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เพื่อ พ่อ-แม่ พี่ชายและเมืองไทยที่รักจ้ะ

ABSTRACT

The aim of this thesis was to explore several important aspects of word problems: the nature of word problems used in school mathematics textbooks and the difficulty level of different types of word problems. The specific goals were to investigate students' performance when solving various types of word problems and to determine whether students' word-problem skills and their beliefs about word problem-solving can be improved by enriching word problems used in mathematics teaching. To achieve the goals, this thesis reports on five original studies, as follows.

Study I showed a comparison between the characteristics of word problems presented in Thai and Finnish school mathematics textbooks. The analyses included 1,565 word problems from a series of second- to fourth-grade Thai and Finnish mathematics textbooks. The overall results show that the nature of word problems used in Finnish textbooks vary from Thai textbooks in many ways. Finnish textbooks contain more multistep word problems, while in Thai textbooks, one-step word problems appear more frequently. Thai textbooks have a smaller percentage of repetitive sections (ones that include only the same type of problems) than Finnish textbooks. In both countries, the percentage of word problems requiring the use of realistic considerations is extremely low, less than five percent of the total.

Studies II and *III* presented the impacts of a Word Problem Enrichment (WPE) programme, developed to encourage teachers to use innovative self-created word problems to improve student mathematical modelling and problem-solving skills. Participants comprised 10 classroom teachers and their 170 students from fourth and sixth grades, from elementary schools in southwest Finland. In *Study II*, the intervention effectiveness on student problem-solving performance was investigated. The results suggested that enriching word problems used in mathematics teaching is a promising method for improving student problem-solving skills when solving non-routine and application word problems. However, it is not known if WPE has an effect on student beliefs about word problem-solving, and how the programme works for students with different initial motivation in learning mathematics. *Study III* examined the effectiveness of WPE on student beliefs about word problem-solving by using latent profile analysis (LPA) and structural equation modelling (SEM) to analyse relationships among the different cognitive, motivation, and belief factors. Results indicated that the impacts of WPE are various depending upon the initial motivation level of students. The effects of WPE on student beliefs appeared only in students with a low initial motivation level, while its impacts on student problem-solving performance were found only in students with a high initial motivation level.

Studies IV and *V* were conducted to examine hypotheses regarding (1) the dimensionality of students' performance on word problems and (2) difficulty level of three types of word problems: routine, non-routine and application word problems by utilizing item response theory (IRT) modelling. The data used in *Study IV* was collected as part of the Word Problem project (*Studies II* and *III*). Participants comprised 170 fourth- and sixth-grade students. Students' problem-solving performance was assessed with a word problem-solving test, including five word problems: one routine, three non-routine, and one application. The results of *Study IV* show that students' performance on word problems can be seen as a unidimensional construct that denies the original assumption. The results of the IRT model indicate that the theoretically demanding application word problem has a higher difficulty level than non-routine and routine word problems.

Nevertheless, the results are obscure if this application word problem (used in *Study IV*) is harder because of its demand for realistic considerations or other possibly relevant factors (e.g. decimal numbers included, division, more problem-solving steps required). Moreover, the sample size of *Study IV* could be considered relatively small for this kind of complicated IRT model. Therefore, *Study V* uses a larger sample size and a bigger set of word problems with more variety in application and non-routine word problems. The data used in *Study V* was collected as part of the Quest for Meaning project. Participants comprised 891 fourth-grade students (446 boys and 445 girls) from different elementary schools situated in cities, small towns, and rural communities in southern Finland. On the same lines as *Study IV*, the results of *Study V* indicated that students' performance on word problems can be seen as a unidimensional construct. Concerning item difficulty level, the results of the IRT model do not show a clear distinction among word-problem types and reject the hypothesis that application word problems have a higher difficulty level than non-routine word problems. Some non-routine word problems appear to be more difficult than the application word problem, even though other characteristics of these two types of word problems were very similar (e.g., they required the same type of operation and the same number of problem-solving steps).

The results of the five studies reveal that even though the mathematics textbooks were highly regarded in Thailand and Finland, most given word problems frequently include a simple goal without demanding any realistic considerations. These results strongly suggest that more innovative application word problems are definitely needed in classroom mathematics. In our study, we developed the WPE to encourage teachers to develop their own meaningful non-routine and applications word problems, and to use these self-created word problems to improve mathematical modelling and students' word problem-solving performance. The results show that WPE is a promising approach to improve not only student problem-solving skills but also student beliefs about word problem-solving. The impacts of WPE are different depending upon students' initial motivation level. The impacts of WPE on student beliefs were found only in students with a low initial motivation level, while its impacts on student problem-solving performance were found only in students with a high initial motivation level. These results suggest that in classroom practice, it is important that teachers provide enough support for students to be more confident and feel less overwhelmed when facing non-routine and application word problems. Teachers should be aware of differences of word-problem types and utilise this information in planning how to scaffold students' word problem-solving by giving word problems based on their difficulty level.

CONTENTS

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Turku, Finland, March 2017

LIST OF PUBLICATIONS

This doctoral thesis is based on the following five studies reported in four original articles, referred to in the text by their Roman numerals:

Study I Pongsakdi, N., Brezovszky, B., Veermans, K., Hannula-Sormunen M., & Lehtinen, E. (2016). A comparative analysis of word problems in selected Thai and Finnish Textbooks. In C. Csikos, A. Rausch, & J. Szitanyi (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education* Vol. 4 (pp. 75–82). Szeged, Hungary: PME.

Study II Pongsakdi, N., Laine, T., Veermans, K., Hannula-Sormunen M., & Lehtinen, E. (2016). Improving word problem performance in elementary school students by enriching word problems used in mathematics teaching. *Nordic Studies in Mathematics Education, 21*(2), 23–44.

Study III Pongsakdi, N., Laakkonen, E., Laine, T., Veermans, K., Hannula-Sormunen M., & Lehtinen, E. (in press). The role of beliefs and motivational variables in enhancing word problem solving. *Scandinavian Journal of Educational Research*.

Studies IV and V Pongsakdi, N., Kajamies, A., Veermans, K., Hannula-Sormunen M., Lertola, K., Vauras, M., & Lehtinen, E. (submitted). Examining dimensionality in word problem performance and difficulty of word problem types.

1. INTRODUCTION

Mathematics has been a central part of school curricula in most countries. Mathematics provides various sets of useful and powerful tools (Muller & Burkhardt, 2007) that can be helpful for students across a range of situations in everyday life if they know how to apply these mathematical tools (Lave, 1992). Nevertheless, for many students, choosing and utilizing these tools appropriately seems to be very challenging (Muller & Burkhardt, 2007). For this reason, word problems have been included in mathematics education to offer practice for students in employing mathematical skills efficiently in various problem situations experienced in everyday circumstances (Verschaffel, Greer, & De Corte, 2000). However, over the last few decades, educators and researchers have criticized this, saying that, in pedagogical practice, word problems are frequently trivialized and do not fully serve their original educational purpose of bridging between school mathematics and real-life mathematics (Verschaffel et al., 2000; Verschaffel, Greer, Van Dooren, & Mukhopadhyay, 2009; Verschaffel, Van Dooren, Greer, & Mukhopadhyay, 2010; Schoenfeld, 1991). Many students have difficulties applying school mathematics to solve word problems that resemble maths-problem situations encountered in daily life (Kajamies, Vauras, & Kinnunen, 2010; Verschaffel et al., 2000, 2009, 2010).

These concerns were corroborated by large-scale empirical evidence when the Director General of the National Board of Education in Finland reported disturbing news related to unexpected results of elementary school students' mathematical knowledge (Pitkälä, 2012) from a study that included almost 5,000 ninth-grade students from 113 elementary schools located in various areas of Finland (Metsämuuronen, 2013). The preliminary results showed that more than half of Finnish ninth-graders could not apply mathematical knowledge to solve word problems that resemble maths-problem situations encountered in everyday life. For example, the students failed to calculate the correct amounts of water and grain to make porridge if not directly instructed on the package. Moreover, less than half of ninth-graders could calculate a new price for electronic home appliances after being given their discount in percentage. These results raise a serious question concerning current pedagogical practice and, especially, a linkage problem between school mathematics and the real world.

In mathematics education (e.g. Lee, 2008; Ministry of Education, 2012; National Council of Teachers of Mathematics, 2010), word problems have been proposed as a means of teaching and learning mathematical modelling and problem-solving to prepare students to apply mathematics effectively in various problem situations confronted in everyday circumstances (Verschaffel et al., 2000). The underlying assumption is that learning mathematics in a meaningful context would enhance the transfer between school mathematics and a wide variety of contexts outside of school (Boaler, 1993). Experience with word problems creates a meaningful linkage for connecting classroom mathematics to real-life applications. However, contrary to these expectations, it has been widely reported that many students are unsuccessful in applying mathematical knowledge to solve word problems that resemble maths-problem situations encountered in their everyday life (Verschaffel et al., 2000, 2009, 2010). Several studies have indicated that many students tend to employ superficial comprehension strategies and do not create a sufficient understanding of the situations (situation model) described in the given problems. Students see the choice of mathematical operations with given numbers to be calculated as important and try to start to calculating immediately without basing the mathematical model on a proper situation model (Reusser & Stebler, 1997; Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005; Van Dooren, De Bock, Vleugels, & Verschaffel, 2010). Even when students use a deeper comprehension approach, they have difficulties in making proper use of realistic thinking when solving word problems that require the use of realistic considerations (Greer, 1993; Verschaffel, De Corte, & Lasure, 1994). For example, when students are requested to solve this plank word problem, “How many planks of 1 m Steve can get if he has bought 4 planks of 2.5 m each?” only 13% of students provided the realistic answer—8 planks (of 1 m)—while 73% answered 10 (Verschaffel et al., 1994). The students showed a strong inclination to suspend realistic considerations when solving the word problems. Several researchers have suggested that the reason for students’ superficial interpretations and unrealistic answers is not a cognitive deficit (Schoenfeld, 1991); it is assumed to originate from student beliefs that gradually developed from everyday practices in the context of schooling (Jiménez & Verschaffel, 2014; Mason & Scrivani, 2004; Reusser & Stebler, 1997; Schoenfeld, 1991) as a result of 1) the nature of word problems included in textbooks and 2) the way in which word problems are conceived and treated by teachers in regular mathematics lessons (Verschaffel et al., 1999, 2000).

To support these claims, there is a substantial amount of research investigating these two influential factors: word problems in textbooks and teachers’ pedagogical practice in mathematics lessons (e.g. Depaepe, De Corte, & Verschaffel., 2010; Gkoris, Depaepe, & Verschaffel, 2013; Joutsenlahti & Vainionpää, 2008), and their plausible impacts on students’ word problem-solving performance and their beliefs about word problem-solving (Jiménez & Verschaffel, 2014; Mason & Scrivani, 2004; Reusser & Stebler, 1997; Schoenfeld, 1991; Verschaffel et al., 2000, 2009, 2010). However, there are three limitations of these earlier studies that need further investigation.

First, previous textbook studies point out that regular mathematics textbooks mainly include word problems that have a straightforward goal and do not ask for any realistic considerations in the modelling process (Gkoris et al., 2013; Joutsenlahti & Vainionpää, 2008). This lack of word problems requiring realistic considerations in regular mathematics textbooks seemingly influences students’ problem-solving behaviour to not apply realistic considerations in the modelling process. Most of these textbook studies have been conducted in Western cultures. An interesting question, therefore, is whether the same issues also occur in textbooks from other cultural and educational contexts and whether they are less severe in textbooks that are considered to be among the highest-quality textbooks in that country. Furthermore, some other aspects of word problems in textbooks (e.g. word-problem types, the sequence of word problems) were mostly investigated during the 1980s (e.g. Stigler, Fuson, Ham, & Kim, 1986), but a more current situation of word problems in school textbooks regarding these particular aspects is unknown. Therefore, one of the studies in this dissertation explores and compares the characteristics of word problems included in highly reputed textbooks from two different cultural and educational contexts: Thailand and Finland.

Second, concerning the plausible impacts of traditional textbooks and pedagogical practice, several researchers have designed experimental programmes aimed at improving students' word problem-solving performance and their beliefs about problem-solving (e.g. Cognition and Technology Group at Vanderbilt [CTGV], 1992; Dewolf, Van Dooren, Ev Cimen, & Verschaffel; 2014; Higgins, 1997; Kajamies et al., 2010; Lee, Yeo, & Hong, 2014; Mason & Scrivani, 2004; Verschaffel & De Corte, 1997; Verschaffel et al., 1999). Verschaffel and De Corte (1997), for example, conducted an intervention study aimed at promoting students' realistic mathematical modelling by using application word problems. The positive effects of the programme suggested that it is possible to improve realistic modelling and reasoning skills by including more application and fewer routine word problems into mathematics lessons. Along the same lines, Higgins (1997) implemented one year of heuristic problem-solving instruction with middle school students. The results indicated that students in the experimental classroom had more sophisticated beliefs about mathematics than the students exposed only to traditional classroom teaching. Although these results demonstrated convincingly that the programmes have positive impacts on students' problem-solving performance and beliefs about word problem-solving, this does not guarantee successful large-scale application in uncontrolled settings (DeAngelis, 2010; Vanderlinde & Van Braak, 2010). Moreover, to investigate the effects of the programmes, these studies examined only students' word problem-solving performance and their beliefs about problem-solving. Other important factors that explain individual differences in word problem-solving performance, such as motivational variables, were not included. In this context, a Word Problem Enrichment (WPE) programme was developed to establish the scaling-up and transfer to uncontrolled settings. The idea of WPE is to encourage teachers to use innovative self-created word problems to improve student mathematical modelling and problem-solving skills in their classroom teaching. Among the goals for the studies in this dissertation was to determine whether it was possible to improve students' word-problem skills and their beliefs about word problem-solving by enriching word problems used in mathematics teaching through the WPE programme, as well as investigating motivational variables as factors in learning to solve word problems.

Lastly, word problems are generally difficult for many students to solve, because the process of word problem-solving requires students to use not only mathematical skills but also other cognitive skills (e.g. reading comprehension) (Daroczy, Wolska, Meurers, & Nuerk; 2015; Kintsch, 1988, 1998; Verschaffel et al., 2000). Not all word problems are the same. Some word problems are harder to solve than others (Cummins, Kintsch, Reusser, & Weimer, 1988). Previously, researchers have distinguished word problems as three different types: routine, non-routine, and application word problems ranging from conceptually simple to conceptually complex. First, a routine word problem is a problem that can be solved straightforwardly by a routine application. In contrast, a non-routine word problem is constructed in such a way that it cannot be solved by straightforward strategies; it requires students to develop an adequate understanding of the situations described in the problem texts (Elia, Van den Heuvel-Panhuizen, & Kovolou, 2009; Lee et al., 2014). Lastly, an application word problem is similar to the non-routine word problem but one additional requirement is the use of non-direct translation of the problem texts on the basis of real-world knowledge and assumptions in the mathematical model (Verschaffel et al., 2000). Accordingly, it seems sensible to assume that students' performance on word problems could be seen as multidimensional constructs as it involves several cognitive skills, and among three types of word problems, the application word problem is assumed to be the most difficult, followed by non-routine and then routine word problems. However, though this may be theoretically plausible, there is a lack of empirical evidence to support these two assumptions.

The present doctoral dissertation is comprised of four main sections providing a theoretical and methodological framework and a summary of five original studies. Firstly is the theoretical background regarding these following topics: difficulties in solving word problems, possible impacts of traditional textbooks and pedagogical practice, the role of beliefs and motivational variables, interventions in word problem-solving, and word problem enrichment (WPE) programme developed in the study. Secondly, the methodological framework of the present studies is explained, followed by a summary of the five original studies. Finally, a discussion of the main findings, educational implications and challenges is presented.

1.1 Difficulties in solving word problems

For many students, word problems are remarkably difficult to solve (Cummins et al., 1988). Word problems are dissimilar from usual mathematical tasks often presented in mathematical notation because the problem is explained through text that describes a situation and a question or questions to be answered by performing mathematical operation(s) derived from the descriptions in the text (Verschaffel et al., 2000). One of the most common explanations (e.g. Cummins et al., 1988; Kintsch, 1988) for students' difficulties in solving word problems is that the process of word problem-solving involves not only mathematical skills but also other cognitive skills. Kintsch (1988) argued that from the viewpoint of knowledge integration word problems are ideal because they not only involve mathematical knowledge but also linguistic and situational knowledge (= knowledge that allows one to understand the situation being described in the word problem) in understanding the problem. The process of word problem-solving is complex as its (complete) process involves a number of phases (Montague, Krawec, Enders, & Dietz, 2014; Verschaffel et al., 2000). Depaepe and colleagues (2015) reviewed different descriptions of the word problem-solving process (e.g. Blum & Niss, 1991; Burkhardt, 1994; Mason, 2001; Verschaffel et al., 2000) and concluded that, basically, they comprise six phases not necessarily sequentially performed: 1) understanding and defining the problem situation leading to a situation model, 2) developing a mathematical model base on a proper situation model, 3) working through the mathematical model to acquire mathematical results, 4) interpreting the results with respect to the original problem situation, 5) examining whether the interpreted mathematical result is appropriate and reasonable for its goal, and 6) communicating the acquired solution of the original word problem.

Not all word problems are the same. Some word problems can be more difficult to solve than others (Cummins et al., 1988). During the 1980s, to understand students' challenges with word problems, most research attention was paid to young children's difficulties in solving specific types of simple arithmetic word problems: change, combine and compare (Briars & Larkin, 1984; Carpenter & Moser, 1984; Carpenter, Moser, & Bebout, 1988; Cummins et al., 1988; Morales, Shute, & Pellegrino, 1985; Nesher, Greeno, & Riley, 1982; Riley & Greeno, 1988). These studies provided convincing evidence that, of these problems, compare problems are the most difficult to solve, despite the underlying maths being formally equivalent to the other types. The solution to compare problems requires children to compare quantities in two sets (Arendasy, Sommer, & Ponocny, 2005). For younger children (kindergarten and first grade), their difficulty in solving such word problems was generally explained by a lack of 1) conceptual knowledge needed to understand the semantic structure of a problem (Riley & Greeno, 1988) and 2) linguistic knowledge required to establish the connection between a given problem and conceptual knowledge (Cummins et al., 1988; De Corte, Verschaffel, & De Win, 1985; Kintsch, 1988). Kintsch (1988), for example, explained that what makes word problems difficult for many students is often not their computational demands but their linguistic expression and how formal mathematics relations map onto the described situation. According to Cummins and colleagues (1988), text comprehension factors have a strong influence in word-problem difficulty. Students' solution strategies appear to be dictated by their quality of comprehension achieved, and in turn, comprehension tends to be influenced by the characteristics of the language used in the problem text. Word problems that include certain linguistic forms such as "altogether" and "How many more X's than Y's?" are particularly hard for children to solve (Cummins et al., 1988).

Recently, Daroczy and colleagues (2015) tried to identify the essential factors that explain the difficulty of different types of word problems from previous existing literature. In the review, they categorized these influential factors on word-problem difficulty into three main aspects: 1) the linguistic complexity, 2) the numerical complexity, and 3) the relation between the linguistic and numerical complexity of the word problem. For the linguistic complexity aspect, Daroczy et al. (2015) elaborated that not only semantic and structural factors (e.g. Cummins et al., 1988) but also other linguistic elements, such as the number of letters and words, or the length of the sentences influence word-problem difficulty (Nesher, 1976; Lepik, 1990). Concerning the numerical complexity aspect, they explained that number properties (e.g. number types: fraction, whole number, decimal number; number of digits; number magnitude) (De Corte, Verschaffel, & Van Coillie, 1988; Haghverdi, Semnani, & Seifi, 2012; Lean, Clemens, & Del Campo, 1990; Nesher, 1976; Raduan, 2010), required operation (type, number) (De Corte et al., 1988; De Corte, Verschaffel, & Pauwels, 1990; Vicente, Orrantia, & Verschaffel, 2007) and the mathematical solution strategy (e.g. Brissiaud & Sander, 2010;

Fuchs et al., 2009; Garcia, Jimenez, & Hess, 2006) most probably impact the difficulty of word problems. For the relation between linguistic and numerical complexity, factors such as the number word structure (e.g. Dowker, Bala, & Lloyd, 2008), the number word system (e.g. Pixner, Kaufmann, Moeller, Hermanova, & Nuerk, 2011) and reading direction (e.g. Fischer & Shaki, 2014) were argued to influence word-problem difficulty. According to Daroczy et al. (2015), word-problem difficulty could be directly explained by the three major aspects. Unfortunately, however, the review mentions very little about non-routine thinking and realistic considerations in solving word problems and how these may influence the difficulty of word problems.

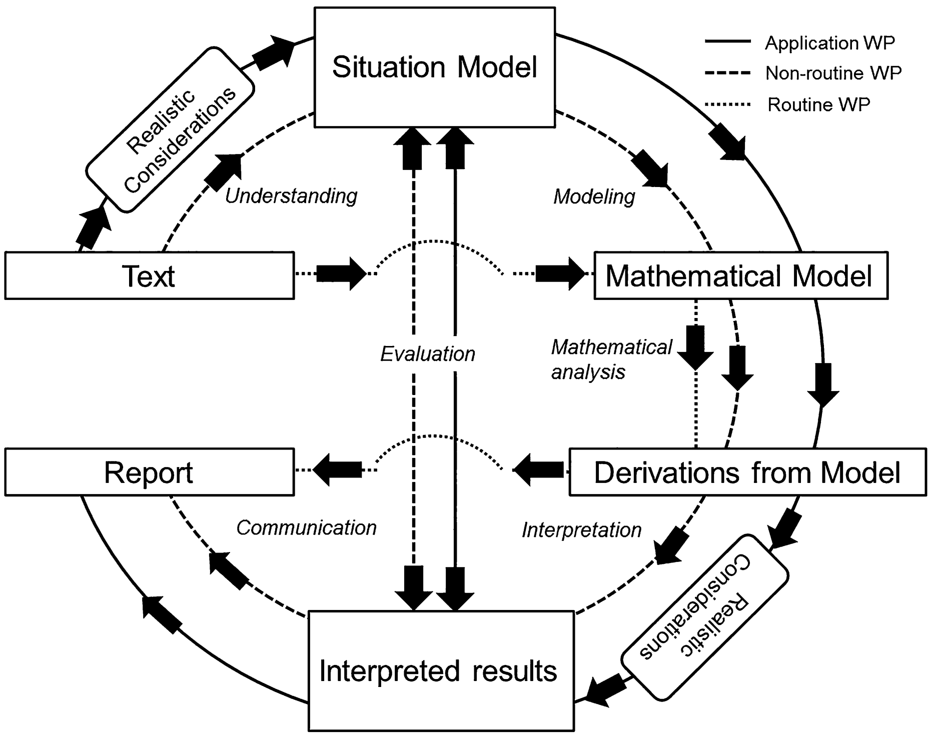

Figure 1. A process of mathematical word problem-solving of routine, non-routine and application word problems (adapted from Verschaffel et al., 2000, p.168)

At the beginning of the 1990s, research interest in students' challenges with word problems shifted to examine older elementary and lower secondary-school students' difficulty in solving word problems requiring non-routine thinking. Several studies reported many students had the habit of applying superficial and non-realistic strategies (see Figure 1), such as a keyword approach (looking for individual words that indicate specific mathematical operations e.g. "in total" = addition) and, as a result, fail to solve word problems requiring a non-routine thinking (De Corte, Greer, & Verschaffel, 1996; Dewolf et al., 2014; Elia et al., 2009, Pantziara, Gagatsis, & Elia, 2009; Van Dooren et al., 2005; Verschaffel et al., 2000, 2009).

Table 1. An example of routine, non-routine, and application word problems

Types	Word problems
Routine	Pekka has 7 adventure books. Pirkko has 6 adventure books more. How many adventure books does Pirkko have? (Correct answer: 13.)
Non-routine	Kalle has 18 euros. He wants to buy two computer games which each of them costs 13 euros. Mother promises that Kalle will get 2 euros every time he takes the trash out. How many times does Kalle have to take the trash out to get enough money to buy both computer games? (Correct answer: 4.)
Application	Paula is preparing some food and drinks for her birthday party. She buys two packets of chips (1 packet costs 2.50 euros), a big packet of mixed candy (1 packet costs 3.60 euros) and 4 bottles of lemonade (1 bottle costs 1.25 euros). Three friends come to the party. How much do the snacks and drinks cost for each participant? (Correct answer: 3.4.)

Non-routine word problems appear to be more difficult than routine word problems because of their unstraightforward solution (Boonen, Reed, Schoonenboom, & Jolles, 2016; Elia et al., 2009; Lee et al., 2014). While the solution to routine word problems can be found through a straightforward translation of the problem text into a mathematical model without the need for developing a proper understanding of the word-problem context, the solution of non-routine word problems requires students to develop an adequate understanding of the situations described in the word-problem texts (situation model) before the mathematical model can be derived, thus making the solution process more complex (Boonen et al., 2016; Lee et al., 2014; Pantziara et al., 2009). There is however, another type of word problem that might presumably be even more complex to solve than non-routine word problems: "non-routine word problems requiring the use of realistic consideration" or in short, "application word problems" (for an overview, see Verschaffel et al., 2000, 2009, 2010). Similar to non-routine word problems, the solution to application word problems requires students to develop a proper situation model but with the additional requirement that it involves a non-direct translation of the word problem texts on the basis of real-world knowledge and assumptions into the mathematical model (= realistic considerations) (Verschaffel et al., 2000). Many studies have shown that most students do not engage in this process of applying real-world knowledge and realistic considerations, and they consequently fail to solve application word problems correctly (Verschaffel et al., 2000, 2009, 2010).

Based on the solution process of these three word-problem types (routine, non-routine, and application word problem) depicted in Figure 1, a) application word problems are presumably the most difficult, followed by non-routine and then routine word problems; and b) students' performance on word problems should likely be seen as multidimensional constructs as it involves several cognitive skills (e.g. mathematical skills and reading comprehension) (De Ayala, 2009, p. 275). However, though theoretically plausible, there is a lack of empirical evidence that supports these assumptions concerning the difficulty in the ordering of the three word-problem types – routine, non-routine, and application word problems – and the dimensionality of students' performance on word problems. One of the major aims of the present set of studies is putting these two hypotheses empirically to the test.

1.2 Possible impacts of traditional textbooks and pedagogical practice

As previously mentioned, the idea behind word problems is to teach and learn mathematical modelling and problem-solving to prepare students to apply mathematics in out-of-school contexts under the assumption is that learning mathematics in a meaningful context enhances transfer. However, it was also mentioned that many students are unsuccessful in applying mathematical knowledge in solving word problems that resemble maths-problem situations encountered in their everyday life, and there is reason to believe that, at least in part, this discrepancy between aim and outcome stems from the nature of word problems in textbooks and the pedagogical practices surrounding their use in the classrooms.

The nature of word problems included in textbooks

For word problems to function as a bridge between school mathematics and real-world mathematics applications, a basic requirement is that word problems presented in textbooks actually do display the characteristics that are thought to be beneficial for establishing this function. Unfortunately, however, educators and researchers have criticized word problems used in regular textbooks in this respect. It has, for instance, been argued that many word problems in common textbooks are too simple or straightforward and the questions can be answered easily by using a superficial approach (Wyndhamn & Säljö, 1997). These word problems mainly require a precise numerical response, which gives little or no room for realistic considerations to be integrated into the solution process (Freudenthal, 1991). Evidence to support this claim was shown in a textbook study done by Gkoris and colleagues (2013). Their findings indicated that around 90 percent of word problems in both old and new fifth-grade Greek mathematics textbooks could be solved by a direct translation of the problem texts into mathematical operations without the demand for any realistic considerations. Joutsenlahti and Vainionpää (2008) presented similar findings, showing that around 94 percent of word problems in fifth-grade Finnish mathematics textbooks are word problems that include a simple goal and always have merely one correct answer. The results of these studies strongly suggest a lack of word problems requiring the use of realistic considerations (application word problems) in elementary school mathematics textbooks.

The nature of problems in textbooks is not only a problem from the mathematical content point of view; it is also a problem because textbooks and their design can trigger certain behaviours in students. First, concerning the contexts of word problems, if word problems presented to students rarely resemble mathematical-problem situations that appear in everyday life, one can hardly expect students to employ realistic considerations when solving the word problems. Second, if word problems are sequenced in such a way that lets students figure out the solution method and the operation(s) needed in advance without reading the text (e.g. giving students whole pages of the same type of word problem) (Jonsson, Norqvist, Liljekvist, & Lithner, 2014), this can be expected to trigger Einstellung (a tendency where students repeat a known procedure even though it is not the optimal one) (Luchins, 1942) rather than comprehensive strategies that would lead to a proper understanding of the situation presented in the problems. Jonsson and colleagues (2014) explained that if word problems are given to students in this particular manner, students do not need to use either conceptual understanding or proper reasoning skills; they merely practice computation skills by recalling facts and imitating a solution procedure illustrated in the textbooks. Lastly, if word problems use graphical representations to describe a word-problem situation that has a direct mapping to the mathematical model, for instance, employing pictures to illustrate how 12 apples can be divided equally between 3 children, it becomes clear to students which mathematical operation they should perform, since the solution procedure is explained within the pictures, which again relieves students from the need to use either conceptual understanding or proper reasoning skills.

The previous section suggests that most of the research (including the textbook studies) regarding (the characteristics of) word problems has been conducted in Western cultures, and there is no clear evidence that this has led to textbooks that address word problems in a way that would be more likely to achieve research goals. The fact that most of the research has been conducted in Western cultures also raises an interesting question regarding other cultures and their approach towards including word problems in the maths textbooks. Therefore, one of the studies in this dissertation explored and compared second-grade to fourth-grade Thai and Finnish mathematics textbooks from this perspective.

Since it is not feasible to analyse all textbooks from a country, and following guidelines proposed in Flyvbjerg (2006), it was decided to focus on highly reputed textbooks, based on the assumption that if word problems are not addressed properly in those, they are either not addressed in less reputed textbooks either, or, reputation is based on wrong arguments.

Teachers' beliefs about word problems and their use of word problems

Teachers' beliefs and their use of word problems in mathematics classrooms are claimed to be another important reason for students' superficial problem-solving strategies and excluding real-world knowledge and realistic considerations in the modelling process. It is presumed that teachers' beliefs and their actions in mathematics lessons play an essential role in encouraging or hindering students to see the importance of learning mathematics in context and take realistic considerations into account (Depaepe et al., 2015; Hiebert et al., 1996).

Several studies have focused on teachers' beliefs. Verschaffel and colleagues (1997), for example, conducted a study on 332 preservice elementary-school teachers to investigate their beliefs about the role of real-world knowledge in mathematical modelling of school word problems. Their results indicated that the preservice teachers showed a strong inclination to exclude real-world knowledge and realistic considerations when dealing with application word problems. The teachers valued students' non-realistic answers when solving application word problems substantially more than realistic ones. Similar results were also found in the replication study by Bonotto and Wilczewski (2007), suggesting that prospective elementary school teachers tend to believe that the role of realistic considerations should not be stimulated in mathematics lessons, which is probably reflected in their practices in mathematics lessons (De Corte et al., 1996; Depaepe et al., 2015)

Chapman (2006) presents evidence to support this assumption by observing classroom culture of teaching word problems of 14 elementary- and high-school teachers. The study adopted the cognitive functioning theory from Bruner (1986) and made a distinction between two different approaches to word problems called *paradigmatic oriented* and *narrative oriented*. The paradigmatic-oriented approach is built on conceptualization, which mainly focuses on the mathematical model or structures that are universal and context-free. Teachers who employed this paradigmatic-oriented approach encouraged students to see how the mathematical structures can be separated from the word-problem context. In contrast, the narrative-oriented approach concerns the context of word problems, concentrating on the context sensitive (e.g. word-problem situations and storyline). Teachers who relied on this narrative-oriented perspective often created opportunities for students to express situations of the problem in which students saw themselves, or real-world experiences, and used that in dealing with the problem.

These two cognitive functioning approaches cannot be treated independently; Bruner (1986) pointed out that "efforts to reduce one mode to other or to ignore one at the expense of the other inevitably fail to capture the rich diversity of thought" (p. 11). However, Chapman's study (2006) showed that teachers employed both a paradigmatic and a narrative approach but the paradigmatic mode was more frequent and was combined with the narrative-oriented mode differently among the teachers. Inspired by Chapman's work (2006), Depaepe and colleagues (2010) conducted an in-depth investigation on how two upper-elementary school teachers handled word problems in their mathematical word-problem lessons over a seven-month period. Similar to the previous study, the results indicated that the lessons given by the two teachers were more categorized by the paradigmatic than the narrative approach. From the perspective of the general goal of word problems to connect school mathematics to real world applications, these findings suggest that in classroom practice, the narrative-oriented approach could be more emphasized when teaching word problems.

1.3 The role of beliefs and motivational variables

The two factors (traditional textbooks and pedagogical practice) described in the previous section may have a direct or indirect influence on two other influential factors that play an essential role in explaining students' word problem-solving behaviour. It has been widely acknowledged that the differences in student performance cannot be explained completely on account of cognitive skills but the role of beliefs and motivational variables must be considered in order to give sufficient explanation of the students' differences in problem-solving performance (Brose, Schmiedek, Lövdén, & Lindenberger, 2012; Maaß & Schlöglmann, 2009; Mason & Scrivani, 2004; McLeod & Adams, 1989; Pepin & Roesken-Winter, 2015; Pintrich & Schunk, 2002; Schoenfeld, 1992; Seegers & Boekaerts, 1993; Wigfield & Cambria, 2010).

Beliefs in learning mathematics

Research interest in the role of beliefs in learning mathematics increased during the 1980s (Hart, 1989), when researchers, who had initially included only cognitive components, failed to explain mathematical problem-solving behaviour and later found that other variables, such as beliefs, play an essential role in constituting elements of problem-solving processes (e.g. Garofalo, 1989; Garofalo & Lester, 1985; Schoenfeld, 1985a). Researchers have described the term “belief” in various ways. Some have seen belief as one of the main variables in an affective construct (e.g. McLeod, 1992), while others have used the term “belief” as overlapping and as synonymous with terms such as attitude, perception and value (see Leder & Forgasz, 2002). Op ’t Eynde and colleagues (2002) reviewed and developed a framework of student mathematics-related beliefs by integrating the major components of different models presented in previous studies (e.g. Kloosterman, 1996; McLeod, 1992; Pehkonen, 1995; Underhill, 1988). Op ’t Eynde et al. (2002) defined beliefs as subjective conceptions that students regard about mathematics education, about themselves as mathematicians and about the mathematics class context. In concordance with other studies, they classified student beliefs into three categories: (1) beliefs about mathematics, mathematical learning and problem-solving, and mathematics teaching, (2) beliefs about self in relation to mathematics, and (3) beliefs about the social norms in class.

Currently, it is commonly accepted that student beliefs have a significant influence on mathematical learning and problem-solving (De Corte, Op ’t Eynde, & Verschaffel, 2002; Schommer-Aikins, Duell, & Hutter, 2005), and there is a general assumption that the impact of student beliefs on their learning and problem-solving behaviour is mediated through three processes: cognitive, conative (motivational and volitional) and affective (Op ’t Eynde et al., 2002). First, several studies have shown that student beliefs about mathematics have an influence on the ways they engage with mathematical activities and how they approach problems (e.g. Garofalo, 1989; Schoenfeld, 1983, 1985a, 1989). For example, students who believe that mathematics involves mostly memorizing facts and formulas tend to handle mathematical problems in a very mechanical fashion, such as attempting to recall the most suitable methods for solving problems (Garofalo, 1989; Schoenfeld, 1989). Moreover, student beliefs about mathematics have an impact on what cognitive strategies or techniques will be used when solving problems. For instance, in Schoenfeld’s study (1985a), students failed to use learned mathematical knowledge because they perceived that the knowledge was not meaningful. It should be noted that the beliefs that underlie both phenomena (mechanically handling and failure to use mathematical knowledge) could be amplified or even be the result of the issues related to textbooks and pedagogical practices that were described in the previous section.

Second, there is also substantial evidence supporting the notion that student beliefs about mathematics and mathematics learning are related to their motivation in learning mathematics (e.g. Kloosterman, 2002; Kloosterman, Raymond, & Emenaker, 1996). It is acknowledged that students will not be highly motivated in learning unless they see the importance of what they learn (Eccles et al., 1983; Schunk, 1991). Lastly, student beliefs about mathematics contribute an important part to the development of their emotional responses to mathematical situations (McLeod, 1991, 1992). For instance, in regular classroom mathematics, students are often asked to solve routine problems. When solving these, student actions are based on previously learned procedures. Students expect that most mathematical problems can be solved in a short period of time and without any obstacles or delays; if there are any obstacles that interrupt problem-solving activities, their emotional responses can become quite intense (Mandler, 1984; McLeod, 1989). Again, the beliefs that underlie both seeing the importance of what they learn and lead to emotional responses to obstacles, illustrate the potential interaction between textbooks and pedagogical practices and beliefs and their joint effects on mathematical learning.

Beliefs about word problem-solving

An important study on student beliefs in the field of mathematical word problem-solving is the in-depth and systematic work by Verschaffel et al. (2000). The study pointed out that the students showed a strong tendency to apply superficial strategies and exclude realistic considerations in modelling processes. This tendency towards responding mechanistically is likely due to student beliefs constructed through the accumulated experience of traditional classroom mathematics (Schoenfeld, 1991; Verschaffel et al., 2000). Reusser and Stebler (1997) presented empirical evidence to support this assumption based on interviews with students who explained their reasons for superficial interpretations and unrealistic responses. Reusser and Stebler (1997) identified assumptions that students typically developed in the culture of traditional school mathematics. For instance, students assumed that every word problem used in the classroom made sense and there was only one correct answer to every problem. Moreover, they believed that looking at keywords or at previously solved word problems would help them to determine mathematical operation(s) when they do not understand the problem.

Several researchers have conducted intervention studies on mathematics education in realistic and powerful learning environments (e.g. CTGV, 1992; Kajamies et al., 2010; Verschaffel & De Corte, 1997). These studies, however, did not specifically investigate the change in student mathematical beliefs. One of the few studies that addressed the possibility of developing appropriate beliefs in the new classroom culture was the study by Verschaffel and colleagues (1999), who set up a design experiment in which a learning environment for solving application word problems was developed and implemented in fifth-grade classes. Student beliefs about the role of real-world knowledge in mathematical modelling and problem-solving were examined. The results indicated that students in the experimental group had more positive beliefs about the learning and teaching of mathematical word problem-solving. However, the effect of the programme on student beliefs was quite small. Unlike previous studies, Mason and Scrivani (2004) conducted a small-scale intervention study aimed specifically at ascertaining student beliefs about mathematics and mathematical learning. Over three months, 46 fifth-graders received instruction that focused on the development of student beliefs by changing the traditional learning environment. The results showed a positive impact of the intervention on student mathematical beliefs and performance in solving word problems.

The studies mentioned above have shown that it is possible to foster appropriate beliefs about mathematics and problem-solving. However, researchers in the field of mathematical problem-solving have shown that the differences in student performance also involve motivational variables (Brose et al., 2012; Pintrich & Schunk, 2002; Wigfield & Cambria, 2010), which were not investigated in these studies.

Motivation in learning mathematics

Motivation is another variable that is often used to explain individual differences in problem-solving performance. It is evident that motivation not only has a role in predicting mathematical achievement (Chiu & Xihua, 2008; Shores & Shannon, 2007; Singh, Granville, & Dika, 2002) but also a crucial role in predicting advancement during mathematics-education studies (Hannula, Kaasila, Pehkonen, & Laine, 2007). Even though there are several theories of motivation that are relevant to student learning, in this study, student motivation in learning mathematics was investigated through the lens of expectancy-value theory, since it has been widely utilised by several studies on mathematics learning (e.g. Berger & Karabenick, 2011; Greene, DeBacker, Ravindran, & Krows, 1999).

Expectancy-value theory

Eccles et al. (1983; Wigfield & Eccles, 2002) developed a modern expectancy-value model that emphasises the crucial influence of an individual's judgement on his or her ability to succeed in a task, as well as the incentive value of an outcome, as proximal determinants of achievement performance, choice, and persistence. The model consists of two main constructs: a) expectancies of success and b) task values.

Expectancies of success are represented by self-efficacy, which is defined as one's perception of his or her own capability to accomplish a specific task (Bandura, 1997). It is evident that students who perceive themselves capable of doing well on the task are much more likely to be motivated with respect to effort, persistence and behaviour than those who have a lower sense of efficacy (Bandura, 1997; Eccles, Wigfield, & Schiefele, 1998; Pintrich & Schunk, 2002). For example, students who view themselves as competent in maths are more willing to confront challenging, non-routine problems. In contrast, students who are not confident, or who view themselves as incompetent, tend to avoid solving tasks that seem complex or difficult.

Task values comprise four major components: interest, attainment value, utility and cost. Intrinsic or interest value is the enjoyment an individual experiences in doing the task – for example, students choose to learn mathematics because maths is exciting to them – while attainment value involves a sense of personal importance in doing well on the given task. Students who hold this value believe that it is important to be good at maths. Utility value, or usefulness of the task, refers to how useful the task is for one's future plans – for instance, students choose to learn maths because it will help them in the future. Finally, cost is defined as opportunities lost due to engagement in the task (e.g. I have to give up a lot to do well in maths), as well as the effort that one needs to make in order to complete the task (see Eccles et al., 1983, for discussion of these components).

1.4 Interventions in word problem-solving

Recent intervention programmes have focused on training students to learn how to apply metacognitive strategies when solving word problems (De Kock & Harskamp, 2014; Lee et al., 2014; Montague et al., 2014; Orosco, 2014). Lee and colleagues (2014), for instance, developed an intervention study aimed at improving students' understanding of the problem posed and their solution planning by using metacognitive-based instruction. The results indicated that intervention has positive effects not only on students' performance but also on their confidence in and personal control of problem-solving behaviour and emotions. Montague et al. (2014) conducted a study using a randomized controlled trial with 1,059 seventh-graders to replicate a result of an intervention programme called *Solve It!* (Montague, 2003). *Solve It!* is a cognitive strategy instructional intervention that teaches students cognitive processes and metacognitive strategies for successful problem-solving use while solving word problems. In agreement with previous study from 2003, the results indicated that the programme had positive impacts on students' problem-solving performance and maths achievement, and its effectiveness was found across students of varying maths ability (Montague et al., 2014). Apart from this metacognitive-based instruction, there is also another approach that has been proved to enhance students' problem-solving skills. Moran et al. (2014) examined the effectiveness of paraphrasing inventions on 72 third-grade students at risk of maths disabilities. The results showed that paraphrasing relevant and complete positions significantly improved students' problem-solving performance.

Unlike other recent intervention studies described here, Dewolf et al. (2014) designed an experimental programme aimed particularly at enabling students to develop a situational model when solving application word problems by using illustrations and warnings. However, contrary to their expectations, the findings showed that neither the illustration nor the warning, nor the combination of both manipulations, influenced the number of realistic reactions.

1.5 Word Problem Enrichment (WPE) programme developed in the study

The suggestion of Dewolf et al.'s (2014) findings is that students' tendency to exclude realistic considerations is deeply entrenched and cannot be changed simply by using an illustration or warning, as it has developed from the mathematics classroom practice and culture over many years. This implies that in order to successfully improve students' realistic mathematical modelling, we need to directly influence teachers' pedagogical practice. The WPE programme was developed to encourage teachers to use innovative self-made word problems to enhance student mathematical modelling and problem-solving skills. The design of WPE was inspired by the works of Verschaffel and De Corte (1997), Verschaffel et al. (1999) and CTGV (1992).

Verschaffel and De Corte (1997) conducted an experiment aimed at improving realistic mathematical modelling by using application word problems. The participants included 54 fifth- and sixth-grade students from three different classes of the same school. A teacher in the experimental class followed planned procedures in how to introduce these application word problems in mathematics lessons. The results of Verschaffel and De Corte's (1997) study indicated that it is possible to enhance realistic mathematical modelling by integrating more application and fewer routine word problems into mathematics classrooms. A few years later, Verschaffel and colleagues (1999) set up a bigger design experiment in which a learning environment for solving application word problems was developed and implemented in fifth-grade classes. A total of 232 fifth-grade students participated in the study. Students' performance on word problems and their beliefs about the role of real-world knowledge in mathematical modelling and problem-solving were examined. The results indicated that students in the experimental group outperformed, and had more positive beliefs about the learning and teaching of mathematical word problem-solving.

A study in the U.S. (CTGV, 1992) used new information technologies called "The Jasper Series" to promote problem posing, problem-solving, reasoning and effective communication in students. The Jasper Series was developed based on the idea of anchored instruction using real-world situations, including challenges to provoke thoughtful engagement that helps in the development of critical thinking and realistic reasoning skills. Together in small groups, students explore the word problem, search for extra information required to solve the word problem, discuss possible options and develop solutions. The findings indicated that the programme had a positive impact on problem-solving skills and planning word problems. Although these programmes proved to have positive effects on students, this does not guarantee successful implementation in a large-scale setting. Literature from various fields of study has revealed a gap between research and practice (Vanderlinde & Van Braak, 2010). It is argued that studies conducted in controlled settings miss the range of "messy variables" that might occur in real life (DeAngelis, 2010). In pedagogical practice, the challenges of integrating research findings into practice could include difficulties in implementation and limitation of resources.

Concerned about these issues and in an attempt to find effective and widely applicable pedagogical methods that could improve mathematical modelling and problem-solving skills, the WPE was developed, inspired by the works of Verschaffel and De Corte (1997), Verschaffel et al. (1999) and CTGV (1992) with the idea to provide examples of non-routine and application word problems for teachers accompanied by guidelines on how to create innovative word problems themselves or together with their students. As part of the programme, teachers were encouraged to use more non-routine and application word problems with meaningful contexts related to the real world in their mathematics classes. The present dissertation presents a number of studies that investigated the impacts of the WPE on students' word problem-solving performance and their beliefs about word problem-solving, and relations among cognitive, motivation and beliefs factors that explain individual differences in word problem-solving performance.

2. AIMS

The previous section outlined research related to the nature of word problems in school mathematics textbooks and pedagogical practice towards word problems in classroom mathematics and identified plausible impacts of issues related to these two factors on students' word problem-solving, their beliefs about word problems and how these, in turn, impact learning. This review of the literature revealed several aspects that deserve further investigation if the goal to equip students with the ability to apply mathematics effectively in everyday circumstances is to be fulfilled. Thus, the present set of studies aimed to:

1) Explore the contemporary status of word problems used in highly reputed mathematics textbooks.

2) Examine the impacts of a WPE on students' word problem-solving performance and their beliefs about word problems, and investigate how the programme worked for students with different initial motivation in learning mathematics.

3) Provide empirical evidence on the hypotheses concerning the dimensionality of students' performance on word problems and the difficulty level of types of word problems.

To complete these objectives, the present work consists of a textbook study, an intervention study, and two cross-sectional studies including students from the fourth and the sixth grades. Study I aimed to explore various aspects of word problems used in highly reputed mathematics textbooks. Studies II and III attempted to examine the impacts of a WPE on students' word problem-solving performance and their beliefs about word problems. Studies IV and V aimed to test hypotheses concerning the dimensionality of students' performance on word problems and the order of difficulty level of three types of word problems: routine, non-routine and application word problems.

In addition to these general goals, each study had more specific aims. Study I compared the characteristics of word problems used in a selection of Thai and Finnish mathematics textbooks. In particular, this study focused on exploring four different aspects of word problems: word-problem types, repetitiveness of word-problem sequences, graphical representations and the use of realistic considerations. Study II attempted to investigate the impacts of WPE on fourth- and sixth-grade students' word problem-solving performance when facing non-routine and application word problems. This study concentrated on cognitive factors only. Study III extended the focus of Study II by including the role of motivation and beliefs in improving word problem-solving performance. Study IV tried to test hypotheses concerning the dimensionality of students' performance on word problems and difficulty level of three types of word problems: routine, non-routine and application word problems. The data used in this study included 170 fourth- and sixth-grade students with five different word problems. Study V aimed to replicate the findings of Study IV with a larger sample size (891 fourth-grade students) and with more variety of non-routine and application word problems (15 word problems).

3. METHODS

This dissertation comprises five studies. In the following, a short description is given of the research projects that the five studies belong to, followed by a concise overview of the studies and an outline of methods and statistical analyses.

All five studies were conducted at the Centre for Learning and Research, University of Turku. Studies I–IV were part of the Word Problem project, funded by the Academy of Finland and led by Professor Erno Lehtinen. The main objective of the project was to develop a WPE programme designed to encourage teachers to use innovative, self-created word problems to improve mathematical modelling and word problem-solving performance in students. The WPE programme included only the professional development of teachers, with no systematic instruction on how the teachers should implement the new method in the classroom. This approach follows the general idea of teacher autonomy in Finnish comprehensive schools, which holds that the teachers have the freedom to design their own teaching.

The data used in Study V is part of a larger project called the Quest for Meaning project, funded by the Academy of Finland and led by Professor Marja Vauras. The main aim of this project was to promote students' word problem-solving skills, particular targeting low achievers. The project included an intervention programme that used the computer-supported adventure game called the Quest of the Silver Owl (Vauras & Kinnunen, 2003) as a tool for enhancing students' word problem-solving performance (Kajamies et al., 2010).

3.1 Participants and samples

Table 2. Overview of the type of study and sample size used in Studies I–V

Study	Type of study	Sample	Grade	Country	Project
I	textbook	1,565 word problems	2, 3, and 4	Thailand and Finland	Word problem
II and III	Intervention	170 students	4 and 6	Finland	Word problem
IV	cross-sectional	170 students	4 and 6	Finland	Word problem
V	cross-sectional	891 students	4	Finland	Quest for Meaning

Study I

Study I is a textbook study. Unlike typical textbook studies, Study I selected only textbooks that are highly regarded, drawing on the opinions of experts and experienced teachers. A series of second- to fourth-grade mathematics textbooks, used in the spring term, were selected for the purpose of this study. A total of 1,565 word problems were analysed.

Table 3. Number of word problems and sections in Thai and Finnish textbooks expressed by grade level.

Grade	Thai textbook		Finnish textbook	
	No. of word problems	No. of sections	No. of word problems	No. of sections
2	81	13	323	64
3	164	28	314	74
4	324	45	359	75
Total	569	86	996	213

Studies II, III and IV

Participants in Studies II and III consisted of 10 classroom teachers and 170 students, 75 boys and 95 girls, from the fourth and sixth grades. Although the students were drawn from different elementary schools located in socioeconomically varied areas in southwest Finland, the households were predominantly middle-class. The majority of students were Finnish (95.3%) and none of them were reported to have learning disabilities. Based on an open call, five classroom teachers volunteered to participate in the professional development programme (WPE). This group of teachers (n = 5) and their 98 students comprised the experimental group, while other volunteer classroom teachers (n = 5) and their 72 students served as the control group. The control-group teachers were not asked to participate in the WPE training, but they volunteered to participate in this study because of their own interest in the use of word problem-solving in mathematics education. All students completed three test instruments: a word problem-solving test, a motivation questionnaire and a word problem-solving beliefs questionnaire, which were developed as part of the Word Problem project. The ethical guidelines of the University of Turku were followed and the school principals and parents approved the study beforehand.

The cross-sectional sample used in Study IV was the same group of students who participated in the intervention studies (Studies II and III). The data used in Study IV was students' word problem-solving performance, which was measured as a pretest in Studies II and III.

Study V

Participants in Study V were 891 fourth-grade students, 446 boys and 445 girls, from different elementary schools situated in cities, small towns, and rural communities in southern Finland. All of them used Finnish as their mother tongue. All participants completed the word problem-solving test, which was constructed as a part of Quest for Meaning project. The same data was also partly used in a previous study (Kajamies et al., 2010). The ethical guidelines of the University of Turku were followed. Permission from both the school and the parents was obtained.

3.2 Measurements

Analytical framework for word problems

To examine the characteristics of word problems in the textbooks in Study I, an analytical framework for word problems was developed (Pongsakdi, Brezovszky, Hannula-

Sormunen, & Lehtinen, 2013), consisting of four main coding schemes: 1) classification of word-problem types, 2) repetitiveness of word-problem sequences, 3) graphical representations, and 4) the use of realistic considerations.

Classification of word-problem types

The coding scheme for word-problem types (see Appendix I) was constructed based on the classification schemes from Greer (1987). Each word problem in the textbooks was classified as belonging to either one-step addition and subtraction word-problem types (21 different types of *change*, *combine*, *compare* and *equalize* word problems), one-step multiplication and division word-problem types (18 different types of *multiple group*, *iteration of measure*, *rate*, *measure conversion*, *rectangular array*, *combination* and *area*) or one-step word problems that do not belong to either category (e.g. Metinee finished her homework at 11.25. She spent 1 hour 20 minutes doing it. When did she start to do the homework?) or a multistep word problem. The inter-rater agreement for word-problem types between two independent coders was high (κ = .81).

Repetitiveness of word-problem sequences

Repetitiveness of word problems was investigated by determining the type of word problem used in a section of word problems. A section was considered repetitive if it contained only one type of word problem. For sections that included only multistep word problems, whether those multistep word problems could be solved in the same way (even if the given numbers were different) was investigated. Sections in which all multistep word problems could be solved in the same way were also considered to be repetitive.

Graphical representations

Graphical representations used in word problems were classified according to the coding scheme presented in Table 4. The inter-rater reliability between two independent coders was excellent (κ = .93).

Table 4. Classification of graphical representations used in the textbooks developed by Pongsakdi et al. (2012, 2013)

Types	Description	Code
No graphical representation	There is no graphical representation used in the word problem.	0
Picture containing numerical data	The main purpose of using the picture is to provide numerical data.	1
Picture describing the situation	The main purpose of using the picture is to illustrate the situation of the word problem. Although the picture may contain the numerical data, students do not need to use them since all data are already provided in the word problem.	2
Picture representing the object	The main purpose of using the picture is to represent the objects mentioned in that word problem. For example, there are 20  in the basket.	3
Picture for decorative purposes	The picture is related to the word problem but it is used only for decorative purposes.	4
Chart, graph, table	The data were represented in chart, graph or table formats.	5

The use of realistic considerations

This coding scheme for the use of realistic considerations was adapted from Gkoris et al. (2003). If word problems are constructed in a way that requires the use of non-direct translation of the word-problem texts on the basis of real-world knowledge and assumptions in the mathematical model, then they are coded as 1; those word problems that can be answered by direct translation of the word problem texts are coded as 0. For example, the bus problem "304 students must be bussed to their camping area. Each bus can hold 32 students. How many buses are needed?" Instead of the answer "9.5 buses", which derives from a mathematical model translated directly from the problem's statement (304 ÷ 32), students need to consider whether their answer is appropriate for the situation being modelled and provide an alternate more suitable answer (10 buses). Therefore, this word problem was coded as 1. The inter-rater agreement between two independent coders was excellent ($\kappa = .91$).

Measure of word-problem performance

Students' word problem-solving performance was assessed using two sets of word problem-solving test. The first set was developed as a part of the Word Problem project used in Studies II, III and IV and contains five word problems: one routine, three non-routine, and one application (see Appendix II). A routine word problem was adapted from a typical routine word problem presented in textbooks. This routine word problem was not included in the analyses in Studies II and III because it appeared to be too easy (a ceiling effect occurred with the pretest and posttest). Next, non-routine word problems were constructed in such a way that they could not be solved using straightforward strategies. For example, the word problems avoided using keywords and provided meaningful data in the written form instead of numbers. Lastly, an application word problem requiring realistic considerations was adapted from an original word problem in Depaepe et al.'s (2009) study. A parallel version of the word-problem pretest was developed for the posttest. The problems were structurally identical but the problem contexts differed. The number of word problems included in the test was quite small. However, in the present study, it is important for us to understand how students solved the problems when there was no time pressure or an overwhelming number of word problems. The students had around 35 minutes to complete the word-problem test.

For the purpose of Study II, the impacts of WPE on students' word problem-solving performance when dealing with non-routine and application word problems were analysed separately. Two types of scoring systems were used to analyse different types of word problems. For non-routine word problems, 1 point was given for each correct answer and 0 for an incorrect answer or no response. For an application word problem, it appeared that students had difficulties doing calculations, especially multiplication and division with decimal numbers. Many students showed that they understood how to solve the application word problem (e.g. writing the mathematical model, explaining the situation by drawing pictures), but they could not complete the calculation or they made calculation errors. In the study, we focused on how students understood the context of word problems and whether they were able to create a mathematical model derived from a proper situational model; therefore, although students made calculation errors, 3 points were given if they could provide a completed and correct mathematical modelling (either by writing a short description or by drawing pictures) that included the use of realistic considerations. Two points were given to students who provided a completed mathematical modelling without the use of realistic considerations, 1 point was given when students provided incomplete mathematical modelling (partly solving the word problem) and 0 for an irrelevant answer (an answer not at all related to the word problem) or no response. The inter-rater agreement between two independent coders scoring this problem was very high ($\kappa = .89$). For the purpose of Studies III and IV, all word problems were analysed using the same scoring system: 1 point was given for each correct answer and o points for an incorrect answer or no response.

The second set of the word problem-solving test was constructed as part of the Quest for Meaning project used in Study V. It contains 15 word problems: 13 non-routine and 2 application (see Appendix III). Similar to the first set of the word problem-solving test, non-routine word problems were created in such a way that they could not be solved by straightforward strategies, and the application word problems were constructed on the basis of original items used in earlier studies (Verschaffel et al., 2000). The students had no time limit to complete the test. All word problems were analysed by giving 1 point for each correct answer and 0 points for an incorrect answer or no response.

Assessments of motivation in learning mathematics and beliefs about word problem-solving

Motivation in learning mathematics

The 14-item questionnaire was used to measure student motivation. These items were adapted from the original scale used by Berger and Karabenick (2011) and developed based on expectancy-value theory (Wigfield & Eccles, 2002). For the purpose of the present study, the items were framed specifically for mathematics and modified to be suitable for primary school students. Students were asked to respond to all items on a five-point Likert scale ranging from 1 (completely disagree) to 5 (completely agree).

Beliefs about word problem-solving

The 13-item questionnaire was used to measure student beliefs about word problem-solving. Seven items related to typical beliefs about mathematics and beliefs about oneself as a problem solver were adapted from the original scale developed by Schoenfeld (1985b). These items were abbreviated to make them easier for primary school students to comprehend and framed specifically for word problem-solving. The other six items were developed based on important aspects of word problem-solving (e.g. keyword approach, importance of situation model) discussed in previous studies (Verschaffel et al., 2000).

3.3 Statistical analysis

Various types of statistical analyses were conducted in three different statistical programmes: SPSS statistics 22, Mplus version 7.0 (Muthén & Muthén, 1998–2012) and R 3.2.3 with ltm package (Rizopoulos, 2006). Table 5 shows the different statistical analyses used in the studies presented in this dissertation.

Table 5. Statistical analyses used in different studies

Statistical Analysis	Study	Purpose
SPSS		
Cohen's kappa	I and II	To check for inter-rater reliability between two independent coders.
Frequencies and descriptive statistics	I	To describe different aspects of word problems as a percentage.
Variance components analysis	II	To explore the need for multilevel approach when students are from separate classes.
Independent sample t-test	II and III	To compare students' responses in the word problem-solving test and beliefs about word problem-solving.
Repeated measures ANOVA	II	To compare students' responses in the word problem-solving test between two conditions.
Factor analysis	III	To explore underlying factors of students' beliefs about word problem-solving and their motivation.
Mplus		
Latent profile analysis	III	To classify students by their motivation.
Structural equation modelling	III	To investigate relationship between cognitive, beliefs and motivational variables.
R		
Item response theory modelling	IV and V	To investigate level of difficulty of word problems.
Unidimensional test	IV and V	To examine dimensionality of data.

Cohen's kappa

One of the main objectives of Study I was to develop a framework for analysing different important aspects of word problems. A total of 40 word problems were randomly selected from all word problems presented in the selected Thai and Finnish mathematics textbooks. These selected word-problem items were first translated into English by native speakers. Then, other independent native speakers translated the English version back into the two original languages to guarantee that the English texts contained the originally intended relevant aspects. The selected word-problem items were coded based on the developed framework by a Thai and Finnish coder. The coders were familiar with the framework and had an intensive training session around two weeks before they coded the actual items. Cohen's kappa was conducted to investigate the inter-rater agreements between two coders.

Frequencies and descriptive statistics

Frequencies and descriptive statistics were mainly used in Study I. They were conducted to describe the different aspects of word problems (word-problem types, repetitiveness of word-problem sequences, graphical representations and the use of realistic considerations) in a meaningful way. Each aspect of the word problems was presented as a percentage complemented by different types of graphical representation (e.g. table and bar chart).

Variance component analysis

Due to the nested nature of the data used in Study II, there might be a need for a multilevel approach. Therefore, a variance component analysis was conducted to compute intraclass correlation coefficients (ICC) to find out how much of the variation is explained by the class. If the results showed an ICC level lower than the cut-off level (0.25) proposed by Kreft (1996), then the data could be analysed using traditional methods (such as ANOVA) exclusively at the individual level (Kreft, 1996; Sagan, 2013).

Independent sample t-test

In Study II, an independent sample t-test was used to compare students' word problem-solving performance in a non-routine and application pretest between students in the experimental and the control group. For Study III, an independent sample t-test was conducted to determine whether there were significant differences regarding initial beliefs about the situation model between the high and low initial motivation groups.

Repeated measures ANOVA

After checking for the ICC level to confirm that the data in Study II could be analysed using traditional methods, the repeated measures ANOVA was conducted to examine the impacts of the intervention programme on students' word problem-solving performance when dealing with non-routine and application word problems.

Factor analysis

Factor analysis was used in Study III to determine the underlying factors of students' beliefs about word problem-solving and their motivation in learning mathematics.

Latent profile analysis

A latent profile analysis (LPA) was used in Study III to explore the patterns of students' initial motivation in learning mathematics. The LPA is a model-based classification technique that classifies students into homogeneous groups or latent person profiles based on their similarities in observed variables. It differs from other traditional person-oriented methods, such as cluster analysis, since it is model based and has stricter criteria for identifying the number of profiles or clusters (Muthén, 2001; Lubke & Muthén, 2005). The LPA was conducted with Mplus 7.0 (Muthén & Muthén, 1998–2012) with 300 and 30 random start values. The most representative model was selected based on these six main criteria: 1) low values for AIC (Akaike Information Criterion), 2) low values for BIC (Bayesian Information Criterion), 3) high values for entropy, 4) a significant result in the BLRT (Bootstrapped Likelihood Ratio Test), 5) a significant result in the LMR (Lo-Mendell-Rubin test), and 6) the class solution had a meaningful theoretical interpretation (Muthén & Muthén, 2000; Nagin, 2005).

Structural equation modelling

For the purpose of Study III, structural equation modelling (SEM) was employed to test a theoretical model to explain the relationships among the different cognitive, motivation and belief factors. SEM is a statistical methodology that applies a hypothesis-testing method to the analysis of a structural theory supporting on some phenomenon (Byrne, 2012). SEM methodology provides several important features that are improvements over the older generation of multivariate procedures. For example, SEM offers explicit estimates of error variance parameters, while traditional multivariate procedures are unable to either assess or correct for measurement error. This may lead to serious inaccuracies, especially when the errors are sizeable. Moreover, SEM procedures allow us to incorporate both unobserved (i.e. latent) and observed variables, whereas former methods are based on observed measurements only (Byrne, 2012).

Item response theory (IRT) modelling

Item response theory (IRT) was used to investigate the level of difficulty of word-problem items in Studies IV and V. It is widely employed in educational and psychological assessment and evaluation research. IRT attempts to model individual response patterns by determining how underlying latent trait level (i.e. ability) interacts with the item's characteristics (e.g. item difficulties or discrimination ability) in order to form an expected probability of the response pattern (Chalmers, 2012; Embretson & Reise, 2000). All IRT analyses were conducted using R 3.2.3 with the ltm package Latent Trait Models for Item Response Theory Analyses, which has been developed for the analysis of multivariate dichotomous data using latent variable models under the item response theory approach (Rizopoulos, 2006).

4. OVERVIEW OF THE STUDIES

4.1 Study I

Pongsakdi, N., Brezovszky, B., Veermans, K., Hannula-Sormunen M., & Lehtinen, E. (2016). A comparative analysis of word problems in selected Thai and Finnish Textbooks. In C. Csikos, A. Rausch & J. Szitanyi (Eds.). *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp 75–82. Szeged, Hungary: PME.

Word problems used in school textbooks are often claimed to be merely routine tasks with no resemblance to real life situation. This type of word problem has created a great concern in several researchers for the possible negative impact on students' problem-solving skills and beliefs about mathematics. The aim of Study I was to compare the characteristics of word problems used in a selection of Thai and Finnish mathematics textbooks. Unlike typical textbook studies, Study I selected only textbooks that are highly regarded, drawing on the opinions of experienced teachers. A series of second- to fourth-grade mathematics textbooks, used in the spring term, were selected for the purpose of this study. A total of 1,565 word problems were analysed.

To investigate the characteristics of word problems in the textbooks, an analytical framework for word problems was developed (Pongsakdi et al., 2013) consisting of four main coding schemes: 1) classification of word-problem types, 2) repetitiveness of word-problem sequences, 3) graphical representations, and 4) the use of realistic considerations. The results show that the characteristics of word problems used in Thai textbooks differ from Finnish textbooks in many aspects. The majority of word problems in Finnish textbooks are multistep word problems, while in Thai textbooks, one-step word problems are more prominent. Finnish textbooks have a higher percentage of repetitive sections (ones that include only the same type of problem) than Thai textbooks. Despite the textbooks used in this study having a good reputation in Thailand and Finland, the results agree with previous studies that most word problems used in the textbooks usually include a simple goal without the need for any realistic considerations (Gkoris et al., 2013; Joutsenlahti & Vainionpää, 2008). In both countries, word problems requiring the use of realistic considerations are infrequent, making up less than five percent of the total.

4.2 Study II

Pongsakdi, N., Laine, T., Veermans, K., Hannula-Sormunen M., & Lehtinen, E. (2016). Improving word problem performance in elementary school students by enriching word problems used in mathematics teaching. *Nordic Studies in Mathematics Education, 21*(2), 23–44.

Over decades, the way in which word problems are used and learned in mathematics education has been heavily criticized by a number of educators and researchers. It is evident that, after being immersed for many years in the traditional practice of word problems in school, students are inclined to apply superficial strategies and exclude several important steps of the modelling process (Verschaffel et al., 2000). The WPE programme's objective is to encourage teachers to use innovative self-created problems to improve mathematical modelling and problem-solving skills in students and so the purpose of this study was to investigate the effectiveness of the programme on students' problem-solving performance.

The study used a quasi-experimental pretest/posttest design. A total of 10 teachers and 170 students from the fourth and sixth grades participated. The experimental group consisted of 5 teachers and 98 students, and the control group comprised 5 teachers and 72 students. The experimental teachers were those who volunteered to participate in the professional development programme. The other teachers in the control group followed a traditional practice of word problems in mathematics lessons.

The results suggested that enriching word problems used in mathematics teaching has a positive impact on students' problem-solving skills not only with non-routine problems but also with complex application word problems. It is also possible that teaching teachers how to develop more advanced work problems also resulted in teachers' deeper understanding of the nature of high-level problem-solving processes. These results agree with previous intervention studies (CTGV, 1992; Verschaffel & De Corte, 1997) that it is possible to improve students' problem-solving skills and mathematical modelling by integrating more application and fewer routine problems into mathematics classrooms. The results of this study corroborate the view that WPE is a feasible method to enhance word-problem performance in elementary school students, and it could be a much-needed addition to current mathematics textbooks and teaching.

4.3 Study III

Pongsakdi, N., Laakkonen, E., Laine, T., Veermans, K., Hannula-Sormunen M., & Lehtinen, E. (in press). The role of beliefs and motivational variables in enhancing word problem solving. *Scandinavian Journal of Educational Research*.

Word problem-solving and mathematical modelling are widely seen as important aims of mathematics learning, which can prepare students to use mathematics in everyday situations. However, teachers face difficulties in teaching mathematics that goes beyond arithmetic. Several researchers have shown that differences in student performance cannot be explained as purely being the result of differences in cognitive skills; the role of beliefs and motivational variables must also be taken into account (Maaß & Schlöglmann, 2009). This study is about a WPE designed to encourage teachers to use innovative, self-created word problems to improve student word problem-solving performance. A positive impact of WPE on student word problem-solving performance was found in Study II focusing on cognitive factors only. However, it is unknown whether WPE has an impact on student beliefs and how WPE works for students with different mathematics motivation. This study investigated the impact of WPE on student beliefs about problem-solving by using

LPA and SEM to analyse the relationships among the different cognitive, motivation, and belief factors.

As in Study II, participants consisted of 10 teachers and 170 students from the fourth and sixth grades located in southwest Finland. Based on an open call, 5 teachers volunteered to participate in the professional development programme. These 5 teachers joined 98 of their students to comprise the experimental group, while the other 5 volunteer teachers and 72 of their students served as the control group.

The results revealed that WPE has a positive impact not only on student word problem-solving performance but also on their beliefs about the nature of word problem-solving. The effects of WPE are various, depending on initial mathematics motivation. The impacts of WPE on beliefs about the situation model were found only in students with a low level of initial mathematics motivation. In contrast, the effects of WPE on word problem-solving performance were found only in students with a high level of initial mathematics motivation. These results could be explained by expectancy-value theory, suggesting that students who are not confident in maths may avoid engagement in new pedagogical practices, where more demanding word problems are used, because they feel they are too difficult.

4.4 Studies IV and V

Pongsakdi, N., Kajamies, A., Veermans, K., Hannula-Sormunen M., Lertola, K., Vauras, M., & Lehtinen, E. (submitted). Examining dimensionality in word problem performance and difficulty of word problem types.

Word problems are notoriously challenging to solve. The process of word problem-solving requires students to use not only mathematical skills but also other cognitive skills (e.g. reading comprehension). In this study, three types of word problems were discussed. First, a routine word problem is a problem that can be solved straightforwardly by a routine application. In contrast, a non-routine word problem is constructed in such a way that it cannot be solved by straightforward strategies; it requires students to develop an adequate understanding of the situations described in the problem texts. Lastly, an application word problem is similar to the non-routine word problem but one additional requirement is the use of non-direct translation of the problem texts on the basis of real-world knowledge and assumptions in the mathematical model. Accordingly, it seems reasonable to hypothesize that students' performance on word problems can be seen as multidimensional constructs as it involves several cognitive skills and among the three types of word problems, the application word problem is the most difficult, followed by non-routine and the routine word problems. However, there is still a lack of evidence to support these two hypotheses. The aim of this study was to test the two hypotheses by using item response theory (IRT) modelling.

The data used in Study IV was collected as part of the Word Problem project. Participants consisted of 170 fourth- and sixth-graders in southwest Finland. Students' problem-solving performance was assessed using a word problem-solving test, containing five word problems: one routine, three non-routine and one application. The results of Study IV indicate that students' performance on word problems can be considered as a unidimensional construct, which contradicts the original hypothesis (concerning dimensionality of students' performance on word problems). Further, the results of the IRT model show that the theoretically more demanding application word problem has a higher difficulty level than non-routine and routine word problems, respectively. Nevertheless, it is not entirely clear whether this application word problem is more difficult because of its requirement for realistic considerations or because of other factors (e.g. decimal numbers included, division, more problem-solving steps required). Moreover, the sample size of Study IV could be considered rather small for a complicated IRT model. As a result, Study V used a larger sample combined with a larger set of word problems, with more variety in application and non-routine word problems. The data used in Study V was collected as part of the Quest for Meaning project. Participants were 891 fourth-grade students, 446 boys and 445 girls, from different elementary schools situated in cities, small towns, and rural communities in southern Finland. Along the same lines as Study IV, the results of Study V indicated that students' performance on word problems can be seen as a unidimensional construct. Concerning item difficulty level, the results of the IRT model do not show a clear distinction among word-problem types, rejecting the hypothesis that the application word problems have a higher difficulty level than non-routine word problems. Some non-routine word problems appear to be more difficult than the application word problem, even though other characteristics of these two types of word problems were very similar (e.g., they required the same type of operation and the same number of problem-solving steps).

5. MAIN FINDINGS AND DISCUSSION

The main goals of the present dissertation were to investigate students' performance when solving various types of word problems and to determine whether it is possible to improve students' word-problem skills and their beliefs about word problem-solving by enriching word problems used in mathematics teaching. The dissertation also aimed to investigate key aspects of word problems: the characteristics of word problems included in textbooks, dimensionality in word problem performance, and the difficulty level of various types of word problems. To accomplish these goals, the present work included a textbook study comparing Thai and Finnish textbooks (Study I), an intervention study (Studies II and III), and two cross-sectional studies (Studies IV and V) including students from the fourth and sixth grades. The results of Study I indicate that a majority of word problems in contemporary Finnish textbooks are multistep word problems, while in Thai textbooks, one-step word problems are more prominent. The percentage of repetitive sections (those that use only the same type of word problem) in Finnish textbooks is higher than in Thai textbooks. This finding suggests a higher risk of students developing superficial comprehension strategies in Finnish textbooks compared to Thai counterparts. Surprisingly, although the mathematics textbooks used in Study I were highly regarded, more than 95 percent of word problems included a simple goal with no demand for any realistic considerations. Though these results agree with previous textbook studies (Gkoris et al., 2013; Joutsenlahti & Vainionpää, 2008), they are also disappointing in the light of the general goal of including word problems into maths education and strongly suggest that more innovative application word problems are very much needed in classroom mathematics.

Faced with a lack of such problems provided by textbooks, the WPE programme was developed to encourage teachers to develop their own meaningful non-routine and application word problems and to use these self-created word problems to improve mathematical modelling and word problem solving performance in students. The results of Studies II and III show that WPE is a promising approach to enhancing not only student problem-solving skills but also student beliefs about word problem-solving. These results are along the same lines as earlier studies, indicating that it is possible to improve students' realistic mathematical modelling and problem-solving skills by enriching word problems used in mathematics teaching (CTGV, 1992; Verschaffel & De Corte, 1997) and is promising as a scalable solution that transfers to uncontrolled settings. However, according to the results of SEM in Study III, the effects of WPE vary depending on students' initial motivation level. The impacts of WPE on student beliefs were found only in students with a low initial motivation level, while its impacts on student problem-solving performance were found only in students with a high initial motivation level.

Regarding theoretical distinctions of word-problem types, and based on the proposed problem-solving models, it was hypothesized that among the three types of word problems – routine, non-routine and application word problems – application word problems, with their theoretically more demanding solution processes, are the most difficult, followed by non-routine and then routine word problems, and students' performance on word problems should be seen as a multidimensional construct that involves several cognitive skills (e.g. mathematical skills and reading comprehension). Concerning the difficulty level of word-problem types, the results of Study V revealed that the difficulty level of word problems cannot be simply justified by this typology. Even with the added complexity of realistic considerations, application word problems can be easier than some non-routine word problems, depending on the difficulty of the given situations to be modelled. Analysis of dimensionality reveals that contrary to the hypothesis, students' performance in word problems can be regarded as a one-dimensional construct according to the results of Studies IV and V. One possible explanation for this finding could be that if another dimension of students' performance on word problems does exist, it might be weak, and therefore, cannot be distinguished from random variation.

Theoretical implications

Several studies have pointed out that student beliefs are influenced by teachers through their practice in the classroom (Depaepe et al., 2015; Pehkonen, 1998; Pehkonen & Törner, 1996) and that it is important to improve student mathematical performance by changing their beliefs about the domain, as well as changing the beliefs of teachers (Mason & Scrivani, 2004). However, the results of the study showed that is not necessarily the case, especially for students with a low initial motivation level in learning mathematics. Even though their beliefs about the situation model were improved, this did not have an impact on student word problem-solving performance. Our findings suggest that it may not be enough to focus merely on changing student beliefs; student mathematics motivation needs to be considered as well. Students, particularly those with low initial motivation, can feel overwhelmed when dealing with challenging non-routine and application word problems. Future studies should pay more attention to the complex relationships between mathematics motivation, beliefs about mathematics learning and problem-solving, and mathematics performance.

Educational implications

The present study provides several educational implications and suggestions for teaching and learning mathematical word problem-solving. First, regarding mathematics textbooks, it is highly recommended that they should have more innovative application word problems. It is evident that this type of word problem could enhance student mathematical modelling and problem-solving skills (Verschaffel & De Corte, 1997), but, unfortunately, our results showed that, even in the highly regarded school mathematics textbooks, very few application word problems have been included. Moreover, it is suggested to mix word-problem types and avoid repetitive problem sequences in order to prevent students developing superficial comprehension strategies. Particularly in Finnish textbooks, they should try to steer away from the use of the same type of word problem in the same section in order to avoid triggering Einstellung (Luchins, 1942).

Second, concerning WPE, the results showed that WPE (which aims to facilitate teachers enriching word problems used in mathematics teaching) is a feasible method to enhance student word problem-solving performance and beliefs about word problems and is a much-needed addition to current mathematics textbooks and teaching. Moreover, the focus on teachers' pedagogical thinking, without any standardised classroom practice, makes it feasible (in terms of resources) to apply this kind of programme in large-scale training in the future (e.g. preservice and in-service teacher curricula).

The results of the studies featured in this dissertation also reveal that, in classroom practice, it is important that teachers provide adequate support for students to be more confident and feel less overwhelmed when facing non-routine and application word problems.

Lastly, previous studies have highlighted the importance of using varied word problems, including complex non-routine and application word problems, in mathematics education (CTGV, 1992; Verschaffel & De Corte, 1997). The results presented in this dissertation, in combination with the conceptualization of problem types and processes (Figure 1), give additional information about different aspects of difficulty. This can be used to inform teachers and ensure they are aware of the differences between and within word problems so that they can provide their students with the variety of word problems (other than routine word problems), support their students in learning important components of word problem-solving processes (e.g., constructing situation models and interpreting results), and develop their students' understanding of these word problems that can be transferred to situations outside the classroom.

Limitations and challenges for future studies

The studies featured in the present dissertation explored different important aspects of word problems, as well as investigating the impacts of WPE on students' word-problem skills and their beliefs about word problem-solving. Despite their important findings, which fill gaps in the research literature and have educational implications in learning and teaching mathematical word problem-solving, there are some limitations that may impact the conclusions that can be drawn. First, for the purpose of the textbook study (Study I), a series of second- to fourth-grade mathematics textbooks, used in the spring term, were selected; results might have differed if the sampled textbooks covered the whole year and several class levels. For example, the total number of word-problem types (a one-step addition and subtraction word problem, a one-step multiplication and division word problem, a multistep word problem) might be different depending on the curriculum and whether they learn that particular topic in the spring or the autumn term and at different class levels, though the fact that the books cover the spring over three years makes it less likely that these differences would influence the general outcome.

Second, concerning WPE, the programme not only promoted the use of more variety of non-routine and application word problems in the classroom but also emphasized to teachers how the traditional practice of word problems in classroom mathematics impacts students and why the current practice needs to be changed. Practically, it tried to convince teachers to change their beliefs about the educational relevance of word problems. Based on teachers' open-ended writings on how the programme affected the way they use word problems in their teaching, all the teachers gave positive remarks about the programme and reported its usefulness and inspirational effects on their own teaching of word problem-solving, but no data on the impact of the classroom practices was collected. Although evidence from Studies II and III suggested positive outcomes of WPE on student word problem-solving performance and beliefs about the situation model, thus indicating effects from altered practices, direct investigations into the effects of WPE on teacher classroom practices (the use of word problems, instructional and assessment approach) and beliefs about word problems should also be conducted.

This is especially important given the fact that the effects of WPE on word problem-solving performance were found only in students with a high level of initial motivation in learning mathematics. Without data on the exact practices, the interpretations of these outcomes have to remain tentative. One such explanation could be, based on expectancy-value theory (Wigfield & Eccles, 2002), that students who see themselves as competent in maths are more willing to confront challenging word problems. In contrast, students who are not confident may avoid engagement in the new pedagogical practices where more demanding word problems are used because they believe them to be too complex or difficult. Another explanation could be that beliefs (that did change for the lower motivation students) mediate or are a prerequisite for performance improvements and that effects would be visible only in the longer term.

In relation to the first explanation, teaching based on the programme could become more effective if it manages to strengthen student confidence, while the second explanation would require collecting longitudinal data. In this study, motivation was measured only before the intervention. For a future longitudinal study, it would, therefore, be important to develop WPE by highlighting motivational aspects to investigate whether the modified programme has an impact on student motivation in learning mathematics and to study the interaction between motivation, beliefs and performance.

Even though the results suggested WPE's positive impact on student word problem-solving performance and beliefs about the situation model, the limitation of the quasi-experimental design used in this study must be taken into account. The experimental-group teachers were those who volunteered to participate in the professional development programme. This may imply that the level of teacher interest might be different between the two groups. However, even though the control-group teachers were not asked to participate in the WPE training, they volunteered to participate in this study because of their own interest in the use of word problem-solving in mathematics education. For future studies, it could be important to use randomized experimental design and to examine how teachers implement the new approaches in their teaching and how their interest in developing word problems mediates the effect of the professional development programme. An additional limitation is that the results of students' word problem-solving performance are based on four items. Moreover, due to a small sample size, it was not possible to include all belief factors in the structural model. Only the theoretically most important factor stressed in this study, the situation model, was included in the SEM. To clarify these issues, future studies would benefit from a larger set of word problems and a larger sample that would allow more fine-grained modelling (e.g. including all belief factors).

Finally, regarding the results of Studies IV and V, they indicated that students' performance on word problems can be explained by a one-dimensional construct, which contradicts the original assumption. It was explained that if another dimension of students' performance on word problems does exist, it might be too weak, and, therefore, cannot be distinguished from other random variations. However, if that was the case, it is not entirely clear what causes other dimension(s) to become invisible. It may be that unintended features (e.g. similar mathematical difficulty or mathematical difficulty covarying with problem type) of the word problems affect the results. Developing a set of word problems on the basis of the conceptualization of problem types and process in Figure 1, which not only takes problem type but also other aspects (e.g. difficulty of the given situation and mathematical difficulty) into account, may be the key to answering this issue.

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7. APPENDICES

Appendix I. Classification of one-step addition and subtraction word problems (adapted from Greer, 1987)

<table>
<tr><th rowspan="2"></th><th colspan="6">Change (action)</th></tr>
<tr><th colspan="3">Joining</th><th colspan="3">Separating</th></tr>
<tr><td>Result unknown</td><td colspan="3">01
Maria had 3 crayons.
Kyle gave her 5 more.
How many crayons does Maria have now?
(+)</td><td colspan="3">02
Maria had 8 crayons.
She gave 5 to Kyle.
How many crayons does Maria have now?
(-)</td></tr>
<tr><td>Change unknown</td><td colspan="3">03
Maria had 3 crayons.
Kyle gave her some more crayons.
Now Maria has 8 crayons.
How many crayons did Kyle give to her?
(-)</td><td colspan="3">04
Maria had 8 crayons.
She gave some crayons to Kyle.
Now Maria has 3 crayons left.
How many crayons did she give to Kyle?
(-)</td></tr>
<tr><td>Start unknown</td><td colspan="3">05
Maria had some crayons.
Kyle gave her 5 more crayons.
Now she has 8 crayons.
How many crayons did Maria have in the beginning?
(-)</td><td colspan="3">06
Maria had some crayons.
She gave 5 crayons to Kyle.
Now she has 3 crayons left.
How many crayons did Maria have in the beginning?
(+)</td></tr>
<tr><td rowspan="3"></td><th colspan="6">Combine (static)</th></tr>
<tr><th colspan="3">Total Missing</th><th colspan="3">Part Missing</th></tr>
<tr><td colspan="3">07
Maria has 3 crayons.
Kyle has 5 crayons.
How many crayons do they have altogether?
(+)</td><td colspan="3">08
Maria and Kyle have 8 crayons altogether.
Maia has 3 crayons.
How many crayons does Kyle have?
(-)</td></tr>
<tr><td></td><th colspan="6">Compare (static)</th></tr>
<tr><td>Difference unknown</td><td colspan="2">09
Maria has 8 crayons.
Kyle has 5 crayons.
How many more crayons does Maria have than Kyle?
(-)</td><td colspan="2">10
Maria has 8 crayons.
Kyle has 5 crayons.
What is the difference between the number of crayons that Maria and Kyle have?
(-)</td><td colspan="2">11
Maria has 8 crayons.
Kyle has 5 crayons.
How many fewer crayons does Kyle have than Maria?
(-)</td></tr>
<tr><td>Compared quality unknown</td><td colspan="3">12
Maria has 3 crayons.
Kyle has 5 more crayons than Maria.
How many crayons does Kyle have?
(+)</td><td colspan="3">13
Maria has 8 crayons.
Kyle has 5 fewer crayons than Maria.
How many crayons does Kyle have?
(-)</td></tr>
<tr><td>Referent unknown</td><td colspan="3">14
Maria has 8 crayons.
She has 5 more crayons than Kyle.
How many crayons does Kyle have?
(-)</td><td colspan="3">15
Maria has 3 crayons.
She has 5 fewer crayons than Kyle.
How many crayons does Kyle have?
(+)</td></tr>
<tr><td></td><th colspan="6">Equalize (action)</th></tr>
<tr><td>Difference unknown</td><td colspan="3">16
Maria has 3 crayons.
Kyle has 8 crayons.
How many more crayons does Maria need to get to have the same number of crayons as Kyle?
(-)</td><td colspan="3">17
Maria has 8 crayons.
Kyle has 3 crayons.
How many crayons does Maria need to lose to have the same number of crayons as Kyle?
(-)</td></tr>
<tr><td>Compared quality unknown</td><td colspan="3">18
Maria has 13 crayons.
If Kyle gets 5 more crayons, he will have the same number of crayons as Maria.
How many crayons does Kyle have?
(-)</td><td colspan="3">19
Maria has 5 crayons. If Kyle loses 2 crayons, he will have the same number of crayons as Maria.
How many crayons does Kyle have?
(+)</td></tr>
<tr><td>Referent unknown</td><td colspan="3">20
Maria has 5 crayons. If she gets 8 crayons, she will have the same number of crayons as Kyle.
How many crayons does Kyle have?
(+)</td><td colspan="3">21
Maria has 13 crayons. If she loses 5 crayons, she will have the same number of crayons as Kyle.
How many crayons does Kyle have?
(-)</td></tr>
</table>

Appendix I (continued). Classification of one-step multiplication and division word problems

Types of multiplication and division problems (asymmetrical cases)			
Category	**Multiplication**	**Division (Partition)**	**Division (Quotition)**
Multiple groups	3 boys had 4 marbles each. How many marbles did they have altogether? **22**	12 marbles were divided equally among 3 boys. How many marbles did they get each? **23**	12 marbles were divided among some boys. Each boy got 4 marbles. How many boys were there? **24**
Iteration of measure	4 pieces of wood are each 3.2 m long. What is the total length of the wood? **25**	A piece of wood 12.8 m long is cut into 4 equal pieces. How long is each piece? **26**	A piece of wood 12.8 m long is cut into pieces 3.2 m long. How many pieces are obtained? **27**
Rate	A man walks for 4.5 hours at a steady speed of 3.2 m.p.h. How far does he walk? **28**	A man walks 14.4 miles in 4.5 hours. What is his speed in m.p.h.? **29**	A man walks 14.4 miles at a steady speed of 3.2 m.p.h. How long does it take him? **30**
Measure conversion	If the rate of exchange is 1.5 dollars per pound, how many dollars will you get for £3.20? **31**	If you get 4.80 dollars for £3.20, what is the exchange rate in dollars per pound? **32**	If the rate of exchange is 1.5 dollars per pound, how many pounds will you get for $4.80? **33**

Types of multiplication and division problems (symmetrical cases)		
Category	**Multiplication**	**Division**
Rectangular array	If there are 3 rows and 4 columns, what is the total number? **34**	If the total is 12 and there are 3 rows (columns), how many columns (rows) are there? **35**
Combinations	If there is a choice of 3 colours and 4 styles, how many combinations of colour and style are there? **36**	If there are 12 combinations of colour and style and there are 3 choices of colour (style), how many choices of style (colour) are there? **37**
Area	If the length is 3.2 cm and the breadth is 4.5 cm, what is the area? **38**	If the area is 14.4 cm and the length (breadth) is 3.2 cm what is the breadth? **39**
	Multistep word problem **40**	Undefined* **41**

*Undefined: Those one-step word problems that cannot be categorized into this classification are coded as undefined.

Appendix II. Routine, non-routine and application word problems used in Studies II, III and IV

Types	Word problems
R1 Routine	Pekka has 7 adventure books. Pirkko has 6 adventure books more. How many adventure books does Pirkko have?
NR2 Non-routine	There was a bowl full of chocolate pieces on a desk. Liisa took 2 pieces of chocolate every day. After two weeks, all the chocolate pieces were gone. How many chocolate pieces were there at the beginning?
NR3 Non-routine	Children are in the market and they buy some apples. A box of apples weighing 8 kilograms costs 9.6 euros. Mari, Milla, Pekka and Jussi buy the apple box and each one of them gets 2 kilograms of apples. How much do 2 kilograms of apples cost?
NR4 Non-routine	Kalle has 18 euros. He wants to buy two computer games which each of them costs 13 euros. Mother promises that Kalle will get 2 euros every time he takes the trash out. How many times does Kalle have to take the trash out to get enough money to buy both computer games?
AP5 Application	Paula is preparing some food and drinks for her birthday party. She buys two packets of chips (1 packet costs 2.50 euros), a big packet of mixed candy (1 packet costs 3.60 euros) and 4 bottles of lemonade (1 bottle costs 1.25 euros). Three friends come to the party. How much do the snacks and drinks cost for each participant?

Appendix III. Non-routine and application word problems used in Study V

Types	Word problems
NR1 Non-routine	Risto, the neighbour, claims that he can run 53 metres in six seconds. You know that you can run nine metres in a second. How many metres can you run in six seconds?
NR2 Non-routine	One summer day, Ville went to the outdoor swimming pool. Ville liked to pool jump more than anything. During the day, he jumped 24 times from the low platform. He had energy to go up to the high 10-metre platform only seven times and jump from there. In total, he jumped six times feet first, the rest head first. How many times did he jump head first?
NR3 Non-routine	All of Kasia's six daughters are ill. Kaisa makes daughters healthy soup. She boils the soup – in total, 2 litres and 4 decilitres. How much soup does each girl get when Kaisa divides the soup equally to her daughters?
NR4 Non-routine	Your uncle gives you and two of your cousins 7 marks and 20 pennies for buying candies. You equally divide money before going to the shop. How much money does each one of you get?
NR5 Non-routine	Hanna, Elisa and Niina decided to get money for candy by collecting empty bottles whole morning. They went through all the gardens next to nearby houses. Every girl found 27 bottles from the ground of which nine were broken, in total. They threw the broken bottles away. How many bottles were left to girls for bringing to the shop?
AP6 Application	22 congressmen were taken by taxi to a dinner party arranged by the President. One taxi could hold four passengers. How many taxies were needed?
NR7 Non-routine	Kalle, Toni, Lauri and Antti belong to their class 4 x 100 metres relay team. The teacher was happy about the boys' practice run. Kalle used 13.4 s for his section, Toni 14.1 s, Lauri 14.2 s and Antti 13.1 s. What was the team's time in the relay race?
NR8 Non-routine	During the night, cat couple went hunting in the garden. They caught, in total, 6 field mice and 16 forest mice. Each one of them ate first 4 delicious forest mice. How many mice did the cats have left for the morning appetite?
NR9 Non-routine	Hannu's home village can be driven by car via four different roads to Tampere. From Tampere can be driven as many as seven different routes to Hannu's new hometown. How many different routes can Hannu travel when he leaves from his home village through Tampere to his new hometown?
NR10 Non-routine	You play basketball with your friends. Opponent's team can make 24 points in the first period. It makes three points less than your own team. Both teams can make equal number of points in both periods. How many points are made in total during the game?
NR11 Non-routine	When you travel to England, you can see street signs where distances given in miles. One mile is about 1 km 600 m. Imagine that you are near London. On the sign, there is information that London is 11 miles away. How far to London is it in kilometres and metres?

NR12 Non-routine	Jukka, Heikki and Pekka went fishing for a day. They had in total 15 sausages and 32 crackers with them. Jukka and Heikki ate, in total, eight crackers for lunch, but big Pekka needed six crackers. The rest of the crackers, they divided equally for a supper together. How many crackers did each boy get in the evening?
AP13 Application	Risto got seven planks which each of them was five metres long. How many two metre planks could he saw from these?
NR14 Non-routine	The water tank in the garden was completely full when Mum and Kaisa decided to water the rose garden. Mum's watering can could hold 7 litres, and Kaisa's can could hold 4 litres. Both filled their cans 3 times from the big water tank and watered the roses. The water tank was finally left with 14 litres of water. How much water was in the water tank before Mum and Kaisa started to water the rose garden?
NR15 Non-routine	Mum wanted to get a big basket filled with apples, but she was too tired to collect them alone. Dad and Hannu helped Mum to collect until her basket was full. Dad's basket could hold 6 kilos of apples and Hannu's basket could hold 4 kilos of apples. Both filled their baskets four times and emptied them into Mum's big basket. Mum had earlier collected 13 kilos of apples in her basket. How many apples were there in Mum's basket in the end?

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