Energy-Optimal Control of Underactuated Bipedal Locomotion Systems

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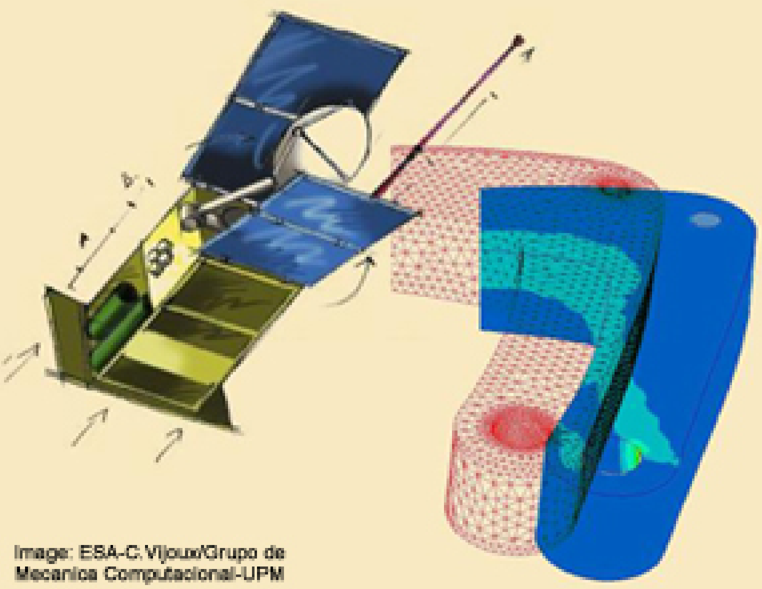

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ENERGY-OPTIMAL CONTROL OF UNDERACTUATED BIPEDAL LOCOMOTION SYSTEMS

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Abstract. *The paper deals with modeling and design of energy-optimal motion of mechatronic system having less number of actuators than degrees of freedom. Such mechatronic system is termed underactuated. We consider an underactuated mechatronic system modeled a bipedal locomotion robot with 11 degrees of freedom. The system comprises nine links and is used to represent the biped's planar dynamics in sagittal plane. The bodies are connected by friction-free hinge joints. It's assumed that the control inputs are torque actuators acting only at hip and knee joints. The ankle and the metatarsal joints of the feet are spanned with springs allowing discrete switching of their stiffness parameters in accordance to varying constraints imposed on the system's motion. The algorithm has been developed for synthesizing the energy-optimal anthropomorphic motion of the bipedal locomotion system with passively controlled feet and discrete switching of their joint stiffness parameters. Algorithm uses the smoothing cubic splines for approximation of variable functions, inverse dynamics approach, extern penalty functions method, and minimization of the nonsmooth objective function in orthogonal directions. The efficiency of the developed algorithm has been confirmed by simulation of human gait like motions for considered underactuated system. Applications of the results obtained can be found in robotics, bioengineering (prosthetics, orthotics), others.*

1 INTRODUCTION

One of the primary goals of mechatronics is to gain as many advantages as possible from the optimal interaction between mechanical, control, electronic and computer subsystems. This requires more fundamental research on a number of topics, e.g. parameter identification and control-structure interaction, contact problems and optimal design of mechatronic systems, structure of actuation and optimal interaction between inherent dynamics and control in advance mechanical systems [1, 2]. The research in the above areas can help to improve performance of existing mechatronic systems and to design new modern products. The important and relevant characteristics of any mechatronic system are its degree and type of actuation. Most systems usually have the same number of actuators as degrees of freedom of their mechanical subsystems, i.e. they belong to the class of fully actuated mechanical systems. If mechatronic system has fewer actuators than joints or more precisely if the dimension of the configuration space exceeds that of the control input space, the system is called *underactuated*. In this paper the dynamic and synthesis of controlled motion of the underactuated bipedal locomotion robot is considered.

The synthesis of bipedal gait is a topic of interest in mechatronics, robotics as well in biomechanics. During the past few decades several approaches have been published, and different bipedal walkers have been constructed in attempts to predict or mimic human gait. Most predictive models are complex, contain many parameters, demand tremendous computational effort, and still do not give much additional insight in the control objectives needed to generate cyclic gait (see, e.g. paper [3] and it references). In the field of robotics, most bipedal locomotion systems require complex control schemes and consume much more energy than human. Different approaches have been used to synthesize energy efficient walking pattern for a bipedal machines (see, e.g. papers [4-17]). The proposed paper deals with modeling and design of energy-optimal motion of underactuated mechatronic system which model bipedal locomotion robot having less number of actuators than degrees of freedom. We consider an underactuated bipedal locomotion system with 11 degrees of freedom. The system comprises nine links and is used to represent the biped's planar dynamics in sagittal plane. The nine links are rigid bodies that model the trunk, the thighs, the shank, and the two-links feet. The bodies are connected by friction-free hinge joints. In contrast to the papers [5, 7, 12-17] it's assumed that the control inputs are torque actuators acting only at hip and knee joints. The ankle and the metatarsal joints of the feet are spanned with springs allowing discrete switching of their stiffness parameters in accordance to varying constraints imposed on the system's motion. The algorithm has been developed for synthesizing the energy-optimal anthropomorphic motion of the bipedal locomotion system with passively controlled feet and discrete switching of their joint stiffness parameters. Algorithm uses the smoothing cubic splines for approximation of variable functions, inverse dynamics approach, extern penalty functions method, and minimization of the no smooth objective function in orthogonal directions. The efficiency of the developed algorithm has been confirmed by simulation of human gait like motions for considered underactuated bipedal locomotion system (BLS).

2 STATETMENT OF THE PROBLEM

Consider a mechatronic system the controlled motion of which can be described by the following equations:

$$\dot{x} = f(x,u,w(x,\xi)) \qquad g(x,w(x,\xi)) = 0, \qquad t \in [0,T] \tag{1}$$

Here $x = (x_1, x_2, ..., x_n)$ is a state vector, $u = (u_1, u_2, ..., u_m)$ is a vector of controlling stimuli (forces, torques) generated by the external powered drives of the system, $w = (w_1, w_2, ..., w_r)$ is a vector of the controlling stimuli of the internal unpowered (passive) drives, and T is the duration of the motion. Vector functions f and g are determined by the structures of the system and unpowered drives, ξ is a vector of design parameters of the unpowered drives. Constraints and restrictions are imposed on the state vector $x(t)$, the controlling stimuli of the unpowered drives $w(x,\xi)$, and the external control $u(t)$ of the system. These restrictions can be written in the following way:

$$x(t) \in Q, \quad w(x,\xi) \in W, \quad u(t) \in U, \quad t \in [0,T] \quad (2)$$

In formulas (2), Q and U are given domains in the state and control spaces of the system, W is a set of addmissible controlling stimuli determined by the structure of the unpowered drives. The differential equations (1) together with the restrictions (2) are called the mathematical model of semi-passively controlled mechatronic system. This model can be used for many purposes, e.g. to study dynamics and control of motion of underactuated systems, to investigate the fundamental questions about the role of inherent dynamics in controlled motion, and how much a mechatronic system should be governed by the external drives and how much by the system's inherent dynamics; and for optimal control and structure optimization of semi-passively controlled mechatronic systems for different applications [7]. The following problem can be formulated.

Problem A. Given a mechatronic system the controlled motion of which is described by equations (1). It is required to determine the vector-function $w_*(t,\xi)$, the motion of the system $x_*(t)$ and the external controlling stimuli $u_*(t, x_*, w_*)$ which alltogether satisfy the equations (1), the restrictions (2), and which minimize the given objective functional $\Phi[u]$.

As a result of the solution of *Problem A* the optimal structure of mechatronic system having both powered and unpowered drives is designed. The external controlling stimuli for the system are also found which minimize the given objective functional.

3 METHODOLOGY

To solve *Problem A* the numerical algorithm has been developed [7]. The algorithm is based on a special procedure to convert the initial optimal control problem with parameters (*Problem A*) into a standard nonlinear programming problem: $F(C) \Rightarrow \min_C, \; h(C) = 0$, where C is a vector of varying parameters. This is made by an approximation of the independently varying functions $q(t)$ (generalized coordinates and/or control) by a combination of the fifth order polynomial and Fourier series. Taking into account the restrictions (2), the list of independently varying parameters can be determined. To solve the nonlinear programming problem different methods can be used, for instance the Rozenbrock's method [5, 7], the sequential quadratic programming method, others. The key features of the methodology developed for solving *Problem A* are its high numerical efficiency, the possibility to satisfy a lot of restrictions imposed on the phase coordinates and/or control of the system automatically and accurately, and its applicability for optimization of semi-passively controlled mechatronic systems with different degree of actuation (underactuation, full actuation, overactuation).

In this paper the *Problem A* will be considered for underactuated BLS.

4 METHEMATICAL MODEL OF UNDERACTUATED BLS

Consider a plane nine-element model of the BLS (Fig. 1). This system comprises a trunk (body G) and two legs. Each leg consists of four elements. Two elements with mass and inertia model the thigh and shank. The elements $H_i A_i M_i$ and $M_i T_i$ that model the feet of the BLS are assumed to be without inertia. The total mass m_{fi} of the foot is located at the ankle joint of the i-th leg.

In addition to the weights of the trunk, thighs, shanks and feet the ground reaction forces and the control moments at the joints of the legs act on the system.

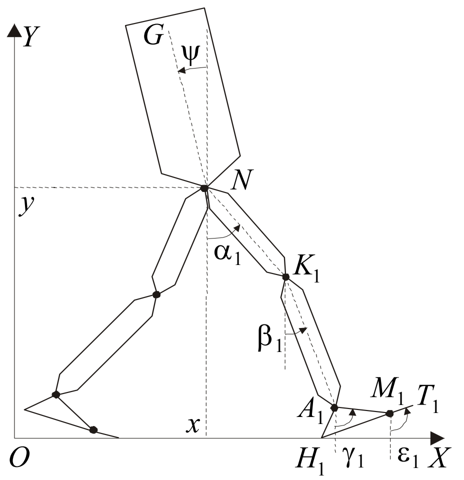

Fig.1 Bipedal Locomotion System

In contrast to the papers [5, 7, 12-17], the considered model of the BLS comprises the two-link feet with both ankle and metatarsal joints modeled by linear springs with switching stiffness parameters (Fig.2):

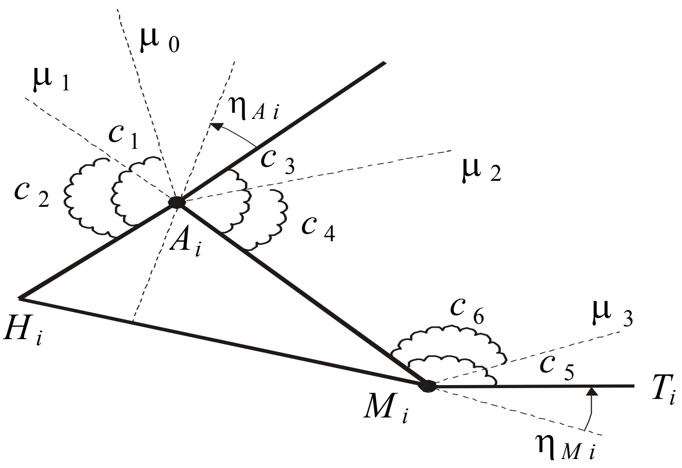

Fig.2 Sketch of the passively controlled foot of BLS

Let OXYZ be a fixed rectangular Cartesian coordinate system. It is assumed that the BLS moves in the OXY plane along the OX axis over a horizontal surface, (X-Z plane).

We will employ the following notations: (x, y, ψ, α_i, β_i, γ_i, ε_i, i=1, 2) is the set of generalized coordinates (Fig. 1); m is the mass of the trunk; r is the distance from the suspension point N of the legs to the center of mass of the trunk; J is the moment of inertia of the trunk relative to the Z axis at point N; $m_{ai}, r_{ai}, a_i, J_{ai}$ are the mass, the distance from N to the center of mass, the length and the moment of inertia of the thigh relative to the Z axis at point N, respectively; $m_{bi}, r_{bi}, b_i, J_{bi}$ are the mass, the distance from K_i to the center of mass, the length and the moment of inertia of the shank relative to the Z axis at point K_i, respectively.

The equations of motion of the system, derived using the technique of the Lagrange equations of the second kind, are as follows:

$$M\ddot{x} + \sum_{i=1}^{2}\left[K_{ai}(\dot{\alpha}_i \cos\alpha_i)^{\dot{}} + K_{bi}(\dot{\beta}_i \cos\beta_i)^{\dot{}}\right] - K_r(\dot{\psi}\cos\psi)^{\dot{}} = R_{1x} + R_{2x} \tag{3}$$

$$M(\ddot{y} + g) + \sum_{i=1}^{2}\left[K_{ai}(\dot{\alpha}_i \sin\alpha_i)^{\dot{}} + K_{bi}(\dot{\beta}_i \sin\beta_i)^{\dot{}}\right] - K_r(\dot{\psi}\sin\psi)^{\dot{}} = R_{1y} + R_{2y}$$

$$J\ddot{\psi} - K_r(\ddot{x}\cos\psi + \ddot{y}\sin\psi) - gK_r \sin\psi = -q_1 - q_2,$$

$$J_i\ddot{\alpha}_i + K_{ai}(\ddot{x}\cos\alpha_i + \ddot{y}\sin\alpha_i) + a_i K_{bi}\left(\ddot{\beta}_i \cos(\alpha_i - \beta_i) + \dot{\beta}_i^2 \sin(\alpha_i - \beta_i)\right) +$$
$$+ gK_{ai}\sin\alpha_i = q_i - u_i + a_i(R_{ix}\cos\alpha_i + R_{iy}\sin\alpha_i),$$

$$J_{ci}\ddot{\beta}_i + K_{bi}(\ddot{x}\cos\beta_i + \ddot{y}\sin\beta_i) + a_i K_{bi}\left(\ddot{\alpha}_i \cos(\alpha_i - \beta_i) - \dot{\alpha}_i^2 \sin(\alpha_i - \beta_i)\right) +$$
$$+ gK_{bi}\sin\beta_i = u_i - p_i + b_i(R_{ix}\cos\beta_i + R_{iy}\sin\beta_i),$$

$$p_i = (x_i - x_{Ri})R_{iy} - y_i R_{ix}, \quad w_i = 0, \quad \text{for} \quad x_{Ri} \in [x_{hi}, x_{mi}], \; y_{hi} = 0 \tag{4}$$

$$p_i = w_i + (x_i - x_{mi})R_{iy} + (y_{mi} - y_i)R_{ix}, \; w_i = (x_{mi} - x_{Ri})R_{iy} - y_{mi}R_{ix},$$
$$\text{for} \quad x_{Ri} \in (x_{mi}, x_{ti}], \; y_{ti} = 0 \tag{5}$$

Here in (3) - (5): q_i, u_i, p_i, w_i are the control moments that act at the hip (point N), the knee (point K_i), the ankle (point A_i) and the metatarsal (point M_i) joints, respectively; R_{ix}, R_{iy} are the horizontal and vertical component of the reaction forces; (x, y), (x_i, y_i), (x_{hi}, y_{hi}), (x_{mi}, y_{mi}), (x_{ti}, y_{ti}), (x_{Ri}, y_{Ri}) are the Cartesian coordinates of the points N, A_i, H_i, M_i, T_i, and of the point of application of the vector of the reaction forces $\mathbf{R}_i$ of the i-th leg, respectively; g is the acceleration due to gravity; " $\dot{}$ " is a derivation with respect to time. In equations (3) we have also used: $M = \sum_{i=1}^{2}\left(m_{ai} + m_{bi} + m_{fi}\right) + m$, $J_i = J_{ai} + a_i^2(m_{bi} + m_{fi})$, $J_{ci} = J_{bi} + b_i^2 m_{fi}$, $K_r = mr$, $K_{ai} = m_{ai}r_{ai} + a_i(m_{bi} + m_{fi})$, $K_{bi} = m_{bi}r_{bi} + b_i m_{fi}$, i=1, 2.

The considered model of the BLS comprises the two-link feet with both ankle and metatarsal joints modeled by linear springs with switching stiffness parameters as follows:

$$p_i = f_{pi}(\eta_{Ai}, \mathbf{C}_p), \qquad f_{pi} \equiv \begin{cases} c_1(\mu_0 - \eta_{Ai}) + c_2(\mu_1 - \eta_{Ai}), \ \eta_{Ai} < \mu_1, \\ c_1(\mu_0 - \eta_{Ai}), \ \eta_{Ai} \in [\mu_1, \mu_0], \\ c_3(\mu_0 - \eta_{Ai}), \ \eta_{Ai} \in [\mu_0, \mu_2], \\ c_3(\mu_0 - \eta_{Ai}) + c_4(\mu_2 - \eta_{Ai}), \ \eta_{Ai} > \mu_2, \end{cases}$$

$$w_i = f_{wi}(\eta_{Mi}, \mathbf{C}_w), \qquad f_{wi} \equiv \begin{cases} -c_5\eta_{Mi}, \ \eta_{Mi} \in [0, \ \mu_3], \\ -c_5\eta_{Mi} - c_6(\eta_{Mi} - \mu_3), \eta_{Mi} > \mu_3, \end{cases} \tag{6}$$

Here in (6): $\mathbf{C}_p = (c_1, c_2, c_3, c_4, \mu_0, \mu_1, \mu_2)$, $\mathbf{C}_w = (c_5, c_6, \mu_3)$ be the vectors of structural parameters of passive drives located at feet joints, $c_j > 0, \ j = \overline{1, 6}$; $\mu_1 < \mu_0 < \mu_2$, $\mu_3 > 0$ – stiffness coefficients and angles of stiffness switching; $\eta_{Ai}(t) \equiv \gamma_i(t) - \beta_i(t) + \varphi_i - \pi/2$, $\eta_{Mi}(t) \equiv \gamma_i(t) - \varepsilon_i(t) - \varphi_i$, $\varphi_i = \angle A_i M_i H_i$, $i = 1, 2$. Angle η_{Ai} describes orientation of the foot $A_i H_i M_i$ about the shank $A_i K_i$.

In what follows we shall show that the proposed model of passively controlled feet makes it possible to synthesize the energy-efficient human gate like motion of the considered underactuated BLS.

5 OPTIMIZATION PROBLEM FOR UNDERACTUATED BLS

The mechanical system under consideration has eleven degrees of freedom. Let us set up the constraints and the restrictions needed for the mathematical statement of the optimization problem for anthropomorphic motions of the considered underactuated BLS based on the analysis of kinematic and dynamic characteristics of human gait [18].

The human motion is periodic. It leads to the following boundary conditions imposed on the phase coordinates:

$$\begin{aligned} &\mathbf{f}_i(\tau + t) = \mathbf{f}_{3-i}(t), \quad \dot{\mathbf{f}}_i(\tau + t) = \dot{\mathbf{f}}_{3-i}(t), \quad \mathbf{f}_i = (y, \psi, \alpha_i, \beta_i, \gamma_i, \varepsilon_i), \quad i = 1, \ 2 \\ &x(\tau + t) = x(t) + L, \quad \dot{x}(\tau + t) = \dot{x}(t), \quad t \in [0, \ \tau] \end{aligned} \tag{7}$$

Here T is the duration of the double step, L is the length of step. Below we shall assume that the motion of one leg of the BLS completely mimics that of the other with the time delay $\tau = T/2$.

Human gait is characterized by a stable sequence of the phases of the leg's action during a double step. We shall assume that there are the following five phases of the first leg action: rotation over the heel during the period of time $t \in [0, \tau_{h1})$, support phase on both heel and metatarsal joint for $t \in [\tau_{h1}, \tau_{m1})$; motion on the phalanges for $t \in [\tau_{m1}, \tau)$; rotation over the ends of the toes for $t \in [\tau, \tau_{s1})$, and the swing phase of the foot over the surface during the period of time $t \in [\tau_{s1}, T)$.

For the second foot we will have: rotation over the ends of the toes for $t \in (0, \tau_{s2})$, the swing phase of the foot over the surface during the period of time $t \in [\tau_{s2}, \tau)$, rotation over the heel during the period of time $t \in [\tau, \tau_{h2})$, support phase on both heel and metatarsal joint for $t \in [\tau_{h2}, \tau_{m2})$, and motion on the phalanges for $t \in [\tau_{m2}, T]$.

For human gait the rhythm parameters should satisfy the following conditions:

$$0 < \tau_{s2} < \tau_{m1} < \tau < \tau_{s1} < \tau_{m2} < T \tag{8}$$

A number of other kinematical and dynamical requirements should be also satisfied for normal human walking [5, 8-12]. Taking into account the above mentioned sequence of the leg action during the time of the double step the anthropomorphic cyclogram of the BLS can be described by the following constraints:

$$\begin{gathered}
x_{hi}(t) \equiv x_{hi}^{0},\ \ y_{hi}(t) \equiv 0,\ y_{ti} > y_{mi} > 0,\ \eta_{Mi}(t) \equiv 0,\quad t \in [\tau_i, \tau_{hi}), \\
x_{hi}(t) \equiv x_{hi}^{0},\ \ y_{hi}(t) \equiv y_{mi}(t) \equiv y_{ti}(t) \equiv 0,\ \eta_{Mi}(t) \equiv 0,\ \ t \in [\tau_{hi}, \tau_{mi}), \\
x_{mi}(t) \equiv x_{mi}^{0},\ \ y_{hi} > 0,\ y_{mi}(t) \equiv y_{ti}(t) \equiv 0,\ \ t \in [\tau_{mi}, \tau_{1+i}], \\
x_{ti}(t) \equiv x_{ti}^{0},\ \ y_{hi} > y_{mi} > 0,\ y_{ti}(t) \equiv 0,\ t \in (\tau_{3-i}, \tau_{si}), \\
y_{hi} > 0,\ y_{mi} > 0,\ y_{ti} > 0,\ \eta_{Ai}(t) \equiv \mu_0,\ \eta_{Mi}(t) \equiv 0,\ \ t \in [\tau_{si}, \tau_{4-i}).
\end{gathered} \tag{9}$$

$$\begin{gathered}
R_{iy} > 0,\ \ t \in [\tau_i, \tau_{i+1}) \bigcup [\tau_{3-i}, \tau_{si}),\ \ R_{iy}(t) \equiv R_{ix}(t) \equiv 0,\ \ t \in [\tau_{si}, \tau_{4-i}), \\
x_{hi} \le x_{Ri} \le x_{si},\ t \in [\tau_i, \tau_{mi}),\quad x_{mi} \le x_{Ri} \le x_{si},\ t \in [\tau_{mi}, \tau_{1+i}) \bigcup [\tau_{3-i}, \tau_{si}), \\
|R_{ix}| \le \mu |R_{iy}|,\quad i = 1, 2,
\end{gathered} \tag{10}$$

Here $\tau_1 = 0,\ \tau_2 = \tau,\ \tau_3 = T$, $x_{h1}^{0} = 0$, $x_{h2}^{0} = x_{h1}^{0} + L,\ x_{mi}^{0} = x_{hi}^{0} + l_{hm},\ x_{ti}^{0} = x_{mi}^{0} + l_{mt}$, $i = 1, 2,$ $l_{hm} = |H_1 M_1| = |H_2 M_2|,\ \ l_{mt} = |M_1 T_1| = |M_2 T_2|$, μ – is the coefficient of friction between the foot and support surface.

Introduce the angles $\eta_{Ni},\ \ \eta_{Ki}$ at the hip and knee joints $N,\ K_i$ as follows: $\eta_{Ni}(t) \equiv \alpha_i(t) - \psi(t),\ \ \eta_{Ki}(t) \equiv \alpha_i(t) - \beta_i(t),\ \ i = 1, 2.$

Assume that the following restrictions impose on these angles:

$$\theta_{Ni}(t) \le \eta_{Ni}(t) \le \Theta_{Ni}(t),\ t \in [0, T]$$

$$\theta_{Ki}(t) \le \eta_{Ki}(t) \le \Theta_{Ki}(t),\ \ \theta_{Ai}(t) \le \eta_{Ai}(t) \le \Theta_{Ai}(t),\ t \in \Lambda_i = (\tau_{3-i}, \tau_{si}) \bigcup [\tau_i, \tau_{i+1}], \tag{11}$$

where the functions $\theta_{Ni,Ki,Ai}$, $\Theta_{Ni,Ki,Ai}$, ($i = 1, 2$) are prescribed using experimental data of normal human gait [18].

Let $Z(t)=\{x,\dot{x},y,\dot{y},\psi,\dot{\psi},\alpha_i,\dot{\alpha}_i,\beta_i,\dot{\beta}_i,\gamma_i,\dot{\gamma}_i,\varepsilon_i,\dot{\varepsilon}_i,\ i=1,\ 2\}$ be a vector of the phase state, $U(t)=\{q_i,u_i,p_i,w_i,\ i=1,2\}$ be a vector of the control stimuli of the BLS, $\mathbf{C}_p\in\Delta_p$, $\mathbf{C}_w\in\Delta_w$. The following problem can be stated.

Problem B. Assume that we are given the step length $L=L_0$ and the duration of the double step $T=T_0$. It is required to determine the control process $\{Z(t),\ U(t)\}$, $t\in[0,T]$, the structural parameters $\mathbf{C}_p^*\in\Delta_p$, $\mathbf{C}_w^*\in\Delta_w$ and the rhythmic parameters of the gait, all of which satisfy the equations (3)-(6), the boundary conditions (7), the given constraints (8)-(11) and which minimize the following functional:

$$E=\frac{1}{2L}\int_0^T\left\{\sum_{i=1}^{2}\left[|\,q_i\dot{\eta}_{Ni}\,|+|\,u_i\dot{\eta}_{Ki}\,|\right]\right\}dt \tag{12}$$

The *Problem B* belongs to the class of more general optimization problem for semi-passively controlled mechatronic systems (see *Problem A* above). From a mathematical point of view the *Problem B* is a nonlinear nondifferentiable optimal control problem with restrictions imposed both on the phase coordinates and the controlling stimuli in which the left and the right-hand end points of the state trajectories are variable.

From mechanical point of view the mechanical system in question has variable number of degree of freedom on the considered sequence of the phases of the leg's action during a double step, and moreover the system is dynamically redundant during the double support phases $t\in[0,\tau_{s2})$, and $t\in[T_1,\tau_{s1})$.

6 ENERGY EFFICIENT CONTROLLED MOTION OF UNDERACTUATED BLS

Central to the proposed approach for solving problem A is the idea that any optimal control problem can be converted into a standard nonlinear programming problem by parameterize each of the free variable functions.

Analysis of the constraints (7)-(9) shows that the following functions can be chosen as free variable functions in *Problem B:*

$$\begin{gathered}\omega=\bigcup_{k=1}^{5}\omega_k,\ \omega_1=\{x(t),x_g(t),\eta_{K1}(t),\eta_{K2}(t)\},\ t\in[0,T],\\ \omega_2=\{\alpha_2(t),\gamma_1(t)\},\ t\in[0,T_1],\ \omega_3=\{\alpha_1(t),\gamma_2(t)\},\ t\in[T_1,T],\\ \omega_4=\{\gamma_2(t),\eta_{M2}(t)\},\ t\in[\tau_{s2},T_1],\ \omega_5=\{\gamma_1(t),\eta_{M1}(t)\},\ t\in[\tau_{s1},T],\end{gathered} \tag{13}$$

where $x_g(t)\equiv x(t)-r\sin\psi(t)$ – is the abscissa of center of gravity of the trunk.

The functions ω_1, ω_2, ω_3 are parameterized by smoothing cubic splines [19]. For every this function the following domain is considered: $\Omega^\omega=\{\sigma_0^\omega,\sigma_1^\omega,...,\ \ \sigma_{n^\omega}^\omega;\sigma_0^\omega<\sigma_1^\omega<...<\sigma_{n^\omega}^\omega\}$. Moreover, for functions $x(t)$, $x_g(t)$ the domains Ω^ω are considered in case when $\tau\in\Omega^x$, $\tau\in\Omega^{x_g}$. It is known [19] that smoothing cubic spline $S^\omega(t)$ is determined by the values of variable function $z_0^\omega,\ z_1^\omega,\ \dots,\ z_{n^\omega}^\omega$, by the weighting coefficients $\rho_0^\omega,\ \rho_1^\omega,\dots,\rho_{n^\omega}^\omega$ at the knots $\sigma_0^\omega,\sigma_1^\omega,...,\sigma_{n^\omega}^\omega$, and also by the following boundary conditions:

$$\dot{S}^{\omega}(\sigma_0^{\omega}) = \dot{S}^{\omega}(\sigma_{n^{\omega}}^{\omega}),\ \ \ddot{S}^{\omega}(\sigma_0^{\omega}) = \ddot{S}^{\omega}(\sigma_{n^{\omega}}^{\omega}), \tag{14}$$

or

$$\dot{S}^{\omega}(\sigma_0^{\omega}) = a^{\omega},\ \ \dot{S}^{\omega}(\sigma_{n^{\omega}}^{\omega}) = b^{\omega}. \tag{15}$$

For every variable functions the values $z_0^{\omega},\ z_1^{\omega},\ldots,\ z_{n^{\omega}}^{\omega}$, and also the a^{ω}, b^{ω} are use as optimization parameters. Some of these parameters are used to satisfy the restrictions (7), (9) for the instance of times $t = 0, T_1, T$. The weighting coefficients $\rho_0^{\omega},\ \rho_1^{\omega},\ldots,\rho_{n^{\omega}}^{\omega}$ are given as the input parameters to the algorithm. Note that at knots σ_l^{ω} with coefficients $\rho_l^{\omega} = 0$ the following equalities are valid: $S^{\omega}(\sigma_l^{\omega}) = z_l^{\omega}$.

The variable parameters z_j^{ω} have been represented by the following expression

$$z_j^{\omega} = \bar{z}_j^{\omega} + C_j^{\omega},\quad j = 0,1,\ldots,n^{\omega}. \tag{16}$$

Here $\bar{z}_j^{\omega}$ are some initial values of the variable function ω at the knots $t = \sigma_j^{\omega}$ which are calculated using the equality constraints, i.e. equality (9); C_j^{ω} are new optimization parameters.

Based on the described methodology of approximation of the free variable functions by cubic smoothing splines and by using inverse dynamics approach, the controlling process $\{Z(t), U(t)\}$ of the BLS can be calculated. Henceforth, the *Problem B* is converted into the following nonlinear programming problem:

$$Q_0(\mathbf{C}) \to \min_{\mathbf{C}},\ \ \mathbf{Q}(\mathbf{C}) = 0, \tag{17}$$

Here $\mathbf{C} = \{C_j^{\omega}, \mathbf{C}_p, \mathbf{C}_w\}$ is a vector of variable parameters. The functions Q_0 and $\mathbf{Q}$ are determined by means of equations (3)-(6), constraints (7)-(11), functional (12), formulae (12)-(16), and the following expressions:

$$Q_{Ni} = \int_0^T [(\eta_{Ni} - \Theta_{Ni})_+ + (\theta_{Ni} - \eta_{Ni})_+]dt,\ \ Q_{Ki} = \int_{\Lambda_i} [(\eta_{Ki} - \Theta_{Ki})_+ + (\theta_{Ki} - \eta_{Ki})_+]dt,$$

$$Q_{Ai} = \int_{\Lambda_i} [(\eta_{Ai} - \Theta_{Ai})_+ + (\theta_{Ai} - \eta_{Ai})_+]dt,\ \ Q_{yi} = \int_0^T [(-y_{Hi})_+ + (-y_{Ti})_+]dt,$$

$$Q_{xi} = \int_{\tau_{s,3-i}}^{\tau_{mi}} [(x_{Hi} - x_{Ri})_+ + (x_{Ri} - x_{Ti})_+]dt + \int_{\tau_{mi}}^{\tau_{i+1}} [(x_{Mi} - x_{Ri})_+ + (x_{Ri} - x_{Ti})_+]dt, \tag{18}$$

$$Q_{Ri} = \int_0^T (-R_{iy})_+ dt,\ \ Q_{\mu i} = \int_0^T (|R_{ix}| - \mu |R_{iy}|)_+ dt,\ Q_{pi} = \int_0^T \left|p_i - f_{pi}\right| dt,\ Q_{wi} = \int_0^T \left|w_i - f_{wi}\right| dt,$$

where the notations are used: $(\nu)_+ = \nu$ for $\nu \ge 0$ and $(\nu)_+ = 0$ for $\nu < 0$.

Using the penalty function approach the problem (17) is reduced to the following problem

$$G(\mathbf{C}) = Q_0(\mathbf{C}) + \sum \lambda_k Q_k(\mathbf{C}) \to \min_{\mathbf{C}}, \tag{19}$$

In (19) $\lambda_k > 0$ are given numbers, Q_k are determined by the formulae (18).

Henceforth, the optimization problem (*Problem B*) has been converted into the unconstrained optimization problem (19). To solve the problem (19) the Rozenbrock's method was used.

The methodology and algorithm described above have been used to solve *Problem B* for underactuated BLS. Below the numerical results are presented for the following anthropomorphic values of the linear and mass-inertia parameters of the BLS:

M = 73.2 kg, m = 46.6 kg, r = 0.39 м, J = 7.1 Nм^2, $m_{a1} = m_{a2}$ = 8.5 kg, $a_1 = a_2$ = 0.47 м, $r_{a1} = r_{a2}$ = 0.26 м, $J_{a1} = J_{a2}$ = 0.26 Nм^2, $m_{b1} = m_{b2}$ = 3.5 kg, $b_1 = b_2$ =0.53 м, $r_{b1} = r_{b2}$ =0.21 м, $J_{b1} = J_{b2}$ =0.22 Nм^2, $m_{f1} = m_{f2}$ =1.2 kg, l_{hm} =0.19 м, l_{mt} =0.1м, $\varphi_1 = \varphi_2 = 38°$.

Coefficient of friction was chosen as $\mu = 0.9$.

Here we describe in detail the resultant energetically optimal motion of the BLS which has been obtained by the solution of *Problem B* for the step length of L =0.76m and duration of the double step T =1.14s, (for so called human gait with natural cadence [18]).

Input parameters for smoothing cubic splines approximation were given as follows: $n^x = n^{x_g}$ =13, $n^{\alpha_1} = n^{\alpha_2}$ =14, $n^{\gamma_1} = n^{\gamma_2}$ =21, $n^{\eta_{K1}} = n^{\eta_{K2}}$ =23, $\rho^x = \rho^{x_g} = \rho^{\alpha_1} = \rho^{\alpha_2} = \rho^{\gamma_1} = \rho^{\gamma_2}$ =0.01, $\rho^{\eta_{K1}} = \rho^{\eta_{K2}}$ =0.0001. Total number of optimization parameters was equal to 76. Accuracies of the numerical solution were equal to 10^{-6} и 10^{-4} with respect to the parameters optimization and the value of the cost function, respectively. Te total CPU time for solving the optimization problem was estimated about 6 minutes by using PC AMD Sempron 2200+ MHz.

The obtained energy-optimal motion is characterized by the following energy and rhythmic parameters: E=95 J/м, τ_{h1} = 0.05T, τ_{m1} = 0.36T, τ_{s1} = 0.58T, τ_{h2} = 0.55T, τ_{m2} = 0.86T, τ_{s2} = 0.08T.

Optimal structural parameters of passive drives located at feet joints: $\mathbf{C}_p$ = (92, 102, 165, 396, 0, -0.04, 5.6), $\mathbf{C}_w$ = (26, 339, 17). Here c_j are given in Nм/rad., μ_k – in degrees.

Figures 3-11 show the ways in which some kinematic and dynamic characteristics change in time over a double step for the obtained energetically optimal motion of BLS (thick curves). Time t is given in percentage of T, angles η_{N1}, η_{K1} and η_{A1} – in degrees, the joint torques $q_1(t)/M$, $u_1(t)/M$, $p_1(t)/M$, $w_1(t)/M$, are given in Nм/kg, horizontal and vertical components $R_{1x}(t)/M$, $R_{1y}(t)/M$ are given in N/кg. In Fig. 3-9 the domains of the values of the respective angular and dynamic characteristics obtained using experimental data for normal human gait [18] are also depicted (the domains are bounded by thin curves). Time histories of control torques at foot joints are shown in Fig. 10-11. Thick curves correspond to feet joint torques which were calculated by using equations (3)-(5). Thin curves were plotted by using formulae (6). The discrepancies between these curves are not larger than 2.3 %.

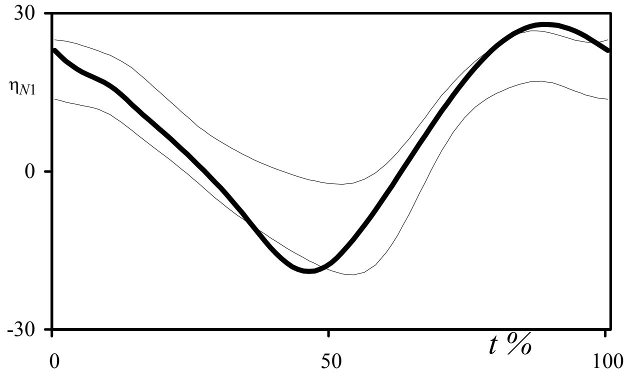

Fig. 3 Hip angle, in degrees

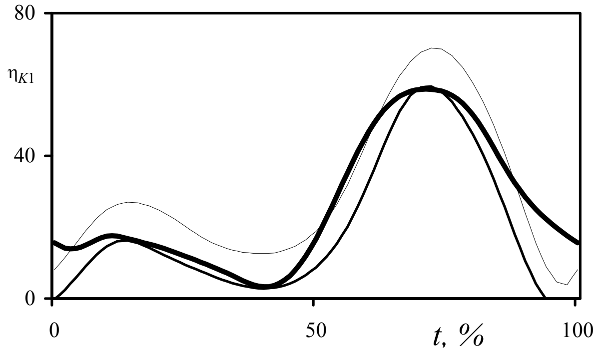

Fig. 4 Knee angle $\eta_{K1}(t)$, in degrees

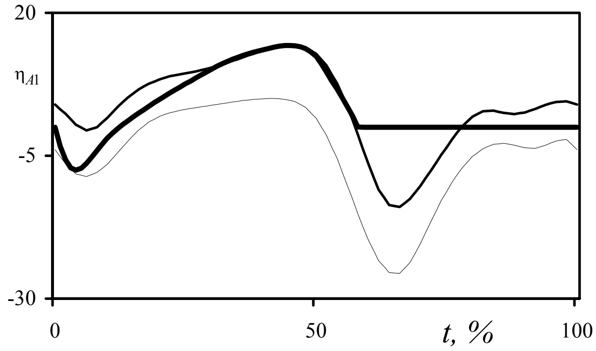

Fig. 5 Angle $\eta_{A1}(t)$, in degrees

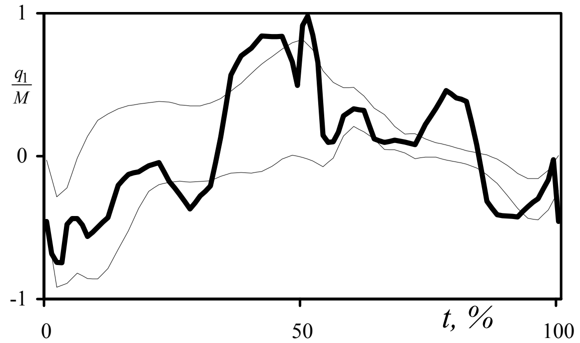

Fig. 6 Hip torque $q_1(t)/M$, in Nm/kg

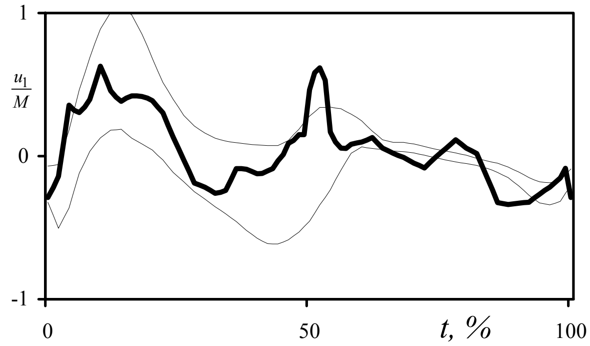

Fig. 7 Knee torque $u_1(t)/M$, in Nm/kg

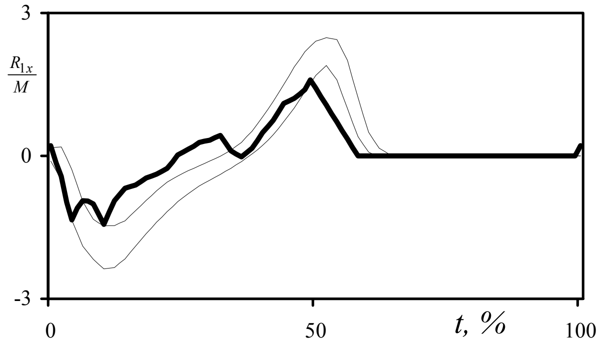

Fig. 8 Horizontal reaction force $R_{1y}(t)/M$, in N/kg

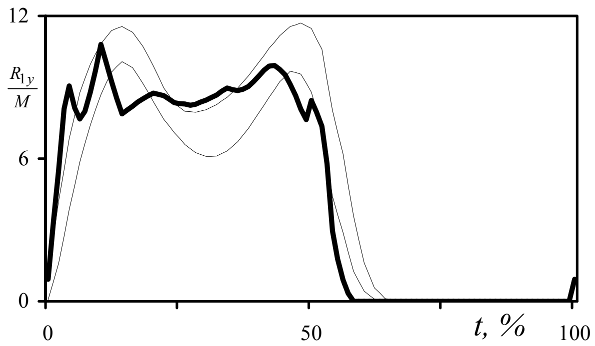

Fig. 9 Vertical reaction force $R_{1y}(t)/M$, in N/kg

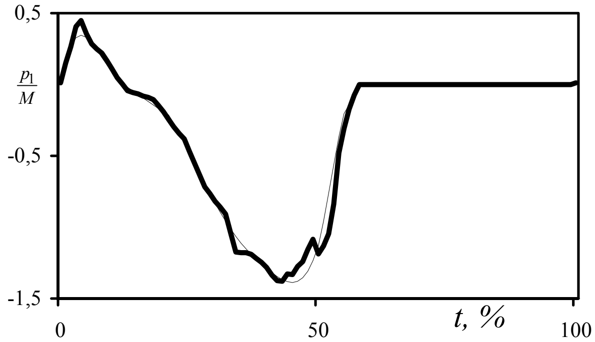

Fig. 10 Ankle torque $p_1(t)/M$, in Nm/kg

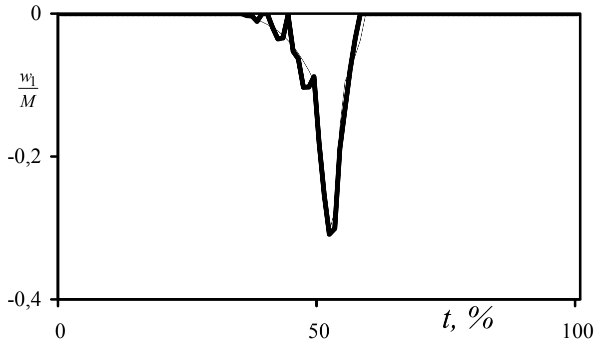

Fig. 11 Metatarsal torque $w_1(t)/M$, in Nm/kg

The analysis of these plots of the Fig. 3-9 indicates that the kinematic and dynamic characteristics of the obtained energetically optimal motion of the considered underactuated BLS are within reasonable proximity to the corresponding characteristics of human gait [18].

7 CONCLUSIONS

The dynamics, control and optimization problems for the BLS are interesting and important for many applications. For instance, to design the optimal legged mobile robots for difficult terrain, to recognize the neuro-system's laws governing the goal-directed motion of human locomotor apparatus, to design the optimal prostheses and orthoses of lower limbs.

In this paper the problem of optimization of the controlled motion of underactuated BLS has been investigated. From a mathematical point of view the considered object is a nonlinear multidimensional controlled system with a lot of constraints and restrictions imposed both on the phase coordinates and the controlling stimuli. The design of optimal control laws for these kinds of systems is a challenging research task that has attracted an increasing interest in recent decades.

A numerical method for the solution of optimal control problems of highly nonlinear and complex BLS has been proposed. The method is based on a special procedure of converting the initial optimal control problem into a standard nonlinear programming problem. This is made by the approximation of the independent variable functions using smoothing cubic splines and by the solution of inverse dynamics problems for the BLS. The key features of the method are its high numerical effectiveness and the possibility to satisfy a lot of restrictions imposed on the phase coordinates of the BLS automatically and accurately. For instance, there were totally 76 variable parameters in *Problem B*. To obtain the solution it was required only about 6 min of CPU time by using standard PC AMD Sempron 2200+ MHz.

An important benefit of recasting the optimal control problem for the BLS *(Problem B)* as a nonlinear programming problem is that it eliminates the requirement of solving a two-point boundary-value problem that must be solved to determine an explicit expression for the optimal control. In contrast to dynamic programming, the proposed method does not require massive computer storage. It thereby offers a streamlined approach for solving different optimal control problems for the BLS.

8 ACKNOWLEDGEMENTS

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