Massive multiuser MIMO downlink with low-resolution converters

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Abstract—In this review paper, we analyze the downlink of a massive multiuser multiple-input multiple-output system in which the base station is equipped with low-resolution digital-to-analog converters (DACs). Using Bussgang's theorem, we characterize the sum-rate achievable with a Gaussian codebook and scaled nearest-neighbor decoding at the user equipments (UE). For the case of 1-bit DACs, we show how to evaluate the sum-rate using Van Vleck's arcsine law. For the case of multi-bit DACs, for which the sum-rate cannot be expressed in closed-form, we present two approximations. The first one, which is obtained by ignoring the overload (or clipping) distortion caused by the DACs, turns out to be accurate provided that one can adapt the dynamic range of the quantizer to the received-signal strength so as to avoid clipping. The second approximation, which is obtained by modeling the distortion noise as a white process, both in time and space, is accurate whenever the resolution of the DACs is sufficiently high and when the oversampling ratio is small. We conclude the paper by discussing extensions to orthogonal frequency-division multiplexing systems; we also touch upon the problem of out-of-band emissions in low-precision-DAC architectures.

I. INTRODUCTION

Nontrivial fronthaul connectivity challenges must be solved if one wants to enable massive multiple-input multiple-output (MIMO) operation over the relatively large bandwidth available in the higher portion of the frequency spectrum assigned to 5G systems. Consider, for example, a base station (BS) equipped with 100 antennas, each one connected to two high-precision (e.g., 10-bit resolution) digital-to-analog converters (DACs) and analog-to-digital converters (ADCs) operating at 1 GS/s. In such a system, 2 Tbit/s of data would need to be transferred to and from the radio unit (typically co-located with the antenna array) to the baseband-processing unit (typically located at the base of the tower hosting the BS). This exceeds by far the rate supported by the common public radio interface (CPRI) used over today's fiber-optical fronthaul links [1].

One promising approach to reduce this fronthaul bottleneck is to lower the resolution of the data converters. Several aspects of massive MIMO systems equipped with low-precision
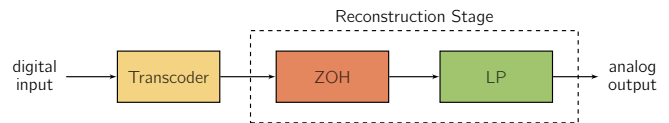
Fig. 1. Block diagram of the basic components of a DAC [17, Fig. 1.1].

converters have been recently investigated in the literature, including achievable rates [2]–[7], channel-estimation and data-detection algorithms [8], [9], precoding design [10]–[14], energy efficiency [15], and out-of-band spectral emissions [16]. In this review paper, we provide an overview of some of the most recent results. Our focus will be exclusively on the *downlink* of a multi-user (MU) massive MIMO system in which a BS serves multiple user equipments (UEs) concurrently in the same frequency band.

II. SYSTEM MODEL AND DIGITAL-TO-ANALOG CONVERTERS

We consider a massive-MIMO BS equipped with B antennas and serving U UEs. Each BS antenna is fed by two DACs, which generate the in-phase and the quadrature components of the transmitted signal. A DAC performs two basic operations: (i) it transforms the digital input sequence into its analog representation (*transcoder* stage) and (ii) it maps the transcoder output to a continuous-time waveform (*reconstruction* stage, typically consisting of a zero-order hold followed by a low-pass filter [17]; see Fig. 1 for an illustration).

Under the simplifying assumption that the digital input to the DAC has infinite precision, we can view the transcoding step of the two DACs as a quantizer, i.e., a nonlinear function $\mathcal{Q}(\cdot)$ that maps a sample in $\mathbb{C}$ to a finite-cardinality set $\mathcal{X} = \{q_0, \ldots, q_{2^Q-1}\} \times \{q_0, \ldots, q_{2^Q-1}\}$. Here, Q is the number of DAC bits. Throughout the paper, we shall consider only symmetric, uniform quantizers and denote their step size by Δ and the number of levels by $L = 2^Q$. Furthermore, we shall assume that the output of the DACs is scaled by a factor α so as to satisfy an average transmit-power constraint.

III. ACHIEVABLE RATES VIA BUSSGANG'S DECOMPOSITION

To begin with, we focus, for simplicity, on the case of transmission over flat-fading channels. We also assume that the

DACs operate at symbol time and that their reconstruction stage involve an ideal rectangular low-pass filter (see [16] for details). Generalizations to more realistic setups are discussed in Section V. Under these assumptions, the input-output relation of the downlink channel can be modeled as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}. \tag{1}$$

Here, the vector $\mathbf{y} \in \mathbb{C}^U$ contains the signal received at the U UEs; $\mathbf{H} \in \mathbb{C}^{U \times B}$ is the fading channel, which is assumed to be perfectly known at the BS. The vector $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_0 \mathbf{I}_U)$ models the additive noise. Finally $\mathbf{x} \in \mathcal{X}^B$ is the output of the transcoder stage of the DACs.

We assume that

$$\mathbf{x} = \mathcal{Q}(\mathbf{P}\mathbf{s}) \tag{2}$$

where $\mathbf{s} \in \mathbb{C}^U$ contains the data symbols intended for the U UEs and $\mathbf{P} \in \mathbb{C}^{B \times U}$ is the precoding matrix, which is a function of the fading channel $\mathbf{H}$. The precoding structure in (2) is referred to in [12] as *linear-quantized precoding*, to distinguish from more general nonlinear precoder structures, which offer superior performance at the cost of additional computational complexity.

By substituting (2) into (1), we see that the presence of the quantizer $\mathcal{Q}(\cdot)$ makes the channel output $\mathbf{y}$ depend on the symbol vector $\mathbf{s}$ in a nonlinear way. We next use Bussgang's theorem [18], a special case of Price's theorem [19], to linearize the input-output relation and to enable a theoretical analysis [20]. Then, we will use the *generalized mutual information* (GMI) [21] to estimate the rate achievable at each UE by scaled nearest-neighbor decoding [22] and a Gaussian codebook ensemble.

Theorem 1 below follows from a simple adaptation of Bussgang's theorem to the quantizer output $\mathcal{Q}(\mathbf{P}\mathbf{s})$.

Theorem 1: Assume that $\mathbf{s} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_U)$. Then for a fixed precoding matrix $\mathbf{P}$, we have that

$$\mathbb{E}\left[\mathcal{Q}(\mathbf{P}\mathbf{s})(\mathbf{P}\mathbf{s})^H\right] = \mathbf{G}\mathbf{P}\mathbf{P}^H \tag{3}$$

where $\mathbf{G}$ is the following real-valued diagonal matrix:[1]

$$\mathbf{G} = \frac{\alpha \Delta}{\sqrt{\pi}} \operatorname{diag}(\mathbf{P}\mathbf{P}^H)^{-1/2}$$
$$\times \sum_{i=1}^{L-1} \exp\left(-\Delta^2 \left(i - \frac{L}{2}\right)^2 \operatorname{diag}(\mathbf{P}\mathbf{P}^H)^{-1}\right). \tag{4}$$

It follows from (3) that, under the assumption that $\mathbf{s}$ is Gaussian, we can rewrite (2) in the linearized form

$$\mathbf{x} = \mathbf{G}\mathbf{P}\mathbf{s} + \mathbf{d} \tag{5}$$

where $\mathbf{d}$ is the zero-mean quantization-noise vector, which is uncorrelated with $\mathbf{s}$. Note that $\mathbf{G}\mathbf{P}\mathbf{s}$ is the linear minimum-mean-square estimate of $\mathbf{x}$ given $\mathbf{P}\mathbf{s}$, and $\mathbf{d}$ is the corresponding estimation error.

Substituting (5) into (1), we obtain a linear input-output relation, with non-Gaussian additive noise $\mathbf{H}\mathbf{d} + \mathbf{n}$. The ergodic

rate[2] achievable over this channel using a Gaussian codebook and scaled nearest-neighbor decoding at the receiver can be established from a GMI analysis similar to the one reported in [22], [24]. Specifically, we have the following result.

Theorem 2: Assume that UE u has knowledge of the channel gain[3] $\mathbf{h}_u^T \mathbf{G}\mathbf{p}_u$, where $\mathbf{h}_u^T$ is the uth row of the channel matrix $\mathbf{H}$ and $\mathbf{p}_u$ is the uth column of the precoding matrix $\mathbf{P}$. Then the GMI R_u achievable with a Gaussian codebook and scaled nearest-neighbor decoding at the uth UE is

$$R_u = \mathbb{E}[\log(1 + \gamma_u)] \tag{6}$$

where the signal-to-interference-noise-and-distortion ratio (SINDR) γ_u is given by

$$\gamma_u = \frac{\left|\mathbf{h}_u^T \mathbf{G}\mathbf{p}_u\right|^2}{\sum_{v \neq u} \left|\mathbf{h}_u^T \mathbf{G}\mathbf{p}_v\right|^2 + \mathbf{h}_u^T \mathbb{E}[\mathbf{d}\mathbf{d}^H] \mathbf{h}_u^* + N_0}. \tag{7}$$

IV. STATISTICS OF THE QUANTIZATION NOISE

Evaluating (7) requires knowledge of the correlation matrix $\mathbb{E}[\mathbf{d}\mathbf{d}^H]$ of the zero-mean quantization noise $\mathbf{d}$. It follows from (5) that

$$\mathbb{E}[\mathbf{d}\mathbf{d}^H] = \mathbb{E}[\mathbf{x}\mathbf{x}^H] - \mathbf{G}\mathbf{P}\mathbf{P}^H\mathbf{G}. \tag{8}$$

For the case $L = 2$ (1-bit DACs), the covariance matrix $\mathbb{E}[\mathbf{x}\mathbf{x}^H]$ of the quantizer output admits a well-known closed-form expression, commonly referred to as the *arcsine law* and reported first by Van Vleck [25]:

$$\mathbb{E}[\mathbf{x}\mathbf{x}^H]$$
$$= \frac{2P}{\pi B}\left(\sin^{-1}\left(\operatorname{diag}(\mathbf{P}\mathbf{P}^H)^{-\frac{1}{2}}\Re\{\mathbf{P}\mathbf{P}^H\}\operatorname{diag}(\mathbf{P}\mathbf{P}^H)^{-\frac{1}{2}}\right)\right.$$
$$+ j\sin^{-1}\left(\operatorname{diag}(\mathbf{P}\mathbf{P}^H)^{-\frac{1}{2}}\Im\{\mathbf{P}\mathbf{P}^H\}\operatorname{diag}(\mathbf{P}\mathbf{P}^H)^{-\frac{1}{2}}\right)\right). \tag{9}$$

Here, P denotes the power constraint. However, for any finite L larger than 2, no closed-form expression is available for $\mathbb{E}[\mathbf{x}\mathbf{x}^H]$ and this matrix needs to be evaluated numerically (see [26], [7]).

Alternatively, one can seek closed-form approximations to $\mathbb{E}[\mathbf{d}\mathbf{d}^H]$. Two such approximations are discussed in [26]. The first one, referred to as *diagonal approximation*, involves neglecting spatial correlation, i.e., assuming that $\mathbb{E}[\mathbf{d}\mathbf{d}^H]$ is a diagonal matrix. Then, one exploits that the entries on the main diagonal of $\mathbb{E}[\mathbf{d}\mathbf{d}^H]$ can be computed in closed form even when $L > 2$. This approximation is accurate only for DACs with medium-to-high resolution (i.e., when $L \geq 4$).

The second one, referred to as *rounding approximation*, involves replacing each DAC by a one-dimensional midrise lattice quantizer (which implies $L = \infty$) with step size Δ, for which the covariance matrix of the quantization error is known in closed form [27]. This approximation is accurate also for low-precision DACs, provided that the step size Δ is chosen so that the distortion due to clipping/saturation is negligible compared to the granular quantization distortion. This requires adapting Δ to the signal strength.

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[1] In (4), the operator $\operatorname{diag}(\cdot)$ returns a diagonal matrix whose main diagonal coincides with that of the matrix it is applied to; furthermore, the exponential function is applied elementwise to the diagonal entries of $\operatorname{diag}(\mathbf{P}\mathbf{P}^H)^{-1/2}$.

[2] We assume that coding can be performed over sufficiently many independent realization of the channel matrix $\mathbf{H}$. See [23] for an analysis of the impact of imperfect channel-state information on the system performance.

[3] This is the scaling factor in the scaled nearest-neighbor decoding rule.

V. EXTENSIONS

Extensions of the analysis described above to the frequency-selective case and to the use of orthognal-frequency-division multiplexing (OFDM) and oversampling DACs are discussed in [26], [28], [16]. In the oversampling case, the diagonal approximation involves neglecting also temporal correlation, and it turns out to be accurate only when the oversampling ratio is small (e.g., less than four for $L = 4$). The rounding approximation does not suffer from this limitation.

The use of low-precision DACs in the massive MIMO downlink causes unwanted out-of-band (OOB) emissions, which may be incompatible with the spectral requirements imposed by regulatory bodies. An extension of the Bussgang's decomposition (3) to OFDM systems with nonideal analog filters is used in [16] to study such OOB emissions. There, it is shown that by an appropriate design of the DACs' low-pass filter and by employing simple digital pre-equalization techniques, one can significantly reduce OOB emissions, at the cost of a small decrease in the SINDR (7) and of a small increase in the peak-to-average power ratio of the transmitted signal.

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