuclear plasmas, i.e. the same numerical techniques and sometimes even the same numerical codes (with minor modifications, e.g. of the boundary conditions) were applied for the study of similar or related MHD phenomena in laboratory plasmas. The results of these investigations are reported in the present section.

5a. Alfvén spectrum of toroidal plasmas

W. Kerner, J.P. Goedbloed, G.T.A. Huysmans, S. Poedts and E. Schwarz

A new normal mode code CASTOR (Complex **A**lfvén **S**pectrum for **TOR**oidal plasmas) was developed and tested. The aim is to compute both the ideal and resistive MHD spectra of axisymmetric toroidal confinement systems and, in particular, to determine the ideal and resistive MHD stability of such systems. The full (unreduced) set of resistive MHD equations is solved in a general non-orthogonal flux co-ordinate system. A combination of cubic Hermite and quadratic finite elements is employed for the discretization in the radial direction and a Fourier expansion in the poloidal co-ordinate.

CASTOR requires a mapping of a 2D equilibrium (*i.e.* with two directions of inhomogeneity) as input. In the initial phase, both the equilibrium and the mapping were obtained with existing Garching codes. Now, the equilibrium and the corresponding mapping are calculated by means of HELENA (see section 2d). The growth rates of unstable internal modes have been computed with an inverse vector iteration and instability curves have been reconstructed successfully for cylindrical plasmas with elliptical cross-section and for analytical (Soloviev) tokamak equilibria. For those cases where there is an unequivocal result available in the literature, convergence studies showed that these growth rates are approximated with great accuracy. Next, the stability of specific JET discharges was analysed. Both ideal and resistive instabilities were found. The normal mode code CASTOR and the first results of these investigations have been presented at the Sherwood conference[5]. A further publication is being prepared at the moment.

5b. External resistive modes in tokamaks

G.T.A. Huysmans, J.P. Goedbloed, W. Kerner

Most studies of resistive instabilities in a general toroidal geometry have been limited to modes which do not perturb the plasma boundary so that a number of interesting experimental and theoretical problems have not been studied. The main theme of our recent research has been the stability of free boundary resistive modes in a fully toroidal geometry. The stability of these modes has not yet been investigated in a fully toroidal geometry, without any ordering in pressure, aspect ratio, or resistivity. The influence of the pressure was investigated and a comparison was made with the stability of fixed boundary resistive modes. It was shown that the stabilising effect of the pressure due to the plasma compression of the fixed boundary modes is lost for the free boundary modes localised near the boundary. Since the stabilisation due to the favourable average curvature in combination with a pressure gradient is small, the influence of the pressure on the stability is much less important for the free boundary modes than for the fixed boundary modes. Subsequently, we have studied the effect of an X-point plasma shape as compared to a circular plasma boundary. The stabilising effect of the X-point, known for ideal free boundary modes, was found even stronger for the resistive modes if the rational q surface lies outside the plasma. The stabilising effect of the X-point is less efficient if the rational surface lies inside the plasma.

In recent years, diagnostics have become available for the measurement of the equilibrium profiles of, for example, the pressure and the safety factor. This allows for the reconstruction of the plasma equilibrium with a reasonable degree of accuracy. This in turn allows for the calculation of the MHD stability properties of the observed MHD instabilities.

The edge localised modes as observed during the H-mode are one example of an observed instability where the free boundary resistive mode can be important. We have analysed the stability of the $n = 1$ resistive free boundary mode of a reconstructed equilibrium of an H-mode discharge in the JET tokamak. It was shown that for the resistive free boundary mode to become significantly unstable, the current density gradient at the edge must be a factor of two larger as compared to the reconstructed equilibrium. Local details of the resistivity and of the current density profile are extremely important for the stability of these modes. The influence of the edge pressure gradient on the stability of the $n = 1$ free boundary resistive mode is small, but it may drive bootstrap currents at the plasma edge causing a local increase of the current density gradient. Knowledge on the current distribution has become available at JET, however, not in sufficient detail at present to prove or disprove the presence of local current gradients large enough to drive external resistive modes. Consequently, rather than confirming experimental results, the calculations presented are necessarily of a predictive nature, deriving conditions under which external resistive modes can be expected in tokamaks.

5c. Marfes

R.A.M. Van der Linden, M. Goossens, W. Kerner

The mechanism of thermal instability may also be invoked for the formation of MARFES, i.e. edge-localised plasma condensations often observed in tokamak discharges. Actually, the result of our astrophysical study (see sections 4b and 4c) also applies to this specific case. Evidently the magnetic field lines are not anchored now to the solar surface. However, limiters constrain the plasma perturbations in a similar way.

The most obvious similarity is to be found in the fine-structure of the condensations. An inspection of the observational data has indeed shown that there is a small-scale structure also inside MARFES. On the other hand, the magnetic data appear to reveal a magnetic perturbation, neglected so far, which coincides with the MARFE phenomenon. This refers to the above statement that magnetic and thermal instabilities should not be studied separately. Since the onset of MARFES has so far been modelled only to a fairly rudimentary extent, it is certainly worthwhile to apply our analytical results (e.g. the conclusions concerning the thermal continuum), as well as the numerical codes CASTOR, LEDA, and POLLUX to the tokamak geometry.

This has opened up a highly interesting line for new research, in which the results of the thermal instability calculations can now be checked to experimental rather than observational evidence.

Taking density and temperature profiles that are typical for the JET configuration, and using the code LEDA (in first instance), the thermal stability of the discharge plasma has been studied. It was found that the tokamak edge plasma (which is where the MARFE is seen to form) is thermally unstable, with the fastest growing mode being poloidally and toroidally symmetric. The time scale of this instability agrees with the observed growth time of marfes.

5d. Continuous spectrum of toroidal, axisymmetric plasmas

S. Poedts, W. Kerner, E. Schwarz, J.P. Goedbloed, G.T.A. Huysmans

We used CASTOR for the spectral analysis of 2D fusion plasmas. During the second year of the twinning project special attention was given to the continuous part of the ideal MHD spectrum of tokamak plasmas. The ideal MHD continuous spectrum is essential for the understanding of the structure of the entire spectrum since it serves as the only location of possible accumulation points. Also, knowledge of the internal structure of the continuous spectrum, and hence of the location of the singularities, is essential for accurately solving the MHD equations. In addition, the resonant absorption of Alfvén continuum waves is a possible scheme for plasma heating and could be used for diagnostic purposes. CSCAS turned out to be an important tool

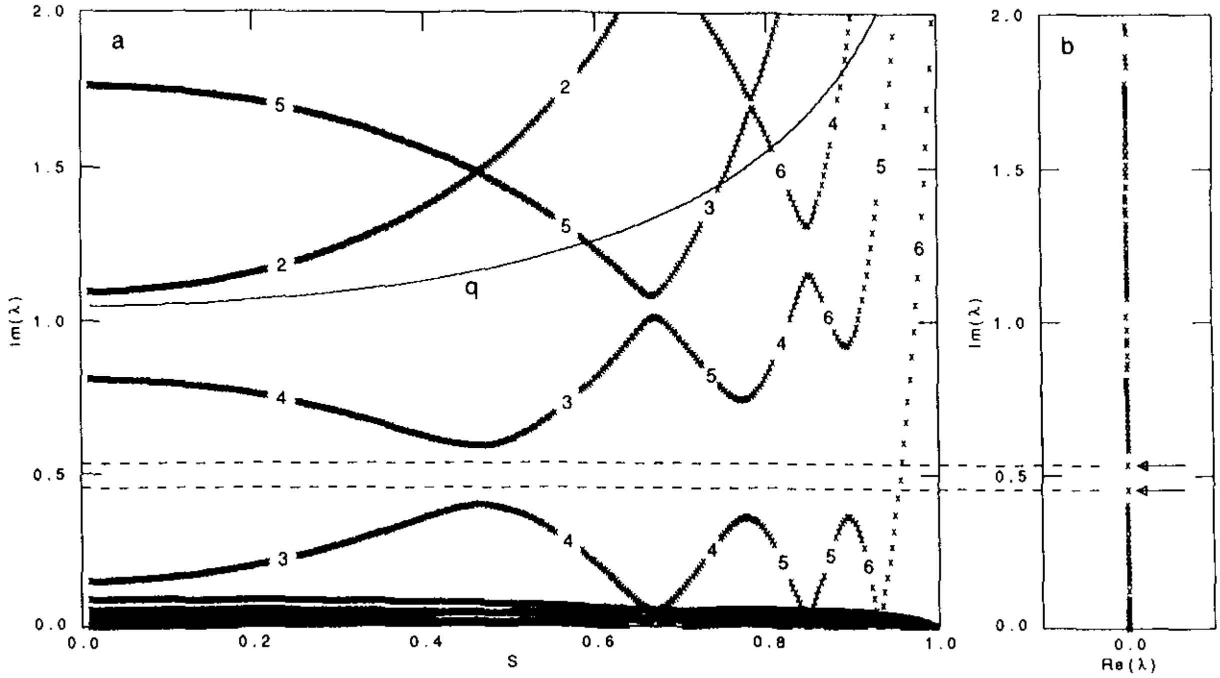

Figure 6: a) Typical structure of the ideal MHD continuous spectrum of a tokamak plasma with a circular cross-section; b) Corresponding part of the full ideal MHD spectrum obtained by using CASTOR.

for the study of the continuous part of the ideal MHD spectrum. Poloidal mode coupling in finite aspect ratio tokamaks causes 'avoided crossings' and this yields 'gaps' in the Alfvén continuum. In other words, these gaps are due to toroidal effects which remove the degeneracies that occur in one-dimensional (cylindrical) plasmas between continuum branches with different poloidal wave numbers m and m' at the rational surface(s) where the safety factor $q = -(m + m')/2n$. The size of these gaps is proportional to the strength of the mode coupling and their appearance stresses the importance of two-dimensional effects, e.g. for Alfvén wave heating, since whole frequency bands that yield resonant absorption in one-dimensional models are not eligible for this heating mechanism in — more realistic — two-dimensional equilibrium models.

5e. Toroidicity-induced Alfvén modes

S. Poedts, W. Kerner, E. Schwarz, J.P. Goedbloed, G.T.A. Huysmans

CSCAS also turned out to be a useful tool for the study of toroidicity-induced Alfvén eigenmodes. When the poloidal mode coupling is strong enough global Alfvén modes are found with a frequency in the above mentioned gaps. These 'gap modes' might play an important role in controlled thermonuclear fusion as they can be destabilised by the interaction with fusion born α-particles. These α-particles — whose confinement is essential for ignition and hence for the possibility of generating controlled fusion energy — are lost by particle-wave resonances. However, the interaction of these gap modes with ideal continuum modes causes phase-mixing so that these modes are damped by the same resonant absorption mechanism that enables Alfvén wave heating. The important question now is which of the two phenomena — destabilisation by interaction with α-particles or damping by interaction with continuum modes — is dominant. Our experience with plasma heating by resonant absorption made it possible to quantify the damping of the gap modes. The internal structure of the ideal Alfvén continuum is very complex in tokamak plasmas and the gaps that occur at the rational surfaces are 'covered' by one or several continuum branches overlaying the gaps. As a consequence, the gap modes interact with the continuum modes with the same frequency and are damped by phase-mixing. Hence, the gap modes become quasi-modes in ideal MHD similar to the quasi-modes we studied in one-dimensional cylindrical plasmas, but different in nature. Upon studying these modes in resistive MHD with CASTOR, and the stationary state code derived from CASTOR, we were able to show that, for asymptotically small resistivity, the damping of the global gap modes is finite and independent of η. We found cases where the ratio of the real (damping) and imaginary (oscillatory) part of the frequency of the gap modes is of the order of 10% in the ideal MHD limit and we made it plausible that the damping can even be stronger. The first results of these investigations have been reported on the Berlin conference on 'Controlled Fusion and Plasma Physics'[18] and a more extensive paper has been published in Plasma Physics and Controlled Fusion[39].

5f. Toroidicity- and elongation-induced Alfvén modes in JET discharges

W. Kerner, B. Keegan, S. Poedts, J.P. Goedbloed, G.T.A. Huysmans, E. Schwarz

Energetic ions, such as fusion born α-particles, can destabilise global Alfvén modes in tokamaks close to break-even conditions. In order to assess this possibility, the Alfvén spectrum of the JET configuration was studied in detail. In particular, the question whether such eigenmodes with small or without any continuum damping exist should be answered. For this purpose CSCAS and CASTOR were again utilised.

The Alfvén continua and corresponding gaps are displayed in Figure 7, the toroidal mode number is $n = 1$. In comparison with the circular cross-section toka-

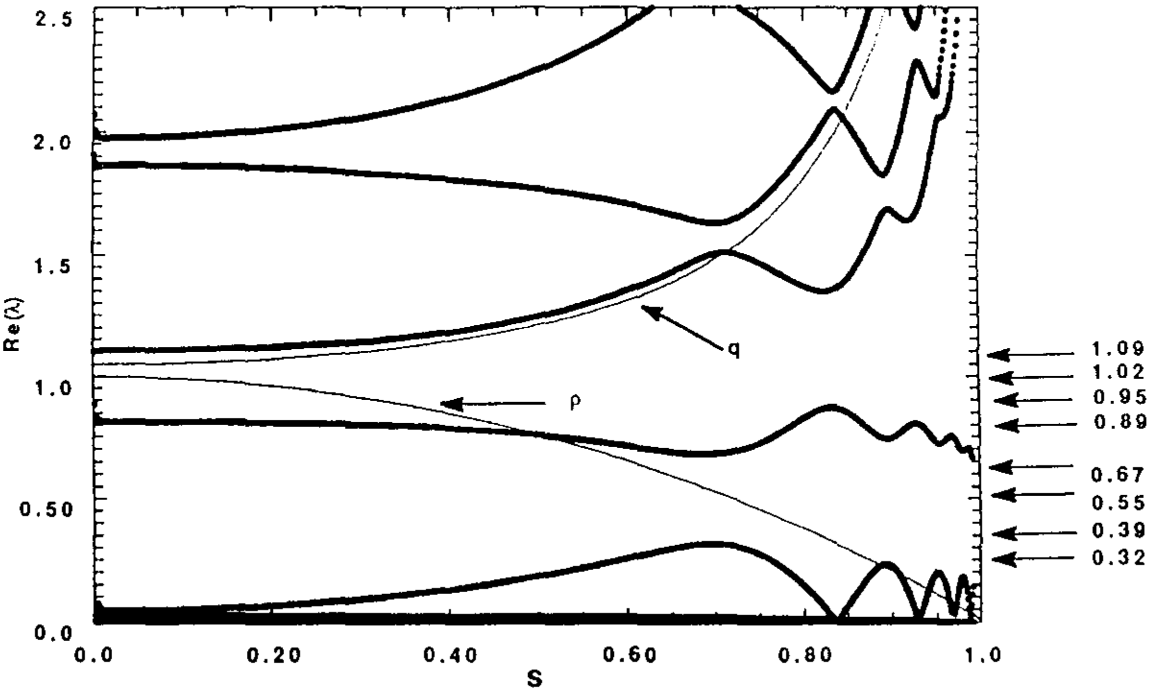

Figure 7: Structure of the ideal MHD Alfvén continuum for a typical JET discharge.

mak it is found that the gaps are wider, especially the elongation and triangularity induced gaps.

For this study of Alfvén eigenmodes in JET discharges we selected a parabolic density profile and reduced the pressure to remove the slow mode continua from the gaps. For each guess of the eigenvalue λ_i, we selected $NG = 101$ radial gridpoints, with nine Fourier harmonics ($m = -1, \ldots 7$) and zero resistivity. Two comprehensive scans of the first and second gap revealed four global modes in each gap. The global modes found are indicated by arrows in Figure 7. It is evident that some of these frequencies overlap with the continua, thereby producing "quasi-modes".

The continuum damping was studied by choosing a constant density profile. The global mode in the lowest gap with $\lambda = 0.3i$ is coupled to the continuum at $s = 0.92$. The damping is determined by introducing resistivity in the limit of asymptotically small η. In this convergence study the number of radial grid points was increased up to $NG = 401$ and up to 13 Fourier harmonics in the poloidal direction were used. The damping $\delta = -Re(\lambda)/Im(\lambda)$ was found to be $\delta = 1.0 \times 10^{-3}$. The damping for the global mode in the second gap with $\lambda = 1.0i$ was found to be considerably higher, namely $\delta = 5 \times 10^{-3}$.

This study is to be continued to higher mode numbers $n = 2, \ldots 10$.

6. Publications

[1] S. Poedts, M. Goossens, W. Kerner: "Numerical simulation of the stationary state of periodically driven coronal loops", *Comp. Phys. Comm.*, **59**, 75–84 (1990).

[2] S. Poedts, M. Goossens, W. Kerner: "Temporal evolution of resonant absorption in solar coronal loops", *Comp. Phys. Comm.*, **59**, 95–103 (1990).

[3] S. Poedts, M. Goossens, W. Kerner: "Coronal loop heating by resonant absorption", *Physics of Magnetic Flux Ropes*, C.T. Russell, E.R. Priest, L.C. Lee (Eds.), 257–262 (1990).

[4] S. Poedts, M. Goossens, W. Kerner: "On the efficiency of coronal loop heating by resonant absorption", *Astrophys. J.*, **360**, 279–287 (1990).

[5] W. Kerner, J. P. Goedbloed, S. Poedts, E. Schwarz: "Resistive modes in tokamaks", *The International Sherwood Theory Meeting*, Williamsburg, Virginia, April 23–25, 1990, poster contribution.

[6] R.A.M. Van der Linden, M. Goossens, W. Kerner: "A Combined Finite Element Fourier Series Method for the Numerical Study of Line-Tied Magnetic Plasmas", *Comp. Phys. Comm.* **59**, 61–73 (1990).

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[22] Poedts, S., and Schwarz, E.: "Computation of the ideal MHD continuous spectrum in axisymmetric plasmas", *J. Comp. Physics*, submitted.

[23] R.A.M. Van der Linden, M. Goossens, and J.P. Goedbloed: "On the existence of a thermal continuum in nonadiabatic magnetohydrodynamic spectra", *Phys. Fluids*, **B3**, 866 (1991).

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7. Visits and Conferences

Visits

H. Buisman

- Department of Mathematical Sciences of the University of St. Andrews, St. Andrews, Scotland, 15/10 - 16/11/90.

J.P. Goedbloed

- IBM Scientific Center, Winchester, U.K., 23 - 24/10/91.
- Max-Planck-Institut für Plasmaphysik, Garching, Germany, 12 - 22/12/89, 28/3 - 20/4/90, 8 - 18/5/90, 10 - 29/03/91, 15 - 16/4/92.
- K.U.Leuven, Heverlee, Belgium, 18 - 19/3/90, 2 - 3/4/91, 12/6/91, 13/9/91, 1 - 2/4/92.

M. Goossens

- Max-Planck-Institut für Plasmaphysik, Garching, Germany, 13 - 15/12/89, and 9 - 11/5/90.
- FOM-Instituut voor Plasmafysica 'Rijnhuizen', Nieuwegein, The Netherlands, 3 - 5/10/90, 24 - 27/11/91.
- Space Science Center, University of New Hampshire, Durham, NH, U.S.A., July 15–August 30, 1991.

G. Halberstadt

- Max-Planck-Institut für Plasmaphysik, Garching, Germany, 12 - 22/12/89, 28/3 - 13/4/90, 12 - 18/5/90, 10 - 29/03/91, 29/07 - 4/08/91.
- K.U.Leuven, Belgium, 18 - 20/3/90, 2 - 3/04/91, 1 - 2/4/92.

G.T.A. Huysmans

- Max-Planck-Institut für Plasmaphysik, Garching, Germany, 12 - 22/12/89.

R. Keppens

- Department of Mathematical Sciences of the University of St. Andrews, St. Andrews, Scotland, October 1–December 23, 1990.

- FOM-Instituut voor Plasmafysica 'Rijnhuizen', Nieuwegein, The Netherlands, 24 – 27/11/91.

B. Keegan

- FOM-Instituut voor Plasmafysica 'Rijnhuizen', Nieuwegein, The Netherlands, 2 – 6/3/92.

W. Kerner

- FOM-Instituut voor Plasmafysica 'Rijnhuizen', Nieuwegein, The Netherlands, 3 – 5/10/90, 24 - 27/11/91, 11 - 12/12/91.

- Max-Planck-Institut für Plasmaphysik, Garching, Germany, 12/2 – 2/3/90, 29 – 30/3/90, 2 – 3/7/90, 11 – 13/7/90, 16/03 – 15/04/91, 22 – 26/07/91, 26 – 27/8/91.

- K.U.Leuven, Heverlee, Belgium, 12 – 14/9/91, 1 – 2/4/92.

S. Poedts

- FOM Instituut voor Plasmafysica, Rijnhuizen, Nieuwegein, The Netherlands, 28/8 – 2/9/90, 3 – 5/10/90, and 16 – 22/12/90.

- JET Joint Undertaking, Culham, England, 20 – 25/5/90, 2 – 7/9/90, and 6 – 20/4/91.

- K.U.Leuven, Heverlee, Belgium, 10 – 22/3/90, 25/5 – 30/6/90, 2 – 3/4/91, 23/6 – 3/7/91, and 1 – 2/4/92.

- High Altitude Observatory / National Center for Atmospheric Research, Boulder, Colorado, U.S.A., 12/10 - 10/11/90.

- Max-Planck-Institut für Plasmaphysik, Garching bei München, Fed. Rep. Germany, 9 – 15/2/92.

E. Schwarz

- FOM-Instituut voor Plasmafysica 'Rijnhuizen', Nieuwegein, The Netherlands, 3 – 5/10/90, 24 – 27/11/91.

- K.U.Leuven, Heverlee, Belgium, 2 – 3/4/91.

H. Stenuit

- Department of Mathematical Sciences of the University of St. Andrews, St. Andrews, Scotland, October 1–December 23, 1990.

- Max-Planck-Institut für Plasmaphysik, Garching, 1 week in January 1991.

- FOM-Instituut voor Plasmafysica 'Rijnhuizen', Nieuwegein, The Netherlands, 24 – 27/11/91.

R.A.M. Van der Linden

- University of St.-Andrews, Scotland, 20–25/11/90.

- FOM-Instituut voor Plasmafysica 'Rijnhuizen', Nieuwegein, The Netherlands, 3 – 5/10/90, 10–21/12/90, 24 – 26/11/91.

- JET Joint Undertaking, Culham, England, 28/1–1/2/91.

- Max Planck Institut für Plasmaphysik, Garching, Germany, 27/11 – 8/12/89, 14 – 15/12/89, 29 – 30/3/90, 10 – 11/5/90, 29/7 – 2/8/91.

- K.U.Leuven, Heverlee, Belgium, 26 – 29/11/91, and 30/3 – 23/4/92.

Conferences

J.P. Goedbloed

- 4th European Workshop on Problems in the Numerical Modelling of Plasmas, Spitzingsee, Germany, 1/10/89 – 5/10/89.

- Conference on Mechanisms of Chromospheric and Coronal Heating, Heidelberg, Germany, 5/6/90 – 8/6/90.

- 16th General Assembly of the European Geophysical Society, Wiesbaden, 21 – 26 April, 1991.

- Gordon conference on Solar Plasma and MHD Processes, Plymouth State College, 4 - 12/8/1991.

- Summer School on non-linear MHD Processes, Goettingen/St. Andreasberg, Germany, 15/9/90 - 21/9/90.

M. Goossens

- 4th European Workshop on Problems in the Numerical Modelling of Plasmas, Spitzingsee, Germany, 1/10/89 - 5/10/89.
- Symposium on Geophysical and Astrophysical MHD, St. Andrews, United Kingdom, 1/4/90 - 5/4/90.
- Heidelberg Conference on Mechanisms of Chromospheric and Coronal Heating, Heidelberg, Germany, 4/6/90 - 8/6/90.
- Summer School on Solar System Plasmas, St. Andrews, United Kingdom, 24/9/90 - 28/9/90.
- 16th General Assembly of the European Geophysical Society, Wiesbaden, 22 - 26 April, 1991.

G. Halberstadt

- Dutch Astronomers Conference, Oost Kapelle, the Netherlands, 10/5/90 - 12/5/90.
- Conference on Mechanisms of Chromospheric and Coronal Heating, Heidelberg, Germany, 5/6/90 - 8/6/90.
- Solar Plasma Physics Summer School, St. Andrews, Scotland, 23/9/90–30/9/90.
- 16th General Assembly of the European Geophysical Society, Wiesbaden, 22 - 26 April, 1991.
- Dutch Astronomers Conference, Lunteren, The Netherlands, 02/05/91 - 04/05/91.
- Summer School on non-linear MHD Processes, Goettingen/St. Andreasberg, Germany, 15/9/90 - 21/9/90.

W. Kerner

- 4th European Workshop on Problems in the Numerical Modelling of Plasmas, Spitzingsee, Germany, 1/10/89 - 5/10/89.

S. Poedts

- 4th European Workshop on Problems in the Numerical Modeling of Plasmas, "Numerical Modeling of Solar and Stellar MHD", Spitzingsee, Germany, 1/10/89-5/10/89

- Heidelberg Conference, "Mechanisms of Chromospheric and Coronal Heating", Heidelberg, Germany, 5/6/90-8/6/90

- European Geophysical Society, XVI General Assembly, Wiesbaden, Fed. Rep. Germany, 22/4/91 - 26/4/91

R. Scheurwater

- Gordon conference on Solar Plasma and MHD Processes, Plymouth State College, August 5 - 9, 1991.

R.A.M. Van der Linden

- 4th European Workshop on Problems in the Numerical Modelling of Plasmas, Spitzingsee, Germany, 1/10/89 - 5/10/89.

- Sup'Eur Fall Meeting 1990, Aachen, Germany, 17/9/90 -19/9/90.

8. Appendices

Appendix 1: Modularization

E. Schwarz, S. Poedts, and J.P. Goedbloed

The codes LEDA, CASTOR and POLLUX have been restructured in modular form, each with its own specific task. The five modules are called MAIN, EQUIL, MAT, SOLV, and DIAG.

- The module MAIN contains the branching calls for the different eigenvalue solvers and it incorporates a built-in test case in the subroutine PRESET.
- In the module EQUIL all equilibrium profiles are calculated.
- The module MAT contains the general subroutines CONAMAT and CONBMAT for the calculation of the matrices **A** and **B**, and the routines MAT1–MAT5 for storage in the appropriate solver-dependent form. All boundary conditions are imposed in the subroutines CONAMAT and CONBMAT.
- The module SOLV contains the various eigenvalue solvers, which are identical for the three codes LEDA, CASTOR, and POLLUX. At present, five solvers are incorporated, viz.:
  SOLV1 (QR-algorithm);
  SOLV2 (Inverse vector iteration, written by J. Steuerwald, with and without separate storage of the matrix **B**);
  SOLV3 (Out-of-core inverse vector iteration, written by E. Schwarz and J. Steuerwald);
  SOLV4 (In-core inverse vector iteration based on the algorithm used in the out-of-core solver, written by R.A.M. Van der Linden and E. Schwarz);
  SOLV5 (Lanczos algorithm, written by J. Cullum of IBM and modified by J. Steuerwald and E. Schwarz).
- The module DIAG contains the subroutines for the diagnostics for the different solvers, for the plotting of the eigenvalues for the QR and Lanczos solvers, and for the plotting of the eigenfunctions for the different inverse vector iteration solvers.

Appendix 2: Eigenvalue solvers

E. Schwarz, R.A.M. Van der Linden, S. Poedts, G. Halberstadt

All three programs now have the five different solvers built-in as well as the corresponding diagnostic subroutines: the QR solver, which solves for the entire spectrum, three inverse vector iteration solvers which differ essentially in the way the matrices are stored, viz. in-core in band storage mode (IC), out-of-core (OOC), and in-core but with the improved efficiency of the OOC solver (ICOOC), and finally the Lanczos solver, which finds all eigenvalues in a given part of the complex plane. The latest versions of these solvers were tested at the three sites (Garching, Leuven, and Nieuwegein). The latest versions of LEDA, CASTOR, and POLLUX are stored in the HADES diskspace at IPP with the following versions of the solvers :

solver	**LEDA 7**	**CASTOR 8**	**POLLUX 8**
QR	Version A (16/7/91) *(real)*	Version C (4/7/91) *(complex)*	Version C (4/7/91) *(complex)*
IC	Version B (4/7/91)	Version B (4/7/91)	Version B (4/7/91)
OOC	Version D (28/8/91)	Version C (4/7/91)	Version E (11/9/91)
ICOOC	Version C (4/7/91)	Version C (4/7/91)	Version C (4/7/91)
Lanczos	Version C (16/7/91)	Version B (4/7/91)	Version C (16/7/91)

Version D of the OOC solver is generalised such that the matrix **B** does not have to be symmetric any more. In addition, in version E of this solver the eigenvector for the plots is stored in EVP. In the Lanczos solver (version C) the call to subroutine DURAND has been changed.

Appendix 3: Parallel I/O

R.A.M. Van der Linden

In its standard version, the practical use of the out-of-core inverse vector iteration solver is jeopardised for systems where the I/O operations produce a substantial time delay. This leads to a dramatic degradation of job performance and to elapse times which are much larger than the actual computing time. Specifically on IBM3090 computers this limits the applicability of this solver. Since also the in-core solvers suffer from severe performance loss for large applications (due to internal 'paging'), using the standard version of the solvers on the IBM3090 can be a fairly uncomfortable task when large matrices are required.

For the IBM3090 in Leuven this problem has been solved by introducing the Parallel I/O Access Method. This creates an excellent environment for jobs doing a lot of I/O operations to external disk space. By using Parallel I/O for all I/O operations in SOLV3, this solver has become very competitive in comparison with its standard version and with the other solvers, even under heavy system load conditions. No longer is the elapse time of the program runs determined by the I/O overhead.

The first tests with Parallel I/O were made in June–August 1990. As these tests turned out to yield very positive results, the method was afterwards introduced as a standard option on the IBM3090 in Leuven. It has been continuously used there since.

The performance improvements obtained by introducing Parallel I/O have been presented at the SUP'EUR Fall '90 meeting, and have been submitted for publication.

Appendix 4: Test cases

E. Schwarz, S. Poedts, G. Halberstadt, R.A.M. Van der Linden

Test cases have been built in for all three programs. These test cases, as well as the solvers, can be selected via a parameter MODE that is read in through the namelist NEWRUN (MODE = number of testcase × 10 + number of solver).

LEDA 7:

nr.	*equilibrium*	*configuration*	*dissipation mechanism*
1	tokamak	plasma-vacuum-wall	resistivity
2	Gold-Hoyle	plasma-wall	thermal cond. (IKAPP=1)
3	Gold-Hoyle	plasma-wall	thermal cond. (IKAPP=2)
4	Gold-Hoyle	plasma-vacuum-wall	thermal cond.
5	Gold-Hoyle	plasma-vacuum	thermal cond.
6	tokamak	plasma-wall	viscosity

CASTOR 8:

nr.	*equilibrium*	*configuration*	*dissipation mech.*
1	Soloviev ($\epsilon = 1/3$, ellip=2)	plasma-wall	none

POLLUX 8:

nr.	*equilibrium*	*configuration*	*dissipation mech.*
1	Gold-Hoyle	plasma-wall	none (RONUL=10)
2	Gold-Hoyle	plasma-wall	none (RONUL=55)

Appendix 5: Extension of CASTOR

G.T.A. Huysmans, J.P. Goedbloed, W. Kerner, S. Poedts, E. Schwarz

In the original set-up, the configuration considered in CASTOR consisted of a toroidal axisymmetric plasma surrounded by a perfectly conducting wall. To enable the calculation of modes which perturb the plasma boundary with the CASTOR code, the plasma-vacuum boundary conditions and the numerical solution of the vacuum magnetic field equations have been implemented in CASTOR 8. The boundary conditions are implemented as natural boundary conditions, with the property that the ideal boundary conditions are retrieved from the resistive ones by putting the resistivity to zero. The vacuum equations are solved independently from the eigenvalue problem of the plasma. The Laplace equation for the scalar potential of the magnetic field perturbation in the vacuum is solved using a cubic finite element/Fourier representation.

Appendix 6: Extension of POLLUX

G. Halberstadt, R.A.M. Van der Linden, J.P. Goedbloed, S. Poedts, E. Schwarz

Versions 1 to 7 of the program POLLUX allowed only calculations of normal modes in one-dimensional equilibria, although the actual numerical analysis is made intrinsically two-dimensional by the line-tying constraint. As there is considerable interest in using two-dimensional equilibrium models, a first step has been made in POLLUX 8 by introducing a longitudinal density variation. Ultimately, this will enable us to include the observed slow variation of equilibrium quantities along the magnetic field lines. Presently, the two-dimensional code enables us to critically assess the physical relevance of the line-tying boundary conditions. This is performed by switching off these boundary conditions and modelling instead the transition region between the photosphere and the corona by a steep density and temperature gradient.