

# On the Impact of Colored Transmitter Noise on mmWave Wave MIMO Systems

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## I. INTRODUCTION

The performance of wireless systems is considerably affected by different radio frequency (RF) impairments such as inphase/quadrature-phase (I/Q) imbalance and distortions due to the power amplifier or mixer nonlinearities. The impact of residual distortion caused by these impairments on the channel capacity of multiple-input multiple-output (MIMO) systems has been studied in a few recent articles. In [1], the authors demonstrate that the residual transmit RF impairments can be modeled as an additive noise. The impact of this noise on the performance of training-based MIMO systems is characterized in [2].

Dissimilar to the existing works where the residual transmit RF impairments are modeled as a zero-mean white complex Gaussian noise, we consider a more general scenario where this noise has an arbitrary covariance matrix. We consider a MIMO system with  $N_t$  transmit and  $N_r$  receive antennas with the following input-output relationship

$$\mathbf{y} = \mathbf{H}(\mathbf{s} + \mathbf{w}) + \mathbf{n}, \quad (1)$$

where  $\mathbf{y} \in \mathbb{C}^{N_r}$  is the receive vector,  $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$  denotes the channel matrix, and  $\mathbf{s}$  stands for the data signal where  $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_{N_t}$ .  $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{K}_w)$  is the transmitter noise and  $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_{N_r})$  denotes the thermal noise at the receiver. Therefore, the residual noise can be written as  $\tilde{\mathbf{n}} = \mathbf{H}\mathbf{w} + \mathbf{n}$  and its covariance is given by  $\mathbf{K}_{\tilde{\mathbf{n}}} = \mathbf{H}\mathbf{K}_w\mathbf{H}^H + \sigma_n^2 \mathbf{I}_{N_r}$ .

In order to evaluate the channel capacity, we pre-multiply the two sides of (1) by  $\mathbf{K}_{\tilde{\mathbf{n}}}^{-\frac{1}{2}}$  and obtain an equivalent input-output relationship as

$$\mathbf{y}' = \mathbf{G}\mathbf{s} + \mathbf{n}', \quad (2)$$

where  $\mathbf{G} = \mathbf{K}_{\tilde{\mathbf{n}}}^{-\frac{1}{2}}\mathbf{H}$  and  $\mathbf{n}' \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_r})$ . Under the assumption that the transmitter knows the channel matrix  $\mathbf{H}$  and the covariance of the transmitter noise, i.e.,  $\mathbf{K}_w$ , the channel capacity is achieved by a singular value decomposition (SVD) precoding/combining along with a waterfilling power allocation. When transmitter noise  $\mathbf{w}$  is white and its covariance matrix has equal diagonal elements (similar to the scenarios considered in [1]-[2]),  $\mathbf{G}\mathbf{G}^H$  resolves into the same eigenmodes as the system without transmitter noise (i.e., (1) with  $\mathbf{w} = \mathbf{0}$ ). However, the optimal waterfilling solution for the scenarios with  $\mathbf{w} = \mathbf{0}$  is not optimal in the cases with colored transmitter noise and the power allocation should be carried out based on the eigenvalues of  $\mathbf{G}\mathbf{G}^H$ .

## II. NUMERICAL RESULTS

We investigate the impact of transmitter noise on the channel capacity over a mmWave MIMO channel. We consider a setup with  $N_t = N_r = 9$ . We consider a general covariance matrix for  $\mathbf{K}_w$  with non-zero off-diagonal elements. The coefficients for our line-of-sight (LOS) channel (with non-optimal inter-antenna distances) is generated according to the

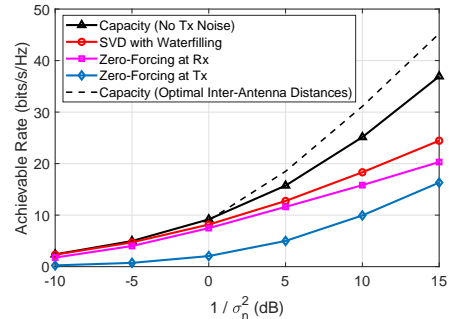


Fig. 1: Achievable rate versus one over the receiver noise variance on a  $9 \times 9$  LOS channel.

specifications of our testbed, i.e., Chalmers mmWave MIMO testbed (MATE) [3], and for this fixed channel, information rates are evaluated. Fig. 1 depicts the performance of different precoding/combining strategies. It can be inferred from Fig. 1 that the impact of transmit noise is more limiting in the high SNR regime. Moreover, the poor performance of the transmit zero-forcing in the presence of transmitter noise is evidenced.

## III. CONCLUSIONS

We have investigated the impact of colored transmitter noise on the capacity of mmWave MIMO channel. Our numerical results reveal the limiting impact of such noise at high SNR regions. Our next objective is to obtain the statistics of the transmitter noise from real-world measurements using the MATE and to develop and implement transmission strategies which minimize the impact of the residual distortion.

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