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DYNAMIC SIMULATION OF A HUMAN GAIT AND DESIGN PROBLEMS OF THE LOWER LIMB PROSTHESES

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Abstract

In this paper the methodology and the numerical algorithm are proposed such as suitable both for the dynamic simulation of a human gait and for solving of the design problems of the lower limb prostheses. The methodology is based on the combination of the optimal control theory and the mathematical modeling with broad utilization of the data obtained from the biomechanical experiments. A special procedure is used for converting the initial optimal control problems for the highly nonlinear and complex bipedal locomotion system into the standard nonlinear programming problems. It is made by approximation of the independent variable functions using the combination of a spline and the Fourier series and the solution of the semi-inverse dynamics problem. The key feature of the algorithm proposed is its high numerical effectiveness and the possibility to satisfy many restrictions imposed on the phase coordinates of the system automatically and accurately. The proposed methodology is illustrated by the computer simulation of a human gait and the numerical results of solution of the design problems of the energy-optimal above-knee prostheses with several types of the structure of the knee mechanisms.

INTRODUCTION

The complexity of the structure and the dynamical instability of a gait of a human locomotor apparatus (HLA) make it very difficult to understand the main features and principles of its control system that provides the goal-directed stable motion. Traditionally, studies of a motion of the HLA have been concentrated on providing a basic information that can be used in different applied areas. For instance, in synthesis of artificial bipedal gait in order to design an active exoskeleton (Vukobratovic', 1975), walking robots (Larin, 1980; Formal'sky, 1982; Beletskii, 1984; Berbyuk, 1989; Morecki, 1997; Pfeiffer et al., 1997; Waldron, 1997; Berbyuk et al., 1998), lower limb prostheses

(Winter and Sienko, 1988; Winter, 1991; Berbyuk, 1994).

The experimental data (Diandelo et al., 1989; Johansson et al., 1993; Öberg et al., 1994) and the theoretical studies (Capozzo et al., 1976; Berbyuk, 1994, 1996) show that the kinematics and the dynamics of the HLA are strongly sensitive to the constructive parameters of a prosthesis (massinertial, elastic, viscoelastic, etc.) and to the parameters of a human gait (cadence, velocity, duration of the leg activity, etc.).

To study the effect of prosthesis design on the kinematic, dynamic and other characteristics of an amputee's locomotion and to improve and create new efficient lower limb prostheses it is expedient to use the mathematical modeling of a human walk process and the dynamic optimization technique.

Many different models of the HLA were proposed in the last decades. Among them the 3-D human musculoskeletal models (Dietrich et al., 1997; Hatze, 1980), the biomechanical model of a human body that is suitable for crashworthiness applications (Ambrósio and Pereira, 1997). The simplest walking models (Garcia et al., 1998) are also very important in trying to understand stability and control principles of the HLA.

Most researchers investigate the dynamic behavior and the control laws of the HLA using the inverse, the semi-inverse or the direct dynamics approach. In recent years the interest in optimal processes of the HLA has increased remarkably (Nuber and Contini, 1961; Beckett and Chang, 1968; Chow and Jacobson, 1971; Hatze, 1976; Larin, 1980; Pandy et al., 1992; Berbyuk, 1997; Berbyuk and Lytwyn, 1998).

In this paper the mathematical model of the HLA, the methodology and the numerical algorithm are proposed that suitable both for the dynamic simulation of a human gait and for solving the design problems of the above-knee prostheses. The HLA is simulated by a plane controlled mechanical system of rigid masses with nine degrees of freedom. The methodology is based on the combination of the optimal control theory and the mathematical modeling with broad utilization of the kinematic and dynamic data obtained from biomechanical experiments.

The nonlinear optimal control problems have been considered to provide insight into the interaction between kinematics, dynamics and control of the HLA. A special procedure has been proposed for converting the formulated optimal control problems for the highly nonlinear and complex bipedal locomotion system into the standard nonlinear programming problems. The computer simulation of a human gait and the numerical results of solution of the design problems of the energy-optimal above-knee prostheses illustrate the effectiveness of the proposed methodology.

DYNAMIC SIMULATION OF A HUMAN GAIT

Mathematical Model

The HLA is simulated by a plane mechanical system of rigid masses (Fig. 1). This system comprises an inertial body G (trunk) and two legs. Each leg consists of three elements. The two elements with mass and rotator inertia model the thigh (link OK_i) and the shank (link K_iA_i), while the third inertia-free element (links $A_iH_iT_i$) models the foot.

In addition to the weights of the trunk, thighs and the shanks, the external forces acting on the HLA include the interaction forces between the feet and the ground, which are replaced by resultant forces $\boldsymbol{R}_i$, ($i=1,2$).

It is assumed that the control moments $q_i(t), u_i(t), p_i(t)$ acting at the hip (point O), knee (point K_i) and the ankle (point A_i) joints, respectively.

Let the NXYZ be a fixed rectangular Cartesian coordinate system. It is assumed that the HLA moves in the NXY plane along the NX axis over a horizontal surface.

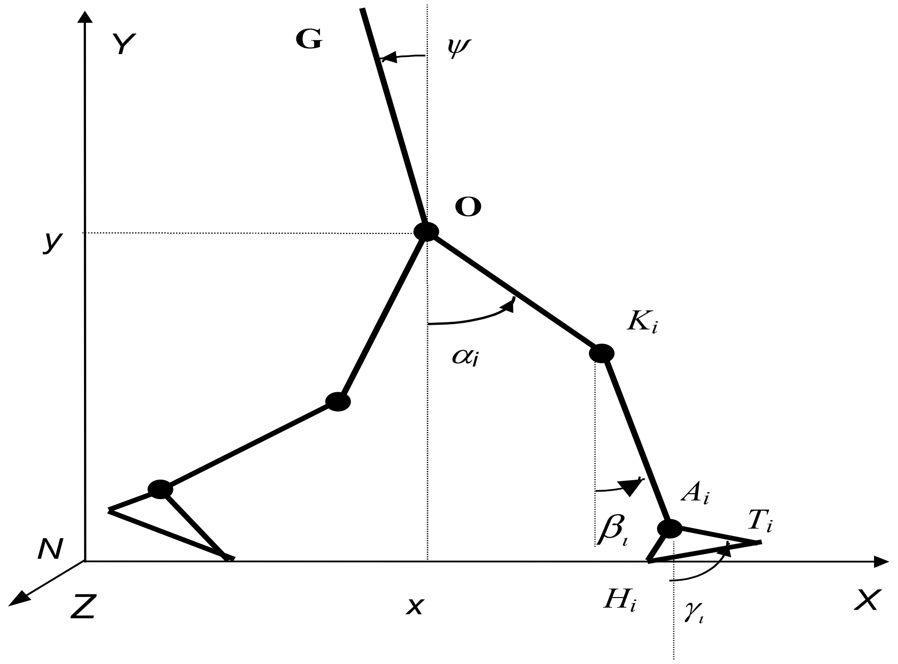

Fig. 1 Model of the Human Locomotor Apparatus

The controlled motion of the HLA can be described by the Lagrange's equations of the second kind and the kineto-static balance conditions for the feet under the action of the ankle moment and the reaction of the support (Berbyuk, 1997):

$$
\begin{aligned}
& f_1(t) - K_r(\ddot{\psi}\cos\psi - \dot{\psi}^2 \sin\psi) = R_{1x}(t) + R_{2x}(t) \\
& f_2(t) - K_r(\ddot{\psi}\sin\psi + \dot{\psi}^2 \cos\psi) = R_{1y}(t) + R_{2y}(t) \\
& f_{3i}(t) = q_i - u_i + a_i(R_{ix}\cos\alpha_i + R_{iy}\sin\alpha_i) \qquad (1) \\
& f_{4i}(t) = u_i - p_i + b_i(R_{ix}\cos\beta_i + R_{iy}\sin\beta_i) \\
& f_5(t) = -q_1 - q_2
\end{aligned}
$$

$$p_i + (y_i - y_{Ri})R_{ix} + (x_{Ri} - x_i)R_{iy} = 0$$

Here

$$f_1(t) = M\ddot{x} + \sum_{i=1}^{2}[K_{ai}(\dot{\alpha}_i \cos\alpha_i)^{\cdot} + K_{bi}(\dot{\beta}_i \cos\beta_i)^{\cdot}],$$

$$f_2(t) = M(\ddot{y} + g) + \sum_{i=1}^{2}[K_{ai}(\dot{\alpha}_i \sin\alpha_i)^{\cdot} + K_{bi}(\dot{\beta}_i \sin\beta_i)^{\cdot}],$$

$$f_{3i}(t) = J_i\ddot{\alpha}_i + K_{ai}(\ddot{x}\cos\alpha_i + \ddot{y}\sin\alpha_i) + gK_{ai}\sin\alpha_i + a_iK_{bi}[\ddot{\beta}_i\cos(\alpha_i - \beta_i) +$$
$$\dot{\beta}_i^2\sin(\alpha_i - \beta_i)],$$

$$f_{4i}(t) = J_{ci}\ddot{\beta}_i + K_{bi}(\ddot{x}\cos\beta_i + \ddot{y}\sin\beta_i) + gK_{bi}\sin\beta_i + a_iK_{bi}[\ddot{\alpha}_i\cos(\alpha_i - \beta_i) -$$
$$\dot{\alpha}_i^2\sin(\alpha_i - \beta_i)],$$

$$f_5(t) = J\ddot{\psi} - gK_r\sin\psi - K_r(\ddot{x}\cos\psi + \ddot{y}\sin\psi),$$

$M=m+m_{a1}+m_{a2}+m_{b1}+m_{b2}+m_{f1}+m_{f2}$, $J_i=J_{ai}+a_i^2(m_{bi}+m_{if})$, $J_{ci}=J_{bi}+b_i^2m_{fi}$,

$K_{ai}=m_{ai}r_{ai}+a_i(m_{bi}+m_{fi})$, $K_{bi}=m_{bi}r_{bi}+b_im_{fi}$, $K_r=rm$, (i=1,2).

In equations (1): x and y are the Cartesian coordinates of the suspension point O of the legs; $\psi, \alpha_i, \beta_i, \gamma_i$ are the angles that specify the position of the elements of the HLA (Fig. 1); m is the mass of the trunk; r is the distance from the suspension point of the legs to the center of mass of the trunk; J is the moment of inertia of the trunk relative to the Z axis at point O; m_{ai} is the mass of the thigh; a_i is the distance from O to the point K_i; J_{ai} is the moment of inertia of the thigh relative to the Z axis at O; r_{ai} is the distance from O to the center of mass of the thigh; m_{bi} is the mass of the shank; b_i is the distance from the knee joint to the point A_i; J_{bi} is the moment of inertia of the shank relative to the Z axis at the point K_i; r_{bi} is the distance from K_i to the center of the mass of the shank; m_{fi} is the mass of the foot located at the ankle joint A_i; $R_{ix}(t)$, $R_{iy}(t)$ are the horizontal and the vertical components of the force $\boldsymbol{R}_i$; (x_i, y_i), (x_{Ri}, y_{Ri}) are the Cartesian coordinates of the ankle joint, and of the point of application of the vector $\boldsymbol{R}_i$ of the i-th leg, respectively; g is the acceleration due to gravity.

Statement of the Problem

It is assumed that there are four phases of the leg action during a double step of a gait of the HLA ($t \in [0, T]$): the *first* double ($t \in [0, t_1]$) and the single ($t \in [t_1, t_2]$) support phases, and the *second* double ($t \in [t_2, t_3]$) and the single ($t \in [t_3, T]$) support phases (Winter, 1991). This rhythm of the double step of a gait leads to the following kinematic constraints:

$$x_{H1}(t) = x_{H1}^0, \quad y_{H1}(t) = 0, \quad x_{T2}(t) = x_{T2}^0, \quad y_{T2}(t) = 0, \quad t \in [0, t_1), \tag{2}$$

$$x_{T1}(t) = x_{H1}^0 + l, \quad y_{T1}(t) = 0, \quad y_{H2}(t) > 0, \quad y_{T2}(t) > 0, \quad t \in [t_1, t_2), \tag{3}$$

$$x_{T1}(t) = x_{H1}^0 + l, \quad y_{T1}(t) = 0, \quad x_{H2}(t) = x_{H1}^0 + L, \quad y_{H2}(t) = 0, \quad t \in [t_2, t_3), \tag{4}$$

$$x_{T2}(t) = x_{T2}^0 + 2L, \quad y_{T2}(t) = 0, \quad y_{H1}(t) > 0, \quad y_{T1}(t) > 0, \quad t \in [t_3, T]. \tag{5}$$

Here $x_{H1}^0, x_{T2}^0, t_1, t_2, t_3, T$ are given parameters (T is called a stride period (Winter, 1991)), l be the length of the foot, L be the length of the single step; $(x_{Hi}, y_{Hi}), (x_{Ti}, y_{Ti})$ are the Cartesian coordinates of the heel and the toe of the i-th leg, respectively. These coordinates are determined by the expressions (Fig.1):

$$x_{Hi}(t) = x_i - l_1 \cos(\gamma_i - \varphi), \; y_{Hi}(t) = y_i - l_1 \sin(\gamma_i - \varphi),$$

$$x_{Ti}(t) = x_i + l_2 \sin(\gamma_i - \varphi), \; y_{Ti}(t) = y_i - l_2 \cos(\gamma_i - \varphi),$$

$$x_i(t) = x + a_i \sin\alpha_i + b_i \sin\beta_i, \; y_i(t) = y - a_i \cos\alpha_i - b_i \cos\beta_i,$$

$$\varphi = a\tan(l_1 / l_2), \; l_1 = A_i H_i, \; l_2 = A_i T_i, \; \angle H_i A_i T_i = \pi / 2.$$

The problem of dynamic simulation of a gait of the HLA can be formulated as the following optimal control problem.

Problem A. Let we are given the parameters of a gait L, T, t_1, t_2, t_3, the angular coordinates

$$\gamma_2(t) - \beta_2(t) - \pi / 2 = \Theta_{a2}(t), \quad t \in [0, T],$$

$$\alpha_2(t) - \beta_2(t) = \Theta_{k2}(t), \quad t \in [0, T],$$

$$\alpha_i(t) - \psi(t) = \Theta_{hi}(t), \quad t \in [0, T], \quad i = 1,2$$

and the components of the ground reaction forces

$$R_{2x}(t) = R_{2x}^e(t), \quad t \in [0, t_1] \cup [t_2, t_3], \quad R_{2y}(t) = R_{2y}^e(t), \quad t \in [0, t_1] \cup [t_2, t_3].$$

It is required to determine the state vector

$$\mathbf{z} = \left\{ x, \dot{x}, y, \dot{y}, \psi, \dot{\psi}, \alpha_i, \dot{\alpha}_i, \beta_i, \dot{\beta}_i, \gamma_i, \dot{\gamma}_i, i = 1,2 \right\},$$

and the vector of the controlling stimuli

$u(t) = \{q_i, u_i, p_i, R_{ix}, R_{iy}, i = 1,2\}$,

which satisfy the equations of motion (1), the kinematic constraints (2)-(5), the boundary conditions

$$\mathbf{z}_x(T) = \mathbf{z}_x(0), \qquad x(T) = x(0) + 2L \tag{6}$$

$$\mathbf{z}_x = \left\{ \dot{x}, y, \dot{y}, \psi, \dot{\psi}, \alpha_i, \dot{\alpha}_i, \beta_i, \dot{\beta}_i, \gamma_i, \dot{\gamma}_i, i = 1,2 \right\},$$

the restrictions on the phase coordinates and the controlling stimuli

$$y(t) \geq h, \quad \alpha_i(t) \geq \beta_i(t), \quad t \in [0,T], \quad i = 1,2, \tag{7}$$

$$R_{iy}(t) \geq 0, \quad t \in [0,T], \quad i = 1,2, \tag{8}$$

$$x_{H1}^0 \leq x_{R1}(t) \leq x_{H1}^0 + l, \quad t \in [0,t_2], \quad x_{R1}(t) = x_{H1}^0 + l, \quad t \in [t_2,t_3], \tag{9}$$

$$x_{R2}(t) = x_{T2}^0, \quad t \in [0,t_1], \quad x_{H1}^0 + L \leq x_{R2}(t) \leq x_{H1}^0 + L + l, \quad t \in [t_2,T], \tag{10}$$

and which minimize the functional:

$$E = \frac{1}{2L} \int_0^T \sum_{i=1}^{2} [|q_i(\dot{\psi} - \dot{\alpha}_i)| + |u_i(\dot{\alpha}_i - \dot{\beta}_i)| + |p_i(\dot{\beta}_i - \dot{\gamma}_i)|]dt. \tag{11}$$

Here the parameters L, T, t_1, t_2, t_3 and the functions $\Theta_{a2}, \Theta_{k2}, \Theta_{hi}, R_{2x}^e, R_{2y}^e$, ($i$=1,2) are given by the biomechanical experiments (Winter, 1991). The boundary conditions (6) reflect the periodic property of a human gait. The restrictions (8)-(10) are due to the requirements of the "non-suction-cup" and stability of a gait.

The objective functional (11) is the integral over a double step of the sum of the absolute values of the mechanical power of all controlling stimuli acting at the joints of the HLA (Beckett and Chang, 1968; Beletskii, 1984; Berbyuk, 1989, 1996, 1997).

Methodology

The analysis of the equations (1) shows that the problem A has one independent variable function during the double support phases and three independent variable functions during the single support phases.

It is suitable to choose the functions:

$$
\begin{aligned}
&x(t),\ t\in[0,t_1]\cup[t_2,t_3],\\
&x(t),\gamma_1(t),x_{H2}(t),\ t\in[t_1,t_2], \qquad (12)\\
&x(t),\gamma_1(t),x_{H1}(t),\ t\in[t_3,T]
\end{aligned}
$$

as the independent variable ones.

Every function (12) in the given interval $t\in[\tau_1,\tau_2]$ was parameterized by the following way (Berbyuk, 1989; Nagurka and Yen, 1990):

$$
\begin{aligned}
&F(t)=P_F(t)+G_F(t), \quad P_F(t)=\sum_{k=0}^{5}C_k(F)t^k,\\
&G_F(t)=\sum_{n=1}^{N(F)}\left(a_n^F\cos n\omega t+b_n^F\sin n\omega t\right), \qquad (13)
\end{aligned}
$$

$\omega=\dfrac{2\pi}{(\tau_2-\tau_1)}$, $F=(x,\gamma_1,x_{H1},x_{H2})$, $N(F)$ are the given numbers.

The coefficients of the function $P_F(t)$ are determined by the following conditions:

$$
\begin{aligned}
&P_F(\tau_1)+G_F(\tau_1)=F(\tau_1), \quad P_F(\tau_2)+G_F(\tau_2)=F(\tau_2),\\
&\dot{P}_F(\tau_1)+\dot{G}_F(\tau_1)=\dot{F}(\tau_1), \quad \dot{P}_F(\tau_2)+\dot{G}_F(\tau_2)=\dot{F}(\tau_2), \qquad (14)\\
&\ddot{P}_F(\tau_1)+\ddot{G}_F(\tau_1)=\ddot{F}(\tau_1) \quad \ddot{P}_F(\tau_2)+\ddot{G}_F(\tau_2)=\ddot{F}(\tau_2).
\end{aligned}
$$

Considering (1), (12)-(14) we convert the problem A into the nonlinear programming problem (Berbyuk et al., 1997):

$$
Q(\xi)\to\min_{\xi}, \qquad f(t,\xi)=0, \quad g(t,\xi)\le 0, \quad t\in[0,T]. \qquad (15)
$$

Here ξ is a vector of the variable parameters; $Q(\xi)$, $f(t,\xi)$, $g(t,\xi)$ are the functions that determined by the equations (1) and the formulae (2)-(11), (13), (14). Note that the procedure of calculating the objective function $Q(\xi)$ includes the solution of the inverse dynamics problem for the mechanical system modeled the HLA (Berbyuk et al., 1997).

The numerical algorithm has been devised to solve the nonlinear programming problem (15) based on the Rozenbrock's method (Bazara and Shetty, 1979).

Numerical Results

The methodology described above has been used to solve a number of variants of the problem A. In the model, a subject height of 1.76 m, mass of 73.2 kg, and the following parameters of the limbs have been considered:

$$m_{ai}\text{=7.08 kg},\ a_i\text{=0.41 m},\ J_{ai}\text{=0.082 kg m}^2,$$
$$m_{bi}+m_{fi}\text{=5.04 kg},\ b_i\text{=0.5 m},\ r_{ai}\text{=0.16 m},\ r_{bi}\text{=0.203 m},\ J_{bi}\text{=0.053 kg m}^2, \qquad (16)$$
$$m\text{=46.7 kg},\ J\text{=7.1 kg m}^2,\ r\text{= 0.39 m}.$$

Lets describe some numerical results of the dynamic simulation of the human gait with natural cadence. The input parameters and functions of the problem A were given by the data of the biomechanical experiments corresponding to normal human gait (Winter, 1991). The stride period T=1.1396 *s*, the stride length L=0.755 m, the duration of the double and the single support phases are equal to $0.14T$ and $0.36T$, respectively.

Figure 2 (dashed curve) shows the way in which the knee angle $\Theta_{k1}(t) = \alpha_1(t) - \beta_1(t)$ of the leg changes in time over the double step of the HLA for the obtained energetically optimal law of motion. The way in which the specific horizontal component $R_{1x}(t)/M$ of the support reaction varies (Fig. 3, dashed curve) indicates that in each single step the support leg successively executes two functions: deceleration of the HLA (time interval in which $R_{1x}(t)$/M<0) and separation (time interval in which $R_{1x}(t)/M$>0). The maximum value of the $R_{1x}(t)$ amounts to 20% of the entire weight of the HLA. The vertical component of the support reaction ($R_{1y}(t)/M$, Fig. 4, dashed curve) exceeds the weight of the HLA by not more than 7%. Figures 5-6 (dashed curves) show the specific control torques $u_i(t)/M, p_i(t)/M$ acting at the knee and at the ankle joint of the legs during the energy-optimal law of motion of the HLA.

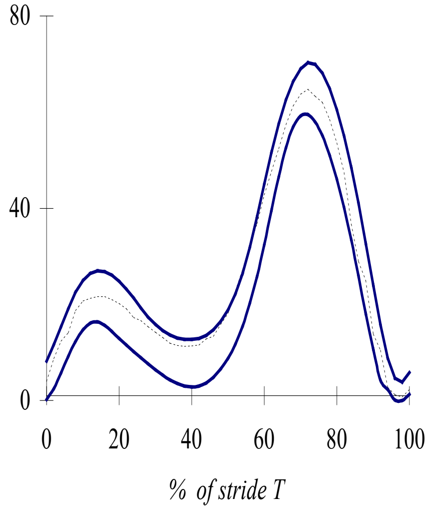

Fig. 2 Knee Angle $\Theta_{k1}(t)$, in degrees

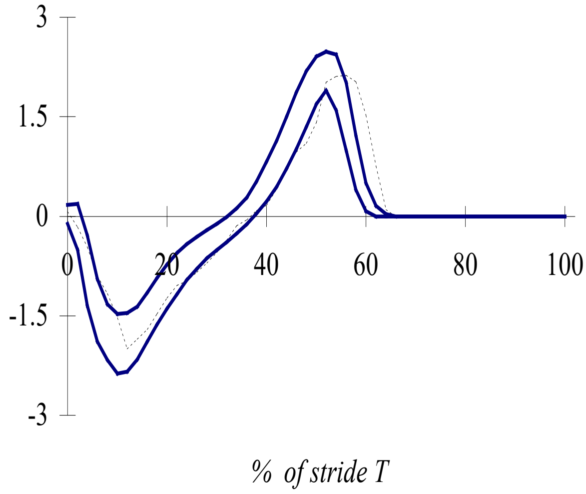

Fig. 3 Horizontal Force $R_{1x}(t)$/M, in N/kg

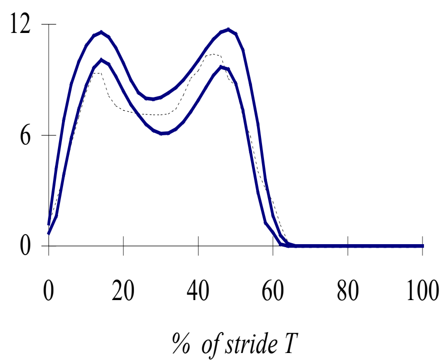

Fig. 4 Vertical Force $R_{1y}(t)/M$, in N/kg

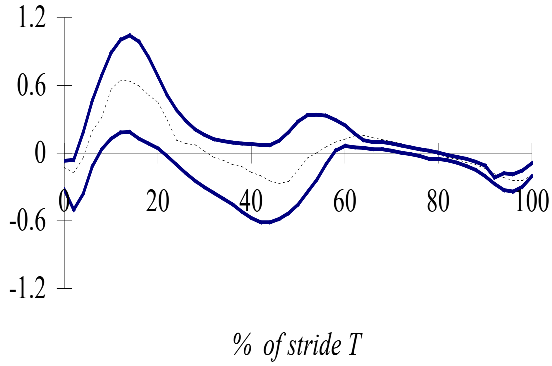

Fig. 5 Knee Torque $u_i\,(t)/M$, in Nm/kg

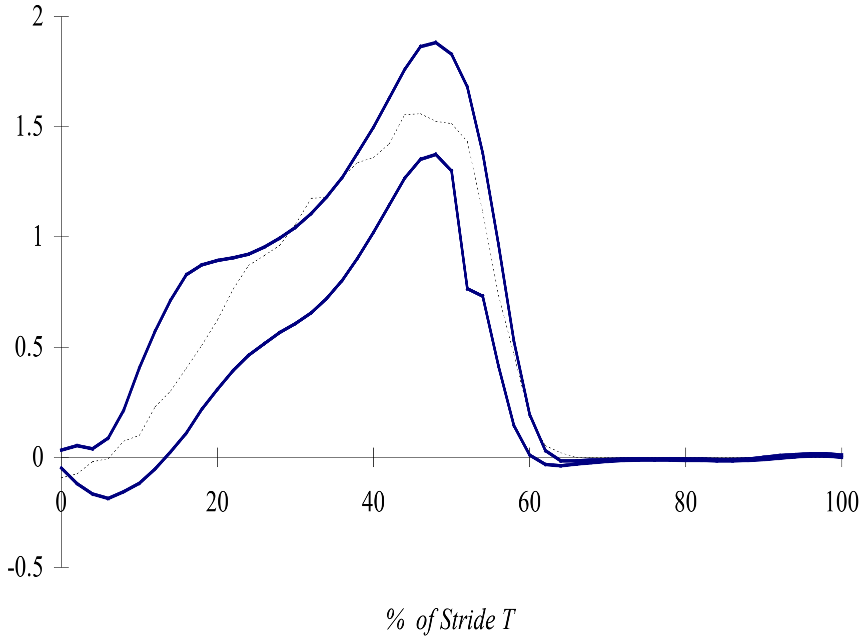

Fig. 6 Ankle Torque $p_i(t)/M$, in Nm/kg

For comparison purposes the domains of the values of the respective kinematic and the dynamic characteristics of a human normal gait that obtained by the biomechanical experiments (Winter, 1991) are shown in Figure 2-6 (the solid curves). The analysis of these data and all above mentioned indicate that the kinematic and the dynamic characteristics of the obtained energy-optimal law of motion of the HLA are within reasonable proximity to the corresponding characteristics of a human normal gait.

DESIGN PROBLEMS OF THE LOWER LIMB PROSTHESES

There is an important difference between the dynamics of an intact limb and a prosthetic limb of an amputee. In the paper the mathematical modeling of a human gait of an amputee with the above-knee prosthesis is considered based on a supposition that the force moments at the knee and at the ankle joints of the prosthetic leg are passive ones. The values of these moments depend not only on the gait pattern, but also on the prosthesis construction.

The model of the amputee locomotor system (ALS) with the above-knee prosthesis is depicted in Fig. 7. It is assumed that the above-knee prosthesis comprises the linear-viscoelastic ankle mechanism and the hydraulic or the pneumatic knee mechanism.

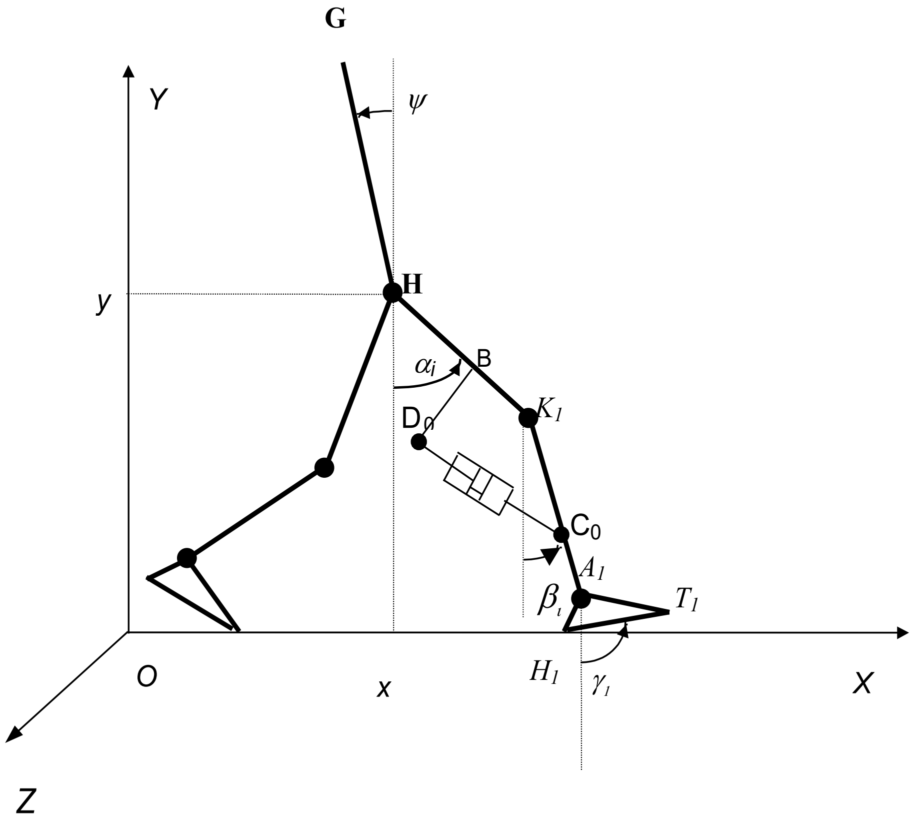

Fig. 7 Model of the Amputee Locomotor System with the Above-Knee Prosthesis

During locomotion of the ALS with the above-knee prosthesis the control torques

$$p_1(t) = C(\beta_1 - \gamma_1 + \pi/2) + K(\dot{\beta}_1 - \dot{\gamma}_1) + D, \tag{17}$$

$$u_1(t) = (P_2 - P_1)S_p d_2 (d_1^2 + l_0^2)^{1/2} \sin(\alpha_1 - \beta_1 + \eta)/l_1$$

are generated at the ankle and at the knee joints of the prosthetic leg, respectively.

Here *C, K* are the torsion spring and the damping coefficients of the ankle mechanism; *D* is determined by the free angle of the spring and torsion spring coefficients; P_1, P_2 are the chamber pressures of the hydraulic or the pneumatic actuator that can be calculated by using the equations of dynamics of the knee mechanism of the prosthesis (Berbyuk and Nishchenko, 1998); S_p is the cylinder piston cross-area,

$$l_1 = (d_1^2 + d_2^2 + l_0^2 + 2d_2(d_1^2 + l_0^2)^{1/2} \cos(\alpha_1 - \beta_1 + \eta))^{1/2}, \ \eta = a\tan(l_0/d_1), \tag{18}$$

$$d_1 = |BK_1|, \quad d_2 = |K_1C_0|, \quad l_0 = |BD_0|.$$

The controlled motions of the ALS with the above-knee prosthesis are described by the equations (1) and the formulae (17), (18).

The design problem of the above-knee prosthesis can be formulated as the following optimal control problem with parameters for the nonlinear mechanical system modeling the ALS with the prosthetic leg (Fig. 7).

Problem B. Let we are given the parameters of a gait L, T, t_1, t_2, t_3, the angular coordinates

$$\gamma_2(t) - \beta_2(t) - \pi/2 = \Theta_{a2}(t), \quad t \in [0,T],$$
$$\alpha_2(t) - \beta_2(t) = \Theta_{k2}(t), \qquad t \in [0,T], \tag{19}$$
$$\alpha_2(t) - \psi(t) = \Theta_{h2}(t), \quad t \in [0,T],$$

and the vertical component of the ground reaction forces

$$R_{2y}(t) = R_{2y}^{e}(t), \qquad t \in [0,t_1] \cup [t_2,t_3]. \tag{20}$$

It is required to determine the state vector

$$\mathbf{z} = \left\{ x, \dot{x}, y, \dot{y}, \psi, \dot{\psi}, \alpha_i, \dot{\alpha}_i, \beta_i, \dot{\beta}_i, \gamma_i, \dot{\gamma}_i, i = 1,2 \right\},$$

the vector of the controlling stimuli of the ALS

$$u(t) = \{q_i, u_i, p_i, R_{ix}, R_{iy}, i = 1,2\},$$

and the vector of the constructive parameters of the above-knee prosthesis

$$C_p = (C, K, D, d_1, d_2, l_0, S_p, S_0)$$

which satisfy the equations of motion (1), the kinematic constraints (2)-(5), the boundary conditions (6), the restrictions on the phase coordinates and the controlling stimuli (7)-(10), the dynamic constraints (17), and which minimize the functional:

$$E_p = \frac{1}{2L} \int_0^T \{ \sum_{i=1}^{2} |q_i(\dot{\psi} - \dot{\alpha}_i)| + |u_2(\dot{\alpha}_2 - \dot{\beta}_2)| + |p_2(\dot{\beta}_2 - \dot{\gamma}_2)| \} dt. \tag{21}$$

Here the parameters L, T, t_1, t_2, t_3 and the functions $\Theta_{a2}, \Theta_{k2}, \Theta_{h2}, R_{2y}^{e}$, are given by the biomechanical experiments (Winter, 1991), S_0 is the cross-area of the hole of the cylinder piston.

The objective functional (21) is used to estimate the energy expenditure per unit of distance traveling of the ALS.

The same approach as described in paragraph 2.3 has been used to solve the problem B. We choose the following functions

$x(t), \gamma_1(t),\ t \in [0, t_1] \cup [t_2, t_3]$,

$\gamma_1(t), x_{H2}(t)$, $t \in [t_1, t_2]$,

$x(t), \gamma_1(t), x_{H1}(t)$, $t \in [t_3, T]$

as independently variable ones.

Due to the dynamic constraints (17) the procedure of converting the problem B into the standard nonlinear programming problem includes the solution of the semi-inverse dynamics problems for the mechanical system that models the ALS with the above-knee prosthesis. It sufficiently increases the time consumption of the numerical algorithm for designing the energy-optimal above-knee prosthesis.

The problem B has been solved numerically for two types of the prostheses: the above-knee prosthesis with the hydraulic actuator at the knee, and the prosthesis with the pneumatic knee mechanism. We used the linear and mass inertial parameters of the ALS determining by the formulae (16) as input data. Rhythm parameters and functions (19), (20) have been chosen corresponding to the biomechanical experiments for the slow (V_s=0.998m/s), normal (V_n=1.325m/s) and the fast (V_f=1.685m/s) human gaits (Winter, 1991).

Optimal values of some of the constructive parameters of the above-knee prostheses obtained by numerical solution of the problem B are presented in the Table (all values are given in SI units). Values C_1, K_1, D_1 correspond to the parameters of the ankle mechanism of the prosthesis (see formulae (17)) for the time of the gait $t \in [0, 0.01 t_* T]$ and values C_2, K_2, D_2 - for the time $t \in [0.01 t_* T, T]$.

Some kinematic and dynamic characteristics of the energy-optimal motion of the ALS with optimal structure of the above-knee prosthesis obtained by the numerical solution of the problem B for the gait with natural cadence are shown in Figures 8 - 11 (solid thin curves correspond to the prosthesis with the hydraulic actuator at the knee, dashed curves - to the prosthesis with the pneumatic knee mechanism).

For the comparison purposes in Figures 8 - 11 the domains of the values of the respective kinematic and dynamic characteristics obtained by the biomechanical experiments for a human normal gait are depicted by heavy solid curves.

Table. The Optimal Values of the Constructive Parameters of the Prostheses

	Pneumatic Knee Mechanism			Hydraulic Knee Mechanism		
C_p	V_s	V_n	V_f	V_s	V_n	V_f
C_1	6.967	6.412	5.971	5.341	4.341	4.217
K_1	0.213	0.131	0.099	0.083	0.062	0.054
D_1	0.359	0.291	0.247	0.433	0.427	0.379
t_*	38	36	36	40	40	38
C_2	4.031	3.582	3.328	2.013	1.517	1.323
K_2	0	0	0	0	0	0
D_2	0	0	0	0	0	0
d_1	0.202	0.185	0.173	0.141	0.133	0.124
d_2	0.216	0.202	0.217	0.201	0.205	0.213
l_0	0.110	0.078	0.085	0.058	0.060	0.062
S_p	0.00001	0.00001	0.00001	0.00010	0.00017	0.00022
E_p	117	104	147	103	96	125

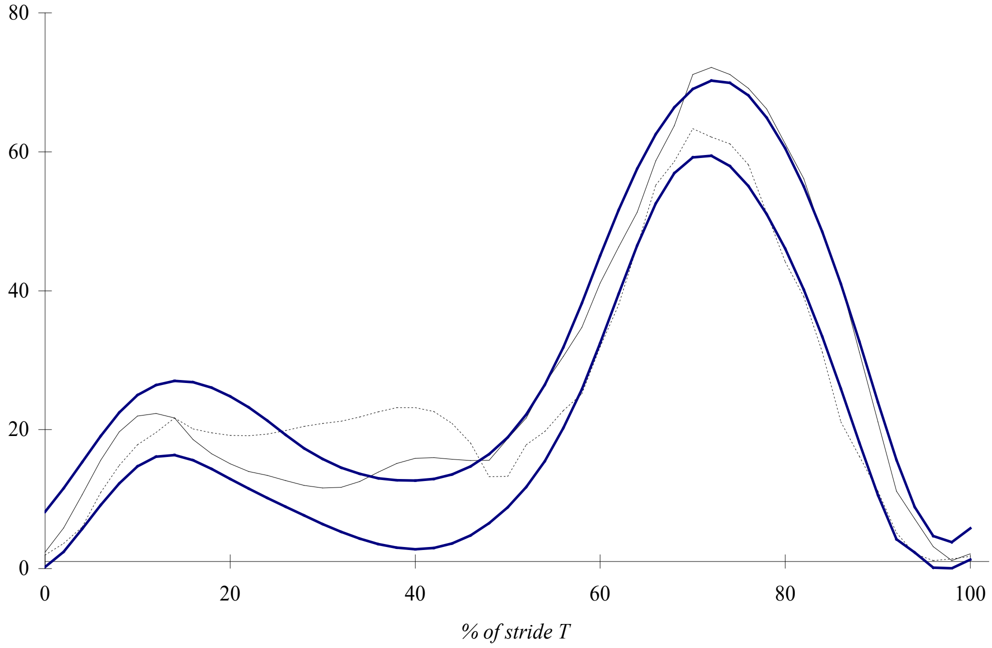

Fig. 8 Knee Angle of the Prosthetic Leg, $(\alpha_1 - \beta_1)$, in degrees

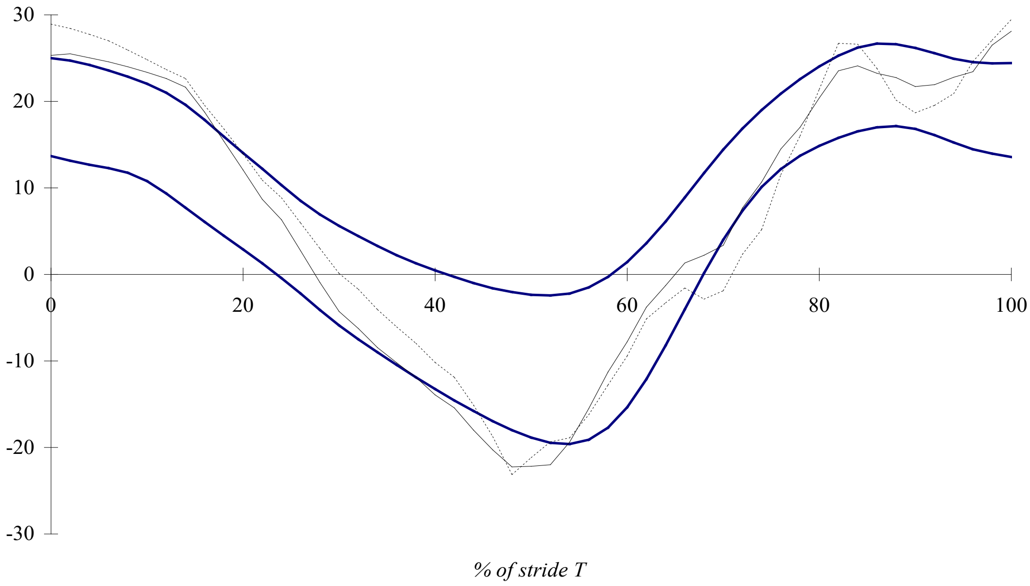

Fig. 9 Hip Angle of the Prosthetic Leg, ($\alpha_1 - \psi$), in degrees

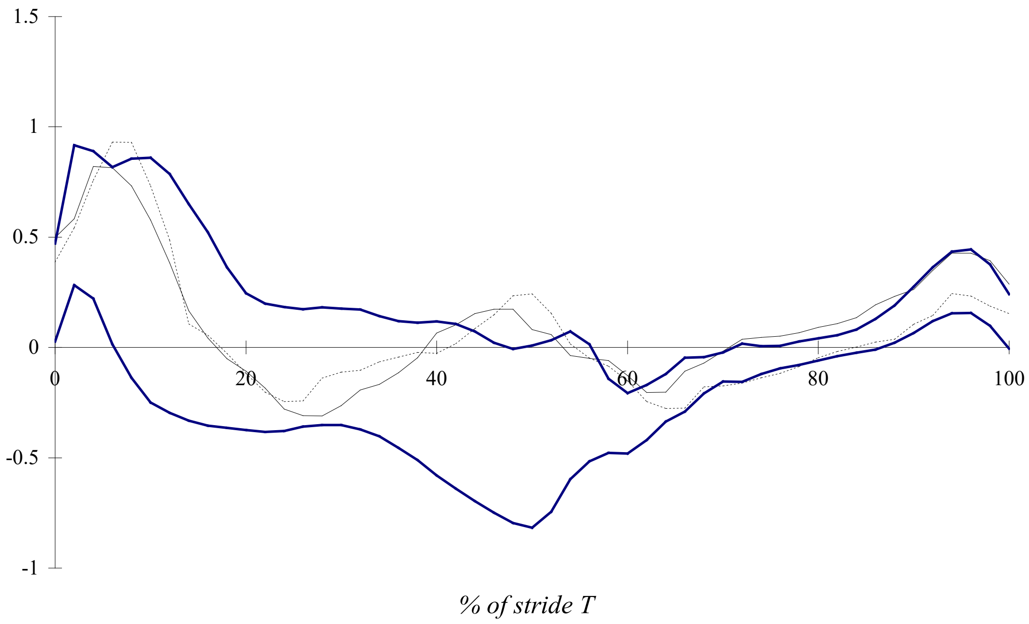

Fig. 10 Hip Torque of the Prosthetic Leg, ($q_1(t)/M$), in Nm/kg

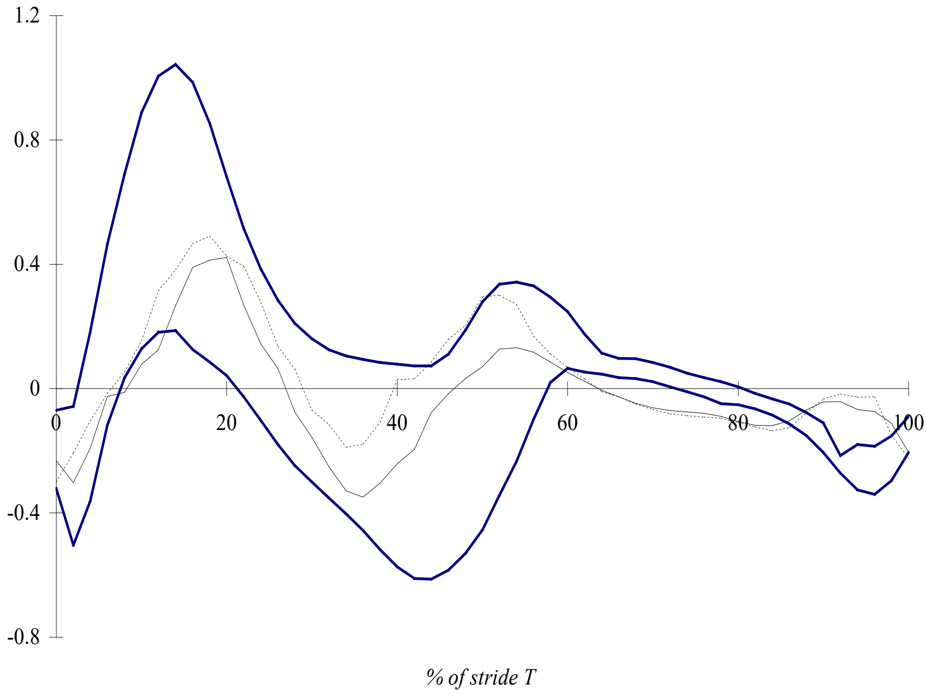

Fig. 11 Knee Torque of the Healthy Leg, ($u_2(t)/M$), in Nm/kg

DISCUSSION AND CONCLUSION

The central idea of the approach that proposed in this paper for the dynamic simulation of a human locomotion is to combine the optimal control theory, the mathematical modeling and the biomechanical experiments. We have formulated the optimal control problem (problem A) with free right-hand and left-hand ends of the phase trajectories, many constraints imposed both on the state variables and the controlling stimuli and with nondifferentiable objective function.

The boundary conditions (6) and the constraints (2)-(5) provide the possibility to model the typical double step of a human gait with the given stride period and the stride length. The data of the biomechanical experiments were used to specify some of the limb's angles and ground reaction forces. It gives possibility to approach the model solution to the respective characteristics of actual human gait and to simplify the numerical procedure of solving the problem A.

We proposed a parameter optimization approach to solve the highly nonlinear optimal control problem under the given boundary conditions, the restrictions on the phase coordinates and on the controlling stimuli (problem A). The approach is based on the special spline and Fourier approximation of the independently variable functions. The key feature of the approach is its possibility to satisfy many constraints imposed on the considered system automatically and accurately.

New prosthetic materials and designs have lead to many prostheses of lower limbs for amputees. As a result, it is becoming difficult for prosthetists and the physicians to choose which prosthesis is the best for the individual amputee. Presently, there is limited information about "optimal" alignment, and how the prosthesis performs dynamically in achieving optimally symmetrical gait for an amputee. Sensory feedback, better control systems, and more energy-efficient devices are strongly needed (Michael and Bowker, 1994). Gait studies, ambulatory physiological monitoring, mathematical modeling of a human controlled motion, and dynamic optimization techniques may be useful tools to improve and create new efficient lower limb prostheses.

In the paper we describe the methodology for solving the design problems of lower limb prostheses. The same as for dynamic simulation of a human gait the methodology is based on utilization of the optimal control theory, the mathematical modeling and the biomechanical experiments. The essential constructive characteristics of the prostheses (stiffness, damping, cylinder piston cross-areas, length of the links, etc.) are used as the important variable parameters of the optimal control problem in question.

In the framework of the considered mathematical model of the ALS wearing the above-knee prosthesis the following conclusions have been drawn.

1. The kinematic, dynamic, and energetic characteristics of controlled motion of the ALS are strongly sensitive to the essential prosthesis parameters. For a given individual and cadence of a gait there exist optimal values of the elasticity and viscoelasticity parameters of the prosthesis ankle mechanism and of the constructive parameters of the knee mechanism. These parameters give minimum energy expended per unit of distance traveled (See Table).

2. The analysis of a number of numerical simulations shows that the natural cadence of the ALS gait gives a minimum to the energy expended per unit of distance traveled comparing to the amount of energy needed for the slow or fast gaits (See Table).

3. The obtained kinematic and dynamic characteristics of the motion of the ALS with optimal above-knee prosthesis structures are within reasonable proximity to the respective characteristics of a human normal gait.

To conclude, this work has demonstrated the effectiveness of the proposed methodology both for dynamic simulation of a human gait and for solving the design problems of the above-knee prostheses.

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REFERENCES

Ambrósio, J. and Pereira, M. (1997) Flexible multibody dynamics with nonlinear deformations: Vehicle dynamics and crashworthiness applications. *Proceeding of the NATO ASI on*

Computational Methods in Mechanisms, J. Angeles and E. Zakhariev, (eds.), **1**, 1-38, Varna, Bulgaria.

Bazara, S. and Shetty, C, (1979) *Nonlinear Programming. Theory and Algorithm*, John Wiley and Sons, New York, Chichester Brisbane, Toronto.

Beckett, R. and Chang, K. (1968) An evaluation of the kinematics of gait by minimum energy, *J. Biomechanics* **1**, No. 2, 147-159.

Beletskii, V. (1984) *Two-Legged Walking: Model Problems of Dynamics and Control*, Nauka, Moscow.

Berbyuk, V. (1989) *Dynamics and Optimization of Robototechnical Systems*, Naukova Dumka, Kiev.

Berbyuk, V. (1994) Modeling of human locomotion and its application in construction of prostheses. *Proc. of the IMACS Symposium on Mathematical Modeling,* Edited by I. Troch and F. Breitenecker, **2**, 352-355, Technical University Vienna, Austria.

Berbyuk, V. (1996) Multibody system modeling and optimization problems of lower limb prostheses, *IUTAM Symposium on Optimization of Mechanical Systems*, D. Bestle and W. Schiehlen (eds.), 25-32, Kluwer Academic Publishers, The Netherlands.

Berbyuk, V. (1997) Dynamics and optimal control of biotechnical systems "Man-Prosthesis", *IUTAM Symposium on Interaction between Dynamics and Control in Advanced Mechanical Systems*, Ed. D. van Campen, 35-43, Kluwer Academic Publishers, The Netherlands.

Berbyuk, V., Krasyuk, G. and Nishchenko, N. (1997) Mathematical modeling of the human gait dynamics in sagittal plane, *Mathematical Methods and Physicomechanical Fields* **40**, (4), 127-138.

Berbyuk, V. and Lytwyn, B. (1998) The mathematical modeling of the human motions based on the optimization of the biodynamic system's controlled processes, *Mathematical Methods and Physicomechanical Fields*, **41**, (3), 153-161.

Berbyuk, V. and Nishchenko, N. (1998) Mathematical design of the energy-optimal above-knee prostheses, *Mathematical Methods and Physicomechanical Fields*, **41**, (4), 110-117.

Berbyuk, V., Peterson, B. and Nishchenko, N. (1998) Linear viskoelastic actuator-based control system of a bipedal walking robot, *Mechatronics' 98*, J. Adolfsson and J. Karlsen, (eds.), 379-384, Elsevier Science Ltd.

Capozzo, A., Figure, F., Leo, T. and Macthett, M. (1976) Biomechanical evaluation of above-knee prostheses. *Biomechanics* V-A, 366-372.

Chow, C. and Jacobson, D. (1971) Studies of human locomotion via optimal programming, *Mathem. Biosciences* **10**, No. 3/4, 239-306.

Diandelo, D.J., Winter, D.A., Ghista, D.N, and Newcomber, W.R. (1989) Perfomance assessment of the Terry- Fox jogging prosthesis for above-knee amputees, *J Biomechanics*, **22**, 6/7, 543-558.

Dietrich, M., Kedzior, K. and Rzymkovski, C. (1997) Some theoretical and practical aspects of modeling and simulation of the human musculoskeletal system, *Human and Machine Locomotion*, A. Morecki and K. J. Waldron, (eds.), CISM Courses and Lectures No. 375, 107-123, Springer-Verlag, Wien New York.

Formal'sky, A. (1982) *Displacement of Anthropomorphic Mechanisms*, Nauka, Moscow.

Garcia, M., Chatterjee, A., Ruina, A. and Coleman, M. (1998) The simplest walking model: stability, complexity and scaling, *ASME, J. of Biomechanical Engineering*, **120**, 281-288.

Hatze, H. (1976) The complete optimization of a human motion, *Mathem. Bioscienses* **28**, 99-135.

Hatze, H. (1980) Neuromusculoskeletal control systems modeling – a critical survey of recent

developments, *IEEE Trans. on Automatic Control,* **AC-25** (5), 375-385.
Johansson, S., Christiansson, B., and Karlsson, D. (1993) Gait analysis when comparing two polycentric prosthetic knee-joint amputees, *Biomechanics Seminar* **7**, 152–156.
Larin, B. (1980) *Control of Walking Robots*, Naukova Dumka, Kiev.
Michael, J.W. and Bowker, J.H. (1994) Prosthetics/orthotics research for the twenty-first century, *J. of Prosthetics and Orthotics* **6**, No.4/Fall, 100–107.
Morecki, A. (1997) Modeling and Simulation of Human and Walking Robot Locomotion, *Human and Machine Locomotion*, A. Morecki and K. J. Waldron (eds.), CISM Courses and Lectures No. 375, 1-78, Springer-Verlag, Wien New York.
Nagurka, M., and Yen, V. (1990) Fourier-based optimal control of nonlinear dynamic systems, *Trans. ASME, J. Dynamic Systems, Meas. and Contr.*, **112**, 3, 19-26.
Nuber, Y. and Contini, R. (1961) A minimal principle in biomechanics, *Bull. Math. Biophys*., **23**, 377-391.
Pandy, M., Anderson, F. and Hull, D. (1992) A parameter optimization approach for the optimal control of large-scale musculoskeletal systems, *ASME, J. of Biomechanical Engineering* **114,** 450-460.
Pfeiffer, F., Rossmann, T. and Steuer, J. (1997) Theory and practice of machine walking, *Human and Machine Locomotion*, A. Morecki and K. J. Waldron, (eds.), CISM Courses and Lectures No. 375, 231-281, Springer-Verlag, Wien New York.
Vukobratovic', M. (1975) *Legged Locomotion Robots and Anthropomorphic Mechanisms*, Mihailo Pupin Institute, Belgrade.
Waldron, K. (1997) Design of walking machines, *Human and Machine Locomotion*, A. Morecki and K. J. Waldron, (eds.), CISM Courses and Lectures No. 375, 283-315, Springer-Verlag, Wien New York.
Winter, D. (1991) *The Biomechanics and Motor Control of Human Gait*, University of Waterloo Press, Canada.
Winter, D., and Sienko, S. (1988) Biomechanics of bellow-knee amputee gait, *J. Biomechanics* **21,** No.5, 361–369.
Öberg T., Karsznia A., and Öberg K. (1994) Joint angle parameters in gait: Reference data for normal subject, 10-79 years of age, *J. Rehabilitation Research and Development*, **31**, 3, 199-213.