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NUMERICAL STUDIES OF DARK ENERGY MODELS AND OBSERVATIONS

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Contents

Abstract

The cosmological standard view is based on the assumptions of homogeneity, isotropy and general relativistic gravitational interaction. These alone are not sufficient for describing the current cosmological observations of accelerated expansion of space. Although general relativity is extremely accurately tested to describe the local gravitational phenomena, there is a strong demand for modifying either the energy content of the universe or the gravitational interaction itself to account for the accelerated expansion. By adding a non-luminous matter component and a constant energy component with negative pressure, the observations can be explained with general relativity.

Gravitation, cosmological models and their observational phenomenology are discussed in this thesis. Several classes of dark energy models that are motivated by theories outside the standard formulation of physics were studied with emphasis on the observational interpretation. All the cosmological models that seek to explain the cosmological observations, must also conform to the local phenomena. This poses stringent conditions for the physically viable cosmological models. Predictions from a supergravity quintessence model was compared to Supernova 1a data and several metric gravity models were studied with local experimental results. Polytropic stellar configurations of solar, white dwarf and neutron stars were numerically studied with modified gravity models. The main interest was to study the spacetime around the stars. The results shed light on the viability of the studied cosmological models.

Tiivistelmä

Kosmologian standardikuvaus perustuu oletuksille avaruuden homogeenisuudesta sekä isotrooppisuudesta ja pohjana on yleisen suhteellisuusteorian gravitaatioteoria. Nämä eivät kuitenkaan yksin riitä kuvaamaan kosmologian nykyhavaintoja, joiden mukaan avaruus laajenee kiihtyvästi. Yleinen suhteellisusteoria vastaa erittäin tarkasti havaintoja, jotka on mitattu aurinkokuntaa pienemmässä mittakaavassa. Nain ollen joko yleisen suhteellisuusteorian materiasisältöä tai sen gravitaatiosektoria täytyy muokata, jotta teoria voisi kuvata myös havaittua avaruuden kiihtyvää laajenemista. Kun Einsteinin yhtälöihin lisätään näkymätön materiakomponentti sekä vakio arvoinen energiakomponentti, voidaan havainnot selittää yleisen suhteellisuusteorian mukaisesti.

Tässä väitöskirjassa on tarkasteltu gravitaatiota, kosmologisia malleja sekä niihin kuuluvia ilmiöitä. Useita pimeän energian malleja, joiden motivaatio tulee fysiikan standardimallin ulkopuolelta, on tarkasteltu painottaen sen tulkintaa havaittavien ominaisuuksien kannalta. Kaikkien teorioiden, jotka selittävät kosmologiset havainnot, tulee myös päteä paikallisesti esimerkiksi aurinkokunnan skaalassa. Tämä vaatimus asettaa tiukkoja ehtoja fysikaalisesti hyväksyttäville kosmologisille malleille. Tässä väitöskirjassa on verrattu mm. supergravitaatio-kvintessenssi -mallin ennustuksia supernovahavaintoihin. Lisäksi useaa metristä gravitaatioteoriaa on tutkittu tähdistä mitattujen havaintotulosten kannalta. Auringon kaltaisia tähtiä, valkoisia kääpiöitä sekä neutronitähtiä mallintavia polytrooppisia tähtimalleja on tutkittu numeerisesti modifioiduissa gravitaatiomalleissa. Pääasiallisena kiinnostuksen kohteena on näiden tähtien ympärillä oleva aika-avaruus. Tulokset liittyvät mallien mahdollisuuksiin selittää fysikaalisia ilmiöitä.

List of papers

This thesis consists of a review of the subject and the following original research articles:

I **Complex supergravity quintessence models confronted with sn Ia data**,
K. Henttunen, T. Multamäki and I. Vilja,
Phys. Lett. B **634**, 5 (2006) (4 pages)

II **Stellar configurations in f(R) theories of gravity**,
K. Henttunen, T. Multamäki and I. Vilja,
Phys. Rev. D **77**, 024040 (2008) (8 pages)

III **Consistency of $f(R)$ gravity models around solar polytropes**,
K. Henttunen and I. Vilja,
Phys. Lett. B **731**, 110 (2014) (8 pages)

IV **Is scalar-tensor gravity consistent with polytropic stellar models?**,
K. Henttunen and I. Vilja,
submitted, [arXiv:1408.6035 [gr-qc]] (7 pages)

Chapter 1

Introduction

The marvels of the night sky have always intrigued mankind. In the field of cosmology the pursue is not so much in the detail, but rather in the overall behavior of the whole universe. In cosmology the averaged phenomena on very large scales is thought to be much simpler and the theoretical description is first aimed at grasping the essence of these phenomena. Cosmology regards the composition and the evolution of the universe. It tries to encompass the particle physics phenomena of the very early universe, the modern universe we live in and everything in between. A physical theory of any kind would have no meaning if it couldn't predict the physical phenomena according to the observations. Cosmological observations are the light beacons out in the dark waters of the universe. Specific events in the course of history of the universe left signals to be detected and interpreted in the current era. Cosmological theories, although may arise purely from mathematical beauty, need to have the power to encompass these observations. Cosmology, a subject as vast as the universe itself cannot be encompassed by this thesis study. Although this thesis deals with very different cosmological models, it is concentrated on small phenomenological details. All the natures laws are in use and sometimes modified in the attempt to describe the observed phenomena correctly and consistently. If a theory is consistent with the measurements within the errors allowed by the experiment, the theory is considered a viable candidate for the description of the measured phenomena.

The art of confronting theoretical models and observations is not usually elegant or graceful, but rather detailed and dirty. There is, for example, no representation-independent approach for generating a unique gravitational theory in which abstract postulates could be turned into practical statements. Therefore, there is no way to relate principles and experiments in a straightforward way. This has lead to the path of trial and error in trying to find an observationally valid theory in cosmology. The theory needs to be peeled layer by layer by making simplifications at each step. Starting from theoretical principles, to modelling, phenomenology and finally to observable parameters that can be measured. In the case of cosmology the variety of observational phenomena is extensive. Cosmological theories or models are usually regarded as models that firstly produce the observed large scale evolution. In the small scales the natural phenomena need to conform to the current physical view, that unfolds according to general relativity and the Standard Model of particle physics.

The studied models do not belong to an unique model class, but rather forms a collection of so called dark energy models that describe the large scale expansion of space. So the unifying subject for the work represented here is the confrontation of these cosmological models with observational data. The extent of the phenomena covered by the studied models, however, inspired the writer to cover the subject of cosmological theories (by scratching its surface) although the actual work was done by numerically studying specific details. The main pursuit in this work has therefore been the confrontation of phenomenological cosmological models with observed quantities. In this thesis a number of dark energy models and derived observables are numerically studied to shed light on the applicability of the considered theoretical model to describe the chosen observed phenomena.

In the very large scales, the assumptions of isotropy and homogeneity, have lead to the very successful ΛCDM cosmological model and to the expanding Big Bang paradigm. The success of these theories lean to the current independent cosmological observations. Although the ΛCDM model (with cold dark matter and the cosmological constant Λ) seems to describe the observations well, there are still many questions left unanswered. By trying to make the model match the observed and also to approach a consistent theoretical construction, new attempts need to be made to find the correct recipe for the state and evolution of the universe. The already well described properties are incorporated into the new modified models, so that at local scales the general relativistic and at large scales the ΛCDM behavior is obtained. The common thing in trying to describe the dark energy is to add new physics or by changing the composition of the gravitational interaction. The modified gravity theories, although designed to explain the large scale accelerated expansion, must agree with the local observations and deviate very little from general relativity in this limit. The bulk of the studied models consider modified gravity. Therefore gravitational theories, their requirements and applications are in the center of examination in the following work. In this thesis the writer also wanted to shed light on the ambiguity of dark energy models that is faced when the model phenomenology is constrained with experiments. It is hard to make distinction between the different types of dark energy models, that can in one representation modify the gravitational action and in another have extra fields in the matter Lagrangian.

With numerical problem solving it is impossible to deal with abstract theories and specific models need to be selected for study when observational results are treated. Although seemingly non-efficient, some progress has been made in ruling out unviable cosmological model classes this way. The work presented in this thesis is in essence studying the viability, although the numerical results present phenomenological details. The results of the studies [1, 2, 3, 4] are contemplated with this in mind. These studies deal with dark energy models basing on supergravity [5], metric $f(R)$ theories [6, 7, 8] and O'Hanlon scalar-tensor gravity [9, 10].

The structure of this thesis is as follows: In Chapter 2 the standard gravitational theory is constructed with the concepts needed to understand the studied phenomena in mind. In Chapter 3 the observational foundation of cosmology is surveyed. The observations that were used in the works [1, 2, 3, 4] are again emphasized. Chapter 4 introduces the standard cosmology and extrapolates to the modifications discussed in the nest Chapter. In Chapter 5 the work represented in the research papers is discussed. This structuring aims at bringing the reader closer to gravitational phenomena and the nature of the dark energy models and tries to clarify the terminology and concepts used later in the text and in the articles [1, 2, 3, 4].

Notation

Natural units $c = \hbar = 1$ are used throughout this work and $\kappa = 8\pi G$, where G is the Newton's gravitational constant. The metric signature is $-+++$. The Ricci tensor is defined as the trace of the Riemann curvature tensor $R^{\nu}{}_{\alpha\nu\beta} = R_{\alpha\beta}$, where the Riemann tensor is $R^{\mu}{}_{\alpha\nu\beta} = \partial_\nu \Gamma^{\mu}{}_{\alpha\beta} - \partial_\beta \Gamma^{\mu}{}_{\alpha\nu} + \Gamma^{\mu}{}_{\sigma\nu}\Gamma^{\sigma}{}_{\alpha\beta} - \Gamma^{\mu}{}_{\sigma\beta}\Gamma^{\sigma}{}_{\alpha\nu}$ with a connection $\Gamma^{\mu}{}_{\alpha\beta}$. The Ricci scalar is $R = R^{\nu}_{\nu} = g^{\mu\nu}R_{\mu\nu}$ and the Einstein summation convention is used throughout the text.

Chapter 2

Gravitational foundations

A consistent and a viable gravitational or cosmological theory must consider matter and energy conditions, causality, initial conditions and stability issues. These will be shortly viewed in the following Sections. The theory is said to be well-formulated and well-posed mathematically if the above is satisfied. All these aim at a physical and realistic description of the gravitational phenomena, that essentially must describe the structure and evolution of the universe as a whole. To really describe the universe we live in, the laboratory measurements and observations must also be explained by the theory. This, by no means, is easy to realize and a careful investigation of the mathematical properties of the theory need to be done to arrive at a model that could represent a physical phenomena. Many layers of simplification are needed even to arrive at an acceptable spacetime. Needless to say even more simplifying assumptions brings the theory close to a phenomenology that accepts physical and measurable concepts and can be tested. General relativity fulfills all these requirements and the general relativistic conditions are shortly considered in the following.

2.1 Requirements for a gravitational theory

In all physical theories the conditions for matter and energy, and the initial values (Cauchy problem) need to be provided self-consistently for a particular physical problem to be well-formulated. The energy conditions concern the matter content and pose requirements for the theory to be physically sound. In a well-formulated theory causality, realistic physical sources and uniquely determined dynamical evolution of the system are guaranteed [11, 12]. The requirement of causality forces constraints for the particles to travel on timelike $ds^2 < 0$ curves. Massless particles like light travels along geodesics or null curves for which $ds^2 = 0$. The values of the matter field must be uniquely determined by the values of the fields and their derivatives in some prior time. This is a requirement posed by the Cauchy problem for the matter fields and is thoroughly dealt with in [11].

A well formulated Cauchy problem in general guarantees that the solutions exist, are unique and depend continuously on the initial and boundary data [13]. A viable theory needs also to be stable. This requires the system to be regular *i.e.* no initially small growing perturbations are allowed. The perturbed system must stay close to the unperturbed system throughout the considered evolution. For Einstein's field equations and suitable matter the Cauchy problem is well formulated, theory is stable and fills the causal requirements. The Bianchi identities ensure the causal structure is preserved for the momentarily comoving reference frames (inertial frames in special relativity) if the initial value problem is well posed [13]. This is not the case in all gravitational theories and care should be taken in the correct formulation of the initial conditions.

Also other obstacles lurk in gravitational theory formulation. The dynamical equations must not possess unobserved short timescale or matter instabilities. These can occur in more complicated gravity theories [9, 14, 15]. General relativity, however, avoids these.

2.1.1 Matter and energy

One essential component of a gravity theory is its non-gravitational energy contents *i.e.* the matter fields and radiation. These are not restricted by the gravitational theory, so the following conditions are usually considered in addition to the requirements of the previous Section.

Throughout this work the term non-gravitational is used to mean energy sources that are defined in the absence of gravity and therefore *live on spacetime* as opposed to gravitation representing the spacetime curvature itself. Describing all the matter fields and their dynamics explicitly would be extremely complicated or even impossible, so a general tensor is chosen to describe the energy and momentum involved. In general relativity the energy-momentum tensor $T_{\mu\nu}$ represents the distribution of energy and momentum of all the non-gravitational fields, *i.e.* matter and radiation. It contains their energy, pressures, stresses and momenta and all these act as sources for the gravitational field [16, 17]

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}\mathcal{L}_m}{\delta g^{\mu\nu}}. \tag{2.1}$$

An often used simplified example in cosmology is the perfect fluid approximation. A perfect fluid is completely described by its density, pressure and its velocity field. In this work only sources of perfect fluid type have been considered. This simple realization of the stress-energy tensor $T_{\mu\nu}$ is characterized by the energy density ρ and pressure p as

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu + pg_{\mu\nu}. \tag{2.2}$$

The quantities ρ and p are bound together with the equation of state that defines the properties of this energy component. For a stationary observer in Minkowski flat space $g_{\mu\nu} = \eta_{\mu\nu}$ the four-velocity of the fluid is $u_\mu = (1, 0, 0, 0)$ and $\eta_{\mu\nu}u^\mu u^\nu = -1$. For a more general metric $g_{\mu\nu}$ with gravitation, the timelike four-velocity tangent vector u^μ can be obtained in a momentarily comoving reference frame (MCRF) by demanding them to be perpendicular to the spatial section by the requirement of isotropy. Now the Minkowskian metric $\eta_{\mu\nu}$ needs to be replaced in (2.2) by the general metric tensor $g_{\mu\nu}$. Also in the presence of gravitation $g_{\mu\nu}u^\mu u^\nu = -1$ needs to hold. The components can then be written in the MCRF with the metric tensor as $u^\mu = ((-g_{00})^{-1/2}, 0, 0, 0)$ for stationary fluid in hydrostatic equilibrium [16].

If the source is not explicitly known one can consult the energy conditions to keep the matter content S_m physical. The energy conditions are formed from the eigenvalues and eigenvectors of the energy momentum tensor. The conditions that need to be fulfilled, are inequalities concerning the components of the energy-momentum tensor. These restrict the properties of energy and matter to hold true with the observed phenomena. Two conditions are mentioned here, the weak energy condition (WEC) and the strong energy condition (SEC). The WEC states that no negative energy densities are observed $T_{\mu\nu}u^\mu u^\nu \geq 0$, for timelike observers moving with a four-velocity $\mathbf{u} = u^\mu \hat{e}_\mu$ [11, 12]. With perfect fluid matter this becomes $\rho \geq 0$, $\rho + p \geq 0$. SEC demands $R_{\mu\nu}u^\mu u^\nu \geq 0$ to hold for all timelike vectors u^μ. With perfect fluid matter SEC reads

$$\rho + p \geq 0 \quad \wedge \quad \rho + 3p \geq 0. \tag{2.3}$$

This condition holds for ordinary matter, but not for the dark energy. Note that the strong energy condition is essentially a geometric statement in GR. Now the matter Lagrangian $\mathcal{L}_m[g_{\mu\nu}, \psi_m]$ depends only on the metric and the matter fields ψ_M and does not couple to other gravitational fields. Also, the freely falling particles will move along metric geodesics as in a metric gravitational theory.

In an isolated system, the total energy and the total momentum need to be conserved. The covariant conservation law for stress-energy is written as

$$\nabla_\mu T^{\mu\nu} = 0. \tag{2.4}$$

2.2 General relativity

Einstein's general relativity (GR) is founded on the *Einstein's equivalence principle* (EEP), that conforms to the three requirements that concern all matter and energy: weak equivalence principle (WEP), local Lorentz invariance (LLI) and local position invariance (LPI). The WEP embodies the equivalence between inertial mass and gravitational mass and LLI and LPI makes general relativity a universal law that holds everywhere in the curved spacetime for all matter fields follow special relativity. In other words, the effects of gravity must be equivalent to the effects of living in a curved spacetime with Minkowskian local neighborhoods for every point $p \in \mathcal{M}$ (and $\mathcal{M}$ is a semi-Riemannian manifold).

The LLI states that the outcome on a non-gravitational experiment at p should not depend on the velocity of the frame and according to LPI this should hold for all p in $\mathcal{M}$. Also, according to the LPI, there is no *preferred* background metric defined but the dynamical equations are generally covariant, *i.e.* true in all coordinate systems if true in one system. A conformal scaling freedom is left for the metric tensor field so that all metrics, conformally related to the Minkowski metric, are equally valid. A theory of gravity that embodies EEP by possessing a metric and a connection that give the paths for a freely falling test body to be the geodesics of the metric, is said to be a metric theory of gravity. This subject is thoroughly discussed in the texts [7, 18, 19]. An even more stringent condition on a gravitational theory can be posed by demanding that also gravitational tests hold between different Lorentz frames and that this is valid for all $p \in \mathcal{M}$. This condition is called the Strong Equivalence Principle (SEP) and is valid only in GR.

In general relativity, gravitation is therefore a curved spacetime phenomenon and is based on semi-Riemannian geometry with a metric tensor $g_{\mu\nu}$ which defines the inner products $g_{\mu\nu}v^{\mu}u^{\nu}$ of tangent vectors $v(p), u(p)$ at each point $p \in \mathcal{M}$. This brings about concepts of length of a curve, angle between vectors, area and volume. Because, the metric of general relativity locally reduces to Minkowski metric $\eta_{\mu\nu}$ in every freely-falling frame, all the frames have the same Lorentz invariant field equations. The local position invariance now guarantees that the laws of physics are the same everywhere. Generally, the metric tensor is a covariant symmetric non-degenerate tensor and it defines the invariant line element ds as

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}. \tag{2.5}$$

Gravitation can, in the weak field limit and for a class of algebraically special metrics, be comprehended as a deformation of the flat Minkowski metric $\eta_{\mu\nu}$:

$$g_{\mu\nu}(x^{\lambda}) = \eta_{\mu\nu} + h_{\mu\nu}(x^{\lambda}) \tag{2.6}$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2.7}$$

The metric tensor also allows the computation of proper time for each observer with four-velocity v^{μ}, determines the shortest distance between two points and determines causality with the speed of light c. The metric and the derived Levi-Civita connection are the dynamical quantities that determine the local acceleration and give the gravitational field and potential over the manifold. In GR the unique connection $\Gamma^{\alpha}_{\mu\nu}$ of a symmetric metric $g_{\mu\nu}$ defines the notion of parallel transport and the covariant derivative on a Riemannian manifold. The covariant derivative ∇_μ can therefore be defined for a contravariant vector field V^ν as

$$\nabla_\mu V^\alpha \equiv \partial_\mu V^\alpha + \Gamma^\alpha_{\mu\nu} V^\nu. \tag{2.8}$$

The covariant derivative makes differentiation a valid tensor operation under a general coordinate transformation in a curved spacetime. This needs to be so, because the only invariant relations on a manifold are tensor relations. By changing partial derivatives to covariant derivatives in the field equations, the laws of physics take the same form in all coordinate systems.

The connection of metric theories of gravity, including GR, is the Levi-Civita connection derived from the metric tensor (the Christoffel symbols)

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\delta} \left(\partial_\mu g_{\nu\delta} + \partial_\nu g_{\mu\delta} - \partial_\delta g_{\mu\nu} \right), \tag{2.9}$$

that is symmetric in its lower indices $\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}$ and the spacetime is torsion free. Note that, for the Minkowski metric in Cartesian coordinates the Christoffel symbol vanishes and covariant derivative reduces to the ordinary partial derivative. By stating that the connection of the spacetime is given by the Levi-Civita connection the metric stays invariant. The metric compatibility is given by

$$\nabla_\alpha g_{\mu\nu} = 0 \tag{2.10}$$

These theories have a symmetric metric, that gives the trajectories of test bodies as geodesics and conform to special relativity for non-gravitational physics [18].

General relativity arises from two founding *postulates* that conform to the SEP. The *first* postulate requires the Lagrangian density for gravity to take the Einstein-Hilbert (EH) form

$$\mathcal{L}_{\rm EH} = \frac{1}{16\pi G} \sqrt{-g} [R - 2\Lambda]. \tag{2.11}$$

The Λ term is not included in the original formulation of Einstein's general relativity, but adding a constant into the action can be accepted and still have second order field equations for the gravitational interaction. This is required, because, a quantity that transforms like a scalar (here R) under Lorentz transformations leaves the action $S = \int \mathcal{L}_{\rm EH} d^4x$ invariant under variation with respect to the metric. The *second* postulate states that the metric $g_{\mu\nu}$ must couple to the matter fields of the Standard Model of particle physics (SM) [20] universally and minimally. This means that in the matter term (2.1)

$$S_m = \int \mathcal{L}_m \sqrt{-g}\, d^4x \tag{2.12}$$

all couplings to Minkowski metric $\eta_{\mu\nu}$ are everywhere replaced by couplings to $g_{\mu\nu}$. Universal coupling entitles the metric therefore a special status as being considered a property of spacetime rather than a field on spacetime. Minimal coupling requirement restricts the gravitational field $\mathbf{g}$ not to couple to any other gravitational fields in the action.

The Einstein-Hilbert action (2.11) leads to the GR field equations in the case of $\Lambda = 0$:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} \tag{2.13}$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor and $T_{\mu\nu}$ is the energy-momentum tensor of the matter fields. The Riemann curvature tensor that is the fundamental embodiment of curvature in Riemannian geometry and Ricci tensor $R_{\mu\nu}$ and the Ricci scalar R encapsulate the curvature effects in the Einstein's equations. Einstein arrived at this form by requiring the gravitational part of the theory to resemble the gravitational Poisson equation of the Newtonian gravity $\nabla^2\Phi = 4\pi G\rho$. The Einstein tensor $G_{\mu\nu}$ is the only divergence-free function of the $g_{\mu\nu}$ and its derivatives of at most second order that is tensorial [21]. Therefore, Einstein field equations generalize the Poisson equation, apply to all coordinate systems and guarantees the local covariant conservation of the energy-momentum tensor [17]. For a correctly formulated initial value problem, the resulting dynamical evolution is unique and agrees with the causal requirement that signals can only be sent between points that can be joined by a curve that is either a timelike $ds^2 < 0$ or a null curve $ds^2 = 0$. From the Einstein equation it can also be seen that the matter-energy content dictates the causal and geodesic structures of spacetime.

Einstein's equations allow physical solutions that preserve causality for matter sources that obey one of the energy conditions discussed in Section 2.1.1. Since the field equations are very complex only solutions for very simple matter content can be analytically found. Only cases of empty space (vacuum) and perfect fluid matter have been considered in this work. However, although not realistic, the simple solutions give an idea how the system behaves qualitatively and therefore reveal possible properties of more complex solutions. Some exact solutions for the Einstein-Hilbert equations with the cosmological constant Λ

$$G_{\mu\nu} = \kappa T_{\mu\nu} - \Lambda g_{\mu\nu} \tag{2.14}$$

that are used in the text are briefly listed here. Three maximally symmetric solutions of (2.14) with no matter (vacuum solutions) are the Minkowski flat spacetime (with $\Lambda = 0$) (2.7), the expanding de Sitter space with positive Λ included and the anti-de Sitter space with constant negative curvature ($\Lambda < 0$). Maximal symmetry guarantees homogeneity and isotropy which are currently observed from the largest scales and from the early universe. Around a spherically symmetric mass distribution with no vacuum energy, the gravitational field is described by the Schwarzschild solution. The Robertson-Walker solution describes an expanding or contracting homogeneous and isotropic universe and is the metric that is used in the cosmological standard model. The following Subsections shed light on the special solutions that have been utilized in the articles [2, 3, 4].

2.2.1 Spherically symmetric solutions

Spherically symmetric spacetimes allow a natural coordinate system for the study of stellar objects

The Einstein-Hilbert field equations have an unique solution for a spherically symmetric vacuum with Λ, the de Sitter solution, and with spherically symmetric matter bodies, the Schwarzchild-de-Sitter (SdS) solution. The SdS solution describes the spacetime in the vicinity of a non-rotating spherically symmetric object. To note, the Newtonian equivalent for the Schwarzschild solution would be the gravitational field of a point particle at infinity.

In a spherically symmetric spacetime all metric components are unchanged under any transformation $\theta \to -\theta$ or $\varphi \to -\varphi$. The metric components are independent of the time coordinate if the spacetime is static and the geometry is unchanged under time-reversal. The most general static, spherically symmetric metric can be written as

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{2.15}$$

and the Schwarzchild metric coefficients read

$$\begin{aligned} B(r) &= \left(1 - \frac{2Gm}{r}\right) \\ A(r) &= \left(1 - \frac{2Gm}{r}\right)^{-1}. \end{aligned} \tag{2.16}$$

The spherically symmetric vacuum solution, the de Sitter solution contains a term with the cosmological constant

$$\begin{aligned} \tilde{B}(r) &= 1 - \frac{\Lambda}{3}r^2 \\ \tilde{A}(r) &= \left(1 - \frac{\Lambda}{3}r^2\right)^{-1} \end{aligned} \tag{2.17}$$

All the major classical tests of GR are based on the spherically symmetric Schwarzchild solution [18], which will be very useful when studying static spherically symmetric spacetimes in more general gravity theories.

2.2.2 Birkhoff's theorem

The *Birkhoff's theorem* of GR states that any $T_{\mu\nu} = 0$ solution of Einstein's equations with continuous first and second derivatives which is spherically symmetric must be static and of the Schwarzchild form (2.16) with $r > 2GM$ [22, 11]. With the Λ term the solution is the Schwarzchild-de Sitter solution. This theorem is not generally true for other gravity theories. It does not hold for example in metric $f(R)$ theories and for scalar-tensor theories it is not generally valid [23, 24]. Ideally one hopes for global solutions, but in many cases only local solution is possible. Therefore, a carefully considered, initial value or boundary value problem may give valuable information on the underlying theory. With this theorem the boundary conditions at the surface for a static spherically symmetric body can be determined. Birkhoff's theorem is broken in higher order theories of gravitation and this condition has been used in the works [2, 3, 4].

2.2.3 Polytropic stars

In thermodynamics the state of a matter system, in the absence of chemical reactions, is completely described by three quantities: pressure p, temperature T and the rest mass density (or mass per volume). This state is described by the equation of state $p = p(\rho, T)$. With the polytropic equation of state (EOS) one can describe a spherical body of non-uniform matter distribution. In a spherically symmetric case the density ρ is a monotonic decreasing function of the radius r and the surface can be found at $p(r_s) = \rho(r_s) = 0$.

The relationship between the star's internal pressure and density is controlled by the energy transport. This is generally a very complicated process and when carefully derived it arises from the nuclear reactions occurring deep within the stellar core. The nuclear densities for the studied stellar objects have been approximated based on nuclear physics by Wakano and Wheeler [17]. By approximating stellar matter with polytropic EOS, that describe the adiabatic processes, the problem is simplified enormously with little lost in the precision of finding realistic masses and radii with physical central densities. The polytropic equation of state reads

$$p(r) = K\rho(r)^{\gamma} = K\rho(r)^{(n+1)/n}, \tag{2.18}$$

here $\gamma = (n+1)/n$ is the adiabatic index and n is the polytropic index. K is a constant that depends on the configuration. Adiabatic processes occur without the exchange of thermal energy with the outside of the system.

For numerical purposes it is useful to use the scaled variables of Lane and Emden [16]

$$\begin{aligned} \rho(r) &= \rho_0\theta(r)^{\frac{1}{(\gamma-1)}} \\ r &= lx \\ l &= \sqrt{K\gamma/(4\pi G(\gamma-1))}\rho_0^{(\gamma-2)/2)}. \end{aligned} \tag{2.19}$$

Here $\theta(r)$ and x are the scaled density and radius of the Lane-Emden configuration. This parametrization was used also in our studies of the local effects of modified gravity in the papers [2, 3, 4].

Stars may be approximated by these processes when the radiation does not carry thermal energy out efficiently enough. However, the surface layers of a realistic star deviate some from the polytropic approximation because

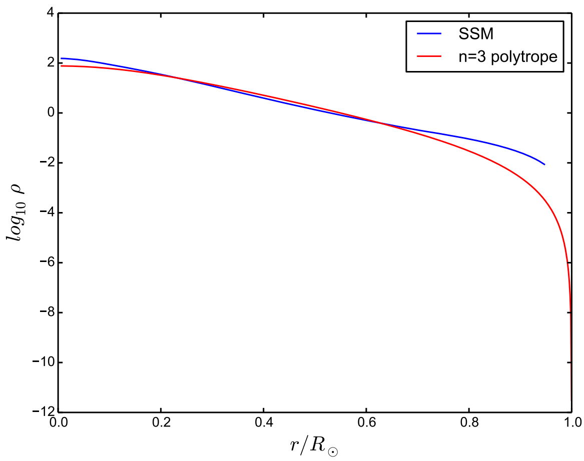

Figure 2.1: The Newtonian Eddington sun ($n = 3$ polytrope) and the Standard Solar Model [25] show a remarkable similarity. The densities deviate in the central core and on the outer layers, where the convection layer is not well described by the Eddington polytropic parameters.

the outer layers no longer conform to the adiabatic approximation. The polytropic model of the Sun with $n = 3$ is called the Eddington model. With Eddington model parameter values fairly good fit for the Standard Solar Model (SSM) is obtained [25]. This can be seen in the Figure 2.1.

Only in the convective regions near the solar surface and in the central core does the polytropic model deviate from the SSM. Therefore, despite its simplicity, the Eddington model does a remarkably good job for describing the sun. Eddington Sun is usually solved for Newtonian gravity, but the parametrization (2.19) is easily upgraded to GR with Tolman-Oppenheimer-Volkoff (TOV) equations and is a valid model for studying the phenomenology of gravitational models. The TOV equation describes the structure of a spherically symmetric body in static gravitational equilibrium.

$$- r^2 p'(r) = G\mathcal{M}(r)\rho(r)\left[1 + \frac{p(r)}{\rho(r)}\right]\left[1 + \frac{4\pi r^3 p(r)}{\mathcal{M}(r)}\right]\left[1 + \frac{2G\mathcal{M}(r)}{r}\right]^{-1}, \tag{2.20}$$

where $\mathcal{M}$ is the integrated mass

$$\mathcal{M}(r) = 4\pi \int_0^{r_s} \rho(r) r^2 dr.$$

Now with the polytropic equation of state and central density for the Eddington sun, mass and the radius can be solved. Perfect-fluid matter with polytropic EOS was utilized in the works [2, 3, 4] for modelling the stellar solutions for various gravity theories. With polytropic equation of state, also white dwarf and neutron star properties can be modelled. Polytropic equations of state offers a powerful tool to describe these compact objects and for example neutron star matter density can be constructed from several layers of polytropic envelopes allowing the separate layers *e.g.* hyperon or quark matter core under the tightly packed neutrons.

2.2.4 The weak-field limit and PPN

Weak field phenomena concern slow motion and weak gravitational potential like in the solar system. In weak-field limit the metric **g** replaces the Newtonian gravitational field. In this limit, it is sufficient to approximate the gravitational field with the parametrized Post-Newtonian formalism (PPN). With this perturbative approximation theories of gravity where the matter responds only to the metric can be compared with each other in terms of dimensionless potentials and their scalar coefficients [18].

In the PPN, gravity is again described by a tensor field $g_{\mu\nu}$ written in terms of the dimensionless gravitational potentials (*e.g.*, U in the spatial part) in flat spacetime. The potentials are constructed from the matter variables like the Newtonian potential [18]. For example, the spatial part of the PPN metric reads to second order

$$\begin{aligned} g_{ij} &= \delta_{ij} + 2\gamma_{PPN} U_{ij}, \\ U_{ij} &= G \int \frac{\rho(x')(x-x')_i (x-x')_j}{|\mathbf{x}-\mathbf{x}'|^3} d^3x'. \end{aligned} \tag{2.21}$$

The parameter γ_{PPN} can be solved by fitting the general spherically symmetric metric to the PPN Schwarzchild solution (2.16). Now

$$\gamma_{PPN} = \frac{1 + \frac{B(r)}{rB'(r)}}{1 - \frac{A(r)}{rA'(r)}}. \tag{2.22}$$

This parameter measures the spatial curvature produced by a unit mass and can be measured very accurately around the sun. It gives the difference in the potential $2GM/r$ when comparing the g_{00} and g_{ii} terms, and is equal to unity for general relativity but not for all gravitational theories.

Chapter 3

Observations and phenomenology

With the precision measurements of today, the experiments play a crusial role in guiding and constraining developement in the gravity theory building. There is a multitude of observational constraints a gravity theory must accommodate [18, 9]. Gravitational interaction of general relativity has been tested on scales ranging from $10\,\mu$m to a few astronomical units and the gravitational action with the cosmological constant (2.14) on cosmological scales [20]. The gravitational theory of the whole universe needs to accommodate the phenomena of all scales, these are the cosmological theories that are introduced in Chapter 4. The large scale observed phenomena can not be described by the GR with Standard Model matter alone, but needs to be explained either by the Einstein's cosmological constant or some other agent. It might be that the universe we live in still posseses strange new features unexplained by the non-standard theory. In this thesis the studied cosmological models and their phenomena are tested against local and supernovae observations.

3.1 Local gravitational measurements

The foundations of GR have been tested in detail. Local Lorentz covariance has been tested to hold true extremely precisely starting from the Michelson-Morley experiment of 1887. The success of LLI is also embodied in the relativistic quantum theories. The Dirac equation and quantum electrodynamics are a staggering example of the power of special relativity. The weak equivalence principle has also been challenged over many

Mission	Measured constraints for the PPN-parameters
Cassini	$\gamma_{PPN} - 1 = (2.1 \pm 2.3) \times 10^{-5}$
VLBI	$\gamma_{PPN} - 1 = (-2 \pm 4) \times 10^{-4}$
LLR	$4\beta_{PPN} - \gamma_{PPN} - 3 = (4.4 \pm 4.5) \times 10^{-4}$
Mercury perihelion	$\lvert 2\gamma_{PPN} - \beta_{PPN} - 1 \rvert < 3 \times 10^{-3}$

Table 3.1: Observational constraints from solar system missions: Cassini spacecraft mission [27], Very Long Baseline Interferometry (VLBI) [28], Lunar Laser Ranging (LLR) tests of general relativity [29] and the Mercury perihelion shift measurements [30].

centuries with the results always being in accordance with WEP.

Local gravitational tests include the laboratory, earth-moon and solar system scale experiments. In the laboratory gravitation has been tested with gravitational redshift, free-fall, rotation and with torsion balances. Also the effects from extra dimensions on gravity have been searched in the Large Hadron Collider [26]. In solar system Mercury perihelion advance, bending of light and the Shapiro time delay effects have been observed and are explained with GR. With earth moon laser ranging experiment, Einstein's theory of gravity has been found to align with observations with high precision.

With these convincing arguments on behalf of GR, it is evident that if Einstein's gravitational theory is not the description of the gravitational phenomena on large scales, the cosmological theory must approach GR on local scales. The post-Newtonian parametrization (PPN) provides an arena for testing modifications to GR. There already exists a wealth of solar system experiments that have provided accurate measures on how much a theory can deviate from GR [18]. In the Table 3.1, a few of the most accurate parameter values deduced from observations, are shown. The most precise measurement for the Shapiro time delay with PPN parameters is the Cassini mission experiment [27]. This mission provided the $\mathcal{O}(10^{-5})$ accuracy measurement for the γ_{PPN}-parameter, which was used as the reference value in the works [3, 4].

3.1.1 Astronomical observations

General relativity has also been contested with compact objects, such as white dwarf and binary pulsar systems [31]. Also, in these strong-field environments GR is very successful. Binary pulsar systems are composed of a pulsating neutron star and a compact companion. Neutron star masses can be detected by studying these systems. The radii of neutron stars are still a highly model dependent property, but the current radius determinations agree well with each other [32].

3.2 Cosmological observations

It seems natural to postulate that we are *not* living in a special position in the universe. This is the Copernican principle, that affects cosmology in a similar fashion than the idea of Copernicus that the sun (not the earth) was the center of the solar system. The idea of thinking of Earth, solar system, and Milky Way galaxy, not sitting in a special point in the universe is guaranteed if the space is isotropic and homogeneous. The current status of the cosmic microwave background, large scale galaxy surveys and the distribution of quasars all agree with the universe being isotropic and homogeneous on very large scales (roughly more than 100 Mpc).

The observations on Hubble's law around 1930 started the quantitative observational testing of cosmological theories. In the 1990's this endeavor resulted in an unexpected interpretation of accelerated expansion of the universe by two teams studying distant supernovae [35]. Spatial volume of the observable universe therefore seems to be expanding according to the Hubble law in megaparsec scales and the current expansion rate seems to be accelerated according to the distant Type Ia supernova experiments. This interpretation fits into the general relativistic picture with a dark component that does not cluster with matter, generally called dark energy. The implications of dark energy are so profound, that complementary tests are needed to find out about the nature of the cause of the accelerated expansion. The supernova observations have been confirmed with different systematic errors and different parameter degeneracies today by many cosmological probes already. The main observational probes confirming dark energy include the very distant supernovae measurements (SneIa) [35], cosmic microwave background (CMB) observations [37, 38], the age estimates of the universe $t_0 \sim H_0^{-1}$ [38, 39], the baryon acoustic oscillation mea-

Parameter	Best Fit	1 σ standard deviation
$\Omega_b h^2$	0.022032	0.02205±0.00028
$\Omega_M h^2$	0.12038	0.1199±0.0027
Ω_Λ	0.6817	$0.685^{+0.018}_{-0.016}$
$H_0 = 100\,h^2$	67.04	67.3±1.2
Age/Gyr	13.8242	13.817±0.048

Table 3.2: Planck 2013 data release of Best fit parameters for the baryonic, matter and vacuum energy density contrasts (Ω_b, Ω_M and Ω_Λ) in the ΛCDM model [38].

surements (BAO) [40] and the large scale structure experiments (LSST) [41, 42].

The current observational value for the Hubble parameter H_0 must conform to the age estimates of the oldest known stellar populations, the globular clusters. The theoretical model for the Type Ia supernovae suggest them to be exploding white dwarf stars in a binary system, where the mass accretion from the companion has caused the white dwarf to collapse. All Type Ia supernovae have, according to this theory, identical luminosities therefore making them useful as astronomical standard candles for cosmological observations. This is, also, believed to be the most common supernova type in the universe. With the Type Ia model assumptions the absolute magnitude of the explosion can be modelled and distances to SneIa explosions be evaluated from the apparent luminosity. Measurements from the luminosity distance $H_0 d_L(z)$ of SneIa suggest constraints for a time-independent equation of state parameter to be in accordance with $w_{DE} = -1$ (with 1σ confidence interval).

The $H(z)$ or SneIa data alone do not constrain the cosmological parameters significantly, this is why combination of independent data sets needs to be used when choosing a good parameter value for a theoretical model. Baryon acoustic oscillations formed before the photons recombined from the baryons and thus reveal the pre-recombination density fluctuations being imprinted on the baryon distribution. BAO data constrain the cosmological parameter space in an orthogonal direction with respect to

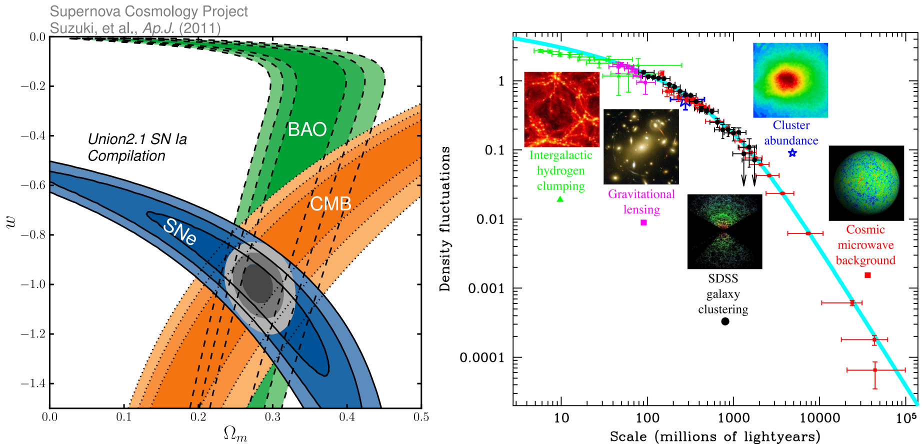

Figure 3.1: The independent combined cosmological data show remarkable consistency. Left: Credits to Supernova Cosmology Project, Right: Copyright Max Tegmark [36].

$H(z)$ data forcing the cosmological parameters to be rather tightly bound.

The current observations of the cosmic microwave background is in agreement with other independent cosmological observations that are SneIa, $H(z)$ and BAO. The combined data set very stringently pins down a density parameter area that accepts the WMAP [37] and Planck best fit results [38] for the cosmological constant and the total matter energy; $\Omega_{\Lambda,WMAP} = 0.7185$ and $\Omega_{\Lambda,Planck} = 0.6711$ and $\Omega_{M,WMAP} = 0.2814$ and $\Omega_{M,Planck} = 0.3175$.

The spatial flatness is currently constrained to deviate less than a percent from $\Omega_k = 0$ [38]. Also, the current most precise measurement made on the CMB, the Planck Mission data [38], is well described by the spatially flat six-parameter Λ Cold Dark Matter (ΛCDM) model with adiabatic perturbations. The current precision observations all suggest DE and are well described by the ΛCDM model. This model has been so successful that it has been standardized as the concordance model of cosmology. This model needs the dark matter component in order to produce the large overdense regions in the universe. With only baryonic matter the observed structures could not have formed. A simulation of the ΛCDM model with dark matter particles [43] show remarkably similar structures that are observed in the large scales. The Millennium run traces the matter evolution in a simulation of more than 10 billion CDM particles set in a cubic region of the universe that is 2 billion light years a side over a time span that corresponds to more than 13 Gyr. The Figure 3.2 at $z = 0$ nicely shows the hierarchy of structure on different scales.

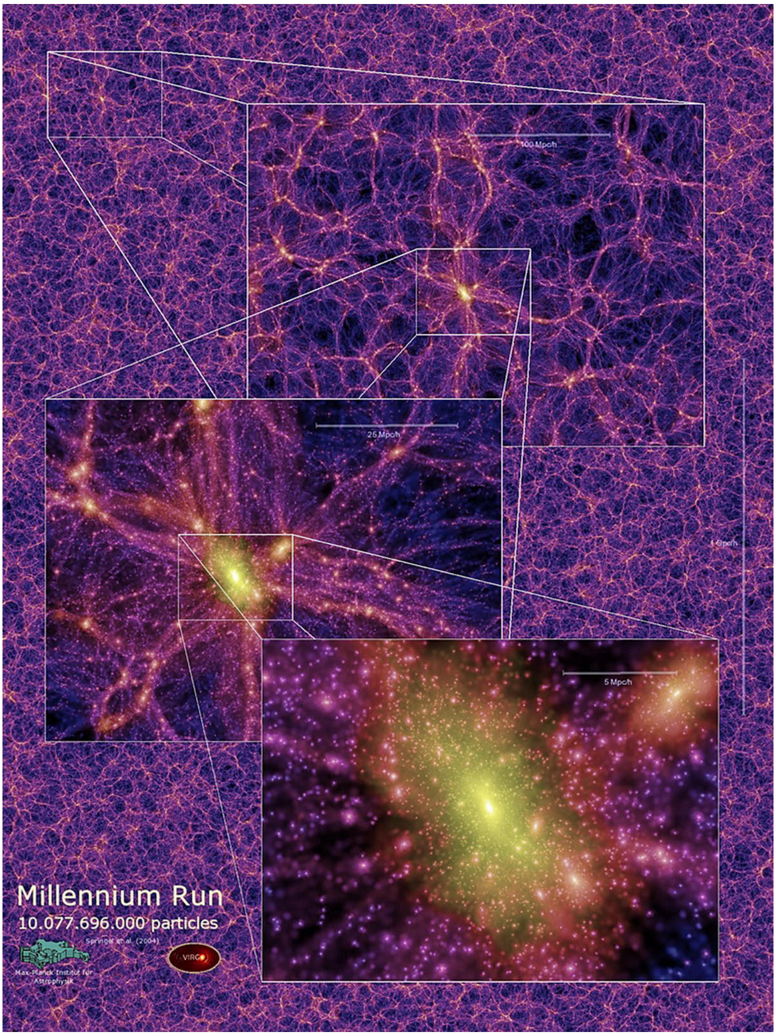

Figure 3.2: Shown is a snapshot of the current universe $z = 0$, with gradually smaller scales in zoomed images [43].

Chapter 4

Cosmological models

The properties of the universe at the largest scales are studied in cosmology. The cosmological scales concerned are even larger than the galaxy superclusters that still span structures around voids, regions of very little matter. At these scales gravitation is the sole effective force. The universe is treated with bulk properties as a homogeneous body composed of different fluid-like components. According to the standard cosmological model, the current variety of cosmological observations coherently point to a composition of energy and matter to be the following. About 70% of the energy budget should be filled with a fluid called dark energy (DE) that is responsible for the exponential acceleration of the spacetime. Almost all the rest of the energy content, about 27%, should be filled with gravitationally interacting matter-type energy, cold dark matter (CDM) and a few percent with that familiar baryonic matter. With this composition the ΛCDM model is able to produce the observed structure growth and to explain the current era of accelerated expansion. A variety of rival cosmological models seeking also to realize the observed large scale phenomena include new forms of energy and matter or modifications to the Einstein's gravitational theory. Dark energy can therefore refer to modified gravity theories that can also give rise to the accelerated expansion.

4.1 The standard cosmology

The standard way to describe the universe is to consider the dynamics to be governed by Einstein's general relativity in spatially homogeneous and isotropic spacetime. The dynamical equations of motion are derived from the Einstein-Hilbert action (2.11) with a cosmological constant Λ, matter, radiation and cold dark matter. The Einstein-Hilbert field equations with the cosmological constant read

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} - \Lambda g_{\mu\nu}. \tag{4.1}$$

If the Λ term is written on the "matter side" of the field equations it is interpreted as the energy of the vacuum. This is the case within the ΛCDM model, that is the standard cosmological parametrization of the Big Bang paradigm. In this text GR refers to the above action without the Λ-term and ΛCDM to the full Einstein-Hilbert action. The ΛCDM model explains well the following observational signatures: the CMB, the large scale galaxy structures, the abundances of the lightest elements and the accelerating expansion of the universe. It also provides an acceptable fit with the observation through the cosmological parameters represented in the Table 3.2.

The energy content of the universe in the standard cosmology is described by several interacting perfect fluids. These include radiation ρ_r (neutrinos and electrons), matter ρ_M (baryons and CDM) and dark energy ρ_Λ. The densities of the energy species are usually denoted by the density parameters that represent their density divided by the critical density $\rho_c \simeq 5$ nucleons m^{-3}. For the matter and radiation species and for the vacuum energy the density contrasts read:

$$\Omega_i = \frac{\rho_i}{\rho_c}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} \equiv \frac{\Lambda}{8\pi G \rho_c}. \tag{4.2}$$

The dark matter sector was initially hypothesized, because apparently there was not enough luminous and observed matter to account for the galactic structures and for the structure formation process within the cosmological general relativistic theory. Also the huge gravitational lenses and colliding galaxies show DM in effect. The viable dark matter particle candidates that are able to address structure formation are thought to have been non-relativistic at the time of recombination and are named cold dark matter (CDM). Many CDM simulations are able to produce many large scales and galactic phenomena well [43, 44].

Dark matter has never been directly observed and there are many possible explanations for it. Several particle physics experiments are making a serious effort in finding the DM particle [45]. The name explains two of the main properties of this energy species. It does not significantly interact with the electromagnetic radiation, so its effects on other matter are only seen through gravitational interaction. The currently dominant models concern non-standard particles, *i.e.* particles that are not described by the Standard Model of particle physics. Other CDM candidates include dark baryonic matter bodies (like dead cold stars, brown dwarf stars, mini black holes) and several varieties of hypothetical non-baryonic particles like weakly interacting massive particles (WIMPs) [46]. Baryonic matter sector, however cannot fill but little of the dark matter energy budget, so in addition also a non-luminous massive CDM particle is required within the ΛCDM . The ΛCDM model with CDM has had a lot of problems that might be explined by baryonic physics [47].

Conservation of energy gives the first law of thermodynamics that dictates how the energy density and pressure evolve over time. This can be represented for the three cosmological fluids ρ_r, ρ_M and ρ_Λ with the equation of state $p_i = w_i \rho_i$. This is for radiation $w_r = 1/3$, for matter $w_M = 0$ and for the vacuum energy $w_\Lambda = -1$. For the ordinary matter and radiation the strong energy condition is fulfilled, but for the dark energy component the SEC condition $\rho + 3p < 0$ does not hold.

The expanding spacetime paradigm complemented with the early universe particle physics is generally called the Big Bang theory [48] (see also article 21 in [20]). The spacetime expansion observed in largest scales therefore points to a very hot beginning. The Big Bang scenario explains the isotropic 2.7 K microwave radiation background. The thermal history, according to this theory, further incorporates the evolution of the observed hierarchical structures of galaxies with initially hot homogeneous plasma being cooled down. This sufficiently cooled plasma then formed gravitationally bound structures in all the observed scales from galaxy super clusters to stellar phenomena. The Big Bang theory includes GR with isotropic and homogeneous spacetime as the gravitational framework and the standard model of particle physics [20] to describe the particle interactions and how the known particles were formed. The very hot beginning additionally needs a period when the spacetime was “initialized” to the state of homogeneity and isotropy. This can be carried out by a period called inflation that occurred in the energy scale of 10^{16}GeV according to the most popular models. This corresponds to time $t \sim 10^{-34}$s after the beginning. The idea is that an initially small smooth and causally coherent patch of the universe was exponentially expanded to fill the comoving volume and this became the entire observable universe today. The curvature radius of the universe is therefore exponentially expanding during the inflation while the energy density remains constant. There is no satisfactory model for inflation available yet, but most inflationary models consider a scalar field called inflaton to source the exponential expansion [48, 49].

4.1.1 The Friedmann model

Although, there is no preferred frame of reference in theories of spacetime that follow the Einstein's Equivalence Principle, some coordinates make the equations of motion easier to deal with. One such choice in cosmology is comoving-coordinates for isotropic solutions. Comoving observers will perceive the cosmic microwave background to be isotropic because they are moving along with the Hubble flow (which is the expansion of the spatial volume of the observable universe). Galaxies, for example, are almost freely-falling and therefore are almost comoving bodies due to their low peculiar velocities.

To explain, what the expansion of the universe means, we can take a look at the Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological model that is behind all the standard observational interpretations. The FLRW metric describes curved isotropic and homogeneous spacetimes

$$ds^2 = -dt^2 + a(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right). \tag{4.3}$$

It describes an expanding or a contracting universe and is an exact solution to Einstein's equations. The spatial curvature k and the relative expansion $a(t)$ define the geometry and evolution of the universe. If $k = 0$ the metric reduces to the Minkowski's flat universe of special relativity. Space that is isotropic in every point is homogeneous and therefore maximally symmetric. FLRW is the most general non-vacuum spacetime metric that has a 3 dimensional maximally symmetric subspace.

The FLRW solution with the FLRW metric is therefore a solution of the Einstein's equations (4.1) that describes a homogeneous and isotropic space with perfect fluid (2.2) source. The time-time $G_{00} = 8\pi G T_{00}$ and the space-space $G_{11} = 8\pi G T_{11}$ components of the Einstein's equations give the first Friedmann equation (4.4) while the second equation is derived by using the trace equation $G^{\mu}_{\mu} = 8\pi G T^{\mu}_{\mu}$.

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{k}{a^2} + \frac{\Lambda}{3}. \tag{4.4}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}. \tag{4.5}$$

In these equations k is the curvature parameter for constant spatial curvature and $a = a(t)$ is the scale parameter of the FLRW metric.

The expansion rate of the universe is determined by the Hubble parameter $H(t) = \dot{a}(t)/a(t)$. The Hubble constant H_0 is the preset value of the expansion rate and is related to the critical density ρ_c of a flat ($k = 0$) universe by

$$H_0^2 = \frac{8\pi G\rho_c}{3}, \qquad H_0 = 100\, h\, \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}, \tag{4.6}$$

where h is called the scaled Hubble constant. The Hubble parameter H_0 is not a constant but varies as t^{-1} and the timescale for the age of the universe is given by the Hubble time H_0^{-1}. The observations are in good agreement that the universe is flat [37, 38] and, therefore, the total energy density is the critical density. Also, inflationary models within the Big Bang scenario generate negligible spatial curvature early in the history of the universe [48].

Now the history of the homogeneous and isotropic universe can be traced back with the Friedmann's equations if the values of the cosmological parameters are known. The energy content of the universe dictates the dynamics of the universe and the evolution of the energy species sets some requirements for the cosmological models. In the Figure 4.1 the time line of the structure evolution of the initially homogeneous matter field into the currently observed matter structures is illustrated.

The continuity equation (2.4) realize the conservation of energy for perfect fluids also in general relativity as

$$\nabla_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \nabla_\mu G^{\mu\nu} = 0. \tag{4.7}$$

Now we can write the energy conservation for a perfect fluid $p_i = w_i\rho_i$ to

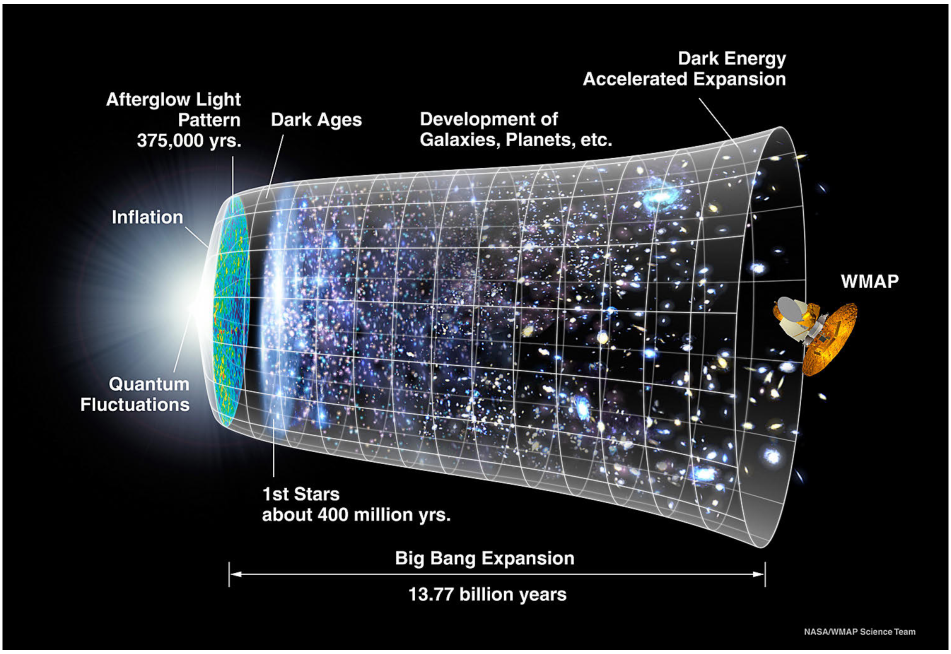

Figure 4.1: Time line of the universe from Big Bang to the current time. Credits to NASA/WMAP Science Team.

get the evolution of an energy species as

$$\rho(t)_i \propto a(t)^{-3(1+w_i)}. \tag{4.8}$$

Here the DE sector with the cosmological constant Λ describes a smooth energy component uncoupled to matter. Generally, the condition $w < -1/3$ for the equation of state parameter, requires a fluid with large negative pressure that can give rise to accelerated expansion.

With the FLRW model it is easy to see how the energy budget has been evolving through the history of the universe. One important piece of any cosmological model is to get the cosmological eras correct to produce sufficient matter structures and to arrive to a situation where the matter and DE energy densities are comparable. The ΛCDM evolution and composition are depicted in the Figure 4.2. The radiation energy density evolves as $\rho_r \sim a^{-4}$ and the cold dark matter component as $\rho_M \sim a^{-3}$, and only

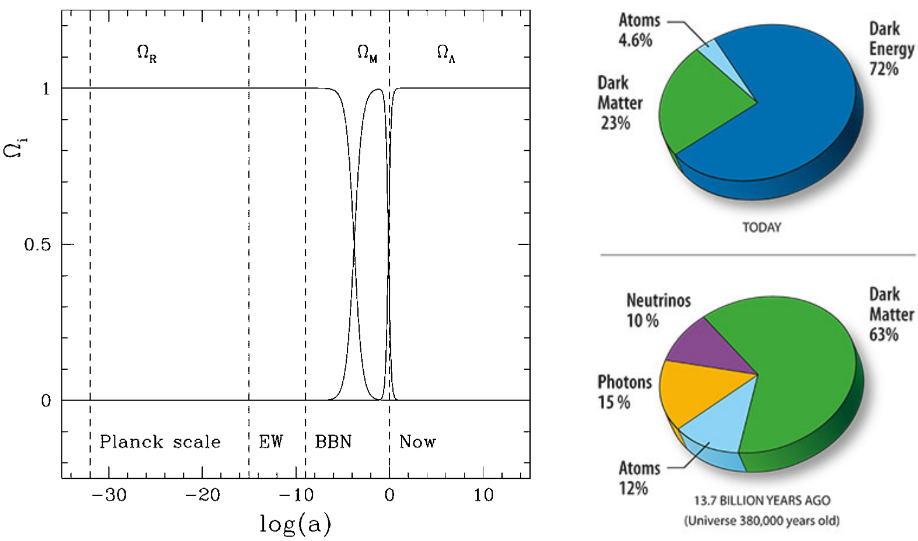

Figure 4.2: Left: Evolution of the energy species according to the FLRW model [50]. Right: Two snapshots of the energy contents, at $t \sim 10^6$ years and at $t_{now} \sim 10^{10}$ years, credits to NASA/WMAP Science Team.

recently has the dark energy with constant density become the dominant component that drives the cosmic expansion. The Friedmann equations provide a way to describe the evolution of the universe, particularly by studying the evolution of the scale factor during different cosmic eras. If the w_i are independent of time, the scale factor evolution can be solved for a single species from the flat Friedmann equation for the dominant fluid with (4.8)

$$a(t) \propto t^{\frac{2}{3(1+w)}}. \tag{4.9}$$

In a radiation dominated universe, the scale factor would grow as $a(t) \propto t^{1/2}$, under matter domination as $a(t) \propto t^{2/3}$ and if the vacuum energy dominates as $a(t) \propto e^{\sqrt{\Lambda/3}t}$. The matter energy fraction is currently comparable to the DE component, so the DE energy domination, and the accelerated expansion, has only recently begun.

4.1.2 The de Sitter universe

The de Sitter solution with no matter fields is the vacuum FLRW solution. De Sitter space is a maximally symmetric, exact solution of Einstein's field equations with Λ and no source. Let's consider a ΛCDM scenario when the expansion of the universe has diluted the matter density to a negligible value. Now the vacuum energy totally dictates the energy budget and the Friedmann equation can be written as

$$\dot{a}(t)^2 = \frac{8\pi G\rho_\Lambda}{3}a(t)^2 - k. \tag{4.10}$$

Now if we assume the curvature term is also diluted away with time and the energy density of the vacuum remains constant, an exponentially expanding solution is found:

$$a(t) \sim e^{Ht}, \quad H^2 = \frac{8\pi G}{3}\Lambda. \tag{4.11}$$

For models of inflationary cosmology, a de Sitter phase of exponential expansion can therefore provide a mechanism that generates the homogeneous conditions observed in the CMB. The universe cannot at all times be in a de Sitter phase, but under certain conditions this mechanism could give rise to exponential expansion. This has been anticipated to occur during inflation and possibly we are approaching this scenario with the currently observed accelerated expansion also.

The Schwarzchild-de-Sitter solution describes the gravitational field far enough from a spherical mass distribution. The SdS metric components in (2.15) read:

$$\begin{aligned} A(r) &= \left(1 - \frac{2GM}{r} - \frac{\Lambda}{3}r^2\right)^{-1}, \\ B(r) &= \left(1 - \frac{2GM}{r} - \frac{\Lambda}{3}r^2\right). \end{aligned} \tag{4.12}$$

With this solution the exterior field can be thought to be independent on the gravitational potentials of the surrounding bodies (or their effect is completely negligible because the realistic distances between stars are sufficiently large) and the field far away is considered to be the de Sitter space. The interior solution of a spherical static star therefore obtains the

SdS solution outside in the standard cosmology.

4.1.3 The ΛCDM model problems

The current standard model of cosmology with dark matter and dark energy gives a good account for the cosmological observations. This model however has several conceptual problems [50, 51, 52]. The biggest problem in the dark energy interpretation is that we don't understand it at the fundamental physical level. Maybe the biggest problem considered today concerns the size of the cosmological constant. If dark energy is described as the zero point energy of the vacuum, its magnitude is problematic to explain. No reasonable explanation for the small observational value, $\rho_\Lambda \approx 7 \times 10^{-30}$g cm^{-3} $\sim (10^{-3}\text{eV})^4$, The approximation for the vacuum energy offered by high energy physics, namely the Planck density, is over 120 orders of magnitude higher [53]. And why is it very small but not exactly zero? There are no other well motivated alternative explanations to the nature of the constant Λ available, unless the anthropic arguments are considered [54]. The naturalness of the cosmological constant, can also be treated from the viewpoint of quantum consistency The treatment of Dvali and Gomez [55] shifts the smallness question into: "*Is cosmological constant a quantum-mechanically-consistent notion?*" These views are the major reason why dark energy has been tried to be explained by non-standard physical theories like rolling scalar fields, higher dimensional or supersymmetric theories [56, 57, 58].

Another puzzling fact concerns the era the universe is observed today. Why do we happen to view the universe at a period when the energy densities of the CDM and vacuum energy are comparable? The large scale structures that we observe set an upper limit to the fraction of the energy densities of DE and CDM. Were it bigger, the structures would not have enough time to grow to the observed state. Were it a few orders of magnitude smaller, it would already be undetectably small. Also some observations are problematic with the CDM+Λ model [59].

Other fundamental concerns arise within the Big Bang theory [48]. Why is the space flat and $\rho_0 \sim \rho_c$? Why is spacetime homogeneous and isotropic as described by the CMB and the large scale data. CMB states that the universe was smooth at the time of recombination with temperature anisotropies from 2.74 K of the order $\delta T/T \simeq 10^{-5}$. Causal effects couldn't have generated the smoothness, because the universe consisted of about 10^5

causally disconnected regions at that time. How does the initially smooth universe grow structures like stars, galaxies, galaxy groups, clusters, voids and the Great Wall?

As already mentioned, an inflationary era within the standard Big Bang scenario can hand an answer to some of these problems. It can smooth out the space to align with the CMB and large scale observations. It also generates structures into the flat background via quantum fluctuations that are "inflated" to become classical density fluctuations that will evolve through gravitational collapse.

Displeased with the cosmological constant as an explanation of some 70% of the energy density in the modern universe, a plethora of models have been considered among theorists. One approach is to consider the gravity to arise from general relativity and the dark energy component to be composed of some fantastic new energy with negative pressure $w_{DE} < -1/3$. The observational constraint suggest that the alternative model still produces the equation of state quite close to the w_Λ case, see Table 3.2. Another approach considers standard matter fields, but changes the gravitational theory such that the current accelerated expansion can be accounted for. For the modified gravity theories only some metric gravity alternatives are studied here although many alternatives can be found in the literature.

Any physical and stable cosmological model needs to provide an explanation for the cosmological and local observations. As the minimum requirement homogeneity, isotropy and the evolution of the energy species is to be correctly included. A complete cosmological model should also include a description of deviations from homogeneity. Some cosmological models that provide the exponential acceleration may also be used in the early universe to describe the inflationary phase. Usually, however, a cosmological model is considered only to provide the late time acceleration and the Big Bang scenario is considered to provide the early universe behavior. With today's precision measurements of GR in the weak field, there is not much room to deviate from general relativity locally. So, the viable of cosmological gravity modifications, that make the expansion of the spacetime accelerate, need to have GR as the local scale approximation. Another line of thought for providing the accelerated expansion without dark energy is called backreaction [60]. It provides the effect of DE via the formation of nonlinear structures. This model is only mentioned in this work and was not studied in the thesis publications.

4.2 Deviations from standard cosmology

If ΛCDM is not the correct way to describe the universe, the currently standardized theories either of spacetime or of the energy content of our universe must radically change. (Although, radical thinking is required already in understanding the CDM and vacuum energy.) Here quintessence dark energy models are briefly considered and the focus is more on gravity modifications. With this distinction, the jungle of cosmological models is not a clear cut case. Gravity sector modifications are usually considered by means of curvature as opposed to having extra fields on spacetime. Here, however, all the models that are able to produce the accelerated expansion are called dark energy models. This can be done because with the Einstein's field equations one can in most cases write the field equations of the cosmological model as

$$G^{\mu\nu} = \kappa \left(T^{\mu\nu} + T^{\mu\nu}_{eff} \right), \tag{4.13}$$

where $T^{\mu\nu}_{eff}$ denotes all non-GR components, effectively making the theory look like GR with DE-fluid in appropriate frame [12].

What also makes a tight DE-model classification vague, is the fact that some of the theory classes can also be described by another formalism. One such example are the $f(R)$ theories that can be described with the Brans-Dicke scalar-tensor gravity formalism and interpreted as chameleon quintessence to describe the local scale phenomena. The discussion on non-standard cosmological models is therefore incomplete in this thesis and tries to provide the reader tools to understand only the models that were studied in the presented papers. Standard cosmology provides an evolutionary history with a coherent interpretation of the observations. This needs to be fulfilled also by any viable cosmological deviation from ΛCDM . All the models should therefore accept early inflation (whether produced by the cosmological model or some additional mechanism), radiation domination (when the hot universe cools down), structure formation era (leading to the observed structures) and the current exponentially accelerated era. This is usually achieved with the FLRW solution deviating only little from the ΛCDM model. Another discriminator is the growth of the cosmological perturbations seen in the CMB and observed in the large scale structures of galaxy clusters [61].

Although the distinction, between having an extra energy species in the action and modifying the gravitational theory, is extremely vague (because these regimes get easily mixed when the theory is represented in another formulation), the cosmological models are divided into these two classes. The first case is therefore discussed as an extra non-gravitational field in the theory (lamely labeled as dark energy models) and the other as modification to Einstein's general relativity.

4.2.1 Dark energy models

Based on the previous Section, let's now take a look at other alternatives that can provide the equation of state $w_{DE} \approx -1$ that may have varied through times and only now act as Λ. In the light of cosmological observations and FLRW cosmology, the unexplained form of energy that pushes the spacetime to expand faster than the Hubble rate needs an explanation. The cosmological constant is hard to justify, so other explanations have been sought for. Sometimes it is useful to include into the theory, in addition to the experimentally observed fields, yet unobserved fields that aid the theory to explain the puzzling features. Theories with fundamental scalar fields include nonlinear theories of gravity, Kaluza-Klein theories, dilatons in superstring theories, models involving varying Newton's constant and the inflationary scenario. Also other higher rank fields, such as vector, tensor or fields of even higher rank could be postulated in gravitational theories. Adding a scalar field is however the simplest alternative and it allows many exact phenomenological solutions to be found. In particle physics scalar fields have for a very long time been used to describe for example the spin 0 particles and, especially, the recently observed Higgs field, that gives the mass to all massive particles. The effects of dark energy (essentially the equation of state $w_{DE} \approx -1$) can be produced if the scalar field action meets certain requirements with a dominant potential term (quintessence, phantom fields with negative kinetic term) or with a special kinetic term (K-essence). For an introductory review on quintessence, K-essence, ghost condensates, tachyonic and dilatonic dark energy see [58] and [62] for phantom models and time varying dark energy equation of state.

Cosmological observations are only currently entering a phase, where the different DE-model classes may be distinguishable [63, 9, 64]. Before we get there, the precision observations at the local scales can provide information of the theory phenomenology or even to rule out some model classes as unviable.

Quintessence

Quintessential scalar fields are considered as homogeneous and dynamical non-gravitational fields that do not effectively couple to matter [56, 65, 66]. These act as non-luminous special fluid that can resist the gravitational collapse over large distances. Note that, in this work for quintessential regime GR is assumed to be the theory of gravitation. The quintessential field is here denoted as ψ for which the action is written as

$$S_Q = \int d^4x\sqrt{-g}\left[\frac{1}{2}g_{\mu\nu}\partial^\mu\psi\partial^\nu\psi - V(\psi)\right]. \tag{4.14}$$

Now the equation of motion for the field $\psi = \psi(t)$ on a FLRW background is given by the Klein-Gordon equation

$$\ddot{\psi} + 3H\dot{\psi} + V'(\psi) = 0, \tag{4.15}$$

where dot represents derivation with respect to time and $'$ is the derivative with respect to the argument. Furthermore, its energy density and pressure can be written as

$$\begin{aligned} \rho_\psi &= \frac{1}{2}\dot{\psi}^2 + V(\psi) \\ p_\psi &= \frac{1}{2}\dot{\psi}^2 - V(\psi) \end{aligned} \tag{4.16}$$

and give $w_\psi = p_\psi/\rho_\psi \approx -1$ in the limit $\dot{\psi}^2 << V(\psi)$. This obviously occurs only if the field moves sufficiently slowly. In the slow roll approximation also the $\ddot{\psi}$ term is considered negligible and the potential can be considered to fulfill

$$3H\dot{\psi} \approx -V'(\psi).$$

Therefore, this slow roll condition, natural in a flat potential plateau, can account for the accelerated expansion of the universe and possibly also produce inflation in the high energy range.

A class of quintessence potentials $V(\psi)$ possess a nice feature called tracking [67, 68, 69]. A tracking field has an attractor-like solution that traces the cosmic evolution of the background equation of state w_B (*i.e.* $w_B = w_R$ for radiation domination and w_M for matter domination) up to present times and for a wide range of initial conditions. For $w_Q < w_B$ the tracking condition can be stated as

$$\Gamma \equiv V''V/(V')^2 > 1 \tag{4.17}$$

and Γ is nearly constant.

Some examples from particle physics have also been studied within the quintessential scenario. One such class is based on effective supergravity (SUGRA) [70] model that suggests sufficiently flat potential for a complex quintessential field to realize the slow-roll regime. For a more general discussion on supergravity inspired models see for example [58].

4.2.2 Modified theories of gravity

Gravitation is the dominant force determining the evolution of the structures in the universe during the matter domination epoch and dictating the current expansion. Could the theory of gravitation itself be modified such that the observed evolution of the universe with no dark energy is reproduced? Einstein's gravity is reasonably well tested in scales smaller than the solar system, so large deviations from GR must occur in the large scales only. This fact is called the infrared modification of GR and it is a design criteria for all viable modified gravity theories. Also, because CMB, nucleosynthesis and the structure formation with dark matter are all in good agreement with the GR, the modification should only be effective at late times. There are numerous ways to deviate from general relativity. Two often considered ways have been studied here, namely the nonlinear $f(R)$ gravity and the scalar-tensor theory. Other models considered in the literature (see *e.g.* [9, 71] for extensive reviews on extended gravity theories) include general higher order gravity theories, bimetric theories, Einstein-Æther, TeVeS [72], ghost condensates, DGP (Dvali-Gabadadze-Porrati) gravity [73] extended dimension Braneworld models [74] and other gravitational extensions arising from higher dimensional theories (*e.g.* from Kaluza-Klein theories).

GR is an example of a well-formulated theory once the Cauchy problem and the sourcing fluids are considered carefully. With the alternative theories of gravity these may present problems because the initial value problem may not be well formulated or some of the sourcing terms might not be realistic [12]. A feature that sometimes makes the phenomenology of a modified gravity theory easier or at other times harder to interpret, is the fact that the procedure of generalizing the E-H action leaves the theorist with a freedom of introducing auxiliary fields, making conformal transformations, performing renormalizations or even redefining fields for convenience. Furthermore, sometimes different formulations may actually be dynamically equivalent. Dynamically equivalent theories give exactly the same results [6]. As an example the mathematical equivalence of metric $f(R)$ theories to a Brans-Dicke type scalar-tensor theory with a potential [75] is considered in this thesis .

Although the current gravity modification scenarios have been studied for a long time, no theory has clearly risen above the others, so all of them ought to be considered as toy models for examining the deviation from GR, or its robustness. The viable modifications to GR have currently gone through a multitude of selection criteria and still many models persist. Currently there are some gravitational experiments and observations that may be used to distinguish metric theories of gravity [9, 76]. The standard theoretical selection criteria must concern unwanted instabilities that higher order gravity modifications may contain. These include ghosts fields [77] with negative energy states (like in the Ostrogradski instability [78, 15]), curvature singularities [79] or matter instabilities (of Dolgov and Kawasaki [14]). Also in the local scales problems may occur. It is actually very hard to modify GR and not to affect the small and intermediate scales via extra forces [80, 9]. One way to mend this is to introduce a mechanism that suppresses the effects of the extra scalar degree of freedom in the local scales.

A screening mechanism solves the fifth force problem by *e.g.* making the field very massive or shielding the local scales from detection with an effective thin-shell. Also other screening mechanisms have been introduced see [81] for a review on screening mechanisms. The best place to measure the effects of a screening mechanism locally would be in space, because the suggested suppression of the fifth force occurring on Earth [9].

Some of the general features of scalar-tensor gravitation theories (like the "physical equivalence/non-equivalence" of the Jordan and Einstein frames, or chameleon screening with the thin-shell mechanism) are not addressed in the included papers at all, because only Jordan frame representation was studied in our phenomenological numerical work. The observables can be straightforwardly used in the code because in the Jordan frame the coupling of the scalar field with matter is minimal. In scalar-tensor gravity weak means no coupling at all between the scalar and the matter fields. In some cases the analytic treatment of the problem is easier in the Einstein frame or one specifically studies phenomena that emerge in the Einstein frame. The choice of the reference frame is not relevant if the frame is not transformed.

In metric theories of gravity [82] *i*) the equations of motion concerning matter involve the metric tensor $g_{\mu\nu}$ to describe the gravitation as curvature on a Riemannian manifold and *ii*) respond to all matter and non-gravitational fields in accordance with $\nabla_\mu T^{\mu\nu} = 0$. The metric, however, can be generated in addition to the matter content by other gravitational fields (such as a gravitational scalar field ϕ that responds to the metric via the field equations). The matter action $S_m = \int \sqrt{-g}\mathcal{L}_m d^4x$ in the metric theories include the matter fields of the particle physical model and the metric $g_{\mu\nu}$ that is used to contract the indices. Therefore, the matter fields cannot directly feel the possible auxiliary long-range gravitational fields that define the curvature in metric theories of gravity and the geodesics are determined by the metric tensor alone. Not all theories of gravity are metric theories like GR. Other formulations can involve curvature where the fundamental gravitational entity is no longer the metric alone [83, 9]. Two non-metric theories include the metric-affine formalism [6] and bimetric theories [84, 85].

Note also, that in many gravity modifications SEC and EEP are violated [86, 12]. Einstein's general relativity conforms to SEP. One other known theory that also conforms to SEP is the Nordström's scalar theory of gravity [87]. Also, the Birkhoff's theorem is not satisfied in higher order gravity theories without additional constraints on the scalar curvature and on the extra degrees of freedom [23, 88]. This ambiguity has been utilized in the works [2, 3, 4] when several modified gravity models have been numerically studied around a static spherically symmetric solution.

The work presented in this thesis concerns some of the already tested models and provides additional or independent tests for the viability of the studied models. The focus of the research has been on the modified gravity models and the local observational parameters, especially on the solar γ_{PPN}-parameter and the observed masses of the sun, white dwarfs and neutron stars. The status of these model with respect to mentioned observables is discussed in the articles [2, 3, 4].

$f(R)$ gravity

This class of gravity modifications provide a fruitful arena for testing the effects deviating from GR. Each $f(R)$ function introduces a class of toy theories for which the gravity formulation is a straightforward generalization of Einstein's gravity $f(R) = R$. A nonlinear function of the Ricci scalar, $f(R)$, is introduced into the gravity Lagrangian

$$S_{\mathrm{E}} = \int \frac{1}{2\kappa}\sqrt{-g}f(R)\,d^4x. \tag{4.18}$$

Although the motivation for higher order invariants (like $R_{\mu\nu}R^{\mu\nu}$ or $R\Box R$) in the gravitational Lagrangian come from high energy physics [89], it is still a guiding line that motivates the study of these theories [6]. Another nice feature of this class of gravity theories is the absence of new physics with respect to the Big Bang scenario. Now the gravitational phenomena is fully explained by the familiar geometrical concepts. This very general class of theories allows a rich cosmological phenomenology to occur. Some $f(R)$ functions can indeed account for the effects of dark energy in the current universe, because it is easy to come up with a term that leaves the action effectively ΛCDM

$$f(R) = \cdots + \frac{\alpha_2}{R^2} + \frac{\alpha_1}{R} - 2C + R + \frac{R^2}{\beta_2} + \frac{R^3}{\beta_3} + \ldots \tag{4.19}$$

here α and β scale the term to be close to the vacuum value and have the correct dimensions and C denotes the constant term in the Laurent expansion (the constant term is not generally used in $f(R)$ gravity models). The polynomial expansion has not provided a viable model that would be stable enough a theory and that can also produce the local phenomena according to the observations. The stability criteria described in the previous Section has proved to rule out several $f(R)$ models [14, 15], while some still persist as viable [90, 91, 92, 93]. These more complicated functions of R can even stand for the solar system constraints [75, 94, 95, 96, 114, 97] and the evolution of large scale perturbations [98, 99, 101, 61, 100, 102]. These criteria are vital when the viability of a $f(R)$ function class is tested. Especially, the solar system experiments offer a good testing ground for any modification of general relativity with the PPN formalism [104, 105, 106, 107, 108]. These $f(R)$ functions effectively reduce to $R + C$, where C vanishes completely in the high curvature regime, but acts as Λ when $R \to 0$.

The Ostrogradski instability is avoided in $f(R)$ theories [15, 9] and de Sitter space can be found for functions $f'(R_0) > 0$ [109]. This is a special feature of these theories. Also the Dolgov-Kawasaki instability can be avoided if $f''(R) > 0$, $R >> R_0$. These properties make $f(R)$ models a special class among gravity modifications and is a good motivation for their study. Many of the functions do not, of course, fulfill all the observational requirements and even the ones that do must still be considered as toy models that survey the possibilities of deviating from GR. Two simple models are discussed: $f(R) = R + \alpha_1/R$ (CDTT) [110] and $R + \alpha_2 R^2$ [111] and models that are designed to to behave like ΛCDM , namely the models of Hu & Sawicki [90] and Starobinsky [91]. These models will be discussed in the next Chapter. Many other trial models can be found in the literature (*e.g.* $f(R) - R = e^{\lambda R}$ or $f(R) - R = \ln(\lambda R)$ [112] and other designer $f(R)$ models like $f'(R) = 1/2[1 + \tanh(aR - b)]$ [92] and [93] have been studied). The simplest models are cast out by the viability criteria due to unstable solutions within the theory or because the model cannot accommodate the solar system tests. The CDTT model for example possesses the matter instability and does not fulfill the observational requirements of the solar system [113, 114, 2], namely $\gamma_{PPN} = 1/2$ making in an unviable candidate. In the designer $f(R)$ models the function is specifically tuned to represent GR in the high curvature regime and to approach ΛCDM in large scales with vanishing R. Also inflationary era may be possible to explain with $f(R)$ modification [115].

Variation of the action (4.18) with respect to the metric gives the field equations

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu f'(R) + g_{\mu\nu}\Box f'(R) = \kappa T_{\mu\nu} \tag{4.20}$$

where differentiation is with respect to the argument and $\Box$ denotes the covariant d'Alembertian The matter part $T_{\mu\nu}$ represents the standard matter (2.1). Consider a function $f(R)$ being any nonlinear function of the Ricci scalar. Even in the linear case, *i.e.* the Einstein-Hilbert case (2.11), the resulting field equations are nonlinear and of the second order with respect to the metric tensor. For the nonlinear $f(R)$ the field equation obtains additional second order covariant derivatives of R resulting in fourth order partial differential equations of the metric. In these fourth order gravity theories there exist an additional dynamical degree of freedom, that in the

$f(R)$ gravity case is the field $f'(R)$. More generally the differential order of the field equations relates to the number of scalar fields needed to describe the system. For every further two orders one needs to introduce one scalar [116, 117]. This family of gravitational theories admits therefore more solutions than in GR. Even the familiar notion in GR of vanishing of the scalar curvature in the vacuum $T = T^\mu_\mu = 0$, with

$$R = -8\pi GT,$$

does not hold. This can be seen from the trace equation of the field equations

$$f'(R)R - 2f(R) + 3\Box f'(R) = 8\pi GT. \tag{4.21}$$

Also, when considering a boundary problem in a $f(R)$ theory, the Birkhoff's theorem does not generally hold due to the additional dependence on the new dynamical degree of freedom in the metric [23]. If the metric is matched to the SdS metric at the boundary, more constraints are needed than in GR [118]. Therefore, for a large class of $f(R)$ models, the SdS metric is an exact solution of the field equations (4.20). These solutions can be found for a set of $f(R)$ functions that satisfy

$$R_0 f'(R_0) - 2f(R_0) = 0 \tag{4.22}$$

with a constant $R_0 = -4\Lambda$. Furthermore, the metric components inside the matter configuration depend the distribution inside making the outside solution matching non-unique [119, 120]. The reason behind this is the higher differential order of the $f(R)$ theory when compared to GR. Also Minkowski, for $R = 0$ in (4.22), and anti-de Sitter vacuum, for $R_0 < 0$, solutions exist in $f(R)$ theories as maximally symmetric vacuum solutions.A criterion for the linear stability of a $f(R)$ model, with the de Sitter background R_0, is derived in [6] and reads for $f_0 = f(R_0)$ as

$$\frac{(f_0')^2 - 2f_0 f_0''}{f_0' f_0''} \geq 0. \tag{4.23}$$

Also the equation of continuity is automatically satisfied [121] in $f(R)$ gravity, so no additional information is gained on top of the field equations.

In this work only the *metric formalism* of $f(R)$ theories of gravity has been studied and used. In the metric (or second order) formalism, the Levi-Civita connection that defines the notion of parallel transport in Riemann geometry, is uniquely associated to the metric and is given by the Christoffel symbol (2.9). In the *Palatini formalism*, the metric and the connection are taken to be independent. Variation of the action is therefore done in non-standard way with respect to both of the variables and result in two field equations. In the *metric-affine formalism* the connection is allowed to depend on the metric, but the variation is done the Palatini way. The interested reader may educate oneself for example with the reviews [6, 9, 122] and with references therein.

Scalar-tensor gravity

Scalar-tensor gravitation is one of the most established way to modify GR. The initial value problem is well posed and a number of exact solutions can be found. Scalar-tensor gravity obtains its name from the fact that the gravitational sector does not consist only of the metric $g_{\mu\nu}$, but there is an additional scalar degree of freedom ϕ involved. Overall, the distinction between a *gravitational* and *non-gravitational* auxiliary field is ambiguous and is used only to distinguish matter and gravitational type energy or behavior [116]. In this work all matter related fields arise from the matter action S_m alone, since all our calculations have been done in the Jordan frame in which the coupling between the auxiliary field and matter is minimal. The studied scalar field ϕ is therefore said to belong to the gravitational sector. In the Einstein frame the matter-gravitational distinction is confised because the scalar field couples to the matter sector. If the scalar field is non-minimally coupled to the matter sector the energy conditions for ϕ are also violated [7].

The field ϕ is a Lorentz scalar that can mediate a so called fifth force if it couples to matter. This is a problem in solar system scales unless the potential depends on the matter distribution in a correct way to be able to screen itself and be undetected. When properly introduced in this context, it can both explain the current exponentially accelerated expansion of the spacetime, and be in coherence with the current accurate measurements in the local neighborhood. The scalar field can also be slowly-rolling down its potential at early history of the universe so it can act as an inflaton and realize the exponential expansion to produce the inhomogenities that will develop into the matter structures of the current universe.

A physical system can sometimes be described with many different theoretical frameworks with a set of field equations that fully describes a system in each frame. In some cases there is a mapping between these frames and the theories can be considered to be dynamically equivalent. Conformal mapping is one such transformation that gives a set of equivalent frames or geometries. The causal structure is preserved in a conformal transformation. As an extreme example, the effect of conformal mapping can cause flat spacetime to be bent in the other frame, as the mapping introduces a scalar that couples non-minimally to matter. Therefore, the matter in this transformed frame feels acceleration. The two frames often considered in scalar-tensor theories are the Jordan frame, where the gravitational scalar field couples with the Ricci scalar, but has no coupling to the matter Lagrangian. This way the WEP of GR is preserved, the covariant conservation law for the stress-energy (2.4) holds and the test particles follow geodesics. In the other, Einstein frame, the scalar field does not couple to the Ricci scalar in the Einstein-Hilbert action (2.11) and the matter is instead coupled to the scalar field with a metric $\tilde{g}_{\mu\nu}$. The transformation from Jordan frame to Einstein frame by reparametrization is

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} \equiv \Omega^2 g_{\mu\nu}. \tag{4.24}$$

Now the scalar field acts as a source for the Einstein frame metric tensor and can be considered as a matter field [75]. Because of the coupling in Einstein frame the two mathematically equivalent frames can be considered physically different (see [75, 123]). The two frames are said to be physically equivalent at the classical level if also the units, derived in Jordan frame, scale with appropriate powers of the conformal factor Ω in the Einstein frame [124]. Now the confomally scaled spacetime $\tilde{g}_{\mu\nu}$ possesses matter couplings also for the field ϕ. The scalar-tensor models in the Jordan frame, where the matter Lagrangian $\mathcal{L}_m$ depends only on the metric $g_{\mu\nu}$ and the matter fields ψ_M, the freely falling particles will move on geodesics of $g_{\mu\nu}$ and the gravitational field ϕ is completely a geometric entity. The metric of this spacetime is affected by the scalar field through the considered field equations.

The most prominent and special scalar-tensor gravitational theory is based on the work of Brans and Dicke [125]. In the so called Brans-Dicke theory (BD) there is one extra auxiliary gravitational field ϕ which is a long range scalar field that is weakly coupled to the trace of the energy-momentum tensor. A general form of the gravitational action of scalar-tensor theories in the Jordan frame can be parametrized by the BD parameter ω_{BD}, as

$$S_{BD} = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega_{BD}}{\phi}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]. \tag{4.25}$$

The metric and Palatini $f(R)$ theories can be described with BD equations with $\omega_{BD} = 0$ and $\omega_{BD} = -3/2$ respectively and the corresponding theories are said to be dynamically equivalent [6, 116]. These classes of BD theories, on the other hand, provide a way to investigate the family of scalar-tensor theories, that historically have been studied on other grounds. In the work [4] BD gravity models with $\omega_{BD} = 0$ were studied. This action with vanishing kinetic term is also called the O'Hanlon action [126] and is dynamically equivalent with metric $f(R)$ theories if $f''(R) \neq 0$ and $\phi = f'(R)$ [6, 127]. Horndeski's theory [128] describes the most general second order scalar-tensor theory [129]. This theory has only recently re-entered into the active study and has also proven to be Ostrogradski stable [77, 130].

The model studied in [4] possesses a non-minimal coupling to the curvature scalar R, a minimal coupling to the matter Lagrangian $\mathcal{L}_m$ and a potential $V(\phi)$. The set of field equations in the Jordan frame are derived from the action (4.25) with $\omega_{BD} = 0$ by variation, first with respect to $g^{\mu\nu}$ and then to ϕ.

$$G_{\mu\nu} = \frac{\kappa}{\phi}T_{\mu\nu} - \frac{1}{2\phi}g_{\mu\nu}V(\phi) + \frac{1}{\phi}\left(\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\Box\phi\right) \tag{4.26}$$

$$R = V'(\phi). \tag{4.27}$$

It can be seen here, that the κ/ϕ term in the Jordan frame can also be interpreted as a varying gravitational coupling. The dynamics for the field ϕ is obtained from the trace equation of (4.26)

$$3\Box\phi + 2V(\phi) - \phi V'(\phi) = \kappa T. \tag{4.28}$$

The expanding de Sitter vacuum solution is a stable attractor solution for many scalar-tensor models, see [131] for an extensive analysis on linear stability in modified gravity theories. The theorem of Birkhoff is not generally valid in scalar-tensor theories although it may apply for some special cases like for the constant background field in the Einstein frame [88].

Chapter 5

The phenomena studied in the papers

Previously, cosmological models and particularly the puzzle of dark energy were discussed. The solution to this open question has been approached from two directions. From models that add a new energy component to produce the observed accelerated evolution for the current universe, or by changing the current scenario of how the existing matter behaves in the spacetime, *i.e.* by changing the gravity. Usually confronting a general principle with the observation demands a lot of simplification, up to a point of non-realistic situation. Yet, only by finding the phenomena that can be expressed by a theoretical construct and is suited for a concrete measurement, can anything be said of its validity. A striving force behind the research represented in this thesis has been the dialog between theoretical phenomenology and measurable parameters inferred from the observations. Our approach has been numerical throughout, so specific models needed to be chosen under study. Because of the nature of the empirical method, these models are now under the magnifying glass and to be judged by the observations.

Numerical solutions for polytropic matter configurations in different gravitational scenarios have been studied. Also a class of quintessence models was constrained with supernova observations in [1]. Polytropic stars, although not fully realistic, provide a good venue to test modified gravity models in local scales because of their importance in GR. All the considered cosmological models have at some point been studied as a potential alternative description for the current evolution of the universe. The solar system tests have proven to be important in casting out unviable models. Therefore, new tests are needed for many of the remaining theories seem indistinguishable from GR in the solar system. In [2, 3] solar system measurements were used when the stellar $f(R)$ gravity configurations were studied.

In the article [4], current astronomical data was utilized in a test of a modified gravity model. The potential of a scalar-tensor gravity model was numerically generated for solar type stars, white dwarfs (WD) and neutron stars (NS). The compact objects were modelled with perfect fluid polytropic matter and the boundary values were required to reach physical conditions. At the center the objects were required to be regular and the initial value of the matter density to align with the standard Wakano-Harrison-Wheeler equations of state [17]. The white dwarf masses and radii and the neutron star masses were bound in this work to the current observational values [33, 34].

Numerical methods

The work was done with self written numerical codes that utilize mainly Mathematica, but also python and Fortran77. In this Section, the common theoretical foundations for modified gravity models, used in the numerical work of [2, 3, 4], is laid out. The results are viewed in the following Sections.

Three cosmological model classes were studied within realistic parameter ranges in the articles. In [1] a quintessence model (5.2) was fitted to supernovae data and model parameters were constrained as a result. In the two articles [2, 3] specific $f(R)$ gravity models were discussed in the context of polytropic stellar models and solar system experimental constraints. And finally in the article [4] the potential of scalar-tensor theories, equivalent to the metric $f(R)$ theories, was studied with solar-type, white dwarf and neutron star polytropic equations of state.

A common theme in [2, 3, 4] was to study the behavior of the gravitational field around a spherically symmetric object. The intuitive notion for Newton's gravitational field is the asymptotic flatness required for an isolated object and this is always obtained for a static spherically symmetric object in GR (Birkhoff's theorem). When the modifications to the EH action (2.11) are small, also the stellar solution deviates only little from the general relativistic TOV solution [118]. The author has studied, in these works, how the polytropic stellar configurations (2.18) behave at the boundary and whether the numerical solutions can come close to the general relativistic SdS solution within the studied gravitational models. This was studied in the PPN formalism by numerically selecting solutions that have γ_{PPN} close enough to unity. In these studies the modified gravitational equations in CDTT, HS and scalar-tensor models were considered.

The equations of motion are the field equations of $f(R)$ (4.20) (or scalar-tensor gravity (4.26)), the trace equations (4.21) (or (4.28)) and the continuity equation (2.4). For the $f(R)$ gravity generally the energy continuity is naturally satisfied and can be omitted if sufficient set of field equations is chosen to solve the system. This equation is, however, used explicitly in composing the independent set of equations in our works, because the reparametrization with respect to the radius breaks the natural degeneracy. The background is solved from the vacuum trace equation (4.22).

The configurations describe static spherically symmetric objects (2.15) of perfect fluid matter (2.2) with the adiabatic equation of state (2.18). In our configurations with the metric (2.15), the continuity equation takes the form

$$\rho' = -\frac{B'}{2B}(\rho + p). \tag{5.1}$$

For numerical purposes, the energy density and radius are scaled for the $f(R)$ configuration like in the Newtonian Lane-Emden case (2.19) as $\rho = \rho_0 \theta^{1/(\gamma-1)}$ and $r = lx$. All the solutions are always separately contrasted to the general relativistic TOV (2.20) counterpart to be able to study the differences of the gravitational models inside the configuration. This serves only as a reference and the TOV equation is not used when the polytropic configuration for the gravity modification is solved.

The higher order theory is more constrained in comparison to GR and more boundary conditions need to be fixed. The amount of free parameters depends on the formulation, see [118] for example, where also $R'' = R_0'' = 0$ and $\rho''(r_s) = 0$ are fixed in $f''(R_0) \neq 0$ theories. The boundary conditions were treated as follows. Regularity at the center is defined by searching an analytic solution at the center. At the surface, the scalar curvature $R = R_0$ and its first derivative $R' = R_0' = 0$ can be fixed with the SdS metric. And the surface is found at $\rho(r_s) = \rho'(r_s) = 0$, $p(r_s) = p'(r_s) = 0$ where the metric is matched to the SdS solution by demanding $|\gamma_{PPN} - 1|$ to conform to the Cassini limits.

In the work represented here, the γ_{PPN}-parameter (2.22) is found directly from the Schwarzschild metric parameters with PPN parametrization. This is used to test CDTT and HS gravity models and the results are presented in the papers [2, 3].

5.1 SUGRA quintessence in the light of supernovae data

In [1] we constrained a supergravity quintessence model with supernovae data and found that the simplest models do not provide a very good fit for this data. The model parameters that best fit the data were found on a flat Friedmannian background. This model class is able to produce the correct cosmological evolution for a wide range of initial conditions, so the coincidence problem can be avoided and to arrive near $\omega_Q \approx -1$ in the current universe. The studied potential contains a large class of potentials, also including the ΛCDM model. This class takes the Planck scale physics into consideration. It is a simple quintessence model in supergravity and is dubbed in the literature as the SUGRA model

$$V_Q(\psi) = \frac{\mathcal{M}^{\alpha+4}}{|\psi|^\alpha} e^{\left(\frac{\kappa}{2}|\psi|^2\right)^{\beta/2}}. \tag{5.2}$$

Here $\kappa = 8\pi G$, $\mathcal{M}$ is the energy scale and α, β are positive integers. The ΛCDM is obtained with $\alpha = \beta = 0$. Other well studied DE models belong to this class as well. The inverse power model ($\beta = 0$), the pure exponential model ($\alpha = 0$) and a special quintessence model of Brax and Martin ($\beta = 2, \alpha = 11$) [70]. The potential is usually studied with a real scalar field, that is a special choice. In our treatment the field $\psi = \phi e^{i\theta}$ is complex, as is more natural in supergravity. The treatment is simplified by a constant of motion $L = \dot{\theta}\phi^2 a^3$ in the cosmological equation of motion and the dynamics can be solved now for the real ϕ only.

The cosmological parameters are consistent with WMAP data [132] and only flat enough models were considered. The parameters were fit to the supernova data [35] with the likelihoods calculated by minimizing the χ^2. The special models that were mentioned do not belong to the 1σ area in the general model parameter space. We conclude that the SUGRA model with ($\beta = 3, \alpha = 4$) fits the supernova data set better than ΛCDM (that lies even outside the 3σ contour). We also deduce the energy scale of the potential to lie below $\mathcal{M} \lesssim 1$TeV. These conclusions can be observed from the Figure 5.1.

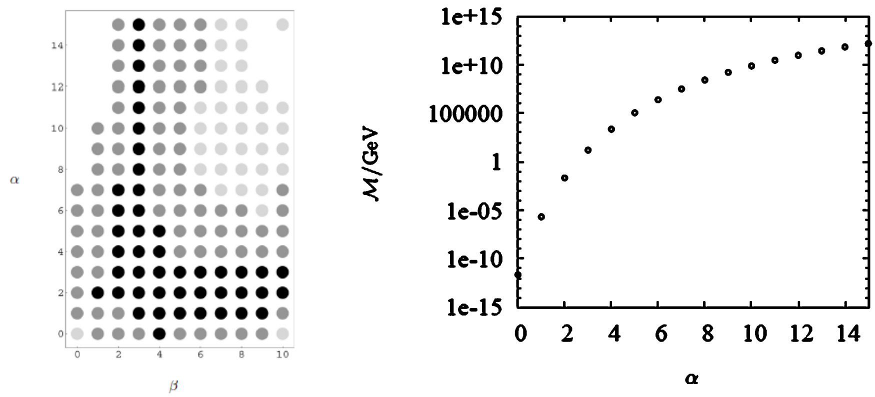

Figure 5.1: Left: The fit of the SUGRA model parameters α and β to the SneIa data set. Flat solutions with 1σ (black dots), 2σ (dark grey) and 3σ (light grey) confidence levels are shown [1]. Right: Unless $\beta = 3$ and $\alpha > 4$, the SneIa data favours energy scales for the potential that lie under $1TeV$ [1].

5.2 Polytropic configurations in the CDTT model

The solar system constraints as well as properties of polytropic stellar configurations in the CDTT model $f(R) = R - \mu^4/R$ were studied. The dynamical system for $f(R)$ gravity with the polytropic equation of state is provided by a highly nonlinear set of equations (4.20). Our study is based on solving the system (ρ, B, A) for the given $f(R)$ function numerically with respect to the stellar radius r, and from these to draw values for observational parameters. An additional independent condition on the field $F(r) \equiv f'(R(r))$ was posed in this paper to lower the differential order of the set of equations. Now we can consider both $f(R(r))$ and $F(r)$ as independent functions of the radius r. This breaks the implicit energy conservation in (4.20) and therefore the continuity equation (5.1) is also used as an independent equation to assure the energy conservation. From the above relation between the $f(R)$ function and the field $f'(R)$, the Ricci scalar can the be algebraically solved for some $f(R)$ models. As an example is the CDTT model, for which this relation is

$$f(r) = \mu^2 \frac{2 - F(r)}{\sqrt{F(r) - 1}}. \tag{5.3}$$

Now the modified field equations, the continuity equation (5.1) and the relation $f(R) = f(F)$ forms the set of independent equations, that are numerically solved together with the polytropic equation of state on a static spherically symmetric spacetime. The regularity condition (8) in [2] was first analytically derived. Then the chosen polytropic configurations were then numerically integrated to yield the density $\rho(r)$ and the metric parameters $B(r)$ and $A(r)$. The stellar configuration was integrated from the center on and some radii outside the star. We also integrated the configuration inwards starting from the boundary and found that the solutions for the same parametrization of the polytropic configuration do not match (see Figure 5.2. Also, the value of the field $f'(R)$ varies very little inside the star and stays near the vacuum value $-\sqrt{3}\mu^2$. We also calculated the γ_{PPN} -constraint for the model directly from the metric parameters (2.22) The solar γ_{PPN} -constraint was straightforwardly found to be $\gamma_{PPN} = 1/2$ till far outside the star, see Figure 5.2. This was the case also for the chosen relativistic WD and non-relativistic NS polytropes in CDTT gravity. The $\gamma_{PPN} = 1/2$ solution holds out to distances where the cosmological background becomes effective (see Figure 5.2). This result is aligned with the previous studies [133, 114]. For $f(R) = R - \mu^4/R$ model the spacetime outside the polytropic sun with regular origin is, therefore, incompatible with the observations from the solar system.

The incompatibility of the model was further proved by starting the integration from the correct SdS boundary value. As a result the numerical solutions were seen to diverge at the origin, leading to an unacceptable solution at the center. The effect of a regular center is of critical importance to the behavior of the metric in these configurations and must be addressed in studies concerning PPN formalism for stellar configurations.

By appropriate initial conditions ($F(r_0) = 4/3$) one finds a solution that reproduces the Lane-Emden solution very well. Furthermore, a simple scaling relation was noted to be present in the radial coordinate for $f(R)$ theories that resemble GR very well within solar densities. The scaling condition suggests that a Lane-Emden -like solution is generally obtainable for such $f(R)$ functions.

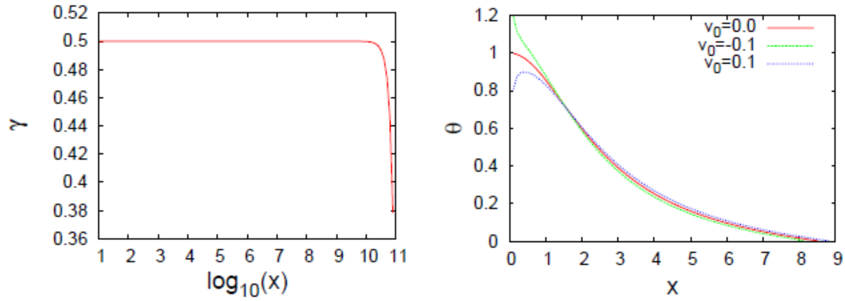

Figure 5.2: Left: Evolution of γ_{PPN} outside the sun in the CDTT model [2]. Right: Solar polytropic CDTT configuration with SdS boundary conditions [2].

5.3 Solar constraint on a chameleon model

By utilizing the recipe described in the Section 5, for the polytropic configurations of $f(R)$ gravitational field equations, we derive the γ_{PPN} solar system constraint for a class of stable $f(R)_{HS}$ models in [3]. These models can both account for the current large scale acceleration and locally behave like general relativity with negligible scalar curvature. All the studied models were constrained, with the field amplitude being $|f_{R0}| = -nc_1/c_2^2/(41)^{n+1} \in [0.1, 10^{-7}]$, to be able produce the observationally viable expansion histories. The Hu and Sawicki chameleon $f(R)$ model [90] reads

$$f(R)_{HS} = R - m^2 \frac{c_1(\frac{R}{m^2})^\alpha}{c_2(\frac{R}{m^2})^\alpha + 1}, \tag{5.4}$$

where $\alpha > 0$, c_1 and c_2 are dimensionless parameters and m^2 is the mass scale of the vacuum today. This model also possesses a thin shell mechanism [134] to suppress the local effects of the fifth force (brought about by the extra degrees of freedom in the theory, the scalar field $\phi = f'(R)$) [76]. Chameleon screening with a thin shell mechanism hides the effect of the field outside the stellar object. In our work we tested the viability of the $f(R)_{HS}$ model around a static stellar polytropic star to find out how well the Cassini results were obtained near the surface by this theory.

With the static spherically symmetric metric and perfect fluid adiabatic matter, we solved the field equations

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)\, f'(R) = 8\pi G T_{\mu\nu} \tag{5.5}$$

for the $f(R)_{HS}$ model to obtain the metric functions $B(r)$ and $A(r)$ inside the configuration.

The chameleon phenomenology is not discussed here analytically nor studied in detail, because the purpose of the work in [4] was to provide a direct numerical result on γ_{PPN} to be compared to the Cassini result. The Cassini spacecraft [27] measured the frequency shift occurring at distance $\lesssim 10r_\odot$ from the solar surface (and the signal peaks at the solar surface), while the Hu and Sawicki [90] γ_{PPN} result was given rather far in the solar system ($r > 100\, r_\odot \approx 1$ AU) where the thin-shell condition was satisfied. We obtained our results at the surface of the polytrope (defined as $\rho(r_s) = 0$). To verify the result, we arrived at the parameter γ_{PPN} from another angle, namely by numerically calculating the metric parameters $B(r)$ and $A(r)$ as in [2]. The parameter had also been previously discussed for other $f(R)$ models in [135, 136] by approximating the γ_{PPN}-parameter in the chameleon $f(R)$ theories from the metric parameters.

We find that although the model admirably arrive near the GR value for most of the polytropic solutions, it does not always find the correct observational γ_{PPN} value (see Table 3.1) near the surface. When we compared the value of (2.22) to the observed value (shown in Table 3.1), we found out that the model value doesn't always stay within the observational bounds with the Eddington model when the central curvature is varied even slightly. The curvature initial value at the center is not known so we chose to present the unstable phenomena with more statistics. We solved some ten thousand polytropes with also varying the central density around the Eddington solution. We find that the effect on metric parameters outside, due to the different initial curvature, is a generic feature in the gravitational model and is not due to the polytropic equation of state. This is seen in the right side Figure 5.3, where the different initial curvature values change the (γ_{PPN}-1). By varying the initial curvature R_0 we effectively test here different initial slopes for the potential $V'(\phi)$ in the corresponding BD-equivalent ($\omega_{BD} = 0, \phi = 1$, (4.26)) in the Jordan frame.

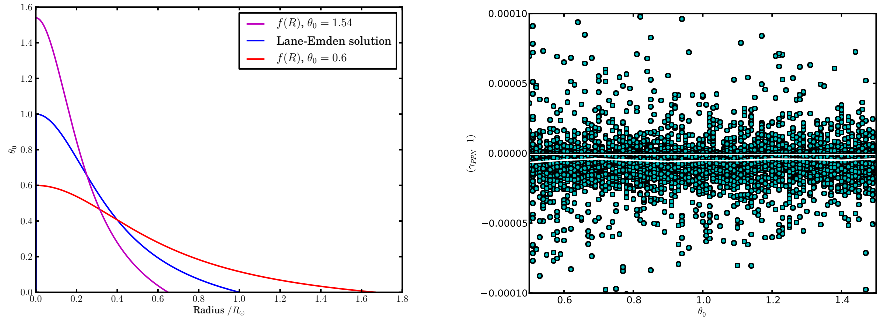

Figure 5.3: Left: Typical density profiles for a polytrope. The density $\rho(r)$ is reparametrized here as $\rho(r) = \rho_o \theta^{1/(\gamma-1)}$ and the Lane-Emden solution corresponds to the SUN [3]. Right: The $f(R)_{HS}$ polytropic solutions for different central densities and initial curvatures. The observational Cassini values are $\in [-0.000002, 0.000044]$ and ΛCDM model gives $\gamma_{PPN} - 1 \simeq 10^{-8}$ [3].

5.4 Scalar-tensor polytropic stars

In the article [4] we studied a representation of a scalar-tensor theory compatible with the ΛCDM model and with the metric $f(R)$ theories. We used the O'Hanlon action

$$S = \int d^4x \sqrt{-g} \left[\phi R - V(\phi) \right] \tag{5.6}$$

and used only the Jordan frame representation (see the discussion of frames in Section 4.2.2). Rather than having the scalar-tensor potential fixed beforehand, we derive the system by using a prior set of equations to first set up the metric according to a ΛCDM $/f(R)$ polytropic configuration and later derive the potential $V(\phi)$ numerically from this information.

As the prior equations, we used the general relativistic ΛCDM (2.14) or $f(R)_{HS}$ (5.4),(4.20) field equations and the energy continuity equation (2.4) with the static spherically symmetric metric (2.15). These give the ΛCDM $/f(R)$ polytropic configuration $(\rho(r), B(r), A(r))$. When searching for the scalar-tensor potential $V(\phi)$, we solve another set of equations (4.26) for the field $\phi(r)$ and the potential $V(r)$ by numerically including the ΛCDM $/f(R)$ prior obtained above. The equations that give the field and the potential can be written as

$$\phi(r)G_{11} = \kappa T_{11} - \frac{1}{2}g_{11}V(r) + \nabla_1\nabla_1\phi(r) - g_{11}\Box\phi(r) \quad (5.7)$$

$$R\phi'(r) = V'(r), \quad (5.8)$$

where the second field equation is used in the (5.8) in the reparametrization of the potential as $V(r)$. The whole configuration then boils down to a system of the variables $(\rho, p, B, A, V, \phi, r)$, where the pressure degree of freedom $p(r)$ can already be reduced with the polytropic equation of state (2.18). We also calculated for comparison the scalar-tensor potentials $V(\phi)_{St}$ for the Starobinsky $f(R)_{St}$ stellar profiles

$$f(R)_{St} = R + \lambda R_\Lambda \left(\left(1 + \frac{R^2}{R_\Lambda^2} \right)^{-n} - 1 \right), \quad (5.9)$$

with $n, \lambda > 0$ and R_Λ of the order of the cosmological constant [91]. There does not exist an extensive amount of numerical studies on scalar-tensor matter configurations. Individual models like rotating neutron stars have been studied [137]. In our study neutron stars were not solvable for neither of the $f(R)$ gravitational models.

The system is, therefore, confined to follow ΛCDM (or $f(R)$) gravity inside the configuration. This gives the $\rho(r), B(r)$ and $A(r)$ as numerical solutions inside the star with a regular center. The physical conditions for the ΛCDM configuration were found by optimizing the polytropic parameters to find physically acceptable configurations. The viable polytropic parameter space (ρ_0, K, γ) in (2.18) and (2.19) shrinks essentially to a point when the solutions are required to obtain physically viable masses and radii (*e.g.* $r_s = R_\odot$ and $\int_{r_i}^{R_\odot} 4\pi G\rho(r)r^2 dr$ in the case for the SUN) that correspond to sensible central densities.

Finally the boundary of the configuration is confined with the SdS vacuum to recover general relativity as

$$V(\phi) = -\left(-\frac{B'}{AB} + \frac{4}{Ar} \right) \phi' + 6H_0^2\phi. \quad (5.10)$$

Here the de Sitter vacuum condition $G_{11} = -g_{11}\Lambda = -3A(r)H_0^2$ was used. Now the potential $V(r)$ and the field $\phi(r)$ can be integrated with appro-

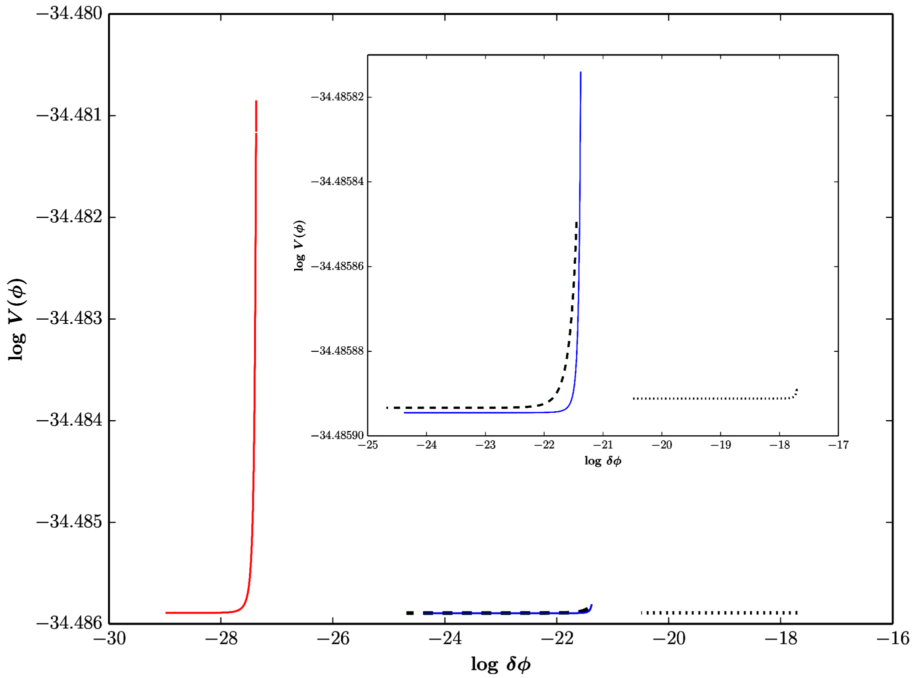

Figure 5.4: Potentials $V(\phi)$ for the NS (red), WD (relativistic - blue, non-relativistic - green, dashed) and SUN (black, dotted) polytropic configurations with small initial field value [4]. The smaller image is a zoom to the WDs and the SUN only.

priate initial conditions. Our solutions do not give a general description of the potential $V(\phi)$, but find the special cases that satisfy the above assumptions.

The potential is required to be initially two times the de Sitter vacuum $V_i(r_s) = 2\Lambda = 6H_0^2$ which now constrains ϕ_i' (5.10). This fully confines the configuration to obtain the typical shape for the polytropic star, shown in the left Figure 5.4. The initial value $\phi_i = 1 + \delta\phi_i$ has affect on the field range that is covered by the configuration, but not on the shape nor the range covered by the potential $V(r)$. The whole potential can be "shifted" towards smaller field values up to a saturation point $\delta\phi_{MIN}$, that is unique per stellar class (SUN, WD, NS). This is depicted in the left Figure 5.4.

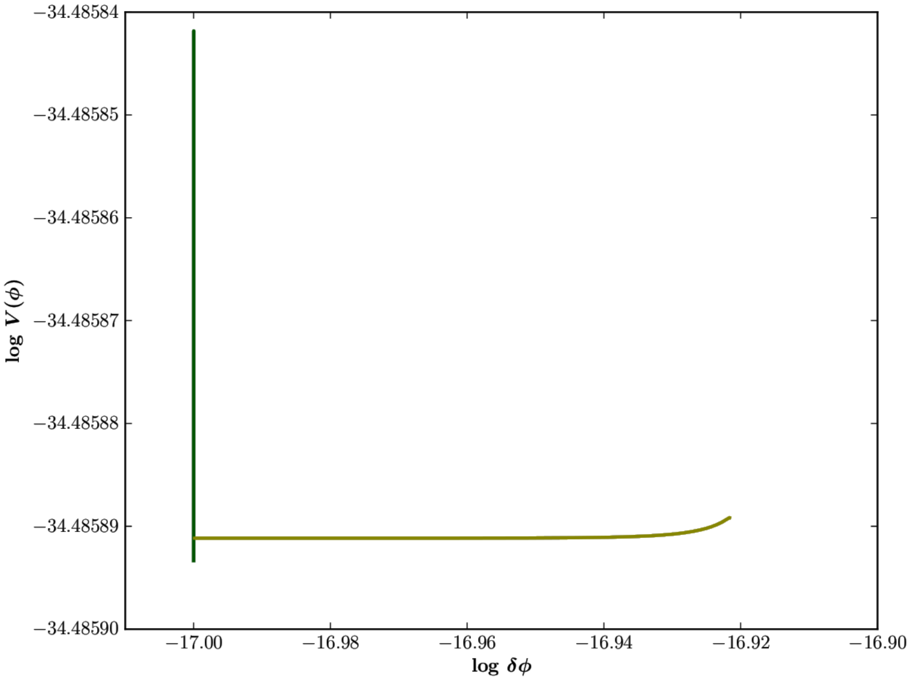

Figure 5.5: Non-relativistic WD (green) and SUN (yellow) with high initial field value.

The shape of the potential remains the same for different $\delta\phi_i$, but scales according to the matter configuration. The more dense object class obtains higher potential values $V_{NS}(\phi) > V_{WD}(\phi) > V_{SUN}(\phi)$ so that all the objects cannot be represented by the same gravitational theory. Even if we choose high a field value initially, the potential cannot describe all the objects. This can be seen also in the right Figure 5.4.

5.5 Concluding remarks

All the studies carried out for this thesis have very general common factors, namely cosmological dark energy models and observations. Common to all these works was the pursuit of numerically studying physically meaningful phenomena within potentially viable cosmological models. The aim was to utilize observational data for constraining the model parameters or viability of the model class under study. The author therefore refrains from extrapolating the individual studies of the articles [1, 2, 3, 4] to one general conclusion and briefly collects the main results here.

New aspects of the studied models were found. The SUGRA quintessence model fits better to the SneIa data than the ΛCDM model. The original supergravity potential of Brax and Martin lies within the 2σ area. Also, small energy scales ($\lesssim 1$ TeV) are favoured unless the potential parameter $\beta = 3$.

The three latter articles concentrate on models that are usually classified as modifications to the gravitational Lagrangean. These models were compared to observations with polytropic matter configurations, and the spacetime outside the configurations was studied. The CDTT model that had already proved to be unstable was found to show the $\gamma_{PPN} = 1/2$ behavior in the metric parameters when the initial conditions were properly considered. Also, a scaling relation for the Newtonian/GR Lane-Emden solution to yield the modified gravity model was found.

The HS $f(R)$ gravity model was found to produce the GR star very well, but a spacetime with $|\gamma_{PPN} - 1| < 10^{-5}$ was not reached for all configurations corresponding to different R_0 with the precision of the Cassini mission.

We solved the scalar field potential for three different matter configurations that are similar to the general relativistic counterparts, and found that all the configurations lead to different theories. The procedure of numerically deriving the potential $V(\phi)$ has not previously been done - to our knowledge - for stellar configurations in scalar-tensor gravity, and therefore it is impossible to draw far reaching or definite conclusions on this result. One can, however, speculate that if the configurations are all stable in this formulation of scalar-tensor gravity, can it be that this theory is not competent with physical systems. The underlying question is: why are the potentials so different? (*e.g.* how can these configurations be produced by a different scalar-tensor theory?). This definitely calls for further study.

Clearly the models of [1, 3] and [4] deserve further investigation with wider object class (*e.g.* red giants and rotating neutron stars) and for different systems (*e.g.* larger structures and clusters in voids) to be able to contemplate on their viability. The new observational initiatives, toward pinning down the number of DE models (*e.g.* [64]) as well as numerical N-body simulations that are gradually pushing towards more general DE implementation [44], are vital in understanding the physical theory of dark energy. Meanwhile, many less ambitious endeavors could help to classify cosmological models to help reduce the number crunching needed in the data reduction from the above massive project. So the way through trial and error does not look futile at all, but can serve as light beacons in the vastness of the cosmological dark energy models.

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