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A quantification of the morphological computations in perception systems

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Abstract

We suggest a straightforward strategy how to quantify how much "computation" a given morphology is performing while transforming a stimuli.

Background

In (Pfeifer, R. and Iida, F., 2005) on p. 54 the following intriguing question was raised:

"One problem with the concept of morphological computation is that while intuitively plausible, it has defied serious quantification efforts. We would like to be able to ask "How much computation is actually being done?" We will discuss a possible way to quantify the morphological computation involved in the perception process, but not the morphological computation at the output action, such as locomotion etc, see for example (Paul, 2006).

The model

In Figure 1, where we depict the process of interest as the transformation between the data presented at the receptor 2, and the presentation at 4 into the central computing unit 5, or "brain" if you wish. That is, we are interested in how much the data-presentation at 4, differs from the very stimuli detection at 2. Suppose that we have the presentation site well defined at 4. The interface has a certain capacity, i.e. the amount information that can be presented. We can view this as a dynamic capacity, e.g. bits per second, or as a static capacity, a vector, matrix or tensor with certain dimension where the elements has a given information capacity. We can also allow combinations; for example a matrix of size m×n which is renewed at a rate of p (Hz). We denote a presentation process at 4 of a given capacity by $P(t)$. We then imagine that this presentation unit was directly mounted at 2, and being able to receive undisturbed, or rather, "unpreprocessed", information directly from the external stimuli at that

Figure 1. External information, 1, is feed to the receptor 2. The morphological computation we want to quantify is the transformation of the detected data in 2 through the channel 3 into the presentation at 4 of the stimuli to the central computation center at 5. We will not discuss any output-actions 6. Note that this separation of such parts in the process is rather arbitrarily, especially seen from an evo-devo perspective.

very specific capacity that is at hand at 4. Let us call this imaginary process the raw input process, $R(t)$. We now define the morphological computation of this system as a functional, F, which maps $R(t)$ to $P(t)$.

Since we are interested in the *amount* of morphological computations preformed we are interested in finding the most efficient (computational efficient) functional which maps *R(t)* to *P(t).* Furthermore, since we are aiming at general definition also including biological processes, we need to be able to allow a certain amount of slack in such a functional. Let us therefore introduce a threshold ϵ. The natural, but not easily quantifiable, choice would be to pick as the limit of conscious separation of different stimuli processes, i.e. if for such an ϵ one would not be able to tell the difference between the stimuli presentation processes $\tilde{P}(t)$ and $P(t)$ if $\left\|\tilde{P}(t) - P(t)\right\|_C < \epsilon$, and where ϵ would be the largest such limit. Here $\|\cdot\|_C$ stands for the capacity-norm, i.e. if the presentation process has a capacity as in the example above with a matrix size of m×n which is renewed at a rate of p Hz, we let the capacity norm of the process be defined as

$$\|P(t)\|_C = \int_0^{\frac{1}{p}} \|P(t)\|_k \, dt,$$

where we might for example let *k=2* for concreteness. Let us now define another norm, the numerical norm $\|F\|_N$ of a functional *F* as the minimal number of arithmetic operations needed to realize, or compute, *F* locally*.* We are now ready to define the amount morphological computation for this system as

$$\varphi := \inf\{\|F\|_N \; ; \; \left\|F(R(t)) - P(t)\right\|_C < \epsilon\}.$$

Let us just comment on two trivial extreme cases. Suppose that the system is perfectly faithful, that is $R(t) = P(t)$. We can then simply find our optimal functional as the identity mapping. For that we need no computations, that is $\varphi = 0$. The other case is even more simple. Suppose now that the sensory transportation is not working at all, i.e. $P(t) \equiv 0$. Then we also have $\varphi = 0$. Note that we have so far not said anything at all about the functionals *F*. We have not even said what space we take the infimum in the above definition. In fact all of the "definitions" above are rather vague. Instead of discuss these issues more at this point, let us look at two examples in order to better illustrate the above technical formulations (together with a few alternative description) in a more concrete setting. Both of those examples are dealing with vision, but the suggested definition of φ, the amount of morphological computation is applicable to all kinds of perceptions, sound, taste, tactile perception, electro-magnetic waves etc

The morphological computation in the visual apparatus
Before we start with our main examples, let me point out that a large amount of highly interesting work has been done in the area relating insect vision with robot vision, essentially introduced in (Franceschini, N.; Pichon, J. M.; Blanes, C., 1992); see for example (Zufferey, J.C. ; Floreano, D., 2006), or Section 2 in (Pfeifer, R. and Iida, F., 2005) for a short overview.

Humans
In (Cowan, 1977) an explicit mapping, called the human retinocortical map, between the retina and the visual (striate) cortex was presented, see (Murray, 2003) p. 628 for a short overview, and (Bressloff, P. C., Cowan, J. D., Golubitsky, M., Thomas, P. J., & Wiener, M. C., 2001) p. 301-303, for more details. This map from a point on the retina with polar coordinates (r_R, θ_R) to the point with the Cartesian co-

ordinate (x, y) on the striate cortex can be expressed in the following way: $x = \frac{\alpha}{\varepsilon} \ln\left(1 + \frac{\varepsilon}{w_0} r_R\right)$, and $y = \frac{\beta r_R \theta_R}{w_0}$, where $w_0 = 0.087$ and $\varepsilon = 0.051$, and where α and β are scaling constants in appropriate units. We will also discuss some important approximation of this map. Now using this retinocortical map and a simple transformation from polar- to Cartesian coordinates, we will be able to estimate φ, the amount of morphological computations in the visual system for humans. By doing that we will also display different possible variants of the "definitions" indicated above.

Cubomedusae

The little box jellyfish, or *Tripedalia cystophora*, has an impressive number of 24 eyes of four different types. Their vision system has been studied in (Nilsson, D.-E., Gislen, L., Coates, M.M., Skogh, C. and Garm, A., 2005), and (Garm, A., Ekström, P., Boudes , M., Nilsson, D.-E., 2006). Eight of these 24 eyes are
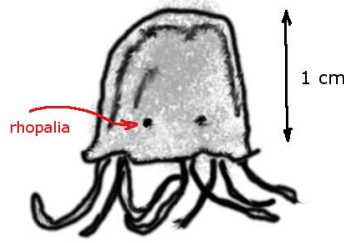
Figure 2. The box jellyfish. Each of the four sensory centers, the rhopalia, consists of six eyes of four morphological types. Two of these eyes have lenses –the upper-, and the lower lens eye.

highly complex and developed with lenses etc. We will study one such eye, and give some estimates of φ for that visual system. We will also comment on how elegantly the little jellyfish computes.

A dual definition:

- The family of "short" algorithms $M_\delta := \{A_i : \mu(A_i) \leq \delta\}$

- We can then compute the dual of the amount of morphological computation preformed as: $\psi = \psi(\delta) := \inf_{A_i \in M_\delta} \nu(A_i)$

THEOREM

If the extremal algorithms, $A^\varepsilon \in F_\varepsilon$ and $B^\delta \in M_\delta$ (i.e. "the book algorithms"), exists for all reasonable ε and δ, so that φ(ε) = μ(A^ε) and ψ(δ)= ν(B^δ), then we have that φ(ψ(δ))=δ and ψ(φ(ε))=ε.

Proof: $A^\varepsilon \in F_\varepsilon \Rightarrow \nu(A^\varepsilon) \leq \varepsilon$ and have $\mu(A^\varepsilon) = \varphi(\varepsilon) = \varphi$. We then have that there exist an extreme algorithm, B^φ, such that $B^\varphi \in M_\varphi$ so that $\mu(B^\varphi) \leq \varphi$ and $\nu(B^\varphi) \leq \psi(\varphi)$ implying $B^\varphi \in F_{\psi(\varphi)}$. Hence $\varphi(\psi(\varphi)) \leq \mu(B^\varphi) \leq \varphi$. Similarly, we have that A^ε in M_φ so that $\psi(\varphi) \leq \nu(A^\varepsilon) \leq \varepsilon \Rightarrow \varphi(\psi(\varphi)) \geq \varphi(\varepsilon) = \varphi$. Thus we have that $\varphi(\psi(\varphi)) = \varphi(\varepsilon)$. If φ is injective, then we can conclude that $\psi(\varphi) = \varepsilon$. The other relation, φ(ψ(δ))=δ, is shown in an analogous way. □

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