Vaasan yliopisto
UNIVERSITY OF VAASA

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Essays on Information and Externalities in Financial Markets

ACTA WASAENSIA 276

BUSINESS ADMINISTRATION 112
ACCOUNTING AND FINANCE

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Julkaisija	Julkaisupäivämäärä	
Vaasan yliopisto	Huhtikuu 2013	
Tekijä(t) Jukka Sihvonen	**Julkaisun tyyppi** Artikkelikokoelma	
	Julkaisusarjan nimi, osan numero Acta Wasaensia, 276	
Yhteystiedot Jukka Sihvonen Laskentatoimi ja rahoitus Vaasan yliopisto PL 700 65101 Vaasa	**ISBN** 978–952–476–439–1 (nid.) 978–952–476–440–7 (pdf)	
	ISSN 0355–2667 (Acta Wasaensia 276, painettu) 2323–9123 (Acta Wasaensia 276, verkkojulkaisu) 1235–7871 (Acta Wasaensia. Liiketaloustiede 112, painettu) 2323–9735 (Acta Wasaensia. Liiketaloustiede 112, verkkojulkaisu)	
	Sivumäärä 177	**Kieli** Englanti

Julkaisun nimike

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Asiasanat

Informaatio, johdannaissopimus, markkinalikviditeetti, markkinatehokkuus, rahapolitiikka, velkakirjamarkkina

Publisher	**Date of publication**	
Vaasan yliopisto	April 2013	
Author(s) Jukka Sihvonen	**Type of publication** Selection of articles	
	Name and number of series Acta Wasaensia, 276	
Contact information Jukka Sihvonen Accounting and Finance University of Vaasa P.O. Box 700 FI–65101 Vaasa Finland	**ISBN** 978–952–476–439–1 (print) 978–952–476–440–7 (online)	
	ISSN 0355–2667 (Acta Wasaensia 276, print) 2323–9123 (Acta Wasaensia 276, online) 1235–7871 (Acta Wasaensia. Business Administration 112, print) 2323–9735 (Acta Wasaensia. Business Administration 112, online)	
	Number of pages 177	**Language** English
Title of publication Essays on Information and Externalities in Financial Markets		

Abstract

The first two essays of this thesis focus on the role of financial markets as a system for aggregating and disseminating information. New information can be assumed to be incorporated into market prices as a result of securities trading triggered by investors' changed return expectations. In the essays, the information content of market prices is utilized to study the fundamentals that change market expectations. The results from derivative markets indicate that market expectations of future interest rate policy rest on the principles of a simple Taylor rule. Furthermore, based on the results it can be shown that central banks can use press briefings to clarify their monetary policy to investors and thereby manage the formation of market expectations of future monetary policy.

The third and fourth essays of the thesis analyze how frictions in the bond market influence the trading behavior of market participants. Due to market frictions, trading concentrates endogenously on bonds that are identified by institutional arrangements or derivative contracts. The coordination of trading generates positive externalities, which benefit the owners of the traded bonds. These benefits include lower transaction and financing costs as well as higher convenience yields.

Keywords
Bond market, derivative contract, information, market efficiency, market liquidity, monetary policy

ACKNOWLEDGEMENTS

I think of research as a path unseen at first and discovered by wandering. Should one wander long enough, and the destination will be found. But not until then can one see what would have been the shortest route. I am most indebted to my friend and supervisor, Professor Sami Vähämaa, for pointing me in the right direction in times I have wandered too far.

Continuing on the matter of my scientific deviations, the detailed feedback from the official pre-examiners of this dissertation improved its quality and provided good ideas for future research. I wish to express my gratitude to Professor Jukka Perttunen from the University of Oulu and Dr. Magnus Andersson from the European Stability Mechanism for their efforts.

The Department of Accounting and Finance is a good place to work at. I thank all my colleagues at the Department, and especially Professor Jussi Nikkinen, for being available when I have needed professional advice and insights. From the Department of Mathematics and Statistics, I gratefully acknowledge the comments and suggestions by Professor Seppo Pynnönen and Dr. Berndt Pape. They have straightened my research settings more than once.

I was recruited to the University of Vaasa by Professor Emeritus Paavo Yli-Olli. Sadly Paavo passed away few years ago, but I will always be grateful for him for guiding me into this profession. I also thank the University for being my long-term employer and providing the financial and scientific bases for my doctoral studies.

My understanding of financial theory originates from the courses provided by the Graduate School of Finance. I thank the director, Dr. Mikko Leppämäki, for establishing a doctoral program of the highest standards, and for having me as a research fellow of the School. Moreover, Dr. Leppämäki organizes national and Nordic workshops that have proved to be most contributive to my research. In this regard, the Department workshops organized by Professor Emeritus Timo Salmi must be acknowledged as well.

In autumn 2008, I had the most enlightening experience of how monetary policy is conducted under severe market circumstances. I wish to thank the people at the Capital Markets and Financial Structure Division for their hospitality during my visit at the European Central Bank. I am especially grateful to Mr. Jacob Ejsing and Dr. Andersson for setting an example of what a good financial economist should be like.

During my doctoral studies, several institutions and foundations have supported my research as well. I thank the Finnish Cultural Foundation, the Finnish Savings Bank Foundation, the Evald and Hilda Nissi Foundation, the OP-Pohjola Group Research Foundation, and the Marcus Wallenberg Foundation for their financial support that allowed me to focus on research and present my results at international conferences.

Finally, I want to express my deepest gratitude to my family. My parents, Tuula and Jorma, have always believed in me and in my abilities, and encouraged me in times of uncertainty. My sister Kirsi has not only offered me the motivation, but also the example how, to achieve things in life and career. I have now lived in the world of research for seven years, and the mental intensity it requires tends to turn me into a serious man. For this reason and many others, I am truly fortunate to have Jenniina in my life to make me smile again.

Vaasa, April 2013

Jukka Sihvonen

Contents

Abbreviations

FOMC	Federal Open Market Committee
OTC	Over-the-counter
Fed	Federal Reserve System

This dissertation consists of an introductory chapter and the following four essays:

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[2] Reprinted with kind permission of Jacob Ejsing and the European Central Bank.

1 INTRODUCTION

Financial assets are contractual claims that derive value from the owner's right to a fraction of the issuer's future wealth. In order for the owner to benefit from the purchase of the asset, the issuer attempts to sustain or accumulate her financial wealth to meet the claim's nominal worth in the maturity. Although there may be social benefits to the issuer's effort, such benefits are irrelevant in the financial market where potential buyers and sellers focus only on whether the issuer is financially sound enough to honor the claim or not.

The market participants express their opinions of the asset in their bid and ask quotes. A quote is a prospective market price at which a participant would transact, and depends on her private valuation of the asset. As market participants observe the competing quotes in the marketplace, they might revalue the asset and quote differently, or just more competitively. Competition and dynamic revaluation enhance the reliability and validity, respectively, of the eventual equilibrium price as a measure of the asset's fundamental value.

Fundamental value depends heavily of the design of the financial claim inherent in the asset. The remoteness and riskiness of the claim depresses the fundamental value of the asset below its nominal worth. The riskiness, in turn, not only depends on the terminal value of the claim under different trajectories of the issuer's wealth, but also on the probabilities of different trajectories. As information helps to assign these probabilities, it plays a pivotal role in the valuation of claims with a terminal value most contingent upon a realization of particular trajectories.

The dynamic interaction between information gathering, valuation, and quoting is generally called price discovery. Price discovery is one of the central purposes of financial markets and, in addition to its economic function, serves academic purposes. By observing market quotes, a financial researcher familiar with the structure of the asset and the workings of the marketplace can make unambiguous predictions about the terminal value of the asset as well as infer the uncertainty associated with these predictions. Likewise, if the exact nature of the asset itself is unclear, the researcher can make an educated guess about its qualities and then adjust this guess by comparing her projections of the asset value to actual quotes under sufficiently divergent market circumstances. Overall, the price mechanism based on bid and ask quotes enables experts to infer increasingly detailed market perceptions by appending the theoretical structure for the inference.

Notwithstanding, the value of information embedded in market quotes is conditional on the functioning of the market itself. If market entry is costly, uncompetitive quotes may drift far apart and no longer pin down the market consensus of the asset value. Or, a market externality may arise that will distort the quotes, so that the market price of the asset does not reflect its fundamental value. In such circumstances, the researcher's predictions become unreliable or even invalid. For this reason, the researcher must be able to recognize the potential externalities in the market and assess its general ability to function before making any more sophisticated projections based on its informational output.

This thesis consist of four essays that focus on two particular features of the financial system as described above. In the first two essays I attempt to identify the pieces of information that alter market expectations of asset values, and then examine how exactly these expectations are altered in response to new information. For these purposes, I use data on the prices of financial derivatives, which as contingent claims tend to convey more information in their prices than assets having less conditional payoffs. In the latter two essays I analyze the origin and effects of certain externalities that arise in markets which differ by organization but are linked by asset design. Specifically, I study intrinsically interconnected markets that do not share the same properties with respect to size, transaction costs, or derivative assets.

The remainder of this introductory chapter is organized as follows. Section 1.1 provides a general framework for asset pricing, and the three following sections extend the framework to introduce the problems examined in the essays. Section 1.2 describes the relationship between asset prices and market expectations. Section 1.3 introduces the concept of market liquidity, demonstrates its effect on asset prices, and describes the role of market coordination in explaining cross-sectional differences in liquidity. Section 1.4 illustrates how coordination, liquidity, and other market externalities can arise as a result of asset design, market design, and institutions. Section 2 summarizes the essays.

1.1 Asset pricing

Following Cochrane (2005), let the price of an arbitrary asset, p, be

$$p = E(mx) \tag{1}$$

where $E(\cdot)$ is the mathematical expectation operator, m is a stochastic discount factor, and x is the payoff of the asset. If m and x are both stochastic, their covariance $\operatorname{cov}(m,x) = E(mx) - E(m)E(x)$ dictates the degree of nonlinearity in the pricing equation. Rewriting Equation eq1 using the definition of covariance, one obtains:

$$p = E(m)E(x) + \operatorname{cov}(m,x). \tag{2}$$

The first component in Equation (2) is the asset's price according to risk-neutral valuation and the second component is a risk adjustment arising from the covariance between x and the discount factor m. If the payoff x is constant and therefore uncorrelated with the discount factor, one can solve for the price of one unit riskless asset,

$$p = E(m), \tag{3}$$

and the instantaneous continuously compounding risk-free rate,

$$r = \ln(p). \tag{4}$$

1.2 Market expectations

A time dimension is included in pricing Equation (1) and rewritten as

$$p_t = E_t(m_T x_T), \tag{5}$$

where subscript t denotes the time of valuation of the payoff received at T, $t \leq T$. $E_t(\cdot)$ is the mathematical expectation conditional on the information set Ω available on time t, $E_t(\cdot) = E(\cdot|\Omega_t)$. As new information arrives, the information set grows, which may alter the conditioning of expectations and induce a change in the asset price. More formally, assume an arrival of new relevant information I_{t+1} on the payoff of the asset, and, for brevity, unit discount factor. Then, ceteris paribus, the resulting price change is

$$\Delta p_{t+1} = E(x_T|\Omega_t, I_{t+1}) - E(x_T|\Omega_t). \tag{6}$$

In the essay titled *"When Bernanke talks, the markets listen: the case of the first FOMC press conference on monetary policy"*, it is examined whether the Federal Open Market Committee's first press conference on monetary policy provided new information (I_{t+1}) about the Federal Reserve's monetary policy stance. If market expectations about the future monetary policy are altered by the topics discussed in press conference, it would induce changes in the values of assets with payoffs depending on the future interest rates.

In addition to conditioning information, changes in the risk adjustment component of Equation (2) change the price of the asset. Yet, in some cases the risk adjustment factor can be ignored. A *contingent claim* is an asset whose payoffs can be replicated with a portfolio of other assets. Therefore, the price of the claim must be equal to the value of the replicating portfolio in order to preclude arbitrage. Since arbitrage does not depend on risk attitudes, no risk adjustments are needed in the pricing of contingent claims. Following the logic, the market expectations of the contingent payoffs can be treated "as if" they are formed by risk-neutral investors. The introduction of *risk-neutral* expectation operator E_t^Q changes Equation (5) to

$$p_t = E_t(m_T) E_t^Q(x_T). \tag{7}$$

Assume that the asset in case is a call option c_t incorporating a contingent claim paying the non-negative difference between S_T, a continuous random value, and K, the strike price. Then, the current risk-neutral value of the payoff is

$$c_t = e^{-r(T-t)} E_t^Q (S_T - K)^+, \tag{8}$$

a product of a discount factor based on the risk-free rate and the expected payoff of the option. If the option is publicly traded, the expectation operator summarizes the market-based probabilities assigned to different outcomes of S_T,

$$c_t = e^{-r(T-t)} \int_K^\infty (S_T - K) f_t^Q(S_T) dS_T, \tag{9}$$

where $f_t^Q(\cdot)$ is the time-t probability density function used by the public to weight the likelihood of different outcomes of S_T. It has to be noted that $f_t^Q(\cdot)$ captures the public expectations as if everyone were indifferent to risk, and thus may differ from the actual probability density function as a function of risk aversion.

Breeden & Litzenberger (1978) show that one can extract risk-neutral market expectations from a continuum of option prices by taking the second partial derivative of the option price in Equation (9) with respect to strike price K,

$$f_t^Q(K) = e^{-r(T-t)} \frac{\partial^2 c_t(K)}{\partial K^2}, \tag{10}$$

where $f_t^Q(K)$ is the probability of value K occurring in the future as implied by the current option prices. The rationality of the option-implied market expectations can be cross-validated by engineering a model for the evolution of the underlying random variable.

In the essay titled *"Forward-looking monetary policy rules and option-implied interest rate expectations"*, it is assumed that the underlying variable is the central bank interest rate, which is set according to a monetary policy reaction function à la Taylor (1993). Therefore, a potential set of factors that shape the option market expectations of the future interest rates are the input variables for the Taylor rule, namely, the current interest rate, expected inflation, and the expected output gap:

$$f_t^Q(S_T) = f^Q(S_T|\Omega_t), \tag{11}$$

where

$$\Omega_t \ni \{i_t, E_t(\pi_T), E_t(x_T)\}, \tag{12}$$

using the notation of the essay.

1.3 Market liquidity

Market liquidity is a general term that summarizes how easily and cheaply an asset can be turned into cash by selling to the market. Usually, liquidity is defined in terms of a percent transaction cost C that has to be paid for an immediate transaction. As will be shown next, market liquidity can be modeled as a result of a coordination game played by the market participants against a market maker. As a result of the coordination game, similar assets end up having different liquidity and price.

Begin with an economy having two agents who trade with probability λ in the next period. If the agents trade, they choose either asset A or B and trade inside or outside the exchange facility. In the exchange, agents have to trade with a maker maker, who charges the transaction cost for providing liquidity. Outside the exchange, the agents trade with each other without any cost. A costless *over-the-counter* (OTC) trade requires that the agents make opposite trades of the same asset at the same time. Without coordination, the probability of trading OTC is:

$$\text{prob(trade = OTC)} = \lambda^2 \times \text{prob(need = buy \& sell)} \times \text{prob(asset = A or B)}, \quad (13)$$

where the first term on the right-hand side is the probability of both agents trading at the same time, the second term is the probability of the agents being on the opposite sides of the trade, and the third term is the probability of both agents trading the same asset. As the probability of over-the-counter trading is arguably small, the market maker is able to set a high transaction cost for both assets

$$C^{A,B} = C\big[1 - \text{prob(trade = OTC)}\big]. \quad (14)$$

If, however, the agents coordinate on when to trade, the probability of finding a counterpart in the over-the-counter market increases to

$$\text{prob(trade = OTC}|t) = \text{prob(need = buy \& sell)} \times \text{prob(asset = A or B)} \quad (15)$$

and the market maker has to lower the transaction costs to

$$C^{A,B} = C\big[1 - \text{prob(trade = OTC}|t)\big]. \quad (16)$$

If the agents further agree on trading asset A only, the probability of trading over-the-counter becomes

$$\text{prob(trade = OTC}|t, i) = \text{prob(need = buy \& sell)}, \quad (17)$$

so that the agents have to trade in the exchange only if they are both buyers or sellers. Thus, when the agents agree on what and when to trade, the transaction cost for the primary trading vehicle A drops to

$$C^A = C\big[1 - \text{prob(trade = OTC}|t, i)\big], \tag{18}$$

while the lack of over-the-counter trading opportunities for the secondary trading vehicle B allows the market maker to charge a full transaction cost,

$$C^B = C. \tag{19}$$

The difference between the transaction costs of assets A and B that arises from the agents' coordination has a direct impact on the relative prices. Consider a simple example in which both assets pay one unit at the maturity time T with certainty. If the agents could coordinate their future trading in the way described above, how much would they quote for the assets at time 0?

Asset A is more liquid than asset B, since coordination ensures that the transaction cost for asset A is less than for asset B. If the agents factor in the future transaction costs when they valuate the assets, the equilibrium price of the assets based on periodic compounding $r = e^r - 1$ is given by

$$p_0^i = p_0\left[1 - C^i(1+r)^{-t}\right], \tag{20}$$

where $p_0 = (1+r)^{-T}$ would be the price of a hypothetical liquid asset that would bear no transaction costs, and time t is when the agents agree to trade, $t < T-1$. As can be seen from Equation (20), the perfectly liquid asset would command a liquidity premium over A and B, because their buyers would demand price concessions to cover the future transaction costs.

In a similar fashion, the agents would also consider the transaction-cost *differential* between A and B when quoting for the assets. Substituting C^A and C^B from Equations (18) and (19), respectively, into Equation (20) gives the relative liquidity premium on asset A over the illiquid asset B:

$$\frac{p_0^A - p_0^B}{p_0} = \frac{\text{prob(trade = OTC}|t, i) \times C}{(1+r)^t} \tag{21}$$

where the liquidity premium increases with the discounted transaction-cost advantage of asset A over B.

Overall, the level of coordination achieved by the agents plays a major explanatory factor in the transaction costs across assets. In the essay titled *"Liquidity premia in German government bonds"*, it is shown that deliverability for a futures contract serves as a mechanism of coordination in the bond market. Coordinated trading lowers the transaction costs for deliverable bonds, which, in turn, has significant effects on the bond prices in the form of liquidity premia.

1.4 Futures market effect

In a market with frictions, the hedging practices by institutional investors have far-reaching implications on the workings of the market. Assume that an agent is endowed with a portfolio $\{A, B\}$ and wants to reduce its size by selling either A or B in the next period. Instead waiting and executing the trade later in the spot market, the agent decides make an outright forward sale using a futures contract on the portfolio. Let the futures contract to be settled by delivering asset A or B at time 1 against the current futures price F_0. Then, the current equilibrium price of the futures contract is the minimum of the forward prices of asset A,

$$F_0^A = (1 + r)p_0^A, \tag{22}$$

and asset B

$$F_0^B = (1 + r)p_0^B + \epsilon, \tag{23}$$

where ϵ_0 is an exogenous pricing error arising from the design of the futures contract. If $\epsilon_0 > 0$, asset A is cheapest to deliver on the futures contract and $F_0 = F_0^A$. This is an arbitrage-based relationship, in that if $F_0 > F_0^A$, other agents in the economy could sell a futures contract and commit to deliver forward, purchase asset A at the forward price F_0^A, and earn riskless arbitrage profit $F_0 - F_0^A$ at the settlement of the contract. Likewise, if $F_0 < F_0^A$, arbitrageurs could buy a futures contract, sell asset A short at the forward price F_0^A, and make an arbitrage profit $F_0^A - F_0$ at the futures settlement.

Futures arbitrage generates positive externalities for the owners of the cheapest-to-deliver asset. First, arbitrage trading enhances the liquidity of the asset and increases its value in the form of a liquidity premium. Second, if enough agents engage in futures arbitrage that necessitates short-selling, Duffie (1996) shows that the arbitrageurs may have to introduce a special rent R, $R \leq r$, to induce ample supply of cheapest-to-deliver assets lent to shorting market. The special rent in the shorting market induces a premium on the cash price of the cheapest-to-deliver asset. Applying the logarithmic transformation, the relative price premium is

$$\ln p_0^A - \ln p_0^B = r - R. \tag{24}$$

Similar results hold if $\epsilon_0 < 0$ and asset B becomes cheapest to deliver on the futures contract. If $\epsilon_0 = 0$, the futures traders are indifferent between delivering asset A or B, and the effects of coordinated trading do not arise. Then, by the law of substitution, $0 \leq |F_0^A - F_0^B| \leq |\epsilon_0|$ provides theoretical bounds for the combined value of the externalities.

In the essay titled *"The cheapest-to-deliver premium: theory and evidence"*, an equilibrium model is derived to describe the technical underpinnings of the cheapest-to-deliver premium in the context of bond and bond futures markets. The main culprit for the emergence of cash-market premia is the conversion factor method used by the futures exchange to calculate bond delivery prices. Market circumstances worsen the conversion factor bias and increase the theoretical upper bound for the cash market premium on the cheapest-to-deliver bond. The linkage between the theoretical upper bound and observed cash market premia is empirically verified.

2 SUMMARY OF THE ESSAYS

2.1 Forward-looking monetary policy rules and option-implied interest rate expectations

Central banks conduct monetary policy to maintain a stable monetary environment. Under the inflation-targeting framework pursued by the major central banks, adverse monetary effects are to be avoided by keeping the changes in the general level of prices at a target rate. According to economic orthodoxy, the inflation target is best achieved through active interest rate policy. Specifically, a central bank can change the interest rate at which its deals with the money market and initiate a change in the market-clearing quantities of savings and borrowing. The desired equilibrium effects may not arise instantly, and the central bank may have to keep changing the policy rate until the market interest rate is at its target level.

The target interest rate level is now widely regarded to be set in accordance with Taylor (1993) type interest-rate rules, which presume a systematic reaction function of interest rate policy to the gaps between inflation and output and their respective target levels. In the forward-looking Clarida et al. (1998) version of the rule, the policy rate is partially adjusted towards the target interest rate, which, in turn, is changed in response to information about prospective future inflation and other economic factors which are expected to affect the future rate of inflation.

The purpose of this essay is to assess whether market participants view a forward-looking policy rule as a guide to the path of future policy rates or, put differently, whether the expectations formation process in financial derivatives markets is consistent with Taylor-type rules. Market expectations are defined in terms of probability distributions of future interest rates, which are computed from cross sections of interest rate option prices. Then, it is empirically examined if the month-to-month movements in these option-implied distributions are related to changes in expectations of the policy rule fundamentals.

The results of the empirical analysis indicate that the changes in interest rate expectations implied by option prices are consistent with the forward-looking policy rule, which suggests market participants perceive the rule as a valid description of the central bank's future interest rate policy and react to its projections in a systematic way. To validate the findings of this study in a more general setting, future extensions of the proposed methodology should employ alternative policy rule specifications as well as incorporate the effect of parameter uncertainty on the policy rule projections.

2.2 When Bernanke talks, the markets listen: the case of the first FOMC press conference on monetary policy

A central bank effects economic activity and inflation by exercising its control over the level of short-term interest rates and by influencing financial market expectations that determine the slope of the term structure. Managing market expectations is, however, a challenging endeavor for the central bank. For example, a contractionary monetary policy shock leading to an unexpected rise in the short-term interest rates might be interpreted as a response to higher economic growth projections or higher inflation expectations. The effect on the term structure of interest rates is ambiguous, which undermines the significance of the expectational channel of monetary policy transmission.

To better align market expectations with its own inclinations regarding future monetary policy, the Federal Open Market Committee (FOMC) of the United States' Federal Reserve System recently changed the way it communicates about monetary policy. In the new framework published in March 2011, the monetary policy statement announced after every FOMC meeting is now four times per year followed by a press briefing held by the Chair of the Committee. In the briefing, the Chair gives a detailed statement of Committee's monetary policy stance and presents its latest economic projections, and then allows members of the media to ask clarifying questions about monetary policy issues.

This essay examines the market adaptation to the Fed's changed communication policy. The research question builds upon the efficient market hypothesis: if the information disseminated in the press briefing adds to the public comprehension of the forces behind monetary policy decisions, asset prices would change in response to such information to better reflect the new understanding of the conduct of monetary policy. Whether or not the Fed's changed communication policy adds market information is tested by comparing the level of market activity before and during the press briefing.[1]

High-frequency analysis of several different asset classes show that the press briefing triggers price discovery in markets for both short- and long-lived assets. The market responses are found to be deterministic and originate from questions and answers pertaining to future monetary policy and state of the economy. Overall, the findings of the study indicate that the Fed's new communication framework serves to achieve the clarification objective of monetary policy communication.

[1] Instead of using the level of market activity in the morning of the press briefing day, an alternative basis of comparison would be the average level of market activity during the same hours of the days without a press briefing.

2.3 Liquidity premia in German government bonds

Variations in liquidity are one reason why yields on otherwise comparable government securities differ. Although the liquidity of a bond can be measured in several ways, the concept essentially captures to what extent the bond can be sold cheaply and easily. Liquidity is thus valuable for market participants, and especially in times of market stress, the most liquid bonds have tended to command a considerable price premium.

Previous studies of liquidity and liquidity premia in government bond markets, based mainly on data from the U.S. Treasury market, have identified pronounced liquidity differences across government securities. For instance, Amihud & Mendelson (1991) document that the most recently issued ("on-the-run") Treasury bills trade at higher prices than seasoned but otherwise similar securities, and attribute the price premium to the better liquidity of in-the-run bills. However, the results from the U.S. Treasury market cannot necessarily be generalized to the German bond market, its euro counterpart. The two markets differ considerably with respect to hedging practices; while dollar interest rate risk is commonly hedged by short-selling the most recently issued ("on-the-run") Treasury bond, the exposure to euro interest rate risk is usually hedged by selling futures contracts on German government bonds. As a result, the turnover in the German bond futures market is many times larger than in the German cash bond market.

In this essay, it is argued that the difference in hedging practice cause trading to be less concentrated on specific bonds in the German market, which, in turn, implies that the differences in liquidity premia are considerably smaller. The empirical results support this conjecture; in sharp contrast to the evidence from the U.S. Treasury market, on-the-run status appears to have only a modest effect on the liquidity and pricing of German government bonds once other factors have been controlled for. However, the existence of a highly liquid German futures market leads to significant liquidity spillovers to the German cash market. Specifically, bonds that are deliverable into the futures contracts are both trading more liquidly and commanding a price premium. The futures market effect has intensified during the recent financial crisis.

2.4 The cheapest-to-deliver premium: theory and evidence

A bond futures contract is a commitment to take a future delivery of an eligible bond at a predetermined delivery price. Because deliverable bond differ in cash flow characteristics, a conversion factor is used to equate their delivery prices. In theory, the conversion factor method makes the eligible bonds perfect substitutes for the delivery and thereby eliminates any adverse delivery effects. In practice, the conversion factor does not equate the delivery prices, but only mitigates the price differences. A bond with certain cash flow characteristics will be cheapest to deliver, and the open futures positions at the contract maturity will be settled by the delivery of this particular bond.

This essay reports on a study of the effects of futures market delivery process on the cheapest to deliver bond. The study concentrates on the German government bond futures market, which overshadows the cash market in size and significance. Following Krishnamurthy (2002), a "pairs" trading test involving short-selling indicates that German cheapest-to-deliver bonds command a price premium, and, at equilibrium, trade at a cheap financing rate in the repurchase market. In order to investigate the issue analytically, a equilibrium model is developed. The model postulates that the price premium is driven by the delivery discount, the amount of deliveries in relation to the amount of outstanding bonds, and the cheapness of financing rates. The reduced form of the model is empirically tested and validated.

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THE JOURNAL OF FUTURES MARKETS

Forward-Looking Monetary Policy Rules and Option-Implied Interest Rate Expectations

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This paper examines the association between option-implied interest rate distributions and macroeconomic expectations in the context of a forward-looking monetary policy rule. We presume that market participants view the policy rule as a guide to the path of future policy rates and price interest rate options in accordance with the policy rule fundamentals. Using data from the UK, we confirm that Libor expectations implied by option prices are consistent with the policy rule variables. The results demonstrate that changes in the distributional form of Libor expectations are strongly associated with changes in the expected inflation and output gaps and financial uncertainty. © 2013 Wiley Periodicals, Inc. Jrl Fut Mark

The authors would like to thank an anonymous referee, Vladimir Gatchev, Markku Lanne, Karl Larsson, Ji-Chai Lin, Leonardo Morales-Arias, Seppo Pynnönen, Larry D. Wall, Henning Weber, Paolo Zagaglia, and seminar participants at the Bank of Finland, the Kiel Institute for the World Economy, Louisiana State University, the University of Central Florida, Stockholm University, the 2010 Graduate School of Finance Research Workshop, the 2010 Nordic Finance Network Workshop, and the 2011 Eastern Finance Association Meeting for helpful discussions and comments. This paper received the Outstanding Paper in Financial Institutions Award at the 2011 Eastern Finance Association Meeting.

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Received April 2012; Accepted December 2012

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The Journal of Futures Markets, Vol. 00, No. 0, 1–35 (2013)

Published online in Wiley Online Library (wileyonlinelibrary.com).
DOI: 10.1002/fut.21596

1. INTRODUCTION

Short-term interest rates are mainly determined by the monetary policy of central banks. Monetary policy, in turn, is now widely acknowledged to be guided by Taylor (1993) type policy rules, which presume a systematic reaction function of monetary policy to inflation and the output gap (see, e.g., Clarida, Gali, & Gertler, 1998; Rudebusch & Svensson, 1999; Bernanke & Gertler, 2000; Woodford, 2001; Nelson, 2003; Orphanides, 2003; Favero, 2006).[1] Under forward-looking policy rules and the inflation-targeting framework pursued by the major central banks, official policy rates are adjusted in response to information about prospective future inflation and other economic factors which may potentially affect the future rate of inflation. Short-term market rates, again, are affected not only by current and prospective inflation and economic output, but also by market participants' expectations about future monetary policy. Given that forward-looking monetary policy operates under considerable uncertainty about future inflation and the state of the economy, general economic uncertainty may also be expected to have a significant effect on short-term market interest rates.

In this paper, we examine the association between option-implied interest rate distributions and macroeconomic expectations in the context of a forward-looking monetary policy rule.[2] In particular, we use market data on the three-month sterling Libor futures options to extract probability distributions of future short-term interest rates, and attempt to relate the month-to-month movements in these implied distributions to changes in inflation expectations, the expected output gap, and perceived financial uncertainty. The purpose of this exercise is to assess whether market participants view the policy rule as a guide to the path of future policy rates or, put differently, whether the expectations formation process in financial derivatives markets is consistent with Taylor-type rules. By focusing on the dynamics of option-implied interest rate distributions, this paper offers new insights into the effects of presumed policy rules and macroeconomic fundamentals on market expectations about future short-term interest rates.

Expected probability distributions implied by option prices have received considerable attention over the past decade. Central banks, in particular, are now increasingly using option-implied probability distributions to assess market expectations of future interest and exchange rates for the purposes of formulating

[1]The original model of Taylor (1993), which was later dubbed "the Taylor rule", is a linear model in which a central bank's target short-term nominal interest rate is defined by the equilibrium real interest rate, current inflation, and the deviations of current inflation and output from their target levels.

[2]It is important to note that our research setting does not require that the central bank would actually set the policy rate on the basis of a forward-looking reaction function, but rather presumes that market participants view the rule as a valid description of the central bank's rate setting behavior. If market participants believe that the central bank's monetary policy is rule-based, we should observe a systematic linkage between interest rate expectations and forecasts about the fundamental variables used in the policy rule.

monetary policy.[3] A number of papers have recently used implied probability distributions to examine the behavior of market expectations around macroeconomic news announcements (e.g., Carlson, Craig, & Melick, 2005; Glatzer & Scheicher, 2005; Vähämaa, Watzka, & Äijö, 2005; Beber & Brandt, 2006), central bank actions (e.g., Bhar & Chiarella, 2000; Carlson et al., 2005; Galati, Melick, & Micu, 2005; Vähämaa, 2005; Morel & Teletche, 2008; Gnabo & Teletche, 2009; Vergote & Puigvert-Gutiérrez, 2012), and other specific events (e.g., Melick & Thomas, 1997; Söderlind, 2000; Coutant, Jondeau, & Rockinger, 2001; Vincent-Humphreys & Puigvert-Gutiérrez, 2010).

Most closely related to our approach are the papers by Bhar and Chiarella (2000), Carlson et al. (2005), Vähämaa (2005), Vähämaa et al. (2005), Beber and Brandt (2006), and Vergote and Puigvert-Gutiérrez (2012). Vähämaa et al. (2005) and Beber and Brandt (2006) show that macroeconomic news announcements related to inflation and unemployment cause significant short-term reactions in the distributional form of expected future bond yields, while Bhar and Chiarella (2000), Vähämaa (2005), and Vergote and Puigvert-Gutiérrez (2012) document systematic movements in the implied probability distributions of short-term interest rates and bond yields in response to policy statements and changes in the monetary policy stance. Finally, Carlson et al. (2005) use federal funds futures options to extract implied probability distributions of the Federal Reserve's target rate decisions, and assess the impact of inflation and employment announcements and monetary policy communication on the market expectations of future policy decisions. Their results indicate that option-implied market expectations of the future monetary policy stance are associated with inflation and employment data releases in a manner consistent with the Taylor rule.

This study extends the prior literature in two main respects. First, in contrast to Bhar and Chiarella (2000), Carlson et al. (2005), Vähämaa (2005), Vähämaa et al. (2005), Beber and Brandt (2006), and Vergote and Puigvert-Gutiérrez (2012), who examine short-run intradaily or daily reactions of interest rate expectations around macroeconomic news announcements and monetary policy events, we focus on month-to-month movements in option-implied interest rate distributions. To the best of our knowledge, this paper is the first attempt to examine the longer-run association between implied interest rate distributions and macroeconomic fundamentals. It is well known that the way prices of financial instruments adjust to changes in fundamentals is often exaggerated in the short run by investor sentiment and other behavioral biases (Shiller, 1981; Shleifer, 1990), and a long-run analysis abstracting from these effects may therefore be more informative about the actual relationship between prices and

[3]All major central banks have publicly acknowledged that option-implied probability distributions are a valuable source of information and provide a useful input to monetary policy decisions.

fundamentals. Second, while recent studies have investigated the role of forward-looking monetary policy rules in analysts' interest rate forecasts (see, e.g., Fendel, Frenkel, & Rülke, 2011, 2013; Frenkel, Lis, & Rülke, 2011), we utilize option-implied probability distributions which provide a comprehensive market-based view of investors' expectations regarding future interest rates. Within this framework, we are able to examine the effects of policy rule fundamentals on the entire distribution of interest rate expectations, and essentially, to assess whether market perceptions are consistent with Taylor-type rules.[4] In general, this analysis provides new information about the formation of short-term interest rate expectations, and may also have important implications for financial market practitioners and monetary policy authorities alike. From the viewpoint of central banks, for instance, it is important to consider to what extent the market expectations of future short-term rates are affected by the expectations regarding policy rule fundamentals.

Our empirical findings indicate that the interest rate expectations implied by option prices are largely consistent with forward-looking monetary policy rules. In particular, the results show that month-to-month movements in the expected level of the three-month Libor rate are related to the expected output gap and perceived financial uncertainty. The expected level of the Libor rate is also substantially affected by shifts in the monetary policy stance. The results further demonstrate that changes in the distributional form of interest rate expectations are significantly influenced by the policy rule fundamentals. We find that the dispersion of market expectations around the expected short-term rate is positively associated with expectations of a widening inflation gap. Moreover, the dispersion of interest rate expectations appears to increase with increasing financial uncertainty and with a widening disparity between the market and policy rates. Asymmetries in interest rate expectations are found to be positively related to the expected inflation and output gaps. Finally, our results indicate that market participants attach higher probabilities for extreme movements in short-term interest rates in response to increasing expected inflation and output gaps. Overall, we interpret these findings as evidence that market participants form and revise their expectations regarding future movements in Libor rates, at least partially, on the basis of Taylor-type policy rules.

The remainder of the study is structured in the following manner. In the second section, we formulate the relationship between macroeconomic expectations that determine the central bank policy rate and its expected future path that, in turn, defines interest rates in the money market. The methodology that is used to extract interest rate expectations from option prices is presented in Section 3. Section 4 describes the data on macroeconomic expectations and

[4]In this regard, Carlson et al. (2005) report exploratory evidence that option-implied probability distributions of the Fed's target rate decisions react to inflation and employment news announcements in a manner consistent with the Taylor rule.

presents our econometric setup. Section 5 reports the empirical findings on the association between option-implied interest rate distributions and macroeconomic expectations. In Section 6, we assess the effects of the financial crisis on the formation of option-implied interest rate expectations. Finally, the last section provides concluding remarks.

2. SHORT-TERM INTEREST RATES AND THE IMPLEMENTATION OF FORWARD-LOOKING MONETARY POLICY

Assume that the central bank implements monetary policy by intervening in the money market in order to achieve the desired level for the short-term interest rate, i_t^*. The short rate is perceived by the market to be adjusted in response to macroeconomic expectations á la a forward-looking Taylor rule of Clarida et al. (1998):

$$i_t^* = r + \pi_t^* + \beta[E_t(\pi_{t+n} - \pi_t^*)] + \gamma[E_t(x_{t+k})] + \xi[z_{t+m}], \tag{1}$$

where r is the equilibrium real rate and π_{t+n} and x_{t+k} are n- and k-period-ahead inflation rate and the output gap; $E_t(\cdot)$ denotes the time t conditional expectation; an asterisk ("*") represents a target value; and z_{t+m} is a measure of financial uncertainty, which is assumed to influence interest-rate setting independently of π_{t+n} and x_{t+k}.

In addition to the inflation and output gaps, which are routinely included in reaction functions such as Equation (1), there are strong reasons to assume that the central bank responds to financial uncertainty.[5] Amongst other things, excessive volatility in the asset markets may seriously hamper the exercise of monetary policy and exacerbate economic downturns, as discussed in many recent studies (see, among others, Bernanke & Gertler, 2000, 2001; Cecchetti, Genberg, Lipsky, & Wadhwani, 2000; Mishkin & White, 2002; Rigobon & Sack, 2003; Bean, 2004; Campbell, 2008; Mishkin, 2009). Perhaps the most direct empirical evidence on the role of financial uncertainty in monetary policy is provided by Jovanovic and Zimmermann (2010), who use an augmented forward-looking Taylor rule similar to ours to establish a negative link between stock market volatility and the policy rate.

With respect to Equation (1), we define $\pi_t \equiv 100 \times log(P_t/P_{t-12})$, $x_t \equiv 100 \times log(Y_t/Y_t^*)$, and $z_t \equiv Var(W_t)^{0.5}$, where P_t is a price index, Y_t is the level of output, and W_t is a measure of financial wealth. Strong expectational channels in Equation (1) establish that optimal monetary policy involves interest rate smoothing, that is, adjusting interest rates only gradually in response to changes in economic environment (Goodfriend, 1987; Woodford, 2003). To capture the

[5]In a survey by Roger and Sterne (1999) of the Centre for Central Banking Studies of the Bank of England, 24 out of 28 central bankers representing industrialized countries find asset price volatility either important or relevant in the setting of monetary policy.

tendency of central banks to smooth interest rate changes, we allow that the *actual* policy rate i_t (henceforth, "instrument rate") partially adjusts to the desired level i_t^*:

$$i_t = \rho i_{t-1} + (1-\rho)i_t^* + \nu_t, \qquad (2)$$

where the smoothing parameter $\rho \in [0, 1]$ captures the extent of monetary policy inertia and ν_t represents an exogenous i.i.d. policy shock. Equation (2) postulates that each period the central bank adjusts the instrument rate i_t to eliminate a fraction $1-\rho$ of the difference between i_t^* and i_t. Empirical applications of the dynamic Taylor rule characterized by Equations (1) and (2) tend to find ρ-coefficients near to one, which is often interpreted as a sign of a rather conservative response to contemporaneous macroeconomic disturbances.

The steering effect of a change in the instrument rate on the economy arises from the central bank's ability to manipulate market interest rates through its monopoly over the monetary base. Thus, the demand for short-term money ultimately depends on the price at which the central bank is willing to supply it, or i_t. Longer-term market rates are then linked to the level of i_t through no-arbitrage relations. Formally, the ability of investors to substitute between different interest rate instruments suggests that the yield to maturity $Y_t^{(\tau)}$ on a τ-period spot market investment is equal to the proceeds from an investment strategy of rolling over one-period central bank loans for the next $\tau - 1$ periods:

$$1 + Y_t^{(\tau)} = E_t\left[\prod_{j=0}^{\tau-1} 1 + i_{t+j}\right]^{-\tau}. \qquad (3)$$

Equation (3) describes the pure expectation hypothesis of the term structure, which states that a τ-period gross market yield is the geometric average of current and expected one-period gross instrument rates. Thus, Equations (1) through (3) indicate that macroeconomic expectations may affect market rates through the expectations of the future short rate independently of the current level and determinants of the short rate.

3. OPTION-IMPLIED INTEREST RATE EXPECTATIONS

We next move to the issue of definition and measurement of interest rate expectations. In this paper, we define "interest rate expectations" in terms of probability distributions of future interest rates, which we compute from cross sections of interest rate option prices using a procedure explained in the first subsection. In the second subsection, we describe a method for calibrating the resulting risk-neutral expectations to actual "real-world" outcomes. The third subsection describes our option price data.

3.1. Recovering Risk-Neutral Interest Rate Expectations from Option Prices

Options are inherently forward-looking financial instruments, and thereby provide a rich source of information about market participants' expectations regarding future price developments of the underlying instrument or asset. In particular, since the price of an option depends on the probability of the underlying asset price exceeding the strike price of the option, a set of option prices with the same maturity but with different strike prices can be used to extract the expected probability distribution of the underlying asset price at the maturity of the option (see, Sherrick, Garcia, & Tirupattur, 1996; Söderlind & Svensson, 1997; Jackwerth, 1999; Bliss & Panigirtzoglou, 2002).

Formally, the price of an option equals the present value of its expected terminal payoff. Let c_t denote the time t value of a European call option with a single expiration date T and a contractual terminal payoff function $\max(S_T - K, 0)$, where S_T and K are the settlement price of the underlying asset and the strike price of the option, respectively. Assuming that the market is arbitrage free, and following the risk-neutral valuation principles of Harrison and Kreps (1979), the time t value of the call option can be written as:

$$c_t = \mathrm{e}^{-r_t(T-t)} E_t^{\mathcal{Q}}[\max(S_T - K, 0)], \tag{4}$$

where $\mathrm{e}^{-r_t(T-t)}$ is a discount factor based on the risk-free interest rate r_t and $E_t^{\mathcal{Q}}[\max(S_T - K, 0)]$ is the conditional expectation of the option payoff under the risk-neutral probability measure $\mathcal{Q}$ (to be distinguished from the physical, or objective, measure $\mathcal{P}$). Cox and Ross (1976) show that the time t value of the call option can be equivalently expressed in terms of a risk-neutral probability density function ("PDF") of the underlying asset price:

$$c_t = \mathrm{e}^{-r_t(T-t)} \int_{-\infty}^{\infty} \max(S_T - K, 0) f_t^{\mathcal{Q}}(S_T)\, dS_T, \tag{5}$$

where $f_t^{\mathcal{Q}}(\cdot)$ denotes the time-t risk-neutral PDF of the underlying asset price at the option maturity date T. Because the option price can be expressed as a function of the probability distribution of the underlying asset price, a set of option prices observable in the market can be used to extract this distribution.

The prior literature has proposed both parametric and nonparametric methods for extracting the expected probability distribution from option prices (for reviews, see Jackwerth, 1999; Bahra, 2002). The parametric methods assume a specific parametric form for the terminal underlying asset price distribution. Perhaps the most commonly used parametric techniques are the lognormalmixture model distributions proposed by Melick and Thomas (1997) and the Gram–Charlier expansion model of Corrado and Su (1996). The nonparametric methods, initiated by Shimko (1993), utilize some flexible function to fit the observed option prices, and then apply the results derived by Breeden and Litzenberger (1978) to extract the implied probability distribution. Campa, Chang, and Reider (1998) show that different methodological approaches lead to virtually similar implied distributions, while the findings reported in Bliss and Panigirtzoglou (2002) and Andersson and Lomakka (2005) indicate that the nonparametric smoothing methods may produce more accurate estimates of implied probability distributions. Galati, Higgins, Humpage, and Melick (2007) compare two alternative nonparametric smoothing methods and report that the resulting implied distributions are essentially indistinguishable from each other.

We estimate the implied probability distributions of future short-term interest rates with the nonparametric volatility-smoothing method proposed in Clews, Panigirtzoglou, and Proudman (2000) and Bliss and Panigirtzoglou (2002).[6] This nonparametric method combines the approaches of Malz (1997) and Campa et al. (1998) by using cubic splines to fit implied volatilities as a function of option deltas. The starting point in the method is the Breeden and Litzenberger (1978) result, which demonstrates that the second partial derivative of Equation (5) with respect to the strike price of the option gives the discounted risk-neutral PDF:

$$f_t^Q(K) = e^{r_t(T-t)} \frac{\partial^2 c(K,T,t)}{\partial K^2}. \tag{6}$$

Unfortunately, Equation (6) as such is of limited use because only a discrete set of option prices can be observed in the market. Thus, in order to extract the implied probability distribution, the discrete option price observations must first be transformed into a continuous pricing function. We begin the transformation by applying the Black–Scholes option pricing model to convert the observed option prices from the price/strike price space into the implied volatility/delta space. Subsequently, we fit a cubic spline to the discrete implied volatilities as a function of option deltas by solving the following minimization problem:

$$\underset{\Theta}{min} \sum_{i=1}^{N} \omega_i \{\hat{\sigma}_i - \hat{\sigma}_i[g(\delta_i, \Theta)]\}^2 + \lambda \int_{-\infty}^{\infty} g''(\delta_i, \Theta)^2 \, d\delta, \tag{7}$$

[6]This nonparametric technique is used by the Bank of England to estimate option-implied distributions and has become a standard technique for estimating implied distributions (see, e.g., Panigirtzoglou & Skiadopoulos, 2004; Nikkinen & Vähämaa, 2010; Vincent-Humphreys & Puigvert-Gutiérrez, 2010; Kostakis, Panigirtzoglou, & Skiadopoulos, 2011; de Vincent-Humphreys & Noss, 2012; Vergote & Puigvert-Gutiérrez, 2012). A detailed description of the technique can be found in the appendices of Clews et al. (2000) and Bliss and Panigirtzoglou (2002).

where $g(\delta_i, \Theta)$ is the cubic spline function, Θ denotes the parameter matrix of the cubic spline, $\hat{\sigma}_i$ and $\hat{\sigma}_i[g(\delta_i, \Theta)]$ are the actual and the spline-fitted implied volatilities (a hat refers to a model-based estimate), δ_i is the option delta corresponding to implied volatility observation i, ω_i is the weighting parameter, and λ is the smoothing parameter. The fitted cubic smoothing spline provides a continuous function of implied volatilities in terms of option deltas. By utilizing the Black-Scholes model for the second time, we then convert the continuous implied volatility function from the implied volatility/delta space into the option price/strike price space to obtain the continuous pricing function. With the continuous pricing function, the Breeden–Litzenberger result given by Equation (6) can be applied to calculate the expected probability distribution of the underlying asset.

One drawback of using raw option price data in estimating the implied distributions is that option prices exhibit time decay, which differs across option moneyness and type. Even if we knew the exact relationship between time to expiration and price, or theta, stale price quotes near expiration would introduce an unnecessary element of noise into the estimation. We circumvent the problem of maturity dependence by constructing a time-series of implied probability distributions with a fixed time-to-maturity of three months. Following the approach detailed in Clews et al. (2000) and Bliss and Panigirtzoglou (2002), the fixed-horizon distributions are obtained by using a cubic spline function to interpolate between the implied volatilities of options with four different maturities but with the same delta. By repeating the interpolation for different values of delta, we obtain a hypothetical implied volatility/delta space with three months to maturity for each point in time. This set of implied volatilities against deltas is then used to estimate constant-maturity implied distributions with the procedure described above.

In order to track changes in the shape of the implied distribution of S_T, we compute the first four moments of the distribution at each point in time:

$$
\begin{aligned}
\mu_{1,t} &= E_t^Q(S_T) = \textstyle\int_0^\infty S_T f_t^Q(S_T)\,dS_T \\
\mu_{2,t} &= \sqrt{E_t^Q(S_T^2) - \mu_{1,t}^2}, \\
\mu_{3,t} &= E_t^Q\left[\left(\frac{S_T - \mu_{1,t}}{\mu_{2,t}}\right)^3\right], \\
\mu_{4,t} &= E_t^Q\left[\left(\frac{S_T - \mu_{1,t}}{\mu_{2,t}}\right)^4\right].
\end{aligned}
\tag{8}
$$

We interpret the moments in the usual way: the implied mean $\mu_{1,t}$ gives the time-t risk-neutral expected value of S_T, and the implied volatility $\mu_{2,t}$ measures the dispersion around its expected value; the implied skewness $\mu_{3,t}$ measures the relative probabilities above and below the expected value, or asymmetry in expectations; and the implied kurtosis $\mu_{4,t}$ captures the tail thickness of the implied distribution, or the probability of extreme outcomes with respect to S_T. As discussed e.g. in Galati et al. (2007), the first and the third moment can be used to make directional predictions of S_T, whereas the second and the fourth moment quantify the uncertainty around these predictions.

3.2. Transformation from Risk-Neutral to "Real-World" Expectations

So far, we have defined interest rate expectations in terms of option-implied distributions under the assumption of risk neutrality. It should be noted, however, that the risk-neutral expectations coincide with "true" market expectations only if the premium for bearing market risk is zero. Otherwise, risk-neutral probabilities include information about marginal investor's subjective assessment of probabilities as well as her risk preferences:

$$risk - neutral\,probability = subjective\,probability \times risk\,aversion\,adjustment,$$

where subjective probabilities, on average, correspond to physical (or, objective) probabilities if the marginal investor is rational. Risk aversion notwithstanding, it is a common practice in the literature to use risk-neutral distributions to infer market expectations. This is partly because Girsanov's theorem states that moving from the risk-neutral to the physical probability measure only changes the *location* of the PDF and not its *shape*; and partly because the risk premium contributing to the change in the location of the PDF is sometimes considered to be small enough to be ignored.

While it may be argued that risk premia are relatively small in the money market,[7] the existence of possible premia may in principle have some implications for the interpretation of the information from risk-neutral densities. In order to assess the validity of risk-neutral expectations in this respect, we employ the empirical calibration method proposed by Fackler and King (1990).[8]

To state the problem formally, define $F_t^{\mathcal{Q}}(\cdot)$ as time t risk-neutral cumulative distribution function ("CDF") of the final settlement price S_T. Once S_T is known, one can go backwards in time and compute $U_T \equiv F_t^{\mathcal{Q}}(S_T)$, the ex-ante risk-neutral probability of S_T occurring. The question whether the hypothesized distribution $F_t^{\mathcal{Q}}(\cdot)$ is the same as the true (physical) distribution $F_t^{\mathcal{P}}(\cdot)$ can

[7]At least, prior to the breakdown of the interbank market in 2008; using three-month Euribor market data from January 1983 to March 2002, Hördahl and Vestin (2005) find that the differences between the risk-neutral and objective three-month-ahead densities are negligible.

[8]An alternative approach is to scale the risk-neutral probabilities with a pricing kernel. The kernel can be calibrated to historical option prices based on an assumed utility function of a representative agent (e.g., Bliss & Panigirtzoglou, 2004) or a time-series model of the underlying asset (e.g., Rosenberg & Engle, 2002). Under certain conditions regarding the dynamics of the asset, Ross (2011) shows that historical data is not necessarily required for defining the kernel; current prices of options with different maturities suffice.

be answered by comparing the empirical CDF of independent U_T's, $C(u) = Prob(U \leq u)$, with that of a uniform distribution: the sequence of probability assessments is uniformly distributed if and only if the hypothesized distribution is correct.[9]

To examine whether the risk-neutral expectations are valid, one can qq-plot $C(u)$ against U_T's on a unit square; the cumulative distribution function should lie close to the diagonal. If, however, the risk-neutral probability distributions produce systematically biased predictions, $C(u)$ bows away from the diagonal. Then, one has to find a suitable calibration function $C'(\cdot)$ (a prime refers to calibration) that maximizes the probability that U_T's are drawn from a uniform distribution and that the risk-neutral probabilities are correct. Specifying $C'(\cdot)$, however, is not always straightforward. We follow Fackler and King (1990) and use the cumulative function of the beta distribution $B(\alpha, \beta)$ to correct the risk-neutral PDF:

$$f_t^{\mathcal{P}}(\cdot) = \frac{F_t^{\mathcal{Q}}(\cdot)^{\alpha-1}\left[1 - F_t^{\mathcal{Q}}(\cdot)\right]^{\beta-1}}{B(\alpha,\beta)} f_t^{\mathcal{Q}}(\cdot), \tag{9}$$

where $f_t^{\mathcal{P}}(\cdot)$ corresponds to the PDF under the physical measure. Since the "calibrated measure" is asymptotically equivalent to the "physical measure" and vice versa, we use them interchangeably from now on, depending on whether the context is empirical or theoretical. The beta function parameters $\{\alpha, \beta\}$ are estimated using maximum likelihood (ML). Since the beta distribution nests the uniform distribution as a special case ($\alpha = \beta = 1$, i.e., no calibration required), a likelihood-ratio test can be used to determine whether the original risk-neutral densities are correctly specified.

3.3. Option Price Data

The implied interest rate distributions used in the empirical analysis are extracted from settlement prices of three-month sterling Libor futures options that are currently traded on the NYSE Liffe and formerly on the London International Financial Futures and Options Exchange. The option price data span the period January 1993–July 2012. We use the data for years 1993–2007 in our main analysis, and utilize the data on the subsequent period of severe market turmoil to perform additional tests.

Better known as "short sterling" futures, the three-month Libor futures contract is based on the spot rate of interest paid on three-month deposits in the London interbank market. Because the Libor rate itself cannot be

[9]See Taylor (2005), pp. 455–456 for proof.

purchased or sold, the short sterling futures are written on a notional asset that equals 100 minus the Libor rate at the maturity of the contract (in percentage points):

$$S_T \equiv 100 - Y_T^{(3)}.$$

Short sterling futures options are American options written on the short sterling futures contracts. However, due to the mark-to-market procedures, the short-sterling futures options are actually priced as European-style contracts. The expiration months for the short sterling futures options are the three nearest calendar months and the following seven months within the quarterly cycle of March, June, September, and December. Both the short sterling futures and futures options expire on the third Wednesday of the expiration month. In general, the short sterling derivatives are widely used and highly liquid contracts, and are therefore ideal for extracting implied probability distributions of expected future interest rates. Figure 1 plots the trading volume and open interest for the short sterling futures option market for the period 1993–2012. The mean (median) daily trading volume during our sample period is 92800 (68900) contracts, while the mean (median) open interest is approximately 2.66 (1.30) million contracts. However, as can be noted from Figure 1, both measures have

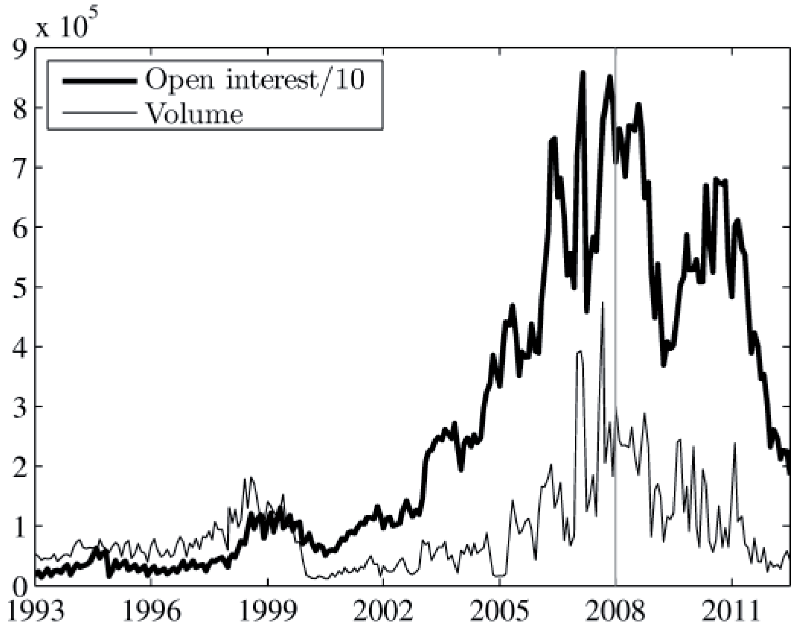

FIGURE 1
Volume and open interest for Libor futures options for the period 1992–2012.

increased consistently throughout the sample period and exhibited considerable variation.

We impose three filters on the option price data to reduce noise in the estimation procedure. First, options with less than five trading days to maturity are eliminated from the sample in order to avoid expiration-related abnormalities in option prices. Second, only at-the-money (ATM) and out-of-the-money (OTM) options are used in the empirical analysis. In-the-money (ITM) options are discarded because they are less liquid than OTM and ATM options, and because by using both OTM call and put options it can be ensured that the complete strike price spectrum is efficiently utilized in the estimation. Finally, we require that the option prices are convex and monotonic functions of the corresponding strike prices, and thereby satisfy the basic theoretical option price conditions.

Table I provides summary statistics for the first four moments of the option-implied PDFs for the period 1993–2007. Each monthly observation is based on a cross-section of the option prices on the 15th day of the month. The first row suggests that the implied distributions are on average slightly right-skewed and fat-tailed, with lots of variation around the mean value of 5.60. A leptokurtic distribution with a longer right tail implies expectations of higher future Libor rates that are more prone to large changes than would normally be assumed. All implied moments exhibit considerable persistence with first-order autocorrelation coefficients ranging from 0.51 to 0.97. However, the Wald and the Augmented Dickey–Fuller (*ADF*) tests indicate that only the implied mean process contains a unit root.

Figure 2A provides a graphical examination of correctness of the risk-neutral interest rate expectations. It plots $C(u)$, the CDF of probability assessments under the risk-neutral measure, for $\{Y_T^{(3)}\}$, a series of independent random draws from the distribution of realized Libor rates at the futures contract expiry dates T,

TABLE I
Summary Statistics on Option-Implied Moments

	Mean	*Volatility*	*Skewness*	*Kurtosis*
Mean	5.60	0.33	0.14	3.52
Median	5.60	0.31	0.09	3.40
Min	3.36	0.11	−0.81	2.94
Max	7.87	0.81	1.44	5.96
Std. Dev.	1.06	0.13	0.47	0.43
$\rho(1)$	0.98	0.85***	0.73***	0.51***
ADF	−2.01	−3.50***	−3.76***	−7.72***

Note. This table provides summary statistics for the moments of risk-neutral probability distributions implied by three-month Libor options. *ADF* and $\rho(1)$ report the augmented Dickey–Fuller test statistic and the first-order autocorrelation coefficient. The null hypotheses for *ADF* and $\rho(1)$ tests are $H_0 : X \sim I(1)$ and $H_0 : \rho = 1$. *, **, and *** denote statistical significance at the 1%, 5%, and 10% levels, based on MacKinnon (1996) and Newey and West (1987) heteroskedasticity- and autocorrelation-consistent (HAC) standard errors. The sample period is January 1993 through December 2007, for a total number of 180 monthly observations.

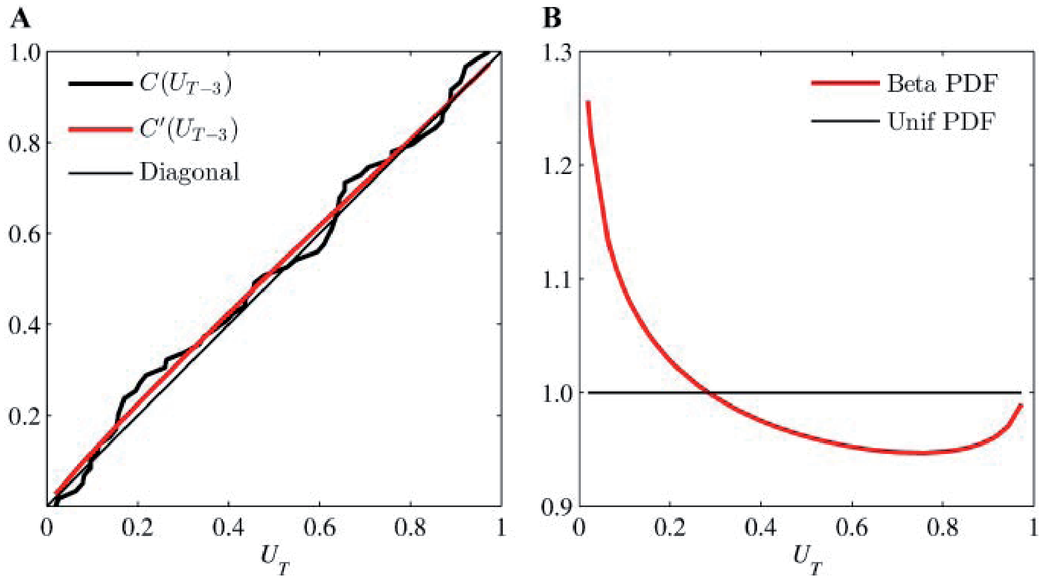

FIGURE 2

A graphical examination of the time-series validity of interest rate expectations under the risk-neutral measure (Figure 2A) and the Beta calibration function (Figure 2B). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com]

$T = T^{(1)}, T^{(2)}, \ldots, T^{(N/3)}$.[10] If the risk-neutral three-month-ahead predictions correspond to the true distribution of final settlement values, $C(u)$ should lie on the diagonal.

Figure 2A indicates that the risk-neutral PDFs generate a $C(u)$-plot that oscillates only slightly above and below the diagonal. This suggests that the risk-neutral probability assessments do not differ significantly from the actual outcomes. Nevertheless, Figure 2B, which plots the ML-estimated Beta distribution used for calibrating the risk-neutral PDFs, shows that the lowest quantile of probability assessments under the risk-neutral measure are too low and need to be scaled upwards. Therefore, we compute a full set of calibrated distributions using a beta function $B(\alpha, \beta)$ with MLE parameters $\{0.95, 0.92\}$.[11]

Figure 3A–D plot the time series of the first four risk-neutral and calibrated moments of the implied Libor distributions. Overall, it can be noted from the figures that the differences between the risk-neutral and calibrated moments are negligible, especially until the onset of the financial crisis. The most notable systematic difference can be observed between the second moments, where

[10]To ensure that the draws are random and independent, only one option-based probability assessment is considered per futures contract. With three-month contracts, the size of the evaluated sample reduces from the original $N = 180$ to $N/3 = 60$.

[11]Both parameters are statistically indistinguishable from one.

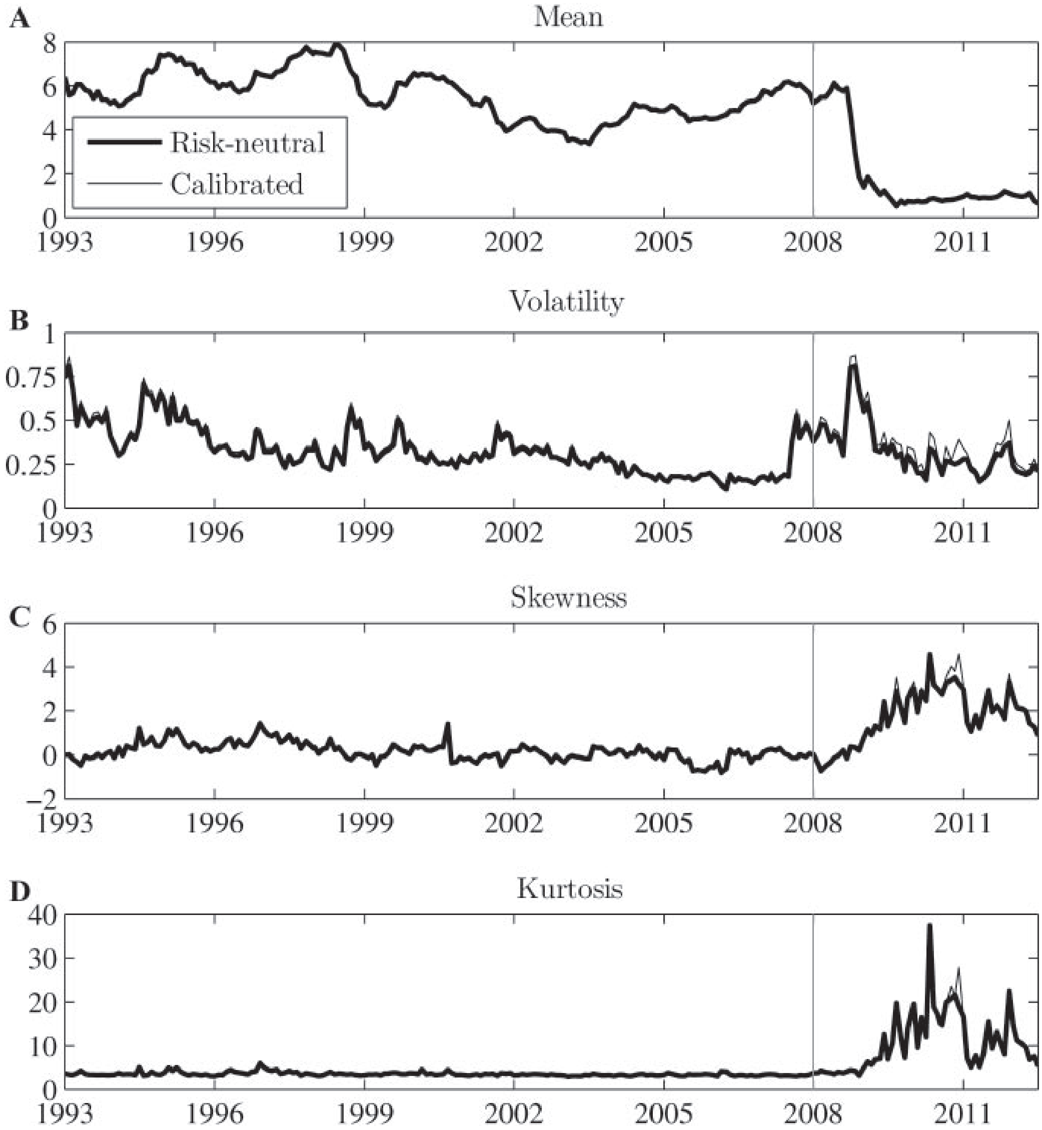

FIGURE 3
The time series of the risk-neutral and calibrated implied moments. The Figure plots the implied mean (A), volatility (B), skewness (C), and kurtosis (D). The vertical line represents the end of the estimation sample.

volatility under the physical measure is consistently above the risk-neutral volatility, which is to be expected given the shape adjustment implied by Figure 2B.

Putting aside the effects of risk neutrality, Figures 3A and B show that rapid changes in the level seem to be associated with heightened level of uncertainty. For instance, when faster-than-expected economic growth compelled the Bank of

England to initiate a series of half percentage-point interest rate hikes in 1994, the implied volatility doubled irrespective of the probability measure. Analogously, consecutive interest rate reductions amid deteriorating economic outlooks in the latter halves of 1998 and 2001 led to 100% and 80% jumps in the perceived Libor volatility. Finally, the rapid decrease in the implied mean after the outbreak of the market turmoil is associated with unprecedented levels of implied volatility.

The implied skewness, plotted in Figure 3C, first increased in 1994 in response to expectations of tightening monetary policy but did not reach its pre-crisis maximum value of 1.44 until early 1997, having remained elevated for two years. Accordingly, the high of 1997 foreshadowed an episode of tightening monetary policy that lasted until 1998, by which the participants in the Libor market had reversed their expectations. Indeed, after a long decline through 1998 the implied skewness dropped below zero in the end of the year, for the first time since late 1993. The negative period, however, did not last long as the market participants quickly adjusted to increasing growth expectations stimulated by rising equity prices in 1999. Following the implosion of the dot-com bubble in 2000, the implied skewness fluctuated close to zero until the beginning of the financial crisis in 2008 when it rose to unprecedented levels.

The times series plot of the implied kurtosis in Figure 3D resembles that of the implied skewness. Amid economic and financial turmoil, the implied probability of extreme movements in interest rates peaks in 1994 and again in 1996, followed by a long period of invariance until times of market turbulence in 2008 when it peaked 10-fold over the sample mean.

4. ECONOMETRIC SETUP

Having established the link between short-term interest rates and macroeconomic expectations in the second section, and defined the measure of interest rate expectations in the third, we next describe our data on macroeconomic expectations from the United Kingdom and then present the econometric specification that we adopt in our empirical analysis.

4.1. Macroeconomic Data

We use monthly macroeconomic survey data to measure real-time expectations about the policy rule fundamentals.[12] For this purpose, we utilize two data

[12]Alternatively, one can assume rational expectations and use actual ex-post values, or generate market expectations endogenously by a term structure model. However, both alternatives are inferior to survey-based expectations; the former approach has been criticized for being unrealistic with respect to data availability and quality, and the latter for producing less accurate forecasts. See Orphanides (2001) and Ang, Bekaert, and Wei (2007) for further details.

sources, HM Treasury (Her Majesty's Treasury) and Consensus Economics, which both poll a substantial number of financial and economic institutions each month for their views on the evolution of principal macroeconomic variables over the current and the next year. The HM Treasury surveys are based on a slightly larger and more diverse survey pool, and hence, we use these data as the primary measure of macroeconomic expectations. However, because the HM Treasury surveys are available only from 1999 onwards, we augment our expectations data with Consensus Economics surveys for years 1993–1998.[13] Previously, survey-based macroeconomic forecasts have been used in the context of Taylor rules, for exapmle, by Fendel et al. (2011, 2013) and Frenkel et al. (2011).

The expected inflation gap $E_t(\pi_{t+n} - \pi^*)$ in Equation (1) is computed using the mean of the respondents' expected year-on-year growth rate of the harmonized index of consumer prices.[14] To obtain a continuous, fixed-horizon forecast series, we merge the current and the next-year point forecasts by weighting them together with respect to the number of months remaining in the current year:

$$E_t(\pi_{t+12}) = \frac{12 - m(t)}{12} E_t\left[\pi_{t+12-m(t)}\right] + \frac{m(t)}{12} E_t\left[\pi_{t+24-m(t)}\right], \tag{10}$$

where $m(t)$ gives the number of month t in the current year.

The output gap x_t in Equation (1) measures the difference between actual output and potential output at full employment. A positive output gap implies overheating of the economy and thus inflationary pressures, whereas a negative gap leads to opposite implications. In the seminal paper of Taylor (1993), the output gap was measured by the deviation of real quarterly gross domestic product (*GDP*) from its linear trend; in a forward-looking context, however, the expected deviation ought to be used. For this we construct a time series of the expected output gap as follows: first, we decompose a time series of quarterly real GDP from UK National Statistics (the "ABMI" series) into trend and cycle components using the Hodrick–Prescott filter with a smoothing parameter of 1,600, and calculate the expected 12-month-ahead potential GDP using the trend component. Then, we use the real GDP series again together with survey data on the 12-month-ahead real GDP growth rate, and calculate the expected

[13]The HM Treasury forecasts are based on a heterogenous survey pool of around 30–40 respondents including e.g. the Bank of America, Deutsche Bank, Goldman Sachs, J.P. Morgan, International Monetary Fund, and the OECD. Surveying both profit and non-profit organizations is expected to balance the element of subjectivity in each other's expressed views.

[14]Retail price index excluding mortgages, RPIX, before June 2003. Prior to June 1995, π_t^* is considered to be the midpoint of the inflation-target range of 1–4%. After that, π_t^* is 2.5% until December 2003, when it was lowered to 2%.

real GDP. With estimates of the expected real and potential GDP in hand, we compute the expected output gap as their log ratio (in percentage terms).

An important nuance of the assumed interest rate rule given by Equation (1) is that the variation in financial wealth is allowed to influence the conduct of monetary policy. Thus, we implicitly assume that market participants perceive the central bank to adjust the policy rate not only in response to inflation and output gaps but also in response to financial uncertainty. Excessive financial uncertainty may lead to a vicious cycle of valuation and macroeconomic uncertainty and initiate an adverse feedback loop often referred to as the "financial accelerator".[15] The financial accelerator mechanism harms the monetary transmission system, and leads to a contraction in the supply of credit as information asymmetry between lenders and borrowers increases. Sharp declines in credit raises the possibility of severe reduction in economic activity, thus inflation, and may therefore influence the appropriate stance of monetary policy. It should be noted that the presumed policy rule does not dictate that the Bank of England would actually react to financial uncertainty, although financial shocks appear to have large effects on output and employment (Bloom, 2009) and recent empirical findings suggest that central banks may respond to such uncertainty (Jovanovic & Zimmermann, 2010).

Given that most of the variation in financial wealth is generated by fluctuations in stock market capitalization (Lettau & Ludvigson, 2004), we use the (log of) implied volatility of three-month FTSE 100 stock index options as a forward-looking proxy for financial uncertainty:

$$E_t(z_{t+m}) = E_t\left[Var(W, t+m)^{0.5}\right] \equiv log\,\hat{\sigma}_{FTSE,t+3}.$$

The implied stock market volatility is sufficiently versatile variable to capture the uncertainty regarding firms' operating environment and the expected cash flows, the risk of financial distress, as well as investors' risk appetite. Furthermore, Adrian and Shin (2008) show that the implied stock market volatility is associated with financial intermediates' ability to provide credit due to the adverse impact of financial uncertainty on their balance sheets.

Finally, we use the official Bank of England rate as a proxy for the instrument rate i_t in order to quantify the monetary policy stance. The time series for the official rate is obtained from the Bank of England (the "IUDBEDR" series). All macroeconomic data span from January 1993 to July 2012.

Table II reports summary statistics for the macroeconomic variables for the period 1993–2007. The first row of the Table shows that inflation expectations

[15]For background on the financial accelerator mechanism, see Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1996), and Bernanke, Gertler, and Gilchrist (1999).

TABLE II
Summary Statistics on Macroeconomic Variables

	$E_t\ (\pi_{t+12}) - \pi^*$	$E_t\ (x_{t+12})$	$\hat{\sigma}_{\mathrm{FTSE},t+3}$	i_t
Mean	0.10	0.13	0.18	5.37
Median	0.00	0.09	0.16	5.50
Min	−0.52	−1.60	0.08	3.50
Max	1.00	1.25	0.38	7.50
Std. Dev.	0.38	0.55	0.06	0.99
$\rho(1)$	0.97	0.97	0.87***	0.98
ADF	−2.20	−3.48***	−3.42**	−2.08

Note. This table provides summary statistics for the macroeconomic variables entering Equations (1) and (2). *ADF* and $\rho(1)$ report the augmented Dickey–Fuller test statistic and the first-order autocorrelation coefficient. The null hypotheses for *ADF* and $\rho(1)$ tests are $H_0 : X \sim I(1)$ and $H_0 : \rho = 1$. *, **, and *** denote statistical significance at the 1%, 5%, and 10% levels, based on MacKinnon (1996) and HAC standard errors. The sample period is January 1993 through December 2007, for a total number of 180 monthly observations.

have averaged about ten basis points above the target, but Figure 4A implies that the gap is mainly driven by high inflation expectations in the 1990s. After the establishment of the Monetary Policy Committee in 1997, however, inflation expectations converged towards the target until the beginning of the financial crisis.

The expected output gap in Figure 4B, with a range of 3% points, has fluctuated much more than the inflation gap during 1993–2007. This is largely due to the overly pessimistic output expectations in 1998 that were not matched by corresponding downward revisions in inflation expectations. As can be seen from Figures 4A and B, both the expected inflation and output gaps deepened drastically in 2008 in response to deteriorating economic sentiment.

The level of implied FTSE 100 volatility in Figure 4C seems to be inversely related to the instrument rate i_t in Figure 4D. For example, the deepening financial crisis towards the end of the 1990s is concurrent with downward-revised output expectations and reductions in the instrument rate; to a lesser extent, a similar chain of events occurred after the terrorist attacks in September 2001. In contrast, increased market uncertainty induced by the looming war against Iraq in mid-2002 did not prompt an instant change in the instrument rate, supposedly because it was not expected to have adverse medium-term effects on the real economy. Finally, it can be noted that the unprecedented increase in the implied FTSE volatility in the Autumn 2008 was followed by exceptionally rapid decrease of the instrument rate from 5.00% to 0.50%.

4.2. Econometric Model

Under the assumption of the pure expectations hypothesis given by Equation (3), the three-month forward rate (essentially, the interest rate underlying the Libor

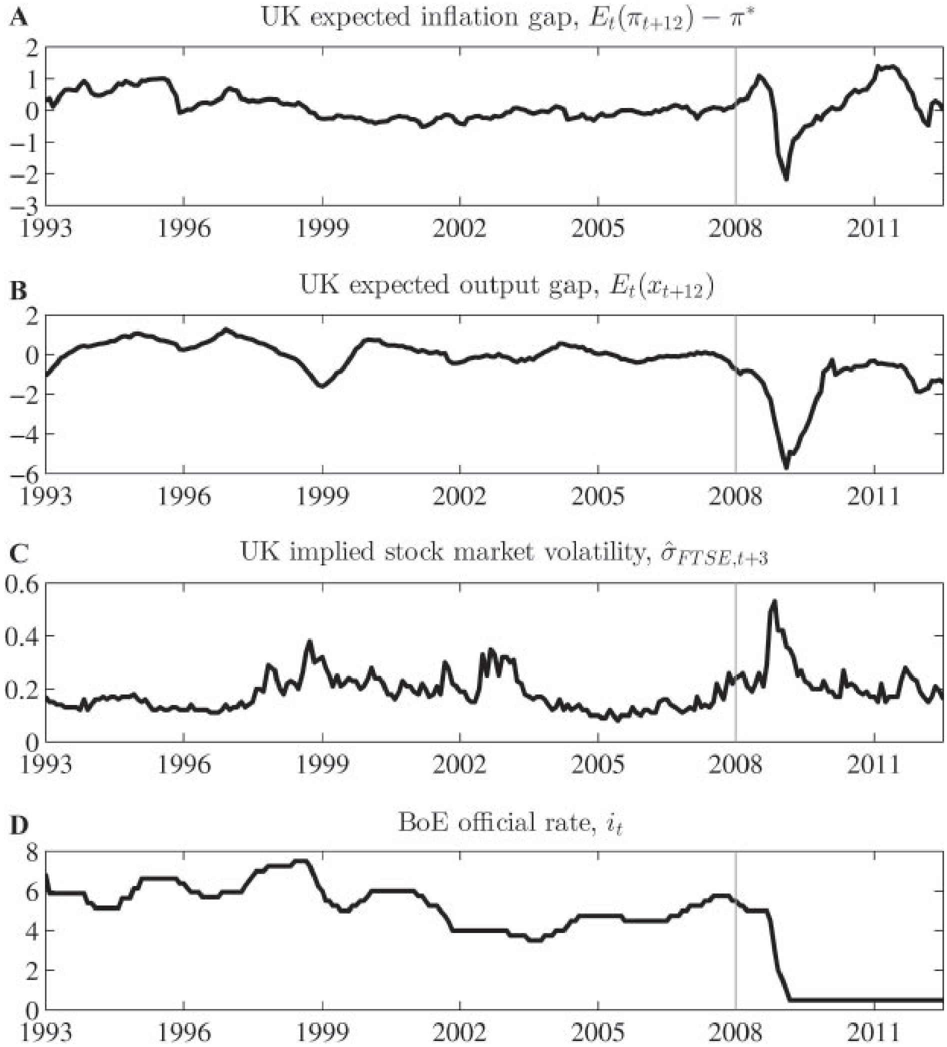

FIGURE 4

The time series of the policy rule fundamentals. The Figure plots the expected inflation gap (A), the expected output gap (B), the expected stock market volatility (C), and the Bank of England interest rate (D). The vertical line represents the end of the estimation sample.

options) can be written as a ratio of six- and three-month spot rates:[16]

$$E_t\left[1 + Y_{t+3}^{(3)}\right] = E_t\left[\frac{1 + Y_t^{(6)}}{1 + Y_t^{(3)}}\right]^{\frac{1}{3}} = E_t\left[\frac{(1 + i_t)(1 + i_{t+1})\ldots(1 + i_{t+5})}{(1 + i_t)(1 + i_{t+1})(1 + i_{t+2})}\right]^{\frac{1}{3}},$$

[16]Longstaff (2000) finds that the pure form of the expectations hypothesis (no risk premia) is a valid description of the short end of the term structure.

which, by taking logarithms and using the $x \approx \log(1+x)$ approximation, can be written in the following continuous compounding form:

$$E_t\left[y_{t+3}^{(3)}\right] = \frac{1}{3}E_t[i_{t+3} + i_{t+4} + i_{t+5}]. \tag{11}$$

From the history-dependence of future instrument rates as implied by Equation (2), it follows that the current instrument rate i_t has substantial explanatory power for the market interest rates with maturities beyond one period. Combining the partial adjustment Equation (2) with (11) yields:

$$E_t\left[y_{t+3}^{(3)}\right] = \frac{1}{3}\sum_{j=1}^{5}\rho^j i_t + \frac{1}{3}\sum_{j=1}^{5}(1-\rho^{6-j})r_{t+j}^* + \frac{1}{3}\sum_{j=1}^{2}\rho^{3-j}r_{t+j}^*. \tag{12}$$

On the assumption that the current desired policy rate i_t^* is a sufficient statistic for near-future desired rates $\{E_t(i_{t+1}^*)\ldots E_t(i_{t+5}^*)\}$, Equation (12) collapses to:[17]

$$E_t\left[y_{t+3}^{(3)}\right] = \Lambda i_t + (1-\Lambda)i_t^*, \tag{13}$$

where $\Lambda = (1/3)(\rho^3 + \rho^4 + \rho^5)$. We substitute $E_t[y_{t+3}^{(3)}]$ based on the pure expectation hypothesis with its risk-neutral counterpart $E_t^Q[y_{t+3}^{(3)}]$, and i_t^* with the interest rate rule given by Equation (1), to adopt the following linear-form econometric model to empirically examine the relationship between the option-implied interest rate distributions and the macroeconomic expectations:[18]

$$\Delta\mu_t = \mathbf{A} + \mathbf{B}(\Delta\mathbf{X_t}) + \mathbf{C}(\mu_{t-1}) + \mathrm{D}(\mathbf{X_{t-1}}) + \mathbf{e_t}, \tag{14}$$

where $\boldsymbol{\mu}_t$ is a 4×1 vector of moments of option-implied distributions defined in Equation (8), $\mathbf{X_t}$ is a 1×4 vector of explanatory variables:

$$\mathbf{X_t} = \left[E_t(\pi_{t+12}) - \pi_t^* \quad E_t(x_{t+12}) \quad log\,\hat{\sigma}_{FTSE,t+3} \quad i_t\right],$$

[17]Given the highly persistent nature of the variables entering interest rate rule (1), and remembering that the average macroeconomic forecasts $E_{t+12}(\cdot)$ are computed from point forecasts spanning the horizon $t+1$ to $t+24$ (with average horizons $t+6$ and $t+18$), there is not too much generality lost in making this assumption. We do, however, conduct overidentification and unbiasedness tests based on GMM forecast errors to assess the validity of the assumption. In the results not presented here for brevity, the Hansen (1982) $T \times J$-test based on publicly available set of macroeconomic information and a Wald test on forecast-error averages both support our original assumption.

[18]The pure expectation hypothesis described in (3) is roughly the same as risk neutrality; risk-neutral arbitrageurs will adjust their positions along the yield curve until the expected one-period returns are equal on all maturities. For further discussion, see Cochrane (2005, pp. 355–356).

or, respectively, the expected inflation gap, the expected output gap, FTSE 100 implied volatility, and the official Bank of England interest rate, and Δ is the first-difference operator. For higher-order moments, two minor modifications to the baseline model are made to facilitate its economic interpretation. First, $|\mathbf{X_t}|$ is used instead of $\mathbf{X_t}$ for the non-directional second and fourth moments. Second, the last explanatory variable i_t is replaced by $\mu_{1,t} - i_t$ for the moments beyond the first. We refer to this difference between the expected Libor rate and the current BoE instrument rate as the expected interest rate gap. This gap controls for currently anticipated shifts in the monetary policy stance.

As implied by Equation (14), we follow a general-to-specific modeling approach in that both the immediate and total effects of $\mathbf{X_t}$ on $\boldsymbol{\mu}_t$ are considered. The resulting four single equation error-correction representations are estimated individually using ordinary least squares. $\{\mathbf{A}, \ldots, \mathbf{D}\}$ are 4×1 parameter vectors, where $\mathbf{A}$ includes the constant terms, and $\mathbf{C}^{-1}$ measures the delay from immediate ($\mathbf{B}$) to total ($-\mathbf{D}/\mathbf{C}$) effects of $\mathbf{X_t}$ on $\boldsymbol{\mu}_t$. In the case $\mathbf{C} = 0$, the total effects, if any, take eternity to realize and are thus irrelevant.

In order to assess how much a change in the probability measure from $\mathcal{Q}$ to $\mathcal{P}$ alters the estimated relationships, Equation (14) is re-estimated using moments from the calibrated PDFs as the dependent variables.

5. INTEREST RATE EXPECTATIONS AND THE MONETARY POLICY RULE

Table III presents the regression results on the association between option-implied risk-neutral Libor expectations and the forward-looking monetary policy rule for the period 1993–2007. The second column of the table reports the estimates for the implied mean of the future Libor rate. The estimated model seems to fit the implied mean relatively well, as the adjusted R^2 of the regression is 0.41, and the F-statistic is statistically significant at the 1% level. Regarding the immediate effects, Table III shows that the changes in the expected inflation gap induce only a modest and statistically insignificant revision of market expectations about future Libor. While several explanations can be adduced to explain this result, perhaps the most important factor is the Bank of England's ability to anchor inflation expectations especially during the latter part of the sample period.[19] The estimated coefficient for the expected output gap is positive and statistically highly significant. The magnitude of the coefficient, 0.49, suggests that the average immediate effect of a 1% point change in the expected output gap is associated with approximately a half percentage point change in the implied mean.

[19]We thank Larry D. Wall of the Federal Reserve Bank of Atlanta for pointing this out.

TABLE III
Implied Risk-Neutral Moments and Policy Rule Variables

	ΔMean	*ΔVolatility*	*ΔSkewness*	*ΔKurtosis*
Immediate effects (**B**)				
ΔE_t (inflation gap)	−0.06 (0.16)	0.12*** (0.04)	0.30 (0.19)	0.91** (0.41)
ΔE_t (output gap)	0.49*** (0.13)	0.02 (0.04)	0.26 (0.20)	0.24 (0.28)
ΔE_t (FTSE volatility)	−0.20** (0.09)	0.13*** (0.03)	0.18 (0.12)	0.06 (0.15)
ΔBoE interest rate	0.80*** (0.10)	0.14*** (0.03)	0.08 (0.13)	−0.20 (0.25)
Effect delay ($\mathbf{C}^{-1}$)				
Level of dep. variable$_{t-1}$	7.66*** (2.80)	4.55*** (0.67)	1.96*** (0.42)	1.58*** (0.24)
Total effects ($-\mathbf{D}/\mathbf{C}$)				
E_{t-1} (inflation gap)	0.05 (0.41)	0.32*** (0.09)	0.22* (0.12)	0.18 (0.23)
E_{t-1} (output gap)	0.51** (0.23)	−0.03 (0.05)	0.31*** (0.11)	0.46** (0.22)
E_{t-1} (FTSE volatility)	0.06 (0.37)	0.16*** (0.05)	0.20 (0.15)	−0.24* (0.13)
BoE rate$_{t-1}$	0.80** (0.41)	0.07 (0.06)	0.28* (0.15)	−0.01 (0.18)
Adjusted R^2	0.41	0.36	0.23	0.32
F	15.05***	12.10***	7.01***	10.46***
DW	2.03	2.28	2.17	2.06
$Q(12)$	12.89	18.42	13.18	5.61

Note. This table reports the estimates from time-series regressions of risk-neutral moments on the policy rule fundamentals: the expected inflation gap, the expected output gap, the FTSE 100 implied volatility, the Bank of England instrument rate. In the regressions for the higher-order moments, the instrument rate is replaced with the interest rate gap, defined as the difference between the implied Libor rate and the instrument rate. Δ is the first-difference operator. HAC standard errors are reported in parentheses. *, **, and *** denote statistical significance at the 1, 5, and 10 per cent levels, respectively. DW is the Durbin–Watson statistic and $Q(12)$ the Ljung–Box test statistic for serial correlation up to order 12. The sample period is January 1993 through December 2007, for a total number of 180 monthly observations.

Consistent with central bank intentions to counter the adverse real effects of financial uncertainty, we find that perceived uncertainty regarding future stock prices is strongly negatively related to the implied short-term rate with an estimate of −0.20. This suggests that increased financial uncertainty is associated with market expectations of cuts in the instrument rate. Given that we have used the log transformation on the FTSE volatility, the estimated coefficient implies that the expected mean of Libor rate should decrease approximately by 20 basis points if the FTSE volatility increases twofold over its average level. Table III further shows that the estimated immediate response of the implied Libor rate to the instrument rate is positive and statistically highly significant. The estimated coefficient of 0.80 is qualitatively similar, albeit slightly lower than the value of 0.92 predicted by the model (Equation 13 with input value $\rho = 0.98$), and suggests that a 100 basis point hike in the BoE policy rate is associated with a 80 basis point increase in the implied Libor rate.

Turning the focus from the immediate effects to total effects, it can be noted from Table III that the adjustment process of the implied mean to the Taylor-rule fundamentals is not completely immediate. In particular, the coefficient for the lagged level of the implied mean suggests the adjustment process may last for over seven months. Nevertheless, the estimated coefficients for the total effects indicate that the adjustments in the implied mean are rather small beyond the immediate effects. The total-effect coefficients are positive and significant for the expected output gap and the instrument rate, while being almost equal in size to the coefficients for the immediate effects. Hence, the effects of the expected output gap and instrument rate on the expected Libor rate appear permanent, and mostly immediate. Interestingly, our estimate for the total effect of the expected output gap (0.52) is very similar to Nelson (2003), who estimates an effect of 0.47. The total-effect coefficient for the FTSE volatility is statistically insignificant, suggesting that the effect of financial uncertainty on the expected Libor rate is immediate but transient. Overall, the estimates for the implied mean indicate that market participants rationally revise their interest rate expectations in response to changes in economic outlook.

The regression results for the implied Libor volatility are reported in the third column of Table III. As can be noted from the table, the dispersion of interest rate expectations is strongly influenced by the policy rule variables. The adjusted R^2 of the regression specification is 0.36, and again, the statistically significant F-statistic implies a good fit of the model. The estimates for the immediate effects in Table III indicate that a widening gap between expected inflation and the inflation target heightens the uncertainty regarding the future short-term rate. More specifically, the estimates suggest that a 1% point increase in the absolute expected inflation gap corresponds to about twelve basis point immediate increase in the implied Libor volatility. The coefficient estimate for the perceived stock market uncertainty is also positive and highly significant, and thus suggests that increasing financial uncertainty tends to diffuse into the Libor market. The coefficient for the logarithmic first difference in the FTSE volatility implies a 0.75 unit effect in the implied Libor volatility in response to a unit shock in the FTSE volatility. In general, this finding is consistent with flexible inflation targeting, which enables the central bank to adapt complementary short-run monetary policy objectives, like, for instance, financial stability as suggested by Bernanke and Gertler (2000). Finally, the estimated immediate effect for the absolute expected interest rate gap is positive and statistically highly significant, thereby suggesting that any anticipated shift in the monetary policy stance increases the uncertainty regarding the future Libor rate.

The estimates in Table III demonstrate that the policy rule fundamentals affect the expected volatility of the Libor rate also after the immediate effects. In particular, the estimated impact of the absolute expected inflation gap increases from the immediate effect of 0.12 to the total effect of 0.32 within a four and half month adjustment period. The magnitude of the total-effect coefficient indicates that a one standard deviation shock in the absolute expected inflation gap leads to a 12% point increase in the expected Libor volatility. Furthermore, the total-effect coefficient for the FTSE volatility is positive and significant, albeit only marginally larger than the immediate-effect coefficient. This suggests that the effect of financial uncertainty on the implied Libor volatility is permanent but mostly driven by the immediate adjustment. Interestingly, the total-effect coefficient for the absolute expected interest rate gap appears insignificant, and thereby reflects the transience of the adjustment.

The estimation results for the higher-order moments of the risk-neutral Libor distributions are reported in the fourth and fifth columns of Table III. As can be seen from Table III, the estimated models for the implied skewness and kurtosis are well specified with adjusted R^2s of 0.23 and 0.32, respectively. The estimates show that there are no statistically significant immediate effects from the policy rule fundamentals to asymmetries in the Libor expectations. Nonetheless, our estimates suggest that option-implied asymmetries are affected by the expected inflation, output, and interest rate gaps within a period of about two months. The positive and statistically significant coefficients for these three variables demonstrate that implied interest rate expectations become more positively skewed or less negatively skewed in response to more optimistic economic outlook. That is, market participants seem to attach higher probabilities for increasing Libor rates (i.e., more restrictive future monetary policy) in response to expectations of an expansionary economy. The estimated total-effect coefficients indicate that a 1% point increase in any of the expected policy-rule gaps leads to approximately 0.25 unit increases in the implied skewness.

As can be noted from the estimates in the fifth column of Table III, the implied kurtosis is positively associated with the absolute expected inflation gap. The immediate-effect coefficient of the inflation gap implies that a percentage point widening of the gap increases implied kurtosis by 0.91 units. Thus, our findings indicate that market participants attach higher likelihood to future extreme movements in short-term interest rates with increasing deviation of expected inflation from the target. Regarding the total effects, our estimates indicate that the implied kurtosis is affected by the absolute expected output gap and FTSE volatility. Somewhat counterintuitively, the weakly significant coefficient for the FTSE volatility is negative, suggesting a lower likelihood for extreme interest rate movements amidst times of elevated financial uncertainty.

Overall, the results in Table III demonstrate that changes in the distributional form of Libor expectations are strongly associated with changes in the expected inflation and output gaps and financial uncertainty. This suggests that financial market participants perceive the monetary policy rule as a valid description of the central bank's rate-setting behavior and react to its projections in a systematic way. These findings are broadly consistent with Carlson et al. (2005) and thereby provide evidence that market participants form and revise their interest rate expectations in a manner described by Taylor-type policy rules.

Table IV reports the estimation results for the calibrated, or "real-world," probability densities. In general, it should be noted that the regression results for the risk-neutral and calibrated interest rate distributions are strikingly similar. This suggests that the role of the risk premium in the short end of the term structure is almost negligible. The adjusted R^2s of the regressions are almost identical, and the signs and magnitudes of the statistically significant coefficient estimates are remarkably consistent across the regressions. The only, and rather marginal, differences across the regression estimates are found in the total effects of the policy rule fundamentals on the directional moments. In particular, the estimated impact of the instrument rate on the implied mean becomes statistically less significant, and the effect of the expected inflation (output) gap on the implied skewness becomes more (less) significant under the calibrated probability distributions. Overall, Table IV provides further evidence on the importance of policy rule fundamentals in guiding the market expectations of future interest rates.

We acknowledge that the estimates reported in Tables III and IV may be biased if inflation and output gap expectations are simultaneously determined with the implied interest rate expectations. Indeed, it is conceivable that

TABLE IV

Calibrated Moments and Policy Rule Variables

	ΔMean	*ΔVolatility*	*ΔSkewness*	*ΔKurtosis*
Immediate effects (**B**)				
ΔE_t (inflation gap)	−0.06 (0.16)	0.13*** (0.04)	0.31 (0.19)	0.87** (0.40)
ΔE_t (output gap)	0.49*** (0.12)	0.02 (0.04)	0.25 (0.20)	0.24 (0.27)
ΔE_t (FTSE volatility)	−0.21** (0.09)	0.14*** (0.03)	0.17 (0.12)	0.07 (0.15)
ΔBoE interest rate	0.80*** (0.10)	0.15*** (0.04)	0.09 (0.13)	−0.19 (0.24)
Effect delay ($\mathbf{C}^{-1}$)				
Level of dep. variable$_{t-1}$	7.82*** (2.93)	4.54*** (0.67)	1.90*** (0.41)	1.64*** (0.26)
Total effects ($-\mathbf{D}/\mathbf{C}$)				
E_{t-1} (inflation gap)	0.04 (0.41)	0.33*** (0.09)	0.23** (0.12)	0.20 (0.24)
E_{t-1} (output gap)	0.52** (0.23)	−0.02 (0.05)	0.30** (0.11)	0.47** (0.23)
E_{t-1} (FTSE volatility)	0.04 (0.37)	0.17*** (0.05)	0.20 (0.14)	−0.26* (0.14)
BoE rate$_{t-1}$	0.80* (0.42)	0.07 (0.07)	0.28* (0.15)	0.01 (0.19)
Adjusted R^2	0.41	0.37	0.24	0.31
F	15.05***	12.65***	7.29***	9.86***
DW	2.03	2.31	2.17	2.06
$Q(12)$	12.24	20.02**	13.67	8.00

Note. This table reports the estimates from time-series regressions of calibrated moments on the policy rule fundamentals: the expected inflation gap, the expected output gap, the FTSE 100 implied volatility, the Bank of England instrument rate. In the regressions for the higher-order moments, the instrument rate is replaced with the interest rate gap, defined as the difference between the implied Libor rate and the instrument rate. Δ is the first-difference operator. HAC standard errors are reported in parentheses. *, **, and *** denote statistical significance at the 1%, 5%, and 10% levels, respectively. DW is the Durbin–Watson statistic and $Q(12)$ the Ljung-Box test statistic for serial correlation up to order 12. The sample period is January 1993 through December 2007, for a total number of 180 monthly observations.

macroeconomic expectations and expected future interest rates are jointly determined in a complex system in which market participants are assessing future central bank policies and the monetary policy authorities, in turn, are assessing the evolution of the economy as well as the market reactions to central bank policies. In such a system, endogeneity is endemic and difficult to address empirically.[20] Nonetheless, we argue that the OLS estimates in Tables III and IV are of interest as the coefficients demonstrate that market participants adjust their beliefs in accordance with the forward-looking monetary policy rule.

6. INTEREST RATE EXPECTATIONS, MONETARY POLICY RULE, AND THE FINANCIAL CRISIS

We next examine whether and how the recent financial crisis affected the association between option-implied interest rate expectations and the policy rule fundamentals. It is important to note that the standard Taylor-type policy rules fail to account for the zero-bound of nominal interest rates that materialized during the crisis, and furthermore, that the period of severe financial turmoil was characterized by unconventional monetary policy tools and actions. In a recent study, Martin and Milas (2013) document a breakdown of the Taylor rule after the onset of the financial crisis. Hence, we re-estimate the regressions reported in Table III using a sample covering the period from January 2008 to July 2012. Moreover, we also perform a dynamic forecasting exercise of the implied PDFs for the crisis period using the pre-crisis parameter estimates from Table III.

Table V reports the coefficient estimates for the crisis period. Overall, it can be noted from the table that the importance of the expected inflation and output gaps in explaining changes in implied distributions has decreased during the crisis. Specifically, none of the estimated total-effect coefficients in Table V are statistically significant, and the significant immediate effects are relatively small in magnitude. The estimates of the specification with the implied mean as the dependent variable demonstrate that the instrument rate has become the dominant factor for explaining market participants' trajectories of the future level of the Libor rate. The increased importance of the instrument rate is most likely related to the constraint imposed by the zero bound and concurrent central bank communication related to the continuation of exceptionally loose monetary policy. Nevertheless, our estimates also indicate that the expected inflation and output gaps have immediate positive effects on the implied mean.

The estimation results for the implied Libor volatility are reported in the third column of Table V. As can be seen from the table, the estimated coefficients for the expected inflation and output gaps appear insignificant, and the only

[20]We thank an anonymous referee for making this observation.

TABLE V
The Impact of the Financial Crisis

	ΔMean	*ΔVolatility*	*ΔSkewness*	*ΔKurtosis*
Immediate effects (**B**)				
ΔE_t (inflation gap)	0.27*** (0.09)	−0.01 (0.03)	−0.45 (0.50)	−3.34 (2.99)
ΔE_t (output gap)	0.19** (0.09)	0.02 (0.02)	0.00 (0.39)	1.27 (2.60)
ΔE_t (FTSE volatility)	−0.08 (0.19)	0.32*** (0.10)	0.51 (0.86)	4.81 (8.58)
ΔBoE interest rate	1.20*** (0.25)	0.07 (0.05)	−0.11 (0.27)	−1.10 (3.27)
Effect delay ($\mathbf{C}^{-1}$)				
Level of dep. variable$_{t-1}$	1.99*** (0.46)	1.99*** (0.47)	5.36** (2.39)	2.19*** (0.56)
Total effects ($-\mathbf{D}/\mathbf{C}$)				
E_{t-1} (inflation gap)	0.05 (0.14)	−0.04 (0.03)	−1.45 (1.86)	−2.74 (4.30)
E_{t-1} (output gap)	−0.13 (0.10)	−0.02 (0.02)	0.19 (0.98)	1.36 (1.70)
E_{t-1} (FTSE volatility)	−0.22 (0.49)	0.51*** (0.15)	−2.75 (2.99)	−9.71 (8.94)
BoE rate$_{t-1}$	1.03*** (0.24)	0.05 (0.07)	0.32 (2.52)	−11.44** (4.69)
Adjusted R^2	0.64	0.43	0.00	0.17
F	11.77***	5.57***	0.85	2.25*
DW	1.62	1.78	2.44	2.27
Q(12)	14.66	6.54	13.92	13.56

Note. This table reports the estimates from time-series regressions of risk-neutral moments on the policy rule fundamentals: the expected inflation gap, the expected output gap, the FTSE 100 implied volatility, the Bank of England instrument rate. In the regressions for the higher-order moments, the instrument rate is replaced with the interest rate gap, defined as the difference between the implied Libor rate and the instrument rate. Δ is the first-difference operator. HAC standard errors are reported in parentheses. *, **, and *** denote statistical significance at the 1%, 5%, and 10% levels, respectively. DW is the Durbin–Watson statistic and Q(12) the Ljung–Box test statistic for serial correlation up to order 12. The sample period is January 2008 through July 2012, for a total number of 55 monthly observations.

statistically significant policy rule variable is the FTSE volatility. This suggests that general financial uncertainty drives the dispersion of the Libor expectations amidst the crisis. Finally, the low and statistically insignificant F-values for the estimated models of the implied skewness and kurtosis imply that changes in the higher-order moments cannot be explained by the policy rule fundamentals. Overall, these findings are broadly consistent with Martin and Milas (2013), and indicate that the importance of the standard Taylor rule variables in explaining market participants' interest rate expectations was sharply reduced during the financial crisis.

Figure 5 plots the actual level and the dynamic forecast of the implied mean of the Libor rate as well as the BoE instrument rate. Specifically, we have used the pre-crisis parameter estimates from Table III with real-time policy rule fundamentals to construct one-month-ahead forecasts of the implied mean for the period 2008–2012. The solid vertical line in the figure represents the end of the estimation sample and the beginning of the forecast sample. Figure 5 demonstrates that the predicted implied mean plummets from about 5.50% near to −1.00% during 2008, whereas the actual implied mean increases 50 basis points in the first half of 2008 and then drops sharply near to the instrument rate

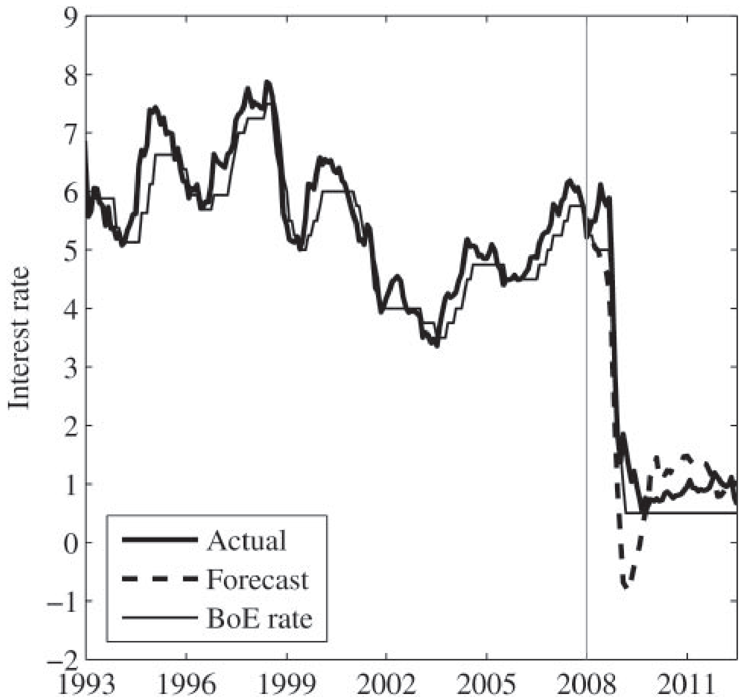

FIGURE 5
Dynamic forecast of the implied mean for the period 2008–2012. The vertical line represents the end of the estimation sample.

level of 0.75%. It is important to notice that the dynamic forecast bluntly violates the zero bound of nominal interest rates for a prolonged period from January 2009 onwards. To further demonstrate the breakdown of the policy rule in explaining interest rate expectations, we plot the actual risk-neutral and forecasted densities for March 2009 in Figure 6. As can be seen from the figure, the actual and forecasted densities resemble reversed counterparts of each other in shape and deviate substantially in location. The actual risk-neutral density has a lower bound at 0%, while the forecasted density is unbounded and assigns considerable amount of probability mass for negative interest rates.

7. CONCLUSIONS

Over the past decade, central banks and investors have increasingly utilized option prices to extract information about financial market expectations and conditions. All major central banks, for instance, have now publicly acknowledged that option-implied probability distributions are a valuable source of information and provide a useful input to monetary policy decisions. Thus far,

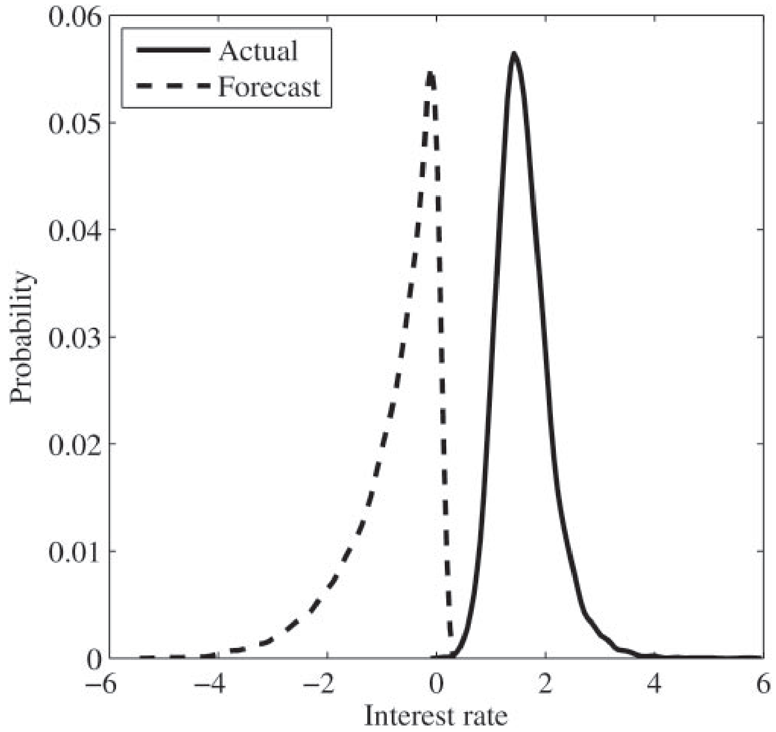

FIGURE 6
A comparison of the actual and forecasted risk-neutral densities in March 2009.

however, the informational content of option-implied distributions has hinged upon theoretical assumptions about the link between option prices and macroeconomic fundamentals. In this paper, we aim to empirically establish this link in the context of a forward-looking Taylor rule. Specifically, we postulate that market participants view the monetary policy rule as a guide to the path of future policy rates and price interest rate options in accordance with the policy rule fundamentals.

The purpose of this study is to examine the association between option-implied interest rate distributions and expectations regarding the policy rule fundamentals. In particular, we use market prices of three-month sterling Libor futures options to extract the expected probability distribution of future interest rates, and attempt to relate the month-to-month movements in these implied distributions to changes in survey-based expectations of the policy rule fundamentals. Through this exercise, we want to assess whether the expectations formation process in financial derivatives markets is consistent with Taylor-type monetary policy rules.

The empirical findings reported in this study indicate that the interest rate expectations implied by option prices are largely consistent with the policy rule fundamentals. In particular, the results show that movements in the expected level of the three-month Libor rate are related to the expected output gap and perceived financial uncertainty. The expected level is also substantially affected by shifts in the monetary policy stance. The results further suggest that changes in the distributional form of interest rate expectations are strongly influenced by the policy rule fundamentals. We find that the dispersion of market expectations around the expected Libor rate increases with increasing deviation of expected inflation from the target level, and also with increasing financial uncertainty. Moreover, the dispersion of interest rate expectations appears to increase with a widening disparity between the market and policy rates. Asymmetries in interest rate expectations are found to be positively related to the expected inflation and output gaps, indicating that the market perceived probabilities of sharp upward movements in interest rates increase with improved economic outlook. Finally, our results suggest that market participants attach higher probabilities for extreme movements in short-term interest rates in response to increasing expected inflation and output gaps.

Overall, our empirical findings demonstrate that financial market participants perceive the monetary policy rule as a valid description of the central bank's rate-setting behavior and react to its projections in a systematic way. More specifically, our results suggest that market participants form and revise their expectations regarding the future movements in Libor rates, at least partially, in a manner described by Taylor-type policy rules. These results are broadly consistent with the recent empirical studies on survey-based interest rate forecasts and thereby provide further support for the view that the private sector's interest rate expectations are formed in accordance with policy rule fundamentals. We believe that our findings may have important implications for financial market practitioners and monetary policy authorities. From the viewpoint of central banks, for instance, the documented linkage between option-implied interest rate distributions and policy rule fundamentals offers a further validation for the use of option-implied information as a tool for monetary policy assessments, decisions, and communication purposes.

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When Bernanke Talks, the Markets Listen: The Case of the First FOMC Press Conference on Monetary Policy

Abstract

This note examines minute-by-minute reactions of the US interest rate and stock markets to the first Federal Open Market Committee press conference on monetary policy. Volatility and volume effects during the press conference are shown to be less pronounced but more lasting than those observed immediately after the release of the monetary policy statement. Market responses during the press conference are found to be deterministic and originate from questions and answers pertaining to future monetary policy and the state of the economy. These findings are in line with the clarification objective of the Fed's new communication framework.

JEL classification: E44, E52, E58, G14

Keywords: monetary policy, Federal Reserve, futures markets, press conference, communication

1 INTRODUCTION

In April 2011, the Federal Open Market Committee (FOMC) of the United States' Federal Reserve System (Fed) introduced a new strategy for monetary policy communication. An essential feature in the new strategy is a press conference that is held four times per year to provide additional context for the monetary policy statement. The press conference comprises two elements: first, the chairman of the FOMC delivers a detailed statement of the Committee's monetary policy stance and presents the latest economic projections. Then, the members of the media are allowed to ask clarifying questions about monetary policy. According to the Federal Reserve, the new strategy is adopted in order to enhance the clarity and timeliness of monetary policy communication, since communication has become an increasingly important aspect of monetary policy (see Blinder et al., 2008, for a review). In financial markets, better communication by the central bank tends to resolve uncertainty about future monetary policy actions and improve the process of price discovery.

Using a wide set of intraday financial data, this note aims to provide the first piece of evidence on the form of market adaptation to the FOMC's changed communication. Specifically, the way market participants respond to the new communication strategy is inferred from intraday patterns in money, bond, and stock index futures markets. Two avenues of research are explored. First, the press conference that aims to explain the FOMC's monetary policy stance is held approximately two hours after the release of the monetary policy statement. The time lapse gives potentially enough time to find a new market equilibrium on the basis of the statement, which makes the press conference seem pointless if the statement already delivers all the relevant information about the policy stance. Thus, the extent to which the FOMC's press conference initiates new price discovery processes can be viewed as a measure of effectiveness of the new communication tool. Second, the press conference is broadcasted live, which makes the chairman's words subject to a real-time rhetorical analysis by the market participants. The participants may draw different price implications from the bits of information forwarded by the chairman, which would induce trading and possible price revisions. Therefore, mapping the topics discussed in the press conference onto synchronous transaction data would allow to identify the exact issues in the press conference that are perceived to be the most relevant from the market viewpoint.

2 DATA AND METHODOLOGY

The Fed scheduled the implementation of the new communication strategy for the FOMC meeting in April 2011. The publication of the new strategy received considerable media attention; investment banks, for example, circulated speculations about the content of the forthcoming press conference that were later reported in the financial press. The uniqueness, investor awareness, and media coverage of the event put it into a special context that guides the design of the empirical analysis.

The analysis focuses on one particular day, April 27th 2011, when the FOMC executed its new communication strategy for the first time. The monetary policy statement (released at 12:30 PM EST) and live footage of the press conference (held at 2:15 PM) were obtained from the Federal Reserve's Web site.[1] Traded volume and prices from the Chicago Mercantile Exchange's Globex system were collected at one-minute frequency for the whole maturity spectrum of three-month Eurodollar, ten-year Treasury note, and S&P 500 E-mini futures contracts. These data span from 9:00 AM to 5:00 PM, capturing the most active trading period of the day with a total of 480 observations per contract.

The market responses to the FOMC communication are gauged by price volatility and trading volume, which both are highly sensitive to the arrival of new information. The intraday volatility in a specific futures market at minute t, V_t, is measured by the absolute one-minute change in the log of volume-weighted average price (VWAP), $\{\bar{p}\}$:

$$V_t = 100 \times |\bar{p}_t - \bar{p}_{t-1}|,$$

where

$$\bar{p}_t = \log\left(\sum_i p_{i,t} q_{i,t} Q_t^{-1}\right).$$

In a similar fashion, aggregate trading volume in specific market, Q_t, is the total of individual contract i volume in that market, $q_{i,t}$. Aggregated series are used in an effort to reduce the effects of microstructure noise, and to better capture the market-wide responses to new information. For illustrative purposes, a video of the intraday evolution of volatilities, volumes, and cumulative returns is available on Youtube Web site.[2]

In the European context, Ehrmann & Fratzscher (2009) find that the market responses to the press conference by the European Central Bank (ECB) are related

[1] `www.federalreserve.gov`.

[2] `www.youtube.com/watch?v=weVoRSUZkhA`.

to the novelty of the preceding policy statement. For this reason, it is necessary to evaluate the information content of the FOMC statement released on the day of the press conference, before turning to the empirical findings. As expected, the statement noted no change in monetary policy: the FOMC announced to keep its target range for the federal funds rate at the minimum level, and to complete the second round of quantitative easing (QE2) as scheduled. Although the FOMC downgraded slightly its general assessment of the economy, the overall impact of the statement on asset prices was mildly positive.

3 RESULTS

Figures 1, 2, and 3 show the intraday evolution of trading volume and price volatilities. In each frame, the box in the upper-left corner shows the market average and standard deviation, and the thicker line represents a five-minute moving average. Focusing first on volatilities on the left, all markets experience a jump after the monetary policy statement is released at 12:30 PM. Immediately after the release, volatilities are 8 to 12 times the market average but fall in half within few minutes. Volatilities remain elevated for another 30 to 45 minutes, depending on the interest-rate sensitivity of the market. The post-release decay of volatilities seem to be consistent with the speed of information revelation rule proposed by Vives (1995), which postulates that market participants trade on their private information so that market prices converge to their new equilibrium levels as an inverse square-root function of trading rounds. For example, the Vives:1995 rule implies that three (thirty) minutes of trading on new information reduces volatility by 50 (82) percent, which roughly corresponds to the patterns observed here.

Volatilities rise again after the beginning of the press conference at 2:15 PM, but this time they rise differently: sudden peaks are absent, and volatilities just shift up to a higher regime and remain there approximately until the end of the conference. Trading volume seem to follow a similar intraday pattern, peaking at 12:30 AM and then remaining above the market average for 30 to 45 minutes. In addition, volume seems to rise after 2:15 PM as do volatilities, although this increase is indistinguishable for the E-mini market after taking the normal U-shaped intraday pattern into account. Positive correlation between volatility and volume is a well-established empirical finding after public news events and is often associated with a noisy rational expectation environment, where investors trade informatively on the basis of their own interpretations of the news as well as past prices (eg. He & Wang, 1995).

Table 1 confirms the findings of the graphical analysis. It presents the results of regressing the logs of volatilities and volumes on intraday dummy variables. Each dummy variable represents a phase in the FOMC communication process: the release period ("RLSE") extends from 12:30 PM to 1:00 PM, the intermediate period ("INTERM") from 1:00 to 2:14 PM, the press conference period ("PRESS") from 2:15 to 3:12 PM, and the post-conference period ("POST") from 3:13 to 5:00 PM. The morning period from 9:00 AM to 12:30 PM is set as a baseline level. With this specification, a regression coefficient can be interpreted as a mean percentage change in the dependent variable relative to its level in the morning.

The coefficients in Table 1 indicate that volatility and volume levels increase after the release of the policy statement and during the press conference, subsequently returning to the baseline levels or below. This finding is statistically verified by using a Wald test for the null hypothesis that a sum of a set of coefficients is zero.

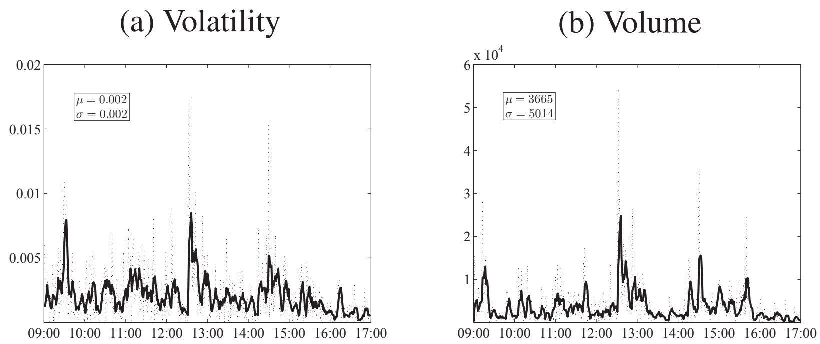

Figure 1. Eurodollar futures

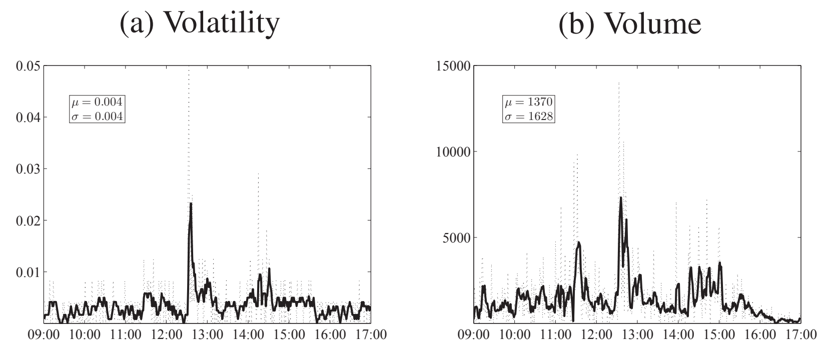

Figure 2. Ten-year treasury note futures

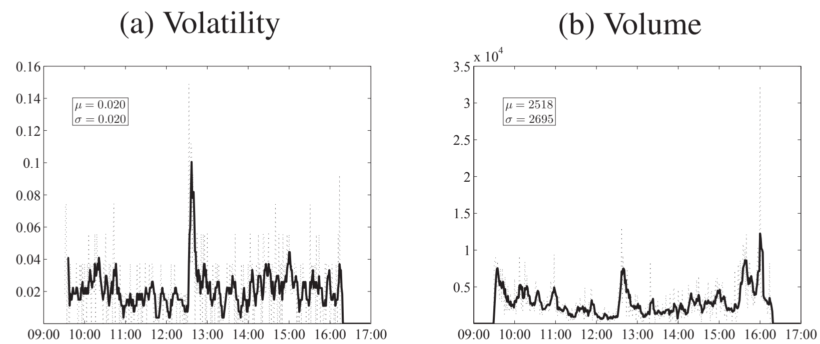

Figure 3. S&P 500 E-mini futures

The Wald statistics on the bottom line show that the sum of RLSE and PRESS coefficients are in fact well above zero (positive effect on volatility and volume), whereas INTERM and POST are zero or below (no or negative effect on volatility and volume).

The F-statistics indicate that the strongest intraday effects are seen in the Eurodollar market, where the statement and the conference increase volatility, respectively, by 63 and 10 percent, and volume by 115 and 61 percent. The volatility effects are at least partly attenuated by high volatility in the morning, and may be better measured

Table 1. Volatility and volume regressions on intraday dummy variables.

log Variable (N)	RLSE (t-stat)	INTERM (t-stat)	PRESS (t-stat)	POST (t-stat)	F-stat DW
Eurodollar V_t	0.63	-0.29	0.10	-0.77	13.87
(478)	(2.78)	(-2.06)	(0.56)	(-4.54)	1.83
Eurodollar Q_t	1.15	-0.25	0.61	-0.89	24.06
(479)	(4.82)	(-1.23)	(3.19)	(-3.22)	1.36
T-note V_t	1.59	0.66	1.04	-0.13	7.31
(453)	(4.63)	(2.49)	(4.35)	(0.51)	1.91
T-note Q_t	1.06	0.03	0.57	-1.33	26.13
(478)	(4.12)	(0.19)	(2.83)	(-3.75)	1.64
E-mini V_t	1.23	-0.12	1.03	0.37	1.66
(405)	(1.88)	(-0.22)	(2.03)	(0.88)	2.09
E-mini Q_t	0.39	-0.46	0.23	0.64	15.61
(405)	(1.66)	(-3.15)	(1.74)	(2.87)	1.48
Wald $\sum_i \beta_i = 0$	6.87	0.39	4.38	-3.62	
	[18.37]	[0.19]	[23.09]	[6.48]	

Note: constant terms included but not tabulated. HAC standard errors.

by *changes* in the coefficients; expressed this way, the Eurodollar volatility increase by 92 and 87 percent during the statement and the conference, respectively.

The effects of FOMC communication are equally significant in T-note and E-mini markets; both show strong variation in volatility and volume levels according to information flow from the Fed. Indeed, the FOMC statement and the press conference at least double the volatility in both markets, with positive but milder effects in volumes as well.

To put these results in context, Andersson (2010) provides a benchmark in his comparison of volatility responses to monetary policy statement under the Fed's former one-stage communication framework and the two-stage framework currently followed by the ECB. Compared to the results of Andersson (2010), volatility responses to the policy statement are now milder in the T-note and E-mini markets than those experienced under the Fed's old framework. In addition, in their pattern and magnitude, the T-note and E-mini volatilities seem like a hybrid of European stock and bond market responses to a statement *with* a policy change, and US market responses *without* one. Specifically, an ECB policy statement is much less informative than the Fed's, and opposite to a Fed statement, only a change in policy rates induces a market reaction. But when it does, the volatility pattern is similar to that observed in the Figures 2a and 3a.

Ehrmann & Fratzscher (2009) study of the volatility and volume effects of ECB communication in the Euribor futures market enables a similar comparison as regards the Eurodollar market. Again, a comparison of the results shows that, with equally peaking volatilities and surging volumes, the two markets remind each other in their responses to policy statements and press conferences. Ehrmann & Fratzscher (2009) identified the market response to the latter event as a product of "clarification" of ECB's monetary policy stance. They hypothesize that the press conference either confirms, reinforces, or causes re-evaluation of the initial market reaction, and show that market turns are indeed more likely during press conferences, especially if the information content of the policy statement is low.

Thus, a logical next step is to find out whether the FOMC's first press conference on its monetary policy served a similar clarification role. Some tentative evidence on this matter can be inferred from first-order autocorrelations: Ehrmann & Fratzscher (2009) note that once new information arrives, earlier price changes are either confirmed (no autocorrelation), reinforced (positive autocorrelation), or reconsidered (negative autocorrelation).

As can be seen from Figure 4, autocorrelations before and at the time of the policy release are negative across asset class, indicating a partial reversal of earlier price changes and hence the market's difficulty in finding new equilibrium prices. Then, some time after the release, return autocorrelations shift up towards zero and prices behave more like a random walk. But once the press conference starts, autocorrelations diverge: the instrument having the smallest duration, namely the Eurodollar contract, exhibits a small increase in the level of autocorrelation. On the other hand, the E-mini contract (having the largest duration) dips to -0.35 in first-order autocorrelation, indicating quite strong re-evaluation of past price changes. The T-note contract with intermediate duration exhibits, accordingly, a level of autocorrelation in between of the two extremes. After the press conference, autocorrelations converge towards zero again.

The key observation in the autocorrelation analysis is the divergent price-process behavior during the press conference. Supposedly, the observed divergence is not random but reflects different asset-class sensitivities to good and bad news about the economy. In order to investigate this possibility, a closer look is taken on the price and volume reactions to different topics discussed in the press conference. Whether or not the chairman's discourse on economic issues cause price adjustments is identified by the variation in the product of minute-by-minute price changes and volumes.

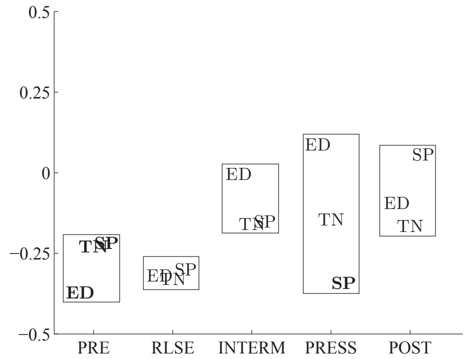

Figure 4. First-order autocorrelations of one-minute VWAP returns during different steps of FOMC's communication process. Note: boldfaced abbreviations denote statistical significance at the five percent level, based on HAC standard errors. Rectangles are for illustrative purposes.

Should any particular content in Chairman Bernanke's answers strike some market participant as unexpected or important, one would expect him or her to trade on that piece of information, resulting in an increase in traded volume. To the extent that the participant trades at the margin, increased volume is accompanied by a change in price. Negatively correlated order flow between interest-rate and equity index futures would further affirm the deterministic behavior of the market participants; one would expect to see negative returns for interest-rate futures in response to news indicative of faster economic growth, higher inflation, and future interest-rate hikes, and opposite effects on stock (index) futures insofar as increased cash-flow expectations dominate the discount rate effect.

Figure 5a, 5b, and 5c present the results for each market. In each Figure, the subjects discussed in the press conference are listed in chronological order on the y-axis, plotted against the mean response in the order-flow proxy (x-axis, scaled by dollar tick size). Looking at the market responses, there is a clear negative reaction at the very beginning of the press conference. This peculiar and particularly strong reaction is not driven by new information from Chairman Bernanke's talk since he was not yet speaking, but may reflect the excitement caused by the novel situation. Other strong responses are seen during Chairman Bernanke's answers about the timing of the next interest-rate hike ("MP") and the growth prospects of the US economy ("ECON"), the latter attracting the most attention in the E-mini markets.

Another regularity in the market responses is the support for the hypothesized negative correlation between asset classes. For example, studentized responses to Bernanke's answer about the Fed's future interest-rate hike are -3.51, -1.70, and 2.58 for the Eurodollar, T-note, and the E-mini contracts, respectively. Seemingly, higher perceived likelihood of interest-rate hikes triggered trading that lowered bond prices and increased expected equity prices as a signal of economic recovery.

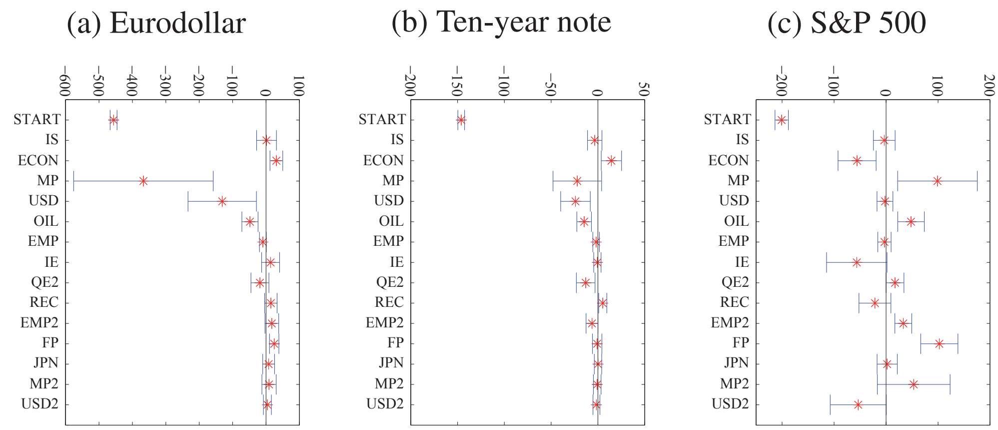

Figure 5. The press conference by topic: VWAP change times volume. Note: START = beginning of the press conference; IS = introductory statement; ECON = economic outlook; MP = monetary policy; USD = US Dollar; OIL = oil price; EMP = unemployment; IE = inflation expectations; QE2 = second round of quantitative easing; REC = economic recovery; FP = fiscal policy; JPN = economic impact of the Fukushima disaster; PRESS = first press conference; ROLE = Fed's role in economic recovery. Stars represent period means in thousands of US dollars and whiskers their 95 percent confidence intervals based on HAC standard errors.

The contrary responses actually reflect the general pattern during the press conference: once the initial reaction ("START") is excluded, the interest-rate/equity correlation of studentized order flow is negative for both the Eurodollar (-0.16) and the T-note market (-0.54).

4 CONCLUDING REMARKS

The purpose of this note is to analyze the market adaptation to the FOMC's new two-stage strategy for communicating monetary policy decisions. In the first stage, the policy statement is released; in the second, the chairman of the Committee provides background information and interacts with the members of the financial press.

A case study focusing on the day of the first two-stage announcement yields interesting insights concerning the market adaptation to the new framework. The findings indicate that both the release of the policy statement and the press conference are important market events but differ in dynamics. Whereas the market response to the former is more short-lived and extreme, the press conference introduces longevity in the adjustment process by stimulating new waves of price discovery after any further clarification of the monetary-policy stance. The chairman's answers pertaining to future monetary policy and economic growth prospects seem to especially trigger simultaneous but opposite reactions in bond and equity futures prices, which is to be expected when discount-rate and cash-flow expectations conflict.

In the light of these findings, it seems that the press conference meets the FOMC's objective for enhanced clarity and timeliness of monetary policy communication. Moreover, the enhanced clarity works both ways; the central bank can now find out which particular topics draw the most attention in financial markets.

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EUROPEAN CENTRAL BANK

EUROSYSTEM

WORKING PAPER SERIES

NO 1081 / AUGUST 2009

LIQUIDITY PREMIA IN GERMAN GOVERNMENT BONDS [1]

by Jacob W. Ejsing [2] and Jukka Sihvonen [3]

In 2009 all ECB publications feature a motif taken from the €200 banknote.

1 The views expressed in this paper are those of the authors and do not necessarily reflect the views of the European Central Bank or Danmarks Nationalbank. We thank the anonymous referee and seminar participants at the ECB and at the GSF Summer Research Workshop in Finance, May 2009, for helpful comments.
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CONTENTS

Abstract

There is strong evidence that on-the-run U.S. Treasury securities trade much more liquidly and at significantly higher prices than their off-the-run counterparts. We examine if the same phenomenon is present in the German government bond market whose market structure differ markedly from that of the U.S. Treasury market. In sharp contrast to the U.S. evidence, we find that on-the-run status has only a negligible effect on the liquidity and pricing once other factors have been controlled for. Instead, the highly liquid German bond futures market, whose turnover is many times larger than in the cash market, leads to significant liquidity spillovers. Specifically, we find that bonds which are deliverable into futures contracts are both trading more liquidly and commanding a significant price premium, and that this effect became more pronounced during the recent financial crisis.

Keywords: *Government bond, liquidity, liquidity premium, futures market*

JEL Classification: *E43, G12, H63*

Non-technical summary

Variations in liquidity are one reason why yields on otherwise comparable government securities differ. Although the liquidity of a bond can be measured in several ways, the concept essentially captures to what extent the bond can be sold cheaply and easily. Liquidity is thus valuable for market participants, and especially in times of market stress, the most liquid bonds have tended to command a considerable price premium.

Liquidity can have important implications for bond yields and the term structure of interest rates. Previous studies of liquidity and liquidity premia in government bond markets, based mainly on data from the U.S. Treasury market, have identified pronounced liquidity differences across government securities. In particular, the most recently issued securities in a given maturity bracket, the so-called on-the-run issues, have been found to trade much more actively and liquidly than their more seasoned counterparts. It has also been found that these differences in liquidity between on-the-run and off-the-run securities have important implications for bond pricing.

To contribute to a better understanding of the underlying determinants of liquidity and liquidity premia, this paper reports on a study of the German government bond market. Such a study is useful particularly because the German and U.S. markets for trading interest rate risk differ considerably. In particular, in contrast to the U.S. market, turnover in the German bond futures market is many times larger than in the German cash bond market. We argue that this difference causes trading to be less concentrated on specific bonds in the German market, which, in turn, helps explain why differences in liquidity premia are considerably smaller.

Our empirical results clearly suggest that the existence of a highly liquid German futures market leads to significant liquidity spillovers to the German cash market. Specifically, we find that bonds which are deliverable into the futures contracts are both trading more liquidly and commanding a price premium. Moreover, we show that this effect has intensified during the recent financial crisis. In sharp contrast to the evidence from the U.S. Treasury market, on-the-run status appears to have only a modest effect on the liquidity and pricing of German government bonds once other factors have been controlled for.

1 Introduction

Previous studies of liquidity and liquidity premia in government bond markets, based predominantly on data from the U.S. Treasury market, have identified pronounced liquidity differences across government securities. In particular, the most recently issued securities in a given maturity bracket, the so-called on-the-run issues, have been found to trade much more actively and liquidly than their more seasoned counterparts. This pattern is usually referred to as the 'on-the-run liquidity phenomenon'. It has also been found that these differences in liquidity between on-the-run and off-the-run securities have important implications for bond pricing, and that - particularly in times of market stress - the on-the-run securities command a significant price premium. For example, the yield discount on the on-the-run ten-year U.S. Treasury note relative to older issues with similar remaining maturity reached over 50 basis points in the autumn of 2008.

With a view to better understand the underlying causes of liquidity and liquidity premia, an examination of the German government bond market can potentially provide new insights. Specifically, the market structures of the U.S. and German government bond markets differ considerably; most notably with regard to the relative sizes of cash and futures markets. Table 1 compares U.S. and German trading volumes in government securities (excluding bills) and corresponding futures contracts. Whereas trading volumes in the German cash bond market is dwarfed by the activity in US Treasury market, the trading volumes in the two futures markets are of the same order of magnitude. This has important implications: whereas benchmark status and on-the-run status are synonymous in the U.S. Treasury market, in the German market, the benchmark status is de facto shared between a number of bonds, namely those bonds which are deliverable into the nearest-to-expiry futures contracts. Figures 1a and 1b show an example of how these differences affect trading volumes throughout the lives of selected ten-year bonds maturing around 2010. The U.S. 'on-the-run liquidity phenomenon' is clearly reflected in the sharp drop-off in traded volumes after the on-the-run period (top panel). For the German bonds (middle panel), however, the initial decline is much less pronounced, and there is a strong resurgence of trading as the bonds become deliverable again for the five-year futures and (albeit to a lesser extent) for the two-year futures.

Table 1: German and US markets for government securities and related futures (2008)

	Amount outstanding (EUR[a] billion)	Total volume 2008 (EUR billion)		Relative size of futures market in %
		Cash market	Futures	
Germany	879	5961	58715	985%
United States	2302	81426	45748	56%

Sources: Eurex, Bundesrepublik Deutschland Finanzagentur,
Federal Reserve Bank of New York, Chicago Board of Trade and the US Treasury Department.

[a]US dollar amounts were converted using the average exchange rate of 2008, 1.4711 USD per EUR.

In this paper, we ask whether the extremely large German futures market (relatively to the cash market) gives rise to significant liquidity spillovers to the cash bond market. In particular, we examine whether deliverable bonds systematically enjoy enhanced liquidity (as measured by higher trading volumes, higher quoted depths and/or tighter bid-ask spreads). Moreover, we investigate whether such liquidity effects are reflected in the prices of German government bonds. There are two main reasons for expecting spillover effects. First, deliverable bonds are easier to hedge using futures contracts, and thus more attractive for dealers (and other market participants with short horizons) to hold. Second, trading of deliverable bonds is directly supported by the strategies of arbitrageurs and speculative investors targeting the bond-future basis.

Our empirical results demonstrate that deliverability into futures contracts - rather than on-the-run status - is the key driver of liquidity and liquidity premia in the German market once other relevant factors have been controlled for. The sizes of the liquidity premia in the German market are found to be much smaller than those previously reported for U.S. on-the-run securities. This is consistent with the more ambiguous

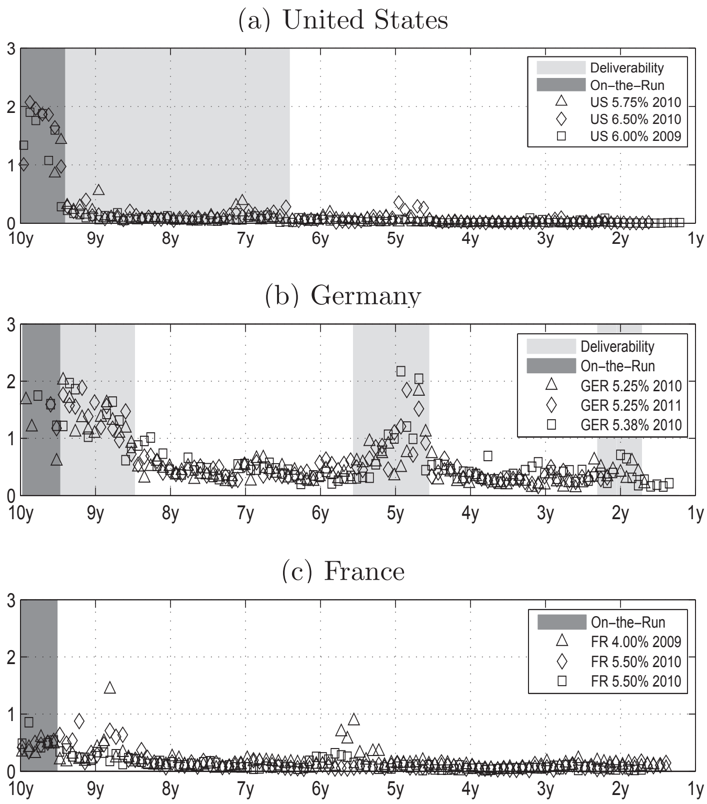

Figure 1: Monthly averages of daily trading volumes (EUR billion, on y-axes) as a function of time-to-maturity (years, on x-axis) for nine 10-year governments bonds. On-the-run and deliverability periods are shaded in darker and lighter colors, respectively. Source: ICMA.

notion of benchmark status in the German market, which diffuses short-horizon trading over a larger set of bonds. We find that the positive effect of deliverability has intensified during the recent financial crisis, probably reflecting that the ability to hedge positions has become even more important amid unusually high volatility.

Our contributions relative to the existing literature on liquidity premia in government bond markets are fourfold. First, we pay closer attention to a key feature of German government bonds, namely their deliverability into extremely liquid futures contracts such as the Euro-Bund future. We find that this feature, which has been largely neglected in most previous studies on euro area bond market liquidity, is key to explaining relative pricing along and vis-à-vis the German yield curve. Our emphasis on market structure also helps explaining the remarkable differences in liquidity premia found between the U.S. Treasury market and other government bond markets.

Second, in contrast to most previous studies conducted on euro area data, which typically have aimed at explaining levels of and variations in sovereign spreads, we take a single-issuer perspective and focus on Germany, the bellwether market for euro-area bond yields. This approach permits a richer cross-sectional analysis, simultaneously considering liquidity and liquidity premia for all outstanding bonds, and allows us to separately identify the effects of deliverability, on-the-run status and other liquidity determinants. Such identification could not have been achieved with the typical approach of comparing, say, ten-year benchmark yields across countries. As a control, we replicate our results with French bonds, which are issued in amounts similar to those of German bonds, but cannot be delivered into futures contracts.

Third, our empirical analysis is based on a very rich data set obtained from a European electronic limit-order market, MTS, containing high quality intra-day measures of liquidity (such as quoted depth and bid-ask spreads) for virtually all outstanding German and French bonds (among other issuers). Our data set covers both the periods before and after the onset of the financial crisis in mid-2007, which allows us to assess whether the determinants of liquidity and liquidity premia changed across these very different market regimes. We use the high-frequency quote data to form robust measures of market liquidity, which are superior to the 'snapshot measures' from a specific time of the day often used in the existing literature on euro area bond market liquidity.

Fourth, since premia related to deliverability contort the German yield curve in subtle ways, which cannot be captured with standard methods (such as the extended Nelson-Siegel specification), we use a flexible approach to yield curve estimation. By allowing for multiple (inverse) humps, our spline-based approach can accommodate the peculiar features of the German yield curve arising from the identified liquidity spillovers from the futures market. Figure 2 preempts the results of our curve estimation analysis. The stars and the circles represent observed spot yields on French and German bonds on a single day in 2008, plotted against their remaining maturity. The figure clearly reveals pronounced inverse humps along the German term structure, which in time-to-maturity terms coincide with the baskets of deliverable bonds for the futures contracts.[1]

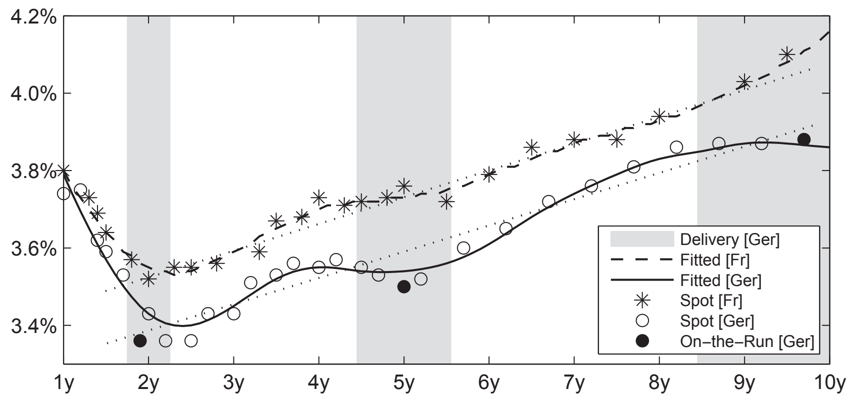

Figure 2: Actual and fitted spot rates for French and German bonds on 11 April 2008 (although plotted, the on-the-run securities are not included in the curve estimation).

The remainder of the paper is organized as follows. The next section discusses the economics of the on-the-run phenomenon, including a brief literature review. The third section presents our data set, and the fourth section examines the determinants of liquidity in the German government bond market. Section 5 examines to what extent liquidity and deliverability is priced. A final section concludes.

[1]The spot rates are bootstrapped from actual market yields according to the no-arbitrage principle. The dashed and the solid line represent the estimated curves for France and Germany, respectively, and as can be seen from the figure, the flexibility of the spline becomes important in capturing the relatively complex shapes of the two term structures. For comparison, we estimated the zero-coupon curves with another popular method, the extended Nelson-Siegel model. Its functional form however turned out to be too restrictive for the yiled curves experienced after August 2007.

2 The economics of the on-the-run liquidity phenomenon

The empirical observation that bond trading and liquidity concentrate on few issues is not necessarily surprising. First of all, it is unnecessary to hold (or short) the entire market portfolio, since a suitable combination of short-, medium-, and long-term bonds captures almost all the variation in the level and shape of the yield curve [Litterman and Scheinkman (1991); Bliss (1997)].[2,3] Once trading in certain maturities becomes customary, positive externalities will tend to reinforce it [Pagano (1989)].

The short-, medium-, and long-term bonds that are the most sensitive to yield curve risk within their maturity segment tend to become *benchmark* bonds [Yuan (2005); Dunne, Moore, and Portes (2007)]. Since benchmark bonds tend to be more liquid [Boudoukh and Whitelaw (1991); Higo (1999)] and therefore trade at lower yields [Boudoukh and Whitelaw (1993)], issuers make efforts to ensure that their bond issues will obtain benchmark status. For example, major sovereign issuers now auction bonds in accordance with an issuance calendar published in advance. This (shorter-term) predictability and transparency of issuance schedules contribute to reduced idiosyncratic price variation in the secondary market by alleviating supply uncertainty. Moreover, concentration of issuance on a few key maturities allows for larger issue sizes, which reduce the price impact of large trades. Related, Brandt and Kavajecz (2004) find that idiosyncratic price variation tends to increase with bond age (often referred to as 'seasonedness'). According to a commonly held view, the relative scarcity of seasoned bonds increases the price impact of trading. For this reason, the most recently issued bond usually becomes the benchmark, and the 'benchmark liquidity phenomenon' becomes indistinguishable from the 'on-the-run liquidity phenomenon'. In the literature, researchers commonly use the latter term to describe the positive liquidity effects (partially) caused by the former. From a theoretical point of view this is mislead-

[2] This is also supported by the sovereign issuance strategies. For example, most new debt issued by the G-10 countries has 2-, 5-, or 10-year maturities.

[3] Hedging or replication of the market return based on three key maturities is common in passive bond portfolio management, see Dynkin, Gould, Hyman, and Konstantinovsky (2006)].

ing because the two phenomena have different origins: benchmark bonds are traded by those who wish to gain or hedge yield curve risk with minimal exposure to idiosyncratic risk, and on-the-run bonds by those who rebalance their portfolios after government auctions [Pasquariello and Vega (2009)] or prefer securities trading near par [Eom, Subrahmanyam, and Uno (1998); Elton and Green (1998)].

Although conceptually distinct, the benchmark and on-the-run liquidity effects are mutually reinforcing because increased liquidity arising from scale is beneficial to all traders. Uninformed trading in the market for on-the-run bonds, like hedging or portfolio rebalancing, attracts informed traders who minimize the price impact of their trades by pooling with the uninformed [Kyle (1985); Chowdhry and Nanda (1991)]. Informed trading fosters price discovery and improves the hedging effectiveness of the on-the-run bonds, which, as a consequence, become benchmarks of their maturity segments.

Intermediaries such as market makers are able to offset their exposure to yield curve risk by short-selling benchmark bonds. Subsequently, however, hedgers have to borrow benchmark bonds from those who own them to cover the short positions in the cash market.[4] To achieve this, hedgers use the repurchase market where they search for bond lenders and bargain over the terms of bond loans. In the repurchase market, hedgers' uninformed demand for benchmark bonds induces bond lenders to increase their supply which, in turn, makes benchmark bonds easier to locate and reduces search costs [Duffie, Garleanu, and Pedersen (2007)]. Vayanos and Weill (2008) show that this virtuous circle arises because short-sellers are contractually bound to a particular bond, which is the one that they initially sold short and eventually will have to buy back and deliver in the repurchase contract. Because of this delivery constraint, market participants typically find it optimal to short the same security as everyone else, i.e. the benchmark bond. As shown by Duffie (1996), superior repurchase-market availability of benchmark bonds increases their value as collateral, leading to an counterintuitive outcome that active short-selling may in fact *inflate* cash prices. Yet the very same phenomenon

[4] Fisher (2002) provides a description of the use of repo markets for bond inventory management.

that causes distortions in benchmark prices, namely their repo-market availability, also facilitates price discovery. This is because informed investors' ability to implement their pessimistic beliefs via shorting benchmarks is key to efficient price discovery process [Diamond and Verrecchia (1987); Cohen, Diether, and Malloy (2007); Boehmer, Jones, and Zhang (2008)] that, ultimately, warrants the retention of the benchmark status itself. Figure 3 illustrates this market coordination process that ultimately leads to the superior liquidity of benchmark on-the-run Treasuries.

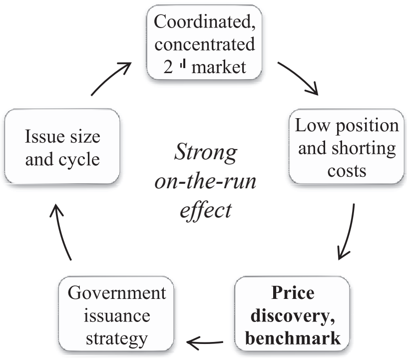

Figure 3: On-the-run effect in the cash market for U.S. Treasury securities.

As discussed above, a well-functioning repurchase market is key to cash market liquidity. On the supply side, market makers are able to lend out bonds and thereby leverage their capital, hold larger inventories, and provide more depth to the market. On the demand side, a large and dispersed investor base that ensures active trading and high liquidity is sustainable only if investors, who want to hedge or speculate with bonds that they do not already own, can take part in the market. For example, hedgers who sell and buy back benchmark bonds on a continuous basis increase the trading volume in the cash market, but are only able to do so using reverse repurchase contracts.

However, due to the multiplicity of markets and market participants involved in creating and maintaining liquidity, it is conceivable that multiple equilibria may occur, some of which may be characterized by low liquidity. Persistent pricing anomalies or market frictions reduce the usefulness of a benchmark for hedging or speculative purposes. For example, frictions in the repurchase market may force market makers to deleverage and cut back their liquidity provision in the cash market for including benchmark bonds. Also, cash market frictions may cause an inflationary spiral of shorting costs whereby investors gradually refrain from short-selling due to its trading intensive nature, and migrate to futures or swap market to create short positions.[5] Consequently, the decline in short selling in response to high shorting costs reduces cash market liquidity and shifts the locus of price discovery towards alternative markets. Brandt, Kavajecz, and Underwood (2007) as well as Mizrach and Neely (2008) provide recent empirical evidence from the U.S. Treasury market.

2.1 The German government bond market

Mainly as a consequence of its relative novelty, the euro-denominated sovereign bond market is still considerably more fragmented than the U.S. Treasury market. This fragmentation remains an impediment for the liquidity and informational efficiency of the European market, as order flow is dispersed over a large number of heterogeneous securities and markets. Consequently, positive externalities that arise when traders come together in space and time, namely better liquidity and/or price discovery, are not realized to the same extent as in the homogeneous U.S. Treasury market. The absence of 'spontaneous' liquidity described above leads to need for more 'artificial' liquidity providers in the form of market makers.

Notwithstanding the considerable widening of sovereign spreads in the course of the financial crisis, euro-area yields have converged dramatically relative to the pre-EMU period. This has created the conditions and the demand for common benchmark securities that accurately reflect the term structure of risk-free euro interest rates. Given that the benchmark status is gained through competition rather than being conferred,

[5]Establishing and maintaining a short position requires more trading than a long position because repurchase contracts are usually very short-term.

the multiplicity of sovereign issuers and the growth of euro-denominated swap market ensure that this implicit definition of benchmark bonds is ongoing in the euro area. In practice, 10-year German government bonds have retained their benchmark status within the euro area, owing to their relative liquidity and credit quality.[6] However, decentralized trading infrastructure in addition to a less well-established repurchase market increase the costs of taking and reversing short-term positions in the German cash market, which is why the bulk of trading and a major share of price discovery take place in the futures market [Bundesbank (2007); Upper and Werner (2007)] .[7] As a consequence, the benchmark status of German government securities may be attributed to both cash and futures markets: the futures contracts are the main instruments for hedging and speculating on euro area interest rates, while the cash instruments are primarily used for asset allocation purposes. This market organization contrasts with that of the U.S. Treasury market, where cash instruments, i.e. the benchmark on-the-run bonds, are used uniformly for pricing, positioning, and hedging.

In a futures-driven cash market, bonds that are deliverable for futures contracts may challenge the benchmark status of the on-the-run securities. This has been shown to be the case in the Japanese government bond (JGB) market, where the market's view of long-term yields is first reflected in the prices of JGB futures [Singleton (1996); Miyanoya, Inoue, and Higo (1999)], and then through arbitrage in the price of key deliverable bond and the rest of the JGBs [Shigemi, Kato, Soejima, and Shimizu (2001)]. Consistent with the arbitrage argument, Shigemi et al. report that the on-the-run and the key deliverable bond are the most actively traded JGBs in the cash market. In addition, Singleton (2004) finds that the key deliverable JGB has the highest sensitivity to changes in the term structure of all off-the-run JGBs, which corresponds to the

[6]Yields on French government BTANs and OATs are occasionally used as reference rates in the intermediate maturities.

[7]Bid-ask spreads in the EUREX futures market are approximately five to ten times smaller than in the MTS cash market. For comparison, the spreads in the cash and futures market for U.S. Treasuries are approximately equal.

argument by Yuan (2005) that benchmark status depends on securities' sensitivity to systematic risk. On the other hand, Singleton's results from the futures-driven JGB market contradicts those from the cash-driven U.S. Treasury market, where Brandt and Kavajecz (2004) find that the sensitivity to market risk declines monotonically in bond seasonedness.

Given the extremely large and liquid futures market for German government securities, one would expect that the relation between the cash and the futures market resembles that of the Japanese market rather than the U.S. Treasury market. As an initial assessment of this conjecture, we estimate the market sensitivities of German on- and off-the-run bonds as a crude measure of benchmark characteristics, and compare these sensitivities to those reported by Brandt and Kavajecz (2004). The bond-specific sensitivity is measured by the amount of yield variation explained by the three first principal components estimated from the term structure of German bonds. The results shown in Table 2 indicate that the German off-the-run bonds, which typically are the key deliverable bonds, reflect to changes in the term structure more precisely than on-the-run bonds. The exact opposite holds for the U.S. Treasury market, where the on-the-run bonds are most sensitive to yield curve risk. Overall, the results in Table 2 and the previous studies on the JGB market suggest that the on-the-run bonds would share the benchmark status with deliverable bonds in the German cash market.

What does the predominance of futures trading in the German market imply for the emergence of liquidity differences between bonds? The more diffuse benchmark status (shared among the bonds in the deliverable basket) contrasts with the unambiguous benchmark status of the on-the-run treasuries, and would suggest that liquidity differentials in the futures-driven German bond market ceteris paribus should be smaller than in U.S. market. Results of Witherspoon (1993) point to a certain threshold level in the informativeness of cash markets relative to futures markets, above which the benchmark status of on-the-run securities (and the liquidity effects thereof) is supported. If the futures market is too dominant with respect to price discovery, it tends

Table 2: The explanatory power of the first three principal components.
This table presents the percentages of yield variation explained by the three first principal components extracted from the correlation matrix of daily changes in German term structure. The sample includes observations on on- and off-the-run bonds in 2-, 5-, and 10-year maturities for the period January 2006-September 2008. The results for U.S. Treasury securities are from Brandt and Kavajecz (2004).

	Adjusted R^2	
Maturity	Germany	United States
2-year		
On-the-run	96.91%	99.57%
Off-the-run	97.27%	99.14%
5-year		
On-the-run	96.65%	99.44%
Off-the-run	97.77%	99.15%
10-year		
On-the-run	98.08%	99.28%
Off-the-run	98.36%	98.72%

to hamper the cash market liquidity due to substitution, but may otherwise enhance the liquidity and price discovery of deliverable bonds through cross-market arbitrage [Holden (1995)].[8]

Indirect evidence of cross-market arbitrage can be seen in the Figure 1 in the Introduction. This figure plots average daily trading volumes for 10-year bonds issued by the United States, Germany and France. The periods during which the bonds are on-the-run and deliverable for futures contracts are shaded with darker color. Maturities where bonds are deliverable, but no longer on-the-run are shaded in a lighter color. As opposed to 10-year U.S. Treasuries in Figure 1a and French OATs in Figure 1c, German Bunds in Figure 1b continue to be actively traded well after the six month on-the-run period and the volume of trading remains high for another year until the bonds are no longer deliverable for the 10-year futures contract. Indeed, the trading activity of

[8]Cross-market arbitrage had grown so popular that in 2003 Eurex launched "basis instruments" for German government bond market, which involve opposite positions in futures and cash markets.

off-the-run Bunds in Figure 1b appears to be governed by deliverability; trading seems to be less active through the periods of non-deliverability, only to become more intense again as seasoned Bunds again become deliverable.

A similar volume pattern does not obtain for U.S. Treasury securities, despite the fact that they are deliverable for the 10-year futures contract traded at the Chicago Board of Trade. A possible explanation is that the simultaneous price discovery in cash and futures markets weakens the cross-market lead-lag effect and thereby makes arbitrage less profitable and trading less worthwhile. Also, the delivery basket for the U.S. 10-year futures contains considerable more securities than in the German case, making arbitrage-based trading less observable in individual securities.

To sum up, costly frictions in the cash market for German government bonds would suggest a diversion of order flow away from the cash instruments and towards futures contracts. Low transaction costs and the ease of taking short positions in the futures market attracts both uninformed as well as informed traders. For this reason, German futures contracts dominate price discovery in euro interest rates over cash bonds.

This is a key difference from the U.S. Treasury market, where trading in the on-the-run bonds and futures contracts are complementary with regard to price discovery. As much an outcome as a cause, the on-the-run U.S. Treasuries are liquid relative to off-the-run securities and actively traded for hedging and speculative purposes. In the absence of such trading, such as for German on-the-run bonds, one would expect the liquidity differentials between on- and off-the-run bonds to be much less pronounced. Indeed, turnover and the related positive liquidity effects may be even greater for German *off-the-run* bonds, since they are typically the cheapest-to-deliver into the two-, five-, and ten-year futures contracts and therefore subject to cross-market arbitrage trading. Figure 4 illustrates this particular relationship between the German cash and futures markets.

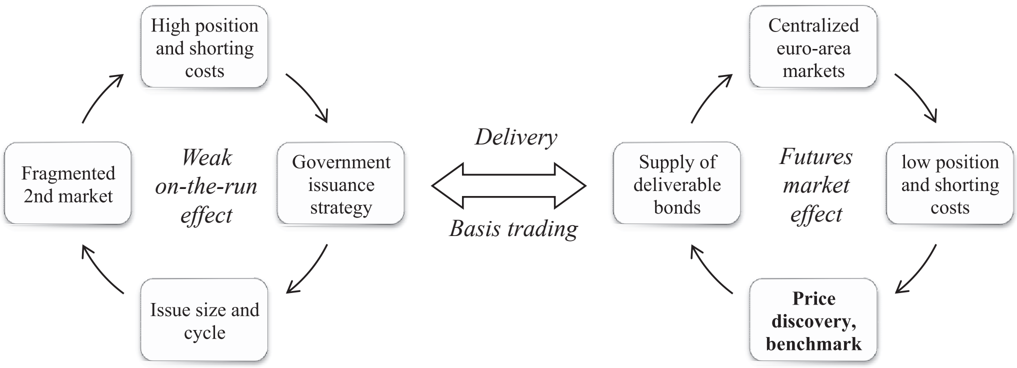

Figure 4: The combined cash and futures market effect in the German market.

3 Data

As is the case with most government bond markets, the secondary market for German government bonds is predominantly an over-the-counter market. Trading takes place mostly between dealers, either using traditional voice brokers and bilateral negotiation, or increasingly through electronic platforms. The source of our data on bond prices, quoted depth and quoted bid-ask spreads is MTS, the largest electronic trading venue for German government bonds, see Bundesbank (2007). MTS is a system of quote-driven platforms with designated market makers who compete for other market participants' order flow. Market makers supply liquidity for the bonds assigned to them by providing two-way proposals of a minimum size for at least five hours a day.

Our sample extends from January 2006 through September 2008. This period is particularly suitable for analysing government bond market liquidity as it covers both the tranquil period before mid-2007 as well as the turbulent period following the onset of the financial crisis.

Overall, our data include approximately ten million quotes and sixty thousand trades on bonds issued by the Federal Republic of Germany. The quote records include three best bid and offer quotes with the associated quote sizes at tick-by-tick frequency. Since quotes on MTS are binding unless withdrawn, the quote records allow us to obtain reliable estimates of the transaction costs that the market participants face as well as the size of the inventory that is available for immediate trade.[9] The transaction records include prices and quantities with an indicator variable of the direction of the trade (buy or sell). Every quote and trade entry in our records is identified by an individual security identification number (ISIN) and a time stamp recorded to the nearest millisecond. Bond issue sizes are provided by the German Finance Agency.

Despite its significant role in electronic trading, the MTS transactions constitute only a small fraction of the overall trading volume in German government bonds. For that reason, we supplement our MTS data with trading volume information provided by International Capital Market Association (ICMA) through Datastream. Analogous to GovPX in United States, ICMA collects and disseminates data on transactions made by its members in the over-the-counter markets. Approximately 400 financial institutions, including the largest dealers in German government bond market, report their trades to ICMA. The sample for traded volumes covers the period January 2002 through February 2009.

Following the findings of previous research, and reflecting the firm-quote nature of our data, we use traded volumes, quoted depths and quoted bid-ask spreads as our measures of liquidity. The quoted spread is defined as the difference between best ask and bid price and is measured in percent of the midpoint price. The bid-ask spread alone, however, does not provide any information about the amounts available for trading at a given time. We therefore also include market depth as a complementary measure of liquidity. Market depth is proxied by the average volume available for trading at the best three bid and offer prices.[10] Both quoted depths and spreads, which are observed at the intra-day frequency, are collapsed into representative daily

[9] To mitigate concerns that quotes are actually not firm, we compare transaction prices to standing quotes. We find that two thirds of the transactions in our sample are made exactly at the quoted prices. For the remaining third of the trades, the differences between quoted prices and transaction prices were small.

[10] Since MTS allows large transactions to be executed as iceberg orders, i.e. partially outside the order book, the market may be actually deeper than the cumulative depth indicates. We do not have data on the iceberg orders, but MTS reports that their share of all orders is less than two percent.

values by taking the median. This is an effective way of removing outliers, which is a serious problem when using end-of-day (or 'snapshot') quotes.

4 Determinants of liquidity in the German bond market

The aim of this section is to empirically assess whether liquidity differences across German government bonds are explicable in terms of deliverability into futures contracts. For this purpose, we consider four different liquidity measures: traded volumes, quoted depths, quoted bid-ask spreads and the 'liquidity index' proposed by Bollen and Whaley (1998). By constructing an (unbalanced) panel consisting of time-series observations (on liquidity measures and potential liquidity determinants) for a large cross-section of bonds, we can separately identify the impact on liquidity of deliverability and 'on-the-run' status. With respect to the impact of deliverability, we distinguish between 'cheapest-to-deliver' (CTD) bonds, and bonds which are merely deliverable.[11] We control for multiple other factors which have previously been found to determine liquidity. The set of control variables includes time to maturity, seasonedness (i.e. bond age) and issue size. Since our main interest is in the cross-sectional variation in liquidity between bonds with different characteristics, we also include time dummies. Time dummies help us overcome the potentially important short-coming that the MTS data reflect activity on electronic trading platforms and not the entire market. Anecdotal evidence suggests that in addition to the general decline in liquidity after July 2007, the market share of electronic platforms have declined.[12] By including time dummies, we minimize the impact of any trend in market share on our results.

To be more confident that any deliverability-related liquidity effects we may de-

[11] Owing to the construction of the so-called conversion factors, during our entire sample, the CTD bonds are consistently the outstanding bond with shortest remaining time to maturity of the bonds in the delivery basket.

[12] As volatility rose precipitously after mid-2007, market participants apparently became increasingly reluctant to supply liquidity to each other in the form of tradeable buy or sell quotes in limit-order markets.

tect are genuine, we conduct identical analyzes for a control country lacking a futures market. For this purpose we use France, as the French government bond market is comparable to the German market in terms of credit rating, currency and amounts outstanding in the individual bonds. In the following, we analyze the determinants of traded volumes, quoted depths, quoted bid-ask spreads and the liquidity index.

4.1 Determinants of traded volumes

To assess the determinants of traded volumes, we regress log average daily volume on time dummies (for each month), deliverability dummies, cheapest-to-deliver dummies, on-the-run dummies, time to maturity (measured in years), seasonedness (also measured in years) and log issue size.

The deliverability dummies reflect the EUREX criteria determining whether a particular bond is eligible for delivery into the 2, 5 and 10-year German bond futures. Eligible bonds for these three contracts have remaining maturity in the ranges 1.75-2.25 years, 4.5-5.5 years and 8.5-10.5 years, respectively. This gives rise to three deliverability dummies.[13] Note that (the compounded value of) the coefficients on these dummies can be interpreted as the percentage increase in trading volume for bonds belonging to the particular maturity bracket (relative to bonds in any of the undeliverable maturity brackets). We also include specific cheapest-to-deliver (CTD) dummies (one for each of the 2, 5 and 10-year futures contracts) taking the value one when a given bond is CTD into the next-to-expire futures contract, and zero otherwise.[14]

The remaining estimated coefficients also have interesting interpretations. The coefficient on the on-the-run dummies gauge the impact on trading volumes related to

[13] A newly issued 10-year bond will first be deliverable into the 10-year futures and then experience a time period where it is not deliverable (from 8.5 to 5.5 years remaining maturity) before it again becomes deliverable into the 5-year futures, and so on. For maturities below 1.75 years, the bond will never again become deliverable.

[14] We use the implied repo rate method to identify the cheapest-to-deliver bonds for each date and futures contract.

a bond being the most recently issued bond of a given original maturity. As mentioned above, studies on U.S. Treasuries typically find very large on-the-run effects on liquidity. The coefficient on the seasonedness variable can be interpreted as the annual percentage decay in trading volume as the bond ages. One would expect that trading volume (and other liquidity measures) decline as a bond ages, because an increasingly large fraction of the issued amount ends up in buy-and-hold portfolios. By controlling for other liquidity determinants in a panel setting (in particular deliverability and on-the-run effects, and developments in overall market liquidity as captured by the time dummies), we can identify the pace of such decay. Finally, the coefficient on the (log) issue size provides the elasticity of trading volumes with respect to issued amounts.[15]

Table 3 displays the results for the determinants of trading volumes for German bonds, and as a control, for French bonds. We first consider the results for Germany. Lines 2-4 of the table show that the impact of deliverability in all cases have the expected positive sign, and the coefficients are all highly statistically significant. The estimated effects of deliverability are economically important, as the estimated coefficients between 0.54 and 1.05 correspond to increases in trading volumes between 72% and 186%.[16] The next three lines in the table reveal that a bond tends to experience an *additional* boost in trading volumes when it is the cheapest-to-deliver bond. The (compounded) increases in trading volume for CTD bonds (relative to comparable non-deliverable bonds) are 148%, 253% and 229% for the 2, 5 and 10-year maturities.[17] On the other hand, on-the-run status *per se* has a somewhat smaller effect, increase trading by around 100%. Although the on-the-run effect on volumes is positive and highly statistically significant, it is smaller than the effects related to being cheapest-to-

[15] Lacking a time series of real-time outstanding amounts, we use outstanding amounts at the end of our sample. This of course ignores changes over time due to tap issues. Therefore we may overstate somewhat the outstanding amounts in some cases, and thus underestimate the true coefficient.

[16] The compounded effects are obtained as the exponential of the relevant estimated coefficients minus one.

[17] In this case, the compounded effects are obtained as the exponential of the sum of the relevant estimated coefficients (e.g. 2-year deliverability and 2-year CTD) minus one.

Table 3: Determinants of trading volumes for German and French bonds
The dependent variable is log trading volume. Asterisks *, **, *** after robust t-values (in parentheses) denote values significantly different from zero at the 10%, 5%, and 1% levels, respectively. Monthly observations from Jan. 2002 through Feb. 2009 (T=86).

	Germany		France	
	Slope	t-value	Slope	t-value
Intercept	-13.12	(-2.62)***	-13.64	(-2.40)**
$1.75 \leq$ maturity < 2.25	0.54	(8.27)***	0.38	(5.08)***
$4.50 \leq$ maturity < 5.50	0.67	(8.75)***	0.33	(3.67)***
$8.50 \leq$ maturity < 10.50	1.05	(7.99)***	0.51	(5.73)***
Cheapest-to-deliver for 2-year future	0.37	(2.78)***		
Cheapest-to-deliver for 5-year future	0.59	(7.45)***		
Cheapest-to-deliver for 10-year future	0.14	(1.19)		
On-the-run status	0.69	(4.88)***	0.59	(5.96)***
Seasonedness (in years)	-0.08	(-4.54)***	-0.12	(-7.19)***
Time to maturity (in years)	-0.02	(-1.79)*	0.01	(1.22)
Log issue size	1.42	(6.64)***	1.38	(5.79)***
Month-fixed effects	Yes		Yes	
Sample	Jan 02-Feb 09		Jan 02-Feb 09	
Number of months	86		86	
Number of bonds	109		66	
Number of month-bond obs.	4427		3024	
Adjusted-R^2	0.73		0.64	

deliver. This comparatively modest on-the-run effect contrasts with the overwhelming effect seen in studies using U.S. Treasury data. The decay related to bond aging (seasonedness) is estimated to be around 8% per year. This implies, for example, that an old 30-year with eight years remaining maturity would attract less than a fifth of the trading volume of a two-year-old 10-year bond with same remaining maturity and issue size. Finally, we find the elasticity of trading volumes with respect to the amount issued to be higher than one. This may reflect that not only are large issues traded more, the resulting enhanced liquidity (in terms of depth and expected transaction costs) may also feed back positively on trading in the bond.

The two rightmost columns of Table 3 display the comparable results for French government bonds. A first thing to note is that all significant coefficients have the same sign as in the German case. Also, all the coefficients of the control variables have very similar magnitudes. There are, however, notable differences in the relative size of the coefficients on the deliverability and on-the-run dummies. In particular, the dummies for the three maturity brackets considered (corresponding to 'deliverability' in the German case) have coefficients which are below that of the on-the-run dummy. This considerably smaller 'deliverability' effect for French bonds probably reflects the absence of a liquid futures market for French government bonds. It should be noted, though, that even these hypothetically 'deliverable' French bonds tend to trade significantly more than their 'non-deliverable' counterparts. A possible explanation is that French bonds which match the maturity requirements for the German futures also can be quite accurately hedged with positions in these futures. Moreover, higher cash-market liquidity for German bonds would make cross-country spread trades cheaper to execute. Therefore, both direct and indirect liquidity spillovers from the German futures market into the French cash bond market are conceivable.

Tables 13-14 in Appendix A show the results when the data set is split in pre-crisis and crisis samples. While the main results remain unchanged, it is notable that the 'deliverability' effect French bonds declined in the crisis sample. This may reflect that the ability to hedge French bonds with German futures was hampered by the dramatic increase in the level and variability of the French-German yield spread. Thus liquidity spillovers to 'deliverable' French bonds may well have declined.

As a robustness check, Table 12 (also in Appendix A) displays the corresponding results for the full-sample panel regressions, but without time dummies. The estimated coefficients remain virtually unchanged and the 85 dummies add relative little to the overall explanatory power of the model. This clearly suggests that the inclusion of time dummies does not drive the results.

4.2 Determinants of quoted depths

Table 4 shows the results of similar panel regressions, but now using quoted debts as the dependent variable. For the pre-crisis sample (the two leftmost columns), quoted depths can be broadly explained by time-to-maturity, seasonedness and log issue size. However, even in this tranquil period, there is evidence that deliverability increases quoted depths. The effects are however smaller than for traded volumes.

In the crisis sample (from July 2007 to September 2008), the importance of deliverability become more pronounced.[18] The amount available for immediate trading at firm quotes was thus significantly higher for deliverable bonds. This holds for all three futures contracts considered (2, 5 and 10 years). Interestingly, the status as 'cheapest-to-deliver' does not appear to add extra depth in this period. This suggests that it is the ability to hedge a given bond with a futures contract which matters for liquidity, rather than the prospects of actual delivery. It is also noteworthy that during the crisis, on-the-run status became insignificant for German bonds. Overall, the coefficients for the remaining controls are quite comparable over the sub-samples: seasonedness and time-to-maturity have the expected signs and are always highly significant. This is in line with the inventory view, where bonds with a long time to maturity (and thus the

[18] This is formally confirmed by a joint exclusion test (F-test) for sub-period dummies interacted with deliverability variables, carried out in regression for the entire sample.

high interest rate risk) are less liquid, as are bond which are 'old' (because the have increasingly ended up in 'buy-and-hold' portfolios). As expected, issue size is important for depth, although much less so than it was for volumes.

For the control country, France (see Table 15 in Appendix A), in the pre-crisis sample, 'deliverability' had a positive effect on depth, but again less than for Germany. The 'on-the-run' status was again found to be quantitatively more important than 'deliverability' for French bonds (in both sub-samples).

4.3 Determinants of quoted bid-ask spreads

Table 5 shows the results for quoted bid-ask spreads. The results for the pre-crisis period are somewhat puzzling: three of deliverability dummies are significant, but have the wrong sign. On-the-run status, on the other hand, has the right negative sign (i.e. spreads are tighter for on-the-run issues), although not strongly significant. One possible explanation is that in the pre-crisis sample, market makers had quoting obligations (i.e. they had to post bid and ask prices which complied with a certain maximum spread). Our results suggests that these spreads were to a very large extent determined by bond characteristics such as time to maturity, seasonedness and issue sizes. Note also that R^2 is as high as 0.93 in this case.[19]

During the crisis sample, where market-maker obligations were suspended most of the time, the picture changed somewhat. Two of the deliverability dummies become significant, and they also have the expected negative sign. Quantitatively, the estimated effects on spreads remain rather small, though. This may indicate that for smaller trade sizes, the distinction between deliverable and non-deliverable bonds may not be particularly important. Market-makers may be willing to provide liquidity in the form of relatively tight bid-ask spreads for small amounts also in non-deliverable bonds. It seems plausible, on the other hand, that if market makers are to provide substantially liquidity in the form of tight bid-ask spreads for large amounts, the ability to hedge

[19]Table 16 in Appendix A shows the comparable results for France, which are broadly similar.

TABLE 4: DETERMINANTS OF QUOTED DEPTH FOR GERMAN GOVERNMENT BONDS
The dependent variable is monthly averages of daily cumulated (log) depth. Asterisks *, **, *** after robust t-values (in parentheses) denote values significantly different from zero at the 10%, 5%, and 1% levels, respectively.

	pre-crisis		crisis	
	Slope	t-value	Slope	t-value
Intercept	4.00	(1.97)**	9.20	(4.60)***
$1.75 \leq$ maturity < 2.25	0.32	(6.51)***	0.34	(7.89)***
$4.50 \leq$ maturity < 5.50	0.04	(0.68)	0.38	(7.90)***
$8.50 \leq$ maturity < 10.50	0.16	(2.18)**	0.47	(10.10)***
Cheapest-to-deliver for 2-year future	-0.06	(-0.97)	-0.06	(-0.94)
Cheapest-to-deliver for 5-year future	0.09	(2.67)***	0.05	(0.67)
Cheapest-to-deliver for 10-year future	0.07	(0.94)	-0.05	(-1.18)
On-the-run status	0.20	(3.26)***	0.09	(1.25)
Seasonedness (in years)	-0.04	(-4.51)***	-0.05	(-5.24)***
Time to maturity (in years)	-0.05	(-13.88)***	-0.03	(-10.46)***
Log issue size	0.58	(6.75)***	0.35	(4.18)***
Month-fixed effects	Yes		Yes	
Sample	Jan 06-Jun 07		Jul 07-Sep 08	
Number of months	18		15	
Number of bonds	75		75	
Number of month-bond obs.	902		748	
Adjusted-R^2	0.78		0.71	

with futures likely becomes more important. To better capture both the depth and spread dimensions of liquidity simultaneously, we finally consider a 'liquidity index' defined as the quoted depth divided by the bid-ask spread.

4.4 Determinants of the liquidity index

The liquidity index is intended to capture the possibility that despite tightly quoted bid-ask spreads, a market may not necessarily be liquid with respect to execution of larger trades. Similarly, although quoted depth is a quite informative measure, it does not take into account the tightness of the market: there may large depth, but if bid and ask prices are far apart, such a situation would not necessarily correspond to a liquid market. To ensure the robustness of our findings against such short-comings of the one-dimensional liquidity measures, we present in Table 6 the results of the same regressions as above, but now using the liquidity index as the dependent variable. On this alternative measure of liquidity, the importance of deliverability clearly rose in the crisis sample for the 5 and 10-year maturities. In the pre-crisis samples, the liquidity index could be explained almost most exclusively by time to maturity, seasonedness and issue size.

Overall, this section has provided three main results. First, the liquidity of German bonds which were deliverable into the nearest-to-expiry futures contracts were found to be superior to non-deliverable bonds, when controlling for relevant bond characteristics such as time to maturity, seasonedness and issue size. Second, the positive impact on liquidity of belonging to the deliverable maturity intervals was consistently found to be higher for German bonds than for the control (French bonds), and - consistent with the more diffuse benchmark notion in the German market - the importance of 'on-the-run' status was found to be correspondingly lower for German bonds. Third, with respect to the comparison across market regimes, i.e. the pre-crisis versus crisis samples, we found that the importance of deliverability generally increased in the crisis sample. We now turn to the question, whether deliverability is also priced into German bond yields,

TABLE 5: DETERMINANTS OF QUOTED BID-ASK SPREADS FOR GERMAN GOVERNMENT BONDS

The dependent variable is monthly averages of daily (log) bid-ask spreads. Asterisks *, **, *** after robust t-values (in parentheses) denote values significantly different from zero at the 10%, 5%, and 1% levels, respectively.

	pre-crisis		crisis	
	Slope	t-value	Slope	t-value
Intercept	10.25	(4.42)***	12.42	(7.41)***
$1.75 \leq$ maturity < 2.25	0.03	(0.55)	-0.10	(-2.11)**
$4.50 \leq$ maturity < 5.50	0.36	(4.89)***	0.03	(0.37)
$8.50 \leq$ maturity < 10.50	0.25	(3.38)***	-0.12	(-1.66)*
Cheapest-to-deliver for 2-year future	0.26	(2.62)***	0.41	(2.57)**
Cheapest-to-deliver for 5-year future	-0.02	(-0.55)	0.05	(0.43)
Cheapest-to-deliver for 10-year future	-0.09	(-1.40)	-0.12	(-0.66)
On-the-run status	-0.25	(-2.36)**	-0.25	(-2.31)**
Seasonedness (in years)	0.03	(3.33)***	0.04	(5.95)***
Time to maturity (in years)	0.09	(16.13)***	0.09	(18.61)***
Log issue size	-0.43	(-4.37)***	-0.52	(-7.32)***
Month-fixed effects	Yes		Yes	
Sample	Jan 06-Jun 07		Jul 07-Sep 08	
Number of months	18		15	
Number of bonds	75		75	
Number of month-bond obs.	846		736	
Adjusted-R^2	0.93		0.91	

TABLE 6: DETERMINANTS OF LIQUIDITY INDEX FOR GERMAN GOVERNMENT BONDS
The dependent variable is monthly averages of daily (log) depth divided by bid-ask spread. Asterisks *, **, *** after robust t-values (in parentheses) denote values significantly different from zero at the 10%, 5%, and 1% levels, respectively.

	pre-crisis		crisis	
	Slope	t-value	Slope	t-value
Intercept	5.83	(2.83)***	13.11	(5.71)***
$1.75 \leq$ maturity < 2.25	0.15	(2.79)***	0.20	(2.87)***
$4.50 \leq$ maturity < 5.50	0.08	(1.06)	0.46	(5.97)***
$8.50 \leq$ maturity < 10.50	0.10	(1.05)	0.57	(6.31)***
Cheapest-to-deliver for 2-year future	-0.33	(-2.94)***	-0.42	(-2.20)**
Cheapest-to-deliver for 5-year future	0.01	(0.34)	-0.06	(-0.59)
Cheapest-to-deliver for 10-year future	0.09	(1.06)	0.12	(0.83)
On-the-run status	0.30	(2.88)***	0.25	(1.58)
Seasonedness (in years)	-0.05	(-5.43)***	-0.07	(-6.54)***
Time to maturity (in years)	-0.13	(-21.83)***	-0.13	(-18.33)***
Log issue size	0.84	(9.53)***	0.52	(5.33)***
Month-fixed effects	Yes		Yes	
Sample	Jan 06-Jun 07		Jul 07-Sep 08	
Number of months	18		15	
Number of bonds	75		75	
Number of month-bond obs.	902		748	
Adjusted-R^2	0.94		0.91	

either directly or indirectly through enhanced liquidity.

5 Price effects of liquidity and deliverability

Having established the positive relation between deliverability and a range of liquidity measures for German government securities, we now examine whether liquidity in general and deliverability in particular are priced. We refrain from using German non-deliverable bonds as pricing benchmarks since their *future* deliverability may affect current prices in the form of liquidity or convenience premium. Instead, we compare the yields on deliverable and non-deliverable German securities to those of France. Besides having monetary policy in common, France is, as mentioned above, a natural choice as a benchmark since the amount of outstanding French debt corresponds to that of Germany in both absolute and relative (% of GDP) terms. Partly as a consequence, the difference in the credit quality are small, which allows us to pin down more precisely the marginal valuation of liquidity. In addition, France does not have a bond futures market, and this allows us to identify the impact of deliverability on German bond yields.

5.1 Variable construction

In order to obtain and easily compare French and German bond yields at different maturities, we estimate continuous zero-coupon yield curves for both countries using smoothed cubic spline interpolation. Cubic splines have been widely used in the literature, and this functional form provides enough flexibility to capture local yield effects that may arise from market segmentation. Once the two curves are estimated from the cross-section of French and German bond prices, we compute the yield spread between the two curves at maturities $m = \{1.0, 1.5, ..., 10.0\}$, with additional observations at 1.75 and 2.25 year maturities.[20] These maturities cover the most relevant part of the

[20]To ensure that the set of securities used in the estimation procedure is homogenous, the following types of securities are excluded: securities with floating rate coupons, securities with remaining maturity

yield curve, and the following maturity subsets 1.75−2.25, 4.5−5.5, and 8.5−10.0 that correspond to the baskets of EUREX German government securities deliverable for 2-, 5-, and 10-year futures contracts, respectively. We calculate these yield differentials for each trading day from January 2006 through September 2008.[21]

We attempt to explain the yield spread across maturities and over time with the difference between French and German bond market liquidity and the deliverability of German securities. To accomplish this, we use the MTS data on French bonds to compute daily liquidity measures similar to those described in Section 3. Once we have the necessary liquidity measures for both French and German bonds, we average them across non-overlapping maturity brackets centered on each maturity m. That is, we use group averages instead of individual values in order to mitigate individual bond effects and non-synchronous maturities. The valuation effects of deliverability are captured by a set of dummy variables that correspond to the ranges of deliverable maturities specified in EUREX futures contracts.

In addition to liquidity and deliverability, several recent studies find that perceived differences in the credit quality of euro-area sovereign issuers have effects on the relative pricing of their bonds.[22] Although we minimize this effect by comparing two sovereign issuers with similar fiscal fundamentals, a market-based measure that is available on daily basis is nevertheless desirable to capture additional aspects of governments' perceived credit quality. For this reason, we augment our empirical model of the interest rate spread with data on sovereign credit default swaps (CDS). [23]

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less than one year, securities with issue size less than EUR 5 billion, securities issued in non-euro currencies, securities originating from a coupon-stripping program, securities issued by government special fund, and inflation- or index-linked securities. Moreover, to disentangle the hypothesized price effect of on-the-run status, the most recently issued securities are excluded as well.

[21] The fit of the spline function is good for both countries, with the mean absolute fitting error being less than one basis point.

[22] See, for instance, Codogno, Favero, and Missale (2003); Bernoth, von Hagen, and Schuknecht (2004); Beber, Brandt, and Kavajecz (2009); Schuknecht, von Hagen, and Wolswijk (2008)

[23] A sovereign CDS is contract that allows the investor to hedge against the event that a particular government defaults on its debt. In exchange for this 'credit protection', the investor agrees to make

A convenient property of CDS premiums is that they provide more direct reflections of the market's assessment of sovereign credit risk. This allows us to compute the credit risk premium on a five-year German government bond simply by subtracting the premium paid on a five-year CDS contract from bond's par yield. In the case of non-integer maturities for which the CDS contracts are not traded, we use observed premiums on nearby contracts to linearly interpolate the missing intermediate ones. Once the credit risk premium is netted out, we can decompose the residual par yield into elements associated with risk-free rate, liquidity, and deliverability. Consequently, French and German spot rates obtained from the spline estimation are transformed into par yields.

Finally, motivated by the results of Krishnamurthy and Vissing-Jorgensen (2007), we control for a potential negative relationship between aggregate supply and pricing of government debt. In particular, we calculate the average sizes of outstanding French and German bond issues on a daily basis and use their logarithmic difference to gauge changes in the relative supply of national debt. We also include year dummies and time-to-maturity as additional control variables to capture any unobserved factors that might affect the relative valuation.

Table 7 presents the summary statistics for yield, liquidity, quality, and issue size differentials between France and Germany. To facilitate comparison across maturities, the statistics are categorized by maturity segments that correspond to permanently non-deliverable maturities as well as 2-, 5-, and 10-year delivery baskets. As shown in Table 7, the differentials of par yields, bid-ask spreads, and credit risk are consistently positive across maturities, whereas log issue size differentials are negative. Taken together, this imply that investors perceive liquidity and credit quality of French bonds to be slightly inferior to their German counterparts, and this may explain that the French securities

periodic payments, known as premiums, to the seller of the contract over its life or until the government defaults. Compiled by Credit Market Analysis Ltd. and provided by Thomson Datastream, our CDS data is based on daily indicative bid premiums quoted by thirty key market participants for contracts on French and German government bonds with average residual maturities from one to ten years.

command higher yields across the yield curve. Several other results should be noted from Table 7. In our sample, the average size of French issues is approximately six per cent smaller than that of German bonds, and the size differential varies from -12% to zero. Yet despite the larger stock of debt, German bonds seem to trade at yield levels that are economically and statistically lower than those required on French securities, which suggests that the positive liquidity effects associated with larger float outweigh the direct supply effects. In addition, the variables in Table 7 exhibit substantial variation both in the cross-sectional and time-series dimensions, which motivates the use of panel estimation techniques.

5.2 Empirical results

To empirically test the conjectured relation between the French-German yield spread and the factors associated with relative liquidity, quality, and deliverability, we pool our data in a panel that includes a time series of daily observations from 2 January 2006 to 30 September 2008 for each maturity m. We split the panel into two sub-samples, using 1 July 2007 as the break to investigate whether the economic importance of our valuation factors change after the onset of the financial crisis. To this end, we estimate the following econometric model both for the pre-crisis period and the crisis period using panel least squares regression:

$$\begin{aligned} R^F_{t,m} - R^G_{t,m} = \alpha &+ \beta_1(LIQ^F_{t,m} - LIQ^G_{t,m}) + \beta_2(CDS^F_{t,m} - CDS^G_{t,m}) \\ &+ \beta_3[\log(AIS^F_t / AIS^G_t)] + \boldsymbol{\delta}(\mathbf{DEL_m}) + \boldsymbol{\lambda}(\mathbf{X_{t,m}}) + \varepsilon_{t,m} \qquad (1) \end{aligned}$$

where

$$\mathbf{DEL_m} = \{NONDEL_{<1.75}, DEL_{1.75-2.25}, DEL_{4.5-5.5}, DEL_{8.5-10.5}\}$$

$$\mathbf{X_{t,m}} = \{m,\ year\ dummies\}$$

and $R^i_{t,m}$ denotes the estimated par yield for country i and maturity m at day t. $CDS^i_{t,m}$

TABLE 7: SUMMARY STATISTICS ON VARIABLES USED IN MULTIVARIATE ANALYSIS

This table presents the summary statistics on the variables used in multivariate analysis for various maturities m. Asterisks *** after robust t-values (in parentheses) denote values significantly different from zero at the 1% level. Daily observations from January 2nd 2006 to September 30th 2008.

	Mean	t-value	Min	Max	SD		Mean	t-value	Min	Max	SD
Par yield differential (*bps*)						Log depth differential					
$m<1.75$	2.17	(4.19)***	-2.00	9.00	1.71	$m<1.75$	-0.19	(-2.01)***	-1.09	0.89	0.33
$1.75\leq m<2.25$	3.61	(17.73)***	-1.00	18.00	3.77	$1.75\leq m<2.25$	0.06	(0.97)	-1.10	0.81	0.38
$4.50\leq m<5.50$	4.28	(15.59)***	-4.00	27.00	5.94	$4.50\leq m<5.50$	-0.42	(-2.96)***	-2.30	0.86	0.48
$8.50\leq m<10.0$	10.79	(10.97)***	2.00	34.00	6.76	$8.50\leq m<10.0$	-0.29	(-3.61)***	-1.94	1.79	0.48
All	5.21	(6.16)***	-4.00	34.00	5.96	All	-0.23	(-4.14)***	-2.30	1.95	0.42
Percentage bid-ask spread differential (*bps*)						Credit risk differential (*bps*)					
$m<1.75$	0.79	(4.62)***	-3.00	67.00	3.47	$m<1.75$	0.60	(17.3)***	-3.95	6.65	1.48
$1.75\leq m<2.25$	1.26	(13.34)***	-3.00	45.00	2.48	$1.75\leq m<2.25$	0.64	(10.24)***	-5.60	7.00	1.77
$4.50\leq m<5.50$	3.48	(11.88)***	-3.00	62.00	6.78	$4.50\leq m<5.50$	0.95	(14.07)***	-3.10	7.10	1.79
$8.50\leq m<10.0$	2.49	(7.64)***	-2.00	65.00	5.25	$8.50\leq m<10.0$	1.43	(27.19)***	-2.20	6.50	1.95
All	2.60	(6.35)***	-3.00	67.00	5.60	All	1.05	(8.74)***	-5.60	7.10	1.83
Log average issue size differential											
All	-0.06	(-50.71)***	-0.12	0.00	0.04						

and $LIQ^i_{t,m}$, and AIS^i_t are the credit default swap premium, liquidity measure, and average issue size, respectively. The liquidity measures are percentage bid-ask spread, log depth, or the log ratio of depth and spread. We adopt log specifications in order to be able to interpret the corresponding regression coefficients as semi-elasticities of the yield spread. $\mathbf{DEL_m}$ is a vector of dummy variables that represent different deliverability conditions and $\boldsymbol{\delta}$ is the corresponding vector of coefficients. In particular, DEL_m takes the value of one if maturity m is deliverable for a German futures contract, that is, it satisfies the maturity condition shown in the subscript. $NONDEL_{<1.75}$ is one for maturities less than 1.75 years that are permanently non-deliverable. $\mathbf{X_{t,m}}$ includes other control variables, namely time-to-maturity and year dummies. In all the regressions, we adjust for both cross-sectional and time effects in residuals $\varepsilon_{t,m}$ using the variance estimators suggested by Thompson (2005).[24]

The estimation results for Equation 1 appear in Tables 8 and 9. Table 8 reports the determinants of the yield spread before the onset the financial crisis. The coefficients for different liquidity measures are not statistically different from zero, suggesting that liquidity was not a key concern for the marginal investors in the pre-crisis period. This result, however, holds only for measures of "artificial liquidity" (i.e. liquidity provided by market makers as opposed to endogenously emerging liquidity): the log ratio of average issue sizes, a measure of relative float, loads negatively on the yield spread and is statistically significant. This means that the indirect liquidity benefits arising from larger issues more than offset the direct supply effects.[25]

[24] Thompson (2005) suggests the following variance estimator $\widehat{Var}(\widehat{\beta})$ for an OLS estimator $\widehat{\beta}$ that is robust to heteroscedasticity and correlation across both distinct maturities m and time t:

$$\widehat{Var}(\widehat{\beta}) = \widehat{\mathbf{V}}_\mathbf{m} + \widehat{\mathbf{V}}_\mathbf{t} - \widehat{\mathbf{V}}_\mathbf{White}$$

where $\widehat{\mathbf{V}}_\mathbf{m}$ and $\widehat{\mathbf{V}}_\mathbf{t}$ are the estimate variances that cluster by maturity and time [Huber (1967); Rogers (1983)], respectively, and $\widehat{\mathbf{V}}_\mathbf{White}$ is the usual heteroskedasticity robust OLS variance matrix [White (1984)].

[25] Although it has to be emphasized that the economic importance of increased float is marginal: coefficient -0.22 implies that a one standard deviation decrease (-0.04) in the issue size ratio increases the yield spread by approximately one basis point.

Table 8: The determinants of sovereign yield spread: Pre-crisis period

This table contains the results contains the results of least squares regression of Equation 1. The dependent variable is $R^F_{t,m} - R^G_{t,m}$, the difference between French(F) and German(G) par yield for maturity m at day t, measured in basis points. $CDS^i_{t,m}$, $LIQ^i_{t,m}$, and AIS^i_t are the credit default swap premium, liquidity measure (bid-ask spread, log depth, or the log ratio of depth and spread), and average issue size, respectively. $NONDEL_m$ and DEL_m are dummy variables that take the value of one if maturity m satisfies the limits shown in the subscripts. Asterisks *, ** and *** after robust t-values (in parentheses) denote values significantly different from zero at the 10%, 5%, and 1% levels, respectively. Daily observations from January 2nd 2006 to June 29th 2007.

	Spread		log Depth		$\log \frac{\text{Depth}}{\text{Spread}}$	
	Slope	t-value	Slope	t-value	Slope	t-value
Intercept	-0.86	(-1.53)	-0.87	(-1.55)	-0.87	(-1.59)
$LIQ^F_{t,m} - LIQ^G_{t,m}$	-0.11	(-0.99)	0.42	(1.33)	0.33	(1.51)
$CDS^F_{t,m} - CDS^G_{t,m}$	0.16	(2.24)**	0.15	(2.20)**	0.15	(2.22)**
$\log(AIS^F_t / AIS^G_t)^\dagger$	-0.22	(-6.34)***	-0.22	(-6.44)***	-0.22	(-6.35)***
$NONDEL_{<1.75}$	0.80	(2.30)**	0.97	(2.34)**	0.91	(2.45)**
$DEL_{1.75-2.25}$	0.83	(5.00)***	0.74	(4.72)***	0.77	(5.21)***
$DEL_{4.5-5.5}$	-0.60	(-1.45)	-0.51	(-1.25)	-0.55	(-1.41)
$DEL_{8.5-10.5}$	3.56	(4.24)***	3.57	(4.22)***	3.51	(4.20)***
m	0.24	(1.71)*	0.25	(1.74)*	0.25	(1.81)*
Year-fixed effects	Yes		Yes		Yes	
Number of obs.	6133		6133		6133	
Adjusted-R^2	0.69		0.68		0.69	

†The regression coefficient is multiplied by 100.

In addition, the deliverability of German bonds appear to be priced, with the convenience yield for holding deliverable bonds being the highest in the 10-year segment at 3.5 *bps*, and less significant for 2- and 5-year segments. Positive, albeit small, coefficient for non-deliverable bonds indicates that deliverable bonds do not lose value once they drop permanently out of delivery basket, which mutes the importance of deliverability especially in the 2-year segment. The coefficients for time-to-maturity, all around 0.25, indicate that the yield spread increases by one basis point for every four-year increment in residual maturity.

Consistent with economic intuition, we also find that the difference in perceived credit quality is positively related to the sovereign yield spread.

Table 9 the results based on the crisis period. All statistically significant coefficients have the expected sign and are larger in magnitude than for the pre-crisis sample. This points to an increased importance of liquidity, quality, and deliverability associated in times of market stress. For example, positive and negative coefficients for bid-ask and liquidity index differentials, respectively, indicate that increased demand for the relatively more liquid German securities depresses the entire yield curve compared to the French one. In particular, a positive coefficient of 0.19 for the bid-ask spread differential implies that a two standard deviation (7.2 in the crisis sample) increase in the relative bid-ask spread is associated with 1.4 *bps* increase in the yield spread across maturities. Also, the liquidity index, which incorporates both spread and depth information, is higher for German securities and thereby loads negatively on the yield spread.

In addition to relative liquidity, the economic importance of relative credit quality increases in the times of market disturbance. Specifically, we find that the coefficients for credit risk differential triple in the crisis sample, ranging from 0.35 to 0.42. Therefore, a ten basis point increase in the CDS spread is associated with approximately four basis point increase in the yield spread.

Consistent with the results in Table 8, the relative issue size is negatively related

TABLE 9: THE DETERMINANTS OF FRENCH-GERMAN YIELD SPREAD: POST-CRISIS PERIOD

This table contains the results of the Panel OLS regression of Equation 1. The dependent variable is $R^F_{t,m} - R^G_{t,m}$, the difference between French(F) and German(G) par yield for maturity m at day t, measured in basis points. $CDS^i_{t,m}$, $LIQ^i_{t,m}$, and AIS^i_t are the credit default swap premium, liquidity measure (bid-ask spread, log depth, or the log ratio of depth and spread), and average issue size, respectively. $NONDEL_m$ and DEL_m are dummy variables that take the value of one if maturity m lies in the maturity range shown in the subscripts. Asterisks *, ** and *** after robust t-values (in parentheses) denote values significantly different from zero at the 10%, 5%, and 1% levels, respectively. Daily observations from 1 July 2007 to 30 September 2008.

	Spread		log Depth		$\log\frac{\text{Depth}}{\text{Spread}}$	
	Slope	t-value	Slope	t-value	Slope	t-value
Intercept	-1.58	(-1.14)	-0.30	(-0.23)	-1.34	(-0.91)
$LIQ^F_{t,m} - LIQ^G_{t,m}$	0.19	(6.33)***	0.37	(0.82)	-0.52	(-1.77)*
$CDS^F_{t,m} - CDS^G_{t,m}$	0.42	(3.82)***	0.35	(2.92)***	0.36	(2.98)***
$\log(AIS^F_t/AIS^G_t)^\dagger$	-0.65	(-7.22)***	-0.61	(-6.1)***	-0.69	(-6.88)***
$NONDEL_{<1.75}$	-2.15	(-3.36)***	-2.19	(-3.91)***	-2.29	(-3.49)***
$DEL_{1.75-2.25}$	1.67	(3.88)***	1.56	(3.39)***	1.75	(3.88)***
$DEL_{4.5-5.5}$	1.37	(2.91)***	1.87	(4.45)***	1.41	(3.43)***
$DEL_{8.5-10.5}$	6.03	(4.25)***	5.36	(4.79)***	5.39	(4.36)***
m	0.39	(1.95)**	0.58	(4.14)***	0.52	(2.88)***
Year-fixed effects	Yes		Yes		Yes	
Number of obs.	5511		5511		5511	
Adjusted-R^2	0.75		0.71		0.71	

†The regression coefficient is multiplied by 100.

to the yield spread in the crisis time as well. However, coefficients from -0.61 to -0.69 are three times larger and suggest that in times of stress, a large float (which makes bonds easier to locate in an OTC market) becomes especially important.

Finally, the deliverability indicators have explanatory power for the yield spread even after controlling for the liquidity differential, which suggests that the value of holding deliverable bonds cannot be completely explained with their superior liquidity as measured by our liquidity indicators. Again, the convenience yield is largest for the 10-year segment, 5.4 to 6.0 *bps* depending on the model specification. Furthermore, it should be emphasized that these figures are over and above the future deliverability premium, which varies from 2.2 to 2.3 *bps*, so that the total convenience yield for 10-year deliverables could well be as high as 8.2 *bps*. For short- and medium-term deliverable bonds the convenience yields are smaller, but still economically significant at around four basis points. We conjecture that wider and deeper delivery baskets in the 2- and 5-year segments reduce the convenience yield attached to individual bonds. The coefficient for time-to-maturity implies that the yield spread in the 10-year segment is ranges from 4 to 6 *bps*, ceteris paribus, depending on the liquidity measure used.

In a recent article, Kuipers (2008) finds similar albeit much smaller deliverability effects for U.S. Treasury bonds deliverable for 30-year futures contract. Kuipers (2008) reports a price premium of less than one basis point on a yield basis, which is in gross terms since the effect of liquidity is not controlled for.

We perform two robustness checks. First, we re-estimate both samples using quantile regression which is less sensitive to distributional assumptions and outliers. In particular, maximum yield and bid-ask spreads presented in Table 7 are quite high compared to their sample averages, which raises concerns that the above results are driven by a few influential outliers. To address this concern, we model the conditional *median* of the independent variables instead of the conditional *mean* reported in Tables 8 and 9. For brevity, we do not report these results in detail but note that they are qualitative similar to those presented in the above tables. Therefore, we conclude that extreme observations do not drive our findings. Second, we address the potential simultaneity bias arising from the joint determination of sovereign yield and CDS spreads by excluding the latter from the regressions. Leaving the CDS differential out of the regressions does not change the subsequent results in any significant way, suggesting that out conclusions from Tables 8 and 9 are robust also in that respect.

5.3 Value of on-the-run status

Having established the negative relation between German bond yields and deliverability, we turn our attention to the most recently issued securities and ask whether the on-the-run status has pricing relevance beyond deliverability. Our approach is straightforward and familiar from the work of Elton and Green (1998) and others. We use the spot curve estimated from the daily prices of German off-the-run bonds to value a synthetic bond with a cash flow schedule similar to that of the on-the-run bond. The reference price is then converted to a yield and subtracted from the actual market yield of the on-the-run bond. If the resulting yield spread is negative, it means that investors are willing to accept lower yields for on-the-run securities relative to similar, off-the-run securities. Table 10 provides summary statistics on yield spreads for German 2-, 5-, and 10-year on-the-run securities.

The yield discount attached to the on-the-run status is surprisingly small for German government bonds. The mean yield concessions that investors are willing to pay in order to own on-the-run securities varies from 1.8 *bps* in the 2-year segment to -1.7 *bps* in the 10-year segment, where the latter value is not statistically different for zero. Since the reference yields in the 10-year segment are based on the prices of deliverable bonds, it can be concluded that investors do not attach additional value to newly issued 10-year bonds over seasoned 10-year bonds which remain deliverable for the 10-year futures contracts. In the 2- and 5-year segments, the on-the-run bonds trade at yields that are 1-2 *bps* below the off-the-run curve, but it should be emphasized that the economic significance of on-the-run status appears trivial compared to Japanese or

Table 10: Summary statistics on yield discounts associated with German on-the-run issues

This table reports summary statistics on yield spreads between actual German on-the-run bonds of various types and reference securities with similar maturity and coupon rate. Reference yields are estimated from the term structure using cubic splines. All values are in basis points. Asterisks *** after robust t-values (in parentheses) denote values significantly different from zero at the 1% level. Daily observations from 2 January 2006 to 30 September 2008.

	Mean	t-value	Min	Max	SD
2-year	-1.75	(-2.72)***	-15.77	3.84	2.81
5-year	-0.67	(-3.25)***	-4.90	4.48	1.36
10-year	1.74	-1.48	-9.54	13.98	3.96

U.S. government bond markets, where yield discounts often are found to be at least an order of magnitude larger.

6 Conclusion

We find no evidence of a significant 'on-the-run liquidity phenomenon' in the German government bond market. Once deliverability into futures contracts and other liquidity determinants are properly controlled for, German on-the-run bonds neither enjoy substantially better liquidity nor trade at an economically significant price premium. In the light of the evidence from the U.S. Treasury market, which documents large liquidity and pricing effects associated with on-the-run status, this is surprising.

Instead, we find clear evidence that the cross-sectional variation in liquidity measures as well as yields across German government bonds is closely related to their eligibility for the two, five and ten-year futures contracts. The yield discounts on deliverable bonds cannot, however, be fully explained by standard liquidity measures and may thus be partly related to a premium for liquidity *risk*.

Our findings suggests that on-the-run securities play very different roles in the German and U.S. Treasury market, in particular with respect to hedging and speculation. In the U.S. market, the benchmark status of the on-the-run securities is indisputable, whereas this status - and the related superior liquidity - appear to be shared among multiple bonds in the Germany market. More generally, our empirical findings highlights the role of a liquid futures markets in supporting the liquidity of the underlying cash market.

Exploiting that our sample covers part of the recent turbulent period in financial markets, we find that the economic importance of liquidity and deliverability increased considerably under severe market stress. Furthermore, our results suggest that the large price premium observed on German bonds during the crisis (relative to other large euro area issuers) may partly be explained by significant liquidity spillovers from the very liquid German futures market. The presence of these effects have implications for studies of euro-area sovereign spreads, which typically are computed relative to German yields.

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Appendix

A Summary statistics and additional tables

Table 11 provides summary statistics for the number bonds, their outstanding amounts, trading volume, and liquidity measures. As for the number of bonds, our sample includes more on-the-run and off-the-run bonds in the two-year segment than in other segments. This reflects the tighter issuance cycle in the short end of the yield curve, as German Finance Agency issues two-year bonds four times a year while five- and ten-year bonds are issued semiannually. Nonetheless, the coverage across maturity and seasonedness ranges is overall quite good, as there are at least five bonds in each category.

Yield spreads between off-the-run and on-the-run bonds appear quite modest, even at the long end of the yield curve. Indeed, the yields for the most recently issued bonds and their immediate predecessors deviate on average less than two basis points. Bid-ask spreads generally get wider with remaining time-to-maturity. For instance, the mean percentage bid-ask spread for the most recently issued 2-year bond is little over two basis points, compared to 3.4 basis points for the 10-year on-the-run bond. This is consistent with market-making models based on inventory management, in which competitive dealers charge wider spreads for securities that have higher price volatility. For bonds with different original but similar residual time-to-maturity, however, the spreads are roughly the same. For example, the bid-ask spreads for original five- and ten-year bonds with approximately three and half years to maturity are virtually identical. The bid-ask spreads between on-the-run and off-the-run bonds do not seem to differ in an economically important way either. This contrasts with the U.S. Treasury market evidence, where Fleming (2002) reports five times wider bid-ask spread for off-the-run bills and Pasquariello and Vega (2009) two times wider spread for off-the-run Treasury bonds. Market depth, while being similar between on-the-run and off-the-run bonds, appears to decline with seasonedness.

Table 11: Sample summary statistics.

This table presents means and standard deviations [in brackets] of yields, trading volumes, percentage price bid-ask spreads and quoted depth for German government bonds over the period January 2006 through September 2008. Bonds are categorized by the original maturity and seasonedness. The on-the-run (off-the-run) bond is the most (next-to-most) recently issued bond in the maturity segment. Sources: MTS Deutschland, ICMA, and German Finance Agency.

	Seasonedness					
Original maturity	On-the-run	Off-the-run	1-2 yrs.	2-5 yrs.	5-7 yrs.	7-10 yrs.
Number of bonds						
2-year	10	10	13			
5-year	6	5	7	11		
10-year	5	5	6	10	10	13
Yield (%)						
2-year	3.660 [0.405]	3.683 [0.415]	3.528 [0.520]			
5-year	3.790 [0.338]	3.781 [0.351]	3.767 [0.360]	3.686 [0.410]		
10-year	3.959 [0.279]	3.941 [0.281]	3.928 [0.286]	3.880 [0.311]	3.784 [0.361]	3.719 [0.411]
Volume (EUR billions)						
2-year	0.907 [0.571]	0.574 [0.456]	0.280 [0.143]			
5-year	1.140 [0.288]	0.446 [0.291]	0.244 [0.082]	0.284 [0.109]		
10-year	2.450 [1.111]	1.630 [0.809]	1.201 [0.571]	0.501 [0.206]	0.502 [0.201]	0.230 [0.118]
Spread (basis points)						
2-year	2.193 [1.335]	2.170 [1.194]	1.785 [1.232]			
5-year	3.103 [0.800]	3.106 [0.712]	2.927 [3.007]	2.318 [1.887]		
10-year	3.444 [0.861]	3.584 [0.802]	3.817 [0.983]	3.430 [1.497]	2.900 [0.885]	2.280 [2.263]
Depth (EUR millions)						
2-year	93.666 [35.638]	101.643 [35.447]	31.369 [7.526]			
5-year	94.740 [29.714]	93.224 [29.524]	79.831 [32.108]	56.806 [28.491]		
10-year	114.709 [45.647]	119.394 [46.698]	82.609 [36.887]	59.664 [22.957]	66.946 [25.229]	55.616 [29.646]

Table 12: Determinants of trading volumes for German and French bonds: Panel regression without time dummies

The dependent variable is log trading volume. Asterisks *, **, *** after robust t-values (in parentheses) denote values significantly different from zero at the 10%, 5%, and 1% levels, respectively. Monthly observations from Jan. 2002 through Feb. 2009 (T=86).

	Germany		France	
	Slope	t-value	Slope	t-value
Intercept	-11.26	(-2.54)**	-13.85	(-2.38)**
$1.75 \leq$ maturity < 2.25	0.57	(8.42)***	0.38	(5.16)***
$4.50 \leq$ maturity < 5.50	0.70	(8.86)***	0.32	(3.60)***
$8.50 \leq$ maturity < 10.50	1.06	(7.91)***	0.51	(5.19)***
Cheapest-to-deliver for 2-year future	0.34	(2.77)***		
Cheapest-to-deliver for 5-year future	0.58	(7.05)***		
Cheapest-to-deliver for 10-year future	0.15	(1.14)		
On-the-run status	0.65	(5.15)***	0.58	(5.70)***
Seasonedness (in years)	-0.09	(-5.58)***	-0.13	(-7.35)***
Time to maturity (in years)	-0.02	(-1.72)*	0.01	(1.21)
Log issue size	1.31	(7.00)***	1.39	(5.66)***
Month-fixed effects	No		No	
Sample	Jan 02-Feb 09		Jan 02-Feb 09	
Number of months	86		86	
Number of bonds	109		66	
Number of month-bond obs.	4427		3024	
Adjusted-R^2	0.70		0.60	

TABLE 13: DETERMINANTS OF TRADING VOLUMES FOR GERMAN AND FRENCH BONDS (PRE-CRISIS SAMPLE)

The dependent variable is log trading volume. Asterisks *, **, *** after robust t-values (in parentheses) denote values significantly different from zero at the 10%, 5%, and 1% levels, respectively. Monthly observations from Jan. 2002 through Jun. 2007 (T=66).

	Germany		France	
	Slope	t-value	Slope	t-value
Intercept	-16.16	(-2.88)***	-11.40	(-1.55)
$1.75 \leq$ maturity < 2.25	0.55	(7.50)***	0.43	(4.33)***
$4.50 \leq$ maturity < 5.50	0.64	(7.02)***	0.34	(3.29)***
$8.50 \leq$ maturity < 10.50	1.01	(6.70)***	0.51	(4.98)***
Cheapest-to-deliver for 2-year future	0.37	(2.60)***		
Cheapest-to-deliver for 5-year future	0.66	(7.85)***		
Cheapest-to-deliver for 10-year future	0.17	(1.32)		
On-the-run status	0.67	(4.34)***	0.54	(5.00)***
Seasonedness (in years)	-0.06	(-2.88)***	-0.14	(-6.30)***
Time to maturity (in years)	-0.02	(-1.95)*	0.01	(1.31)
Log issue size	1.55	(6.47)***	1.29	(4.17)***
Month-fixed effects	Yes		Yes	
Sample	Jan 02-Jun 07		Jan 02-Jun 07	
Number of months	66		66	
Number of bonds	109		66	
Number of month-bond obs.	3426		2294	
Adjusted-R^2	0.74		0.61	

Table 14: Determinants of trading volumes for German and French bonds (Crisis sample)

The dependent variable is log trading volume. Asterisks *, **, *** after robust t-values (in parentheses) denote values significantly different from zero at the 10%, 5%, and 1% levels, respectively. Monthly observations from Jul. 2007 through Feb. 2009 (T=20).

	Germany		France	
	Slope	t-value	Slope	t-value
Intercept	-5.81	(-1.29)	-20.27	(-3.45)***
$1.75 \leq$ maturity < 2.25	0.60	(5.61)***	0.22	(2.28)**
$4.50 \leq$ maturity < 5.50	0.74	(8.43)***	0.28	(1.94)*
$8.50 \leq$ maturity < 10.50	1.07	(8.27)***	0.45	(3.37)***
Cheapest-to-deliver for 2-year future	0.36	(1.69)*		
Cheapest-to-deliver for 5-year future	0.27	(1.74)*		
Cheapest-to-deliver for 10-year future	0.05	(0.43)		
On-the-run status	0.69	(3.49)***	0.73	(5.20)***
Seasonedness (in years)	-0.11	(-7.40)***	-0.10	(-5.69)***
Time to maturity (in years)	-0.01	(-0.94)	0.00	(0.25)
Log issue size	1.08	(5.68)***	1.66	(6.69)***
Month-fixed effects	Yes		Yes	
Sample	Jul 07-Feb 09		Jul 07-Feb 09	
Number of months	20		20	
Number of bonds	109		66	
Number of month-bond obs.	1001		730	
Adjusted-R^2	0.73		0.74	

Table 15: Determinants of quoted depth for French government bonds

The dependent variable is monthly averages of daily cumulated (log) depth. Asterisks *, **, *** after robust t-values (in parentheses) denote values significantly different from zero at the 10%, 5%, and 1% levels, respectively.

	pre-crisis		crisis	
	Slope	t-value	Slope	t-value
Intercept	9.86	(3.85)***	17.00	(2.98)***
$1.75 \leq$ maturity < 2.25	0.17	(2.73)***	0.27	(3.35)***
$4.50 \leq$ maturity < 5.50	-0.09	(-1.50)	-0.07	(-0.63)
$8.50 \leq$ maturity < 10.50	0.16	(3.86)***	0.27	(3.05)***
On-the-run status	0.49	(6.71)***	0.50	(3.95)***
Seasonedness (in years)	-0.05	(-7.71)***	-0.05	(-4.38)***
Time to maturity (in years)	-0.04	(-13.57)***	-0.02	(-4.04)***
Log issue size	0.33	(3.05)***	0.01	(0.06)
Month-fixed effects	Yes		Yes	
Sample	Jan 06-Jun 07		Jul 07-Sep 08	
Number of months	18		15	
Number of bonds	51		51	
Number of month-bond obs.	668		555	
Adjusted-R^2	0.81		0.65	

TABLE 16: DETERMINANTS OF QUOTED BID-ASK SPREADS FOR FRENCH GOVERNMENT BONDS

The dependent variable is monthly averages of daily (log) bid-ask spreads. Asterisks *, **, *** after robust t-values (in parentheses) denote values significantly different from zero at the 10%, 5%, and 1% levels, respectively.

	pre-crisis		crisis	
	Slope	t-value	Slope	t-value
Intercept	7.50	(2.25)**	6.52	(2.47)**
$1.75 \leq$ maturity < 2.25	0.09	(1.70)*	-0.08	(-1.22)
$4.50 \leq$ maturity < 5.50	0.12	(2.47)**	0.22	(3.93)***
$8.50 \leq$ maturity < 10.50	0.06	(0.98)	-0.04	(-0.75)
On-the-run status	-0.23	(-3.35)***	-0.44	(-5.72)***
Seasonedness (in years)	0.01	(1.54)	0.01	(1.39)
Time to maturity (in years)	0.08	(14.84)***	0.08	(15.77)***
Log issue size	-0.30	(-2.16)**	-0.25	(-2.22)**
Month-fixed effects	Yes		Yes	
Sample	Jan 06-Jun 07		Jul 07-Sep 08	
Number of months	18		15	
Number of bonds	51		51	
Number of month-bond obs.	634		553	
Adjusted-R^2	0.91		0.89	

TABLE 17: DETERMINANTS OF LIQUIDITY INDEX FOR FRENCH GOVERNMENT BONDS
The dependent variable is monthly averages of daily (log) depth divided by bid-ask spread. Asterisks *, **, *** after robust t-values (in parentheses) denote values significantly different from zero at the 10%, 5%, and 1% levels, respectively.

	pre-crisis		crisis	
	Slope	t-value	Slope	t-value
Intercept	10.53	(4.10)***	17.57	(3.79)***
$1.75 \leq$ maturity < 2.25	0.13	(2.54)**	0.21	(2.23)**
$4.50 \leq$ maturity < 5.50	-0.09	(-1.37)	-0.31	(-2.54)**
$8.50 \leq$ maturity < 10.50	0.01	(0.29)	0.20	(2.25)**
On-the-run status	0.48	(8.09)***	0.74	(5.74)***
Seasonedness (in years)	-0.07	(-7.67)***	-0.06	(-4.30)***
Time to maturity (in years)	-0.12	(-29.79)***	-0.11	(-15.25)***
Log issue size	0.63	(5.77)***	0.31	(1.59)
Month-fixed effects	Yes		Yes	
Sample	Jan 06-Jun 07		Jul 07-Sep 08	
Number of months	18		15	
Number of bonds	51		51	
Number of month-bond obs.	668		555	
Adjusted-R^2	0.96		0.88	

The Cheapest-to-Deliver Premium: Theory and Evidence

Abstract

This paper provides a theoretical and empirical investigation of the impact of bond futures trading on the price of the underlying bond. Using data from German government bond and futures markets, it is found that cheapest-to-deliver bonds trade on a premium that decreases towards zero as the bonds become ineligible for delivery. Based on this observation, an equilibrium model is developed that describes the theoretical underpinnings of the premium. Consistent with the model, the premium is found to be positively related to bond's relative cheapness in delivery, value as collateral, the amount of physical deliveries, and time to delivery.

JEL classification: G12, G14, G15

Keywords: German government bonds, cheapest-to-deliver, futures markets

1 INTRODUCTION

The markets for bond futures have experienced a decade of rapid growth, and many of the single contracts are nowadays amongst the most actively traded derivatives in the world. Since most of these contracts are designed to be settled with a delivery of an actual bond, it is important to understand how trading in large and active futures markets may affect the functionality of the cash markets. Previous literature has concentrated mostly on the volatility effects arising from speculative futures trading [e.g. Bortz (1984), Hegde (1994), Mayhew (2000) for a review], or on the price effects in instances of futures market manipulation [Merrick, Yaid, and Yadav (2005), Järvinen and Käppi (2004), Jeanneau and Scott (2002, 2001)]. This paper provides a theoretical and empirical assessment of the cash price effects under normal, competitive market conditions.

Typically, a futures contract that is settled by physical delivery specifies a standard par-delivery grade on which the contract is written, but allows several non-par grades to be used to settle the contract.[1] Bond futures are usually written on a notional bond with given maturity and coupon, and the bonds that are eligible for delivery have to meet certain criteria concerning the issuer, residual maturity, or issue size. A deep and uniform basket of deliverable bonds is important for any bond futures market, since it enables a well-functioning and frictionless settlement process. If the size imbalance between the two markets is large, the supply for deliverable bonds may not meet the demand of the futures sellers preparing to make a physical delivery, except at sharply higher cash prices. Moreover, if deliverable bonds are qualitatively different, which is usually the case, the quality option implicit in the futures contract allows the futures sellers to choose the optimal bond for delivery. Under such circumstances, the sellers' demand concentrates on the bond that is cheapest to deliver, making it a natural vehicle for the examination of the cash price effects that futures trading may induce.

This paper examines the effects of the futures trading on the price of the cheapest-to-deliver bond. The focus is on the ten-year German government bonds, which are deliverable for one of the most heavily traded derivative, the Bund futures contract.

[1] The seller of the futures contract has a right to choose the delivered bond. Recent work concerning the quality option include e.g. Nunes and de Oliveira (2007), Henrard (2006), Lin, Chen and Chou (1999), Ritchken and Sankarasubramanian (1995). Chance and Hemler (1993) consider the valuation of the option, and Grieves and Marcus (2005), Chen, Kang, and Yang (2005), and Rendleman (2004) examine its effects on hedging.

These two separate but interrelated markets provide an interesting setting for the analysis, since the total size of open positions in the futures markets is typically much greater than the size of the cash markets.[2] In addition, the cheapest-to-deliver bond for Bund futures is typically very predictable, since the costs of delivering the second-best bond are high. Using daily prices from January 2001 to March 2007, convergence trade tests show that German ten-year cheapest-to-deliver bonds trade on a premium, which decreases with the remaining time that the bond is expected to be cheapest-to-deliver. In other words, the premium is highest (lowest) for current cheapest-to-deliver bonds that are expected to remain cheapest (become non-deliverable) in subsequent futures settlements.

Based on the empirical observations, a general equilibrium model is developed to describe the dynamics of the price premium. The comparative statics of the model yield four testable predictions: first, the price premium is positively related with bond's relative cheapness in delivery, as it becomes increasingly suboptimal to deliver the second-cheapest bond. Second, the premium increases with the expected amount of physical deliveries to be made as a direct result from expected futures-market demand. Third, the premium increases with the relative rental value of the cheapest-to-deliver bond, as the bond becomes "special" collateral in the repurchase markets. Fourth, the premium is positively related with the number of periods that the bond is expected to be cheapest-to-deliver, since the rental value of the "special" collateral accumulates with the rental time. Using a panel dataset on German government bond and bond futures markets from January 2001 to March 2007, the model predictions are empirically tested and supported.

Futures exchanges may find the results useful in designing a new futures contract or an optimal settlement process. The design of the quality option plays an important part in the success of a futures contract, as the value of the option is effectively the upper limit for bias in the futures price. Johnston and McConnell (1989) report that the quality option embedded in the GNMA bond futures contract became so valuable that the hedging effectiveness of the contract was severely hampered. The results reported in this paper show that the quality option is related to a bias in cash prices as well. The reported futures market effect may have practical relevance also for bond

[2] For example, in 2005 the average open interest was equivalent to €153 billion worth of underlying bonds. Yet, the amount of outstanding deliverable bonds was only around €80 billion, and the average stock for the cheapest-to-deliver issue was only €25 billion. From 2001 to 2007, on average 16 percent of the outstanding cheapest-to-deliver bonds were delivered at every futures contracts expiry.

investors, who consider forward rates for the assets inside and outside the delivery basket.

The remainder of the paper continues as follows. In Section 2 the German Bund futures contract and the concept of the cheapest-to-deliver bond are introduced, and some particular features of cheapest-to-deliver bonds are discussed. The existence of cheapest-to-deliver premium is tested using a convergence trade test. In Section 3 an equilibrium model for the cheapest-to-deliver premium is developed and discussed, and in Section 4 the predictions of the theoretical model are tested with empirical data. Section 5 presents the summary and conclusions drawn from the findings of the study.

2 CHEAPEST-TO-DELIVER BOND

The Bund futures contract is written on a notional ten-year German government bond with a face value of €100.000 and an annual coupon of six percent. The Bund contract is traded in Eurex derivatives exchange and, like all major bond futures contracts, is settled by physical delivery. On the delivery date, the contract allows a government bond with residual maturity from 8½ to 10½ years and minimum issued amount of five billion euro to be used for delivery. Since the German government bonds are issued semiannually, there are typically four deliverable bonds with total issue size close to €80 billion. However, both the number of bonds and the depth of the Bund delivery basket fall short of those for the ten-year U.S. Treasury Note futures. The Treasury Note contract, which is approximately equal to Bund in volume of traded contracts, has typically 8 – 12 deliverable bonds with total issue size of $150 – $300 billion.

Since deliverable bonds for Bund futures usually have different maturities and coupons, thus cash prices, the exchange provides bond-specific conversion factors that are used to calculate the actual delivery prices.[3] In essence, the delivery price is the price at which bond would trade if its yield-to-maturity was equal to the notional six percent coupon. The conversion factor is supposed to make all eligible bonds equally attractive for delivery, though the conditions under which this objective is met are very rare. Namely, it requires that market yields are at the level of the notional coupon. Since a flat, six-percent yield curve is unlikely, bias in the conversion factor tends to make a bond with suitable cash flow characteristics cheaper to deliver than the others [e.g. Benninga and Wiener (1999), Oviedo (2006)]. Nonetheless, the closer the market yields are to the notional six percent level, the lesser is the price discrimination between deliverable bonds.

The actual cheapest-to-deliver bond is not known with certainty until on the delivery date. A shift in the level or slope of the yield curve may change the cheapest-to-deliver bond, especially if the yields are in the vicinity of notional coupon and the deliverable bonds are qualitatively similar. One effective measure of such uncertainty is the difference between bond delivery prices. If the initial delivery prices of the cheapest and the second cheapest bond are close to each other, only a small change in yields may cause a rotation in the cheapest grade. Figure 1 plots the difference in

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[3] Eurex (2007) provides the formula for the conversion factor.

delivery prices between the cheapest and the second-cheapest bond for every Bund contract from March 2001 to March 2007. As can be seen from the figure, the relative price advantage (ΔP) in favor of the cheapest grade is typically quite significant, varying from €0.20 to €1.25 per €100. This implies that the cheapest-to-deliver bond for the expiring Bund futures is very predictable, since the probability of a rotation to any other deliverable bond is small. Even a relatively large, half percentage point parallel shift in the market yields (ΔP* bounds in Figure 1), does not seem to be enough to change the cheapest-to-deliver bond.

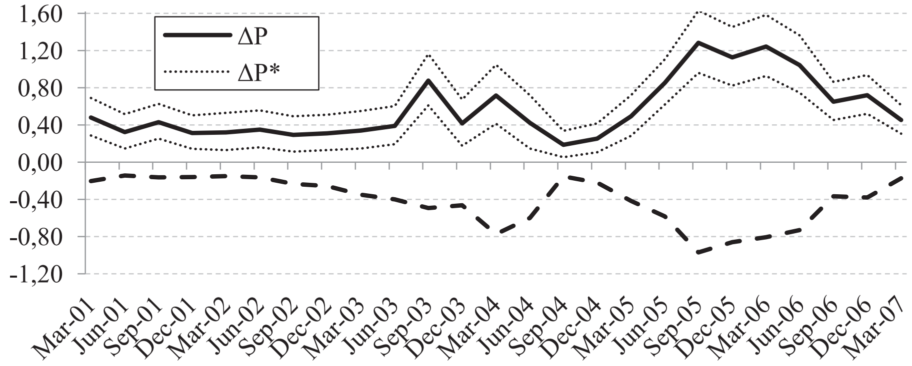

Figure 1. Marginal cost for delivering the second-cheapest bond (ΔP), and alternative costs after a half percentage point shift in market yields (ΔP*). ΔDΔr plots the joint effect of the difference in modified durations between the cheapest and the second-cheapest bond (ΔD), and the difference between actual yield and notional yield (Δr).

There are two features in economic environment of the Bund contract that explain the price advantage, thus predictability, of the cheapest-to-deliver bond. First, euro-area interest rates after the 1990s have been considerably lower than the notional six percent level, which increases the bias in the conversion factor pricing in favor of the cheapest-to-deliver bond. Second, the difference in interest rate sensitivities between the cheapest-to-deliver bond and the second-best alternative is typically large enough to prevent the cheapest grade to change when market yields change. The joint effect of these two attributes is illustrated by ΔDΔr variable in Figure 1. ΔDΔr is the product of ΔD, the difference between interest rate sensitivities measured by modified duration, and Δr, the difference between the actual and the notional yield. As can be observed from Figure 1, the (inverted) ΔDΔr variable moves hand in hand with the relative price advantage ΔP.

The predictability of the cheapest-to-deliver bond is evidenced in the delivery records held by Eurex. The records show that for given Bund futures settlement, the sets of delivered bonds comprise solely of the actual cheapest grade (on average, one-sixth of the total outstanding issue), which is contradictory if the cheapest-to-deliver bond changed very frequently. Under the latter circumstances, the lack of predictability would result in a variety of delivered bonds, which are projected as cheapest and purchased in advance by the precautionary shorts.[4]

2.1 Convergence Trading Strategy

Based on the examination of Bund delivery environment above, the following issues arise: first, the lack of cost-effective alternatives makes the cheapest-to-deliver bond for the Bund futures contract predictable. Second, Eurex records show that the futures-induced demand concentrates solely on the cheapest grade. Third, the size difference between the cash and the futures markets for Bund bonds is exceptionally great. The central question of this paper is: how do these three issues affect the cash price of the cheapest grade?

Intuitively and economically, it makes sense that the markets appreciate the possession of the cheapest-to-deliver bond due to its importance in the delivery process. Accordingly, this generates a premium on the spot price against the future delivery, insofar as the advantage in delivery costs remains. On the other hand, futures-market initiated trading should not have any impact on the price of a bond that is not deliverable, since it cannot be used in the settlement process. Therefore, the price relationship between cheapest-to-deliver and non-deliverable bond should reveal information concerning the price premium.

In order to extract the premium from the cash price of the cheapest-to-deliver bond, an approach similar to Krishnamurthy (2002) is chosen. Krishnamurthy (2002) tested the effect of liquidity on the prices of U.S. Treasury bonds by short-selling more liquid on-the-run bond against less-liquid off-the-run bond. Continued on a daily basis until the on-the-run bond becomes off-the-run as well, Krishnamurthy (2002) reports an average annualized profit of \$0.33 per \$100 bond, which could be

[4]The delivery records are available at http://www.eurexchange.com.

associated with the markets' appreciation for the superior liquidity of the on-the-run bond.[5]

The on-the-run/off-the-run continuum in Krishnamurthy's (2002) analysis closely resembles the one in question. First, the cyclical nature of bond issuances makes the remaining lifetime of the on-the-run bond perfectly predictable, allowing the markets to price the liquidity accordingly. Second, the similarity between the on-the-run and off-the-run bonds gives little fundamental reason for the bonds to be priced differently. The same type of convergence strategy is may be applied on the relationship between the current and preceding cheapest-to-deliver bond. If the futures markets value the current cheapest-to-deliver bond for its special status in futures delivery, it trades on higher price than the "old" cheapest-to-deliver bond.

The convergence strategy is expected to generate profit once the current cheapest-to-deliver bond becomes "old" as well and the price premium disappears. Thereby, the strategy it is "reversed" in the way that the profits do not stem from the relative appreciation itself, but from the discontinuation of it. In addition, it takes advantage of the predictability, discussed above, by assuming that the projected cheapest-to-deliver remains cheapest until the delivery day. The following notation is similar to Krishnamurthy (2002):

- Let t_n, $n = \{0, \ldots, N\}$ correspond to subsequent business days on which bonds are traded. t_0 is the first business day following the delivery day of the preceding futures contract; or, equivalently, the first trading day of the near-month futures contract. N is either 66 or 132, depending on whether the constituents in the convergence strategy change after the futures contract matures. If the constituents remain the same, the strategy is rolled into the next quarter as such.

- Let $P(t_n)$ correspond to the price of a non-deliverable bond that was deliverable at t_{-1}.

- Let $\omega(t_n)$ correspond to the number of units of non-deliverable bond at time t_n. Trades are settled two business days later ($n + 2$ settlement), which means that any cash flow, negative or positive, is received two business days after the trade.

[5] It should be emphasized that the main result of Krishnamurthy's (2002) convergence trades is that the profit from the price convergence is mitigated by the differences in financing costs. However, the concern here is not the actual profits but the relative prices and hence the financing rates are assumed to be equal.

- Let $R(t_n)$ be time t_n general repo rate for a (reverse) repo agreement with duration of $t_{n+3} - t_{n+2}$. The repo agreement is settled on the same business day.

- Let $C(t_{n+1})$ be the possible interim coupon payment paid at t_{n+1}. The coupon is compensated automatically to the current bond holder.

- The variables above are analogous for the cheapest-to-deliver bond, which is distinguished with a hat (^).

The data for the convergence trades extend from January 2001 to March 2007, for total 1534 trading days. It comprises end-of-day prices and general repurchase rates for German government ten-year Bundesanleihen ("Bund") bonds, both from ICMA, and end-of-day prices for near-month Eurex-Bund futures contracts from Thomson Datastream. The cheapest-to-deliver bond is identified by calculating the implied repurchase rate on all deliverable bonds:

$$CTD(t_n) \equiv \max_i 365 \frac{F(t_n, T)\beta^i + AI(i, t_n) + C(i, t_{n+c}) - P(i, t_n)}{P(i, t_n)(T - t_n) - C(i, t_{n+c})(T - t_{n+c})} \tag{1}$$

where

$i =$ Index for a set of deliverable bonds
$t_{n+c} =$ Time index for interim coupon payment, $t_n \leq t_{n+c} \leq T$
$T =$ Futures delivery date
$F(t_n, T) =$ Futures price maturing at T
$\beta^i =$ Eurex conversion factor
$AI(i, t_n) =$ Accrued interest

A single convergence trade involves entering into two repurchase agreements; in the first and standard one, $\widehat{\omega}(t_n)$ units of the projected cheapest-to-deliver bonds are delivered to the security borrower at time t_n. The security borrower pays cash amount of $\hat{P}(t_n)$ per unit in exchange, which is received at t_{n+2} and reinvested at the general repo rate for $t_{n+3} - t_{n+2}$ days. At t_{n+1}, the bonds are bought back for $\hat{P}(t_{n+1})$ per unit. In the second one, which is a reverse repo agreement on non-deliverable bonds, the exact opposite holds; ω_t units of non-deliverable bonds are bought at time *t*, and the purchase is financed at the repo rate. At t_{n+1}, the non-deliverable bonds are sold back for $P(t_n)$ per unit. The corresponding mathematical representation becomes:

$$\mathbb{P}(t_{n+3}) = \quad \omega(t_n)\left[\Delta_t P\left(1 + R(t_{n+2})\frac{t_{n+3} - t_{n+2}}{365}\right) + C(t_{n+1})\right] \\ - \widehat{\omega}(t_n)\left[\Delta_t \hat{P}\left(1 + R(t_{n+2})\frac{t_{n+3} - t_{n+2}}{365}\right) + \hat{C}(t_{n+1})\right] \tag{2}$$

where $\mathbb{P}(t_{n+3})$ is the profit from a single convergence trade and Δ_t is the time–series difference operator. The weights $\omega(t_n)$ and $\widehat{\omega}(t_n)$ are set so that the profit is unaffected by parallel shifts in the bond yields, which makes the strategy essentially market-neutral; Pérignon and Smith (2007) show that 85% of the variance in German government bond yields is due to parallel changes in the term structure. The weights are obtained by solving

$$\omega(t_n) m_d(t_n) = \widehat{\omega}(t_n)\,\widehat{m_d}(t_n) \tag{3}$$

where $m_d(t_n)$ is the modified duration of the non-deliverable bond at time *t*. The product $\omega(t_n) m_d(t_n)$ is held equal to seven, so that the exposure of the profit to changes in the yield spread is similar every time the strategy is established. The scale factor is chosen to be seven according to the average modified duration of the related bonds; this leads to weights that correspond approximately to €100 face value of bonds.

The convergence strategy consists of a continuum of similar trades, which are repeated until the bond ceases to be cheapest-to-deliver, or becomes non-deliverable. The profit from the complete strategy k, $\mathbb{P}(S_k)$ consists of cumulative profits from individual convergence trades:

$$\mathbb{P}(S_k) = \sum_{n}^{N} \mathbb{P}(t_{n+3}) \tag{4}$$

For analytical purposes, the daily proceeds are not reinvested.

Table 1 presents the summary statistics for all 17 convergence strategies. The number of strategies is low compared to the number of matured futures contracts, since most bonds that become cheapest-to-deliver against a Bund futures contract remain cheapest for two consecutive contracts. Consequently, the average duration for a single strategy is closer to two quarter years than one. The cheapest-to-deliver bonds have an average time-to-maturity of 8.82 years, ranging from 8.56 to 9.06 years.

Given that deliverability requires time-to-maturity between eight and half and ten and half years, the summary statistics suggest the following: first, the cheapest-to-deliver bonds are amongst the ones with least time-to-maturity in the delivery basket. This, and the result that the bonds examined remain cheapest-to-deliver for two contracts, imply that bonds cease to be cheapest-to-deliver due to the lower time-to-maturity bound rather than losing the advantage in delivery costs.

Table 1. The summary statistics for the convergence strategies.

Number of matured futures contracts	32
Number of convergence strategies	17
Average duration (trading days)	119
Average profit, per €100 (range)	€0.21 (-0.26 to 1.06)
Total profit from all strategies, per €100	€3.53

The average profit from a convergence strategy is €0.21 per 100 euro, but actual profits fluctuate from €0.26 loss to €1.06 profit per €100. The cumulative outcome of all 17 convergence strategies yields a profit of €3.53 per €100.[6]

Figure 2 provides a graphical illustration on the accumulation of profits; the average cumulative profit on y-axis, as a function of remaining time on x-axis. According to Figure 2, the cumulative profits are on average positively sloped, which indicates the convergence effect discussed above. The strategy, however, could be separated into two periods on the basis of profit-making ability: Figure 2 shows that the strategy is not particularly profitable on the cheapest-to-deliver period ending to the first delivery day ("DD I"). In fact, all statistically significant profits are made on the period ending to the second delivery day ("DD II").

This finding may reflect markets' expectations concerning the retention of the cheapest-to-deliver status; as the cheapest-to-deliver bond usually remains unchanged after DD I, the hypothesized cheapest-to-deliver premium remains unchanged on the first period as well. However, after the second delivery day the cheapest-to-deliver bond is inevitably to change due to the maturity bounds. As a consequence, the

[6] It should be emphasized noted that the results from Cornell and Shapiro (1989) and Krishnamurthy (2002) suggest that allowing for market-driven, individual financing rates would probably zero out the convergence profits. However, the concern here is not the actual profits but the relative prices and hence the financing rates are assumed to be equal.

cheapest-to-deliver premium diminishes towards DD II and the convergence strategy becomes profitable.

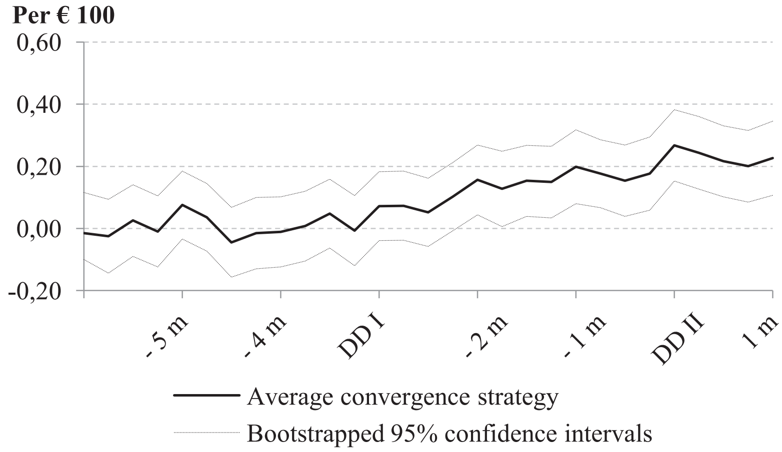

Figure 2. Average cumulative profit from convergence strategies.

3 A SIMPLE MODEL OF THE PRICE PREMIUM

The analysis in the previous section demonstrates that the short-selling strategy generates profits that are on average positive and statistically significant, and upward-sloping until the bond ceases to be the cheapest-to-deliver. The results suggest that the cheapest-to-deliver bond trades on a premium that decreases gradually before, rather than suddenly after, the last possible delivery date. Based on the results of Cornell and Shapiro (1989) and Krishnamurthy (2002), it is likely that the cheapest-to-deliver bond trades on a special financing rate as well.[7]

Next, a simple model is developed in order to reconcile these results and analytically investigate the dynamics of the price premium. Based on the general equilibrium theory, the model introduces futures sellers who, preparing for physical delivery, express price-inelastic demand for the value-maximizing deliverable bond. Futures market participants inflate the price of the cheapest bond, given that at least some bonds are actually delivered, a marginal cost of delivering another grade exists, and the shorts' aggregate demand dictates the general demand. Adopted from Krishnamurthy (2002), the no-arbitrage violations indicated by the convergence profits are accounted for by allowing the cheapest bond to trade on a special financing rate. By this augmentation, the multi-period version of the model generates a gradually decreasing price premium suggested by the convergence strategy.

3.1 Economy

Consider a two-period economy, where $t_n = \{t_0, t_1\}$ defines the trading periods. There are two types of agents in the economy: the investor ("I") and the arbitrageur ("A"). The agents in the economy face three competitive markets: the cash market, the futures market, and the financing market.

Two similar bonds $i = \{c, d\}$ trade at the cash markets, namely the "cheap" bond and the "dear" bond. Let $P(c, t_n)$ and $P(d, t_n)$ denote the cash prices of these bonds and $\Delta_i P(t_n) = P(c, t_n) - P(d, t_n)$. Concerning the prices, assume the following:

[7] Even though improbable, the results do not rule out the possibility that the non-deliverable bond trades on a gradually disappearing discount, which is unrelated to the futures markets. For now, however, the cheapest-to-deliver premium is assumed, and the authenticity of this assumption is addressed later on.

Assumption A.1: Bond prices $P(c,\ t_n)$, $P(d,\ t_n)$ converge so that $P(c,\ t_n) = P(d,\ t_n),\ \ \forall n \geq 1$. Thus the cash prices may differ on the first period, but not afterward.

The only traded asset in the bond futures markets is a standard contract, which is an agreement on a future delivery of an eligible bond. Let t_1 be the delivery date and both bonds to be deliverable. In exchange for every bond the futures seller delivers, the contract obligates the buyer of the contract to pay the futures price $F(t_0)$, multiplied by bond-specific conversion factors $\{\beta^C, \beta^D\}$. Not all futures contracts, however, are settled by physical delivery. At the futures expiration, some the futures positions are closed by making an offsetting transaction ("liquidation") instead of delivering a bond. Table 2 illustrates the cash flows generated by the alternative ways from the futures seller's perspective:

Table 2. Futures seller's alternative transactions.

Settlement	Cash flows
Delivery	$-P(i,\ t_n)$ $+\ \beta^i F(t_n)$
Liquidation	$-\Delta_t F$

where $\Delta_t F = F(t_1) - F(t_0)$. Note that the cash flows are similar in the case of liquidation, but different if the contract is settled by physical delivery. In the latter case, short's delivery proceeds depend on the cash price and the conversion factor of the delivered bond.

Finally, the financing markets consist of two varieties of collateralized loans. A general repurchase agreement is used to finance a long bond position. It is a single transaction involving a cash market sale and a forward repurchase of the bond. The proceeds of the cash sale are thus collateralized by the bond itself. The difference between the cash and repurchase prices implies a general repurchase rate $R(\cdot)$ for the bond. Table 3 illustrates the general repurchase agreement.

Table 3. The transactions generated by a general repurchase agreement on bond i.

Time	Borrower (bond owner)	Lender (cash owner)
t_0	Delivers bond i	Pays $P(i,\ t_0)$
t_1	Pays $P(i,\ t_0)[1+R(t_0)]$	Returns bond i

A special repurchase agreement is similar to the general one with the exception that the bond acting as collateral is considered "special" by the lender. For this reason, the lender is willing to accept a lower, special repurchase rate $r(t_n)$, $r(t_n) \leq R(t_n)$, on the proceeds of the cash market sale. The monetary benefit for the borrower is then $\Delta r(t_n) = R(t_n) - r(t_n)$ in financing costs.

3.2. Agents

The investor is assumed to smooth her consumption by transferring wealth from period t_0 to the next period t_1 by purchasing δ^I units of either deliverable bond. The t_1–wealth is hedged against interest rate risk by selling the corresponding amount of standard futures contracts. At t_1, a proportion of bonds equal to π is delivered against the bond-adjusted futures contract price $\beta^i F(t_1)$. The remaining $1-\pi$ bonds (futures) are sold (bought). Concerning π, assume the following:

> **Assumption A.2** A positive proportion of futures contracts are settled by physical delivery; $0 < \pi \leq 1$.

The investor has an initial endowment $\mathbb{E} \gg 0$, and the general rate for one-period financing is $R(t_0)$. Supposing that the i-th bond is chosen as the investment asset, investor's expected cash flows become:

$$\mathbb{E} - \delta^I \left\{ P(i,t_0)[1+R(t_0)] - E_0[\,\pi \times F(t_1) \times \beta^i\,] - E_0\big[\,(1-\pi)[-\Delta_t F + P(i,t_1)]\big] \right\} \tag{6}$$

where E_0 corresponds to the expectations made at t_0. In the above expression, the first term inside the braces denotes the purchasing and financing costs; the second term stands for the expected delivery proceeds; and the third term is the expected proceeds from the capitalization of remaining futures and bonds. The investor's problem of choice could be pinned down to the difference in cash flows that bonds c, d generate:

$$\delta^I \left\{ -\Delta_i P(t_0)[1 + R(t_0)] + E_0[\, \pi \times \Delta_i \beta \times F(t_1)\,] \right\} \tag{7}$$

where $\Delta_i \beta = \beta^C - \beta^D$, and the t_1–bond prices offset each other by Assumption A.1. Expression (7) states that the investor's choice depends on the difference between the initial purchasing prices and on the expected difference in delivery proceeds. Even though the deliverable bonds and thus the delivery proceeds ought to be similar, let us assume that the conversion factor system fails to characterize the bonds perfectly, and the delivery prices differ:

Assumption A.3 The functional form of conversion factor fails to characterize the deliverable bonds, and $\Delta_i \beta > 0$.

By A.2 and A.3, the latter term in expression (7) is strictly positive, and hence the investor is expected to profit from using the cheap bond in delivery instead of the dear. Henceforth, let these proceeds to be called "marginal delivery profit". Consequentially, the utility-maximizing investor expresses excess demand for the cheap bond, which changes the initial cash prices so that they solve

$$\Delta_i P(t_0) = \frac{v[\pi \times \Delta_i \beta \times F(t_1)]}{1 + R(t_0)} \tag{8}$$

where strictly increasing function $v[\cdot]$ defines investor's utility for expected marginal delivery profit. Then, Eq. (8) states that the equilibrium difference between $P(c,\ t_0)$ and $P(d,\ t_0)$ equals investor's utility from the expected marginal delivery profit, discounted to t_0.

The arbitrageur trades on relative prices, and has no wealth constraints, commitments, or transaction costs. The arbitrageur establishes one-period, self-financing strategy, by a (reverse) repo agreement and an outright purchase (sell) of the dear (cheap) bond. On the next period, the bond positions are reversed and the repo commitments are fulfilled. Valued at t_0, the strategy is expected to generate the following cash flows to the arbitrageur:

$$\Delta_i P(t_0)[\,1 + R(t_0)\,] - E_0[\,\Delta_i P(t_1)\,] \tag{9}$$

which, in order to prevent arbitrage, must sum up to zero. Assuming A.1 holds, setting the arbitrage profits to zero, and combining with Eq. (8), yields

$$v[\,\pi * \Delta_i \beta * F(t_1)\,] \neq 0 \tag{10}$$

which, under A.2 and A.3, is contradictory.

3.3 Economy with Special Repo Rates

Because investor's excess demand for the cheap bond results in contradictory Eq. (10), the cheap bond is allowed to trade on a special financing rate $r(t_n)$. Then, Exp. (6) is restated for bond c :

$$\mathbb{E} - \delta^I \begin{Bmatrix} P(c,t_0)[\,1 + R(t_0)\,] - E_0[\,\pi \times F(t_1) \times \beta^C\,] \\ -E_0\big[\,(1-\pi)[-\Delta_t F + P(c,t_1)]\big] \end{Bmatrix} + \phi^I\big[\Delta r(t_0) - \lambda[\phi^I]\big]P(c,t_0) \tag{11}$$

where the last term represents the adjustment for lower financing cost: ϕ^I corresponds to the amount of cheap bonds supplied onto the financing markets as special collateral, $\Delta r(t_0)$ is the spread between general and special financing rates, and $\lambda[\phi^I]$ ($\lambda'[\cdot] > 0$, $\lambda''[\cdot] > 0$) is the search cost for borrowing against special collateral.

Assumption A.4 The investor supplies at least some, but not all, cheap bonds onto the financing markets as special collateral: $0 \le \phi^I < \delta^I$.

The different financing rates restates investor's problem of choice [Exp. (7)] into the following form:

$$\delta^I \left\{ -\Delta_i P(t_0)[\,1 + R(t_0)\,] + E_0[\,\pi \times \Delta_i \beta \times F(t_1)\,] \right\} + \phi^I[\,\Delta r(t_0) - \lambda[\phi^I]]P(c,\, t_0) \quad (12)$$

The arbitrageur's strategy in Exp. (9) is restated as well:

$$\phi^A \left\{ \Delta_i P(t_0)[\,1 + R(t_0)\,] - \Delta r(t_0)P(c,\, t_0) - E_0[\,\Delta_i P(t_1)\,] \right\} \quad (13)$$

where $0 < \left|\phi^A\right|$ is the arbitrageur's demand for collateral, and the second term inside the braces is the adjustment for different financing rates. Remark that due to search costs associated with collateralized borrowing, the investor's optimal financing ratio is may be less than 1; that is, the amount of special collateral supplied onto the financing markets may be less than the amount of bonds purchased by the investor, implying $\left\{ \left|\phi^A\right|,\ \phi^I \right\} < \delta^I$.

3.4 Equilibrium

Expressions (12) and (13) describe a joint equilibrium of two interrelated markets and two agents. First, from investor's point of view, the cash market equilibrium is defined by

$$\Delta_i P(t_0) = \frac{\phi^I[\Delta r(t_0) - \lambda[\phi]]P(c,\, t_0) + E_0[\,\pi \times \Delta_i \beta \times F(t_1)\,]}{1 + R(t_0)} \quad (14)$$

Then again, from the arbitrageur's point of view the cash market equilibrium solves

$$\Delta_i P(t_0) = \frac{\Delta r(t_0) P(c,\ t_0)}{1 + R(t_0)} \tag{15}$$

Therefore, solving for the equilibrium price spread $\Delta_i P(t_0)$ requires the definition of agents' utilities. Investor's preferences and choices are based on the maximization of the following utility function:

$$U^I = \max_{\delta^I,\,\phi^I} c_0 + v[c_1] \tag{16}$$

where

$$\begin{aligned} c_0 &= \delta^I \times \Delta_i P(t_0)[1 + R(t_0)] + \phi^I \times \left[\Delta r(t_0) - \lambda[\phi^I]\right] P(c,\ t_0) \\ c_1 &= \delta^I \times \pi \times \Delta_i \beta \times F(t_1) \\ v &= v'[\cdot] > 0, v''[\cdot] < 0 \end{aligned}$$

$v[\cdot]$ implies that the investor is risk-averse with respect to future outcome. The arbitrageur maximizes

$$U^A = \max_{\phi^A} c_1 \tag{17}$$

where

$$c_1 = \phi^A \times [\ \Delta_i P(t_0) \times [1 + R(t_0)] - \Delta r(t_0) P(c,\ t_0)\ ]$$

Thus the arbitrageur is risk-neutral with respect to future outcome. Investor's first-order conditions with respect to choices $\{\delta^I, \phi^I\}$ are:

$$\frac{\mathrm{d}U^I}{\mathrm{d}\delta^I} = \Delta_i P(t_0)[\ 1 + R(t_0)\ ] + \pi \times \Delta_i \beta \times F(t_1) \times v'[\delta^I] = 0 \tag{18}$$

$$\frac{\mathrm{d}U^I}{\mathrm{d}\phi^I} = \left[\Delta r(t_0) - \lambda[\phi^I]\right] P(c,\ t_0) - \phi^I \times \lambda'[\phi^I] \times P(c,\ t_0) = 0 \tag{19}$$

and the first-order condition concerning the arbitrageur's choice ϕ^A is

$$\frac{\mathrm{d}U^A}{\mathrm{d}\phi^A} = \Delta_i P(t_0)[\,1 + R(t_0)\,] - \Delta r(t_0) P(c,\, t_0) = 0 \tag{20}$$

Remark that the partial derivatives in F.O.C.'s (19) and (20) are both taken with respect to ϕ. Therefore, F.O.C. (18) is combined with (20) in order to find a common criterion upon which the agents maximize utility:

$$\Delta r(t_0) = \frac{\pi \times \Delta_i \beta \times F(t_1)}{P(c,\, t_0)} v'[\delta^I] \tag{21}$$

where the right-hand side of the equation is positive by A.2 and A.3, and strictly increasing with $v'\left[\delta^I\right]$. Rearranging F.O.C. (19) gives

$$\Delta r(t_0) = \frac{\lambda[\phi^I] + \lambda'[\phi^I]\phi^I}{P(c,\, t_0)} \tag{22}$$

where the right-hand side of the equation is positive and strictly increasing with $\lambda'\left[\phi^I\right]$. Given that $v''\left[\delta^I\right] < 0$ and $\lambda''\left[\phi^I\right] > 0$, and clearing condition for financing markets $\phi^I = \phi^A$ satisfied, Equations (21) and (22) define equilibrium for interest rate spread $\Delta r(t_0)$.

Figure 3 illustrates the situation where supply and demand for special collateral reach an equilibrium level ϕ^*, defining the equilibrium repurchase rate spread $\Delta r(t_0)$. Then, by Walras' Law, the cash market clears as well, and F.O.C. (20) gives the equilibrium price spread:

$$\Delta_i P(t_0) = \Delta r(t_0) \frac{P(c,\, t_0)}{1 + R(t_0)} \tag{23}$$

where the equilibrium price and repurchase rate spreads are positively related by a scale factor.

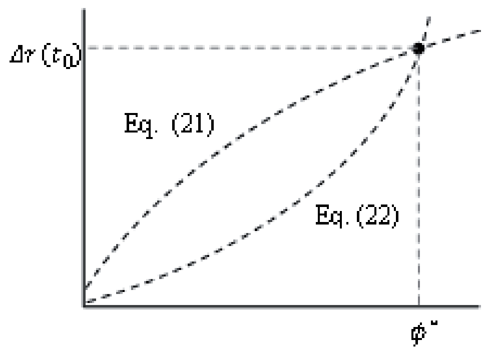

Figure 3. Equilibrium repurchase rate spread.

In a three-period framework, where the same bond remains cheapest-to-deliver and thus special for two consecutive deliveries, the arbitrageur is able to reestablish the strategy two times until the prices converge at t_2. In order to prevent arbitrage, the equilibrium price spread widens accordingly to counterbalance the arbitrageur's increased profits:

$$\frac{P(c,\, t_0)}{P(d,\, t_0)} = \left[\frac{1 + R(t_0)}{1 + r(t_0)}\right] \times \left[\frac{1 + R(t_1)}{1 + r(t_1)}\right] \tag{24}$$

Eq. (24) is a rearranged, three-period form of Eq. (23). Generalizing to N periods, Eq. (24) becomes

$$\frac{P(c,\, t_0)}{P(d,\, t_0)} = \prod_{n=0}^{N} \frac{1 + R(t_n)}{1 + r(t_n)} \tag{25}$$

Taking logarithms and rearranging gives

$$\ln\left[\frac{P(c,\, t_0)}{P(d,\, t_0)}\right] = \sum_{n=0}^{N-1} (\ln\, [1 + R(t_n)] - \ln\, [1 + r(t_n)]\,) \tag{26}$$

and

$$\ln\left[P(c,\, t_0)\right] - \ln\,\left[P(d,\, t_0)\right] \approx \sum_{n=0}^{N-1} R(t_n) - r(t_n)\; = \sum_{n=0}^{N-1} \Delta r(t_0) \tag{27}$$

where the current equilibrium (log) price spread increases according to the sum of (future) equilibrium repo spreads over $[t_0,\, t_N]$, and a time trend effect in $\Delta_i P(t_0)$ results.

3.5 Discussion of the Model

The model developed here is a simplified one in order to focus attention on the dynamics, and generate empirically testable predictions, of the cheapest-to-deliver premium. Four simplifying assumptions are made: first, it is assumed that the prices converge, which is a common assumption in periodical models such as the Amihud and Mendelson (1986) model for liquidity and the Duffie (1996) model for repurchase rates. It ensures that the contemporaneous equilibrium spread does not reflect any structural differences between bonds. Second, the model assumes that at least some futures buyers have positive marginal utility for accepting physical delivery, implying $\pi > 0$. This assumption corresponds to Fackler (1993) and Pirrong (1993) equilibrium models for futures markets, and is supported by empirical evidence. Assumption A.3 is based on the well-known bias of conversion factor method, discussed for example by Oviedo (2006), and assumption A.4 with search cost $\lambda[\phi^I]$ defines the natural equilibrium discussed in Duffie (1996).

The model provides several testable hypotheses concerning the equilibrium price spread, which are briefly discussed next. The positive relationship shown in Eq. (23) enables to write Equations (21) and (22) in terms of equilibrium price spread $\Delta_i P(t_0)$. Then, Eq. (21) states that the equilibrium price spread increases with the marginal delivery profits; that is, as product of the amount of physical deliveries and the relative cheapness in delivery. The arguments for this result are straightforward. First, if none of the bonds were to be delivered, then $\pi = 0$ and there would be no excess demand for the cheap bond. Second, if the conversion factor equalized the delivery prices completely, then $\Delta_i\beta = 0$ and there would be no cheap bond. Evidently, either of the two arguments is sufficient to take the marginal delivery profits and thus the equilibrium spread to zero. Eq. (22) states that the equilibrium price spread increases with the costs of borrowing against special collateral, $\lambda[\phi^I]$. This is the equilibrium effect of decreased collateral supply ϕ^l, and not directly futures markets initiated. Equations (23) and (27) relate to the no-arbitrage relationship between the cash and the financing markets á la Duffie (1996). The former equation states that the equilibrium price spread increases with the repurchase rate spread. Thus, the cash market price of the cheapest-to-deliver bond rises to offset the monetary benefit for borrowing with lower interest rate, indicated by the right-hand side of Eq. (23). If the bond owner is able to borrow with special rates for multiple periods as in Eq. (27), the contemporaneous price spread widens accordingly.

4 EMPIRICAL ANALYSIS OF THE MODEL

The comparative statics discussed in Section 3.5 provide several testable hypotheses concerning the equilibrium spread. For instance, the model implies that the investor affects the equilibrium price spread directly by choosing to purchase the cheapest-to-deliver bond for future delivery, and indirectly through financing markets by borrowing against the cheapest-to-deliver bond with special rates.

Unfortunately, not all implications are directly testable due to data restrictions. First, the equilibrium price spread is not observable as such; in order to quantify the spread, the benchmark bond price $\bar{P}(c,\ t_0)$ is estimated from the term structure of interest rates. Likewise, the repurchase agreements on special collateral are usually traded over-the-counter, which effectively limits the availability of data on equilibrium repo spread $\Delta r(t_n)$ and demand-supply flows indicated by $\left\{\phi^A, \phi^I\right\}$. Instead, the implied values of $\Delta r(t_n)$ are used, as well as the qualitative features of repurchase rate spread like the trend effect in Eq. (27).

In the next subsections, the data and the methodology for empirical tests are described more comprehensively. The empirical variables are defined, and the reduced-form model of Equations (21) and (27) is presented and its parameters are empirically estimated using fixed-effect least squares regression.

4.1 Data and Methodology

The data used in empirical testing is similar to Section 2, augmented with data on physical deliveries from Eurex and bond trading volumes and bid-ask-spreads from ICMA. The data comprises the period from January 2001 to March 2007.

The model in Section 3 specifies the equilibrium price spread being the difference between the price of the cheapest-to-deliver bond and a similar bond that is not deliverable. Since such bond does not usually exist nor is it traded, one is created and priced according to the term structure of interest rates. The approach is similar to the one used in studies analyzing tax effects [e.g. Litzenberger and Rolfo (1984); Green and Odegaard (1997)], liquidity [e.g. Elton and Green (1998); Jankowitsch, Mosenbacher, and Pichler (2006); Díaz, Merrick, and Navarro (2006)], and corporate bond pricing [e.g. Elton, Gruber, Agrawal, and Mann (2004)].

Suppose that the market price for the cheapest-to-deliver bond is

$$P(c,\ t_0) = \delta_m C_m + e \tag{28}$$

where δ_m is a $1\times\ M$ vector of discount factors derived from the term structure of risk-free interest rates, and C_m is a $M \times 1$ vector of coupon payments paid in m years, $0 \leq m \leq M$. The residual term e includes all idiosyncratic elements, such as the cheapest-to-deliver premium, included in $P(c,\ t_0)$. However, the vector δ_m is not directly observable, but has to be replaced with estimated values $\overline{\delta}_m$. For further description of the estimation process, see Appendix 1. Then, the price of a similar, synthetic bond with no idiosyncratic price elements is

$$\overline{P}(c, t_n) = \overline{\delta}_m C_m \tag{29}$$

Replacing $\delta_m C_m$ in Eq. (27) by $\overline{P}(t_n)$ gives the estimated equilibrium spread:

$$P(c,\ t_0) - \overline{P}(c,\ t_0) = \bar{e} \tag{30}$$

The theoretical model implies that the equilibrium spread is governed by the marginal delivery profits, the equilibrium spread between general and special repurchase rates, and the remaining time as special collateral. First, the marginal delivery profits comprise two components: the amount of delivered cheapest-to-deliver bonds π and the relative cost advantage $\Delta_i\beta$. The former is defined as $UTIL$, the utilization rate of cheapest-to-deliver bonds in near-month delivery at T_1:

$$UTIL \equiv \frac{OI(T_1)}{AOS(c)} \times 100000 \times \beta^C \tag{31}$$

where $OI(T_1)$ is futures market open interest on the delivery date, $AOS(c)$ is the outstanding amount of cheapest-to-deliver bonds, and $100000 \times \beta^C$ is the futures contract size times the bond conversion factor. The relative cost advantage $\Delta_i\beta$ is derived from difference between the implied repurchase rates, Eq. (1) of the cheapest and second-cheapest bonds:

$$IRR\ [c,\ \beta^C \times F^+(t_n, T_1)] = IRR\ [d,\ \beta^D \times F(t_n, T_1)] \tag{32}$$

where $F(t_n, T_1)$ is the actual futures price, and $F^+(t_n, T_1)$ is an artificially inflated futures price that equalizes the two IRR's. Then, the projected benefit in delivery costs is defined as:

$$MARGIN \equiv F^+(t_n, T) - F(t_n, T) \tag{33}$$

The equilibrium repurchase rate spread $\Delta r(t_0)$ is related to the equilibrium price spread by the no-arbitrage condition stated for example in Eq. (23). General repurchase rates are directly observable from the markets, but unfortunately the data sources for bond-specific special repurchase rates are scarce. Instead, the implied repurchase rate spread is used, which is calculated from well-known no-arbitrage relationship between the prices of the cheapest-to-deliver bond and the bond futures contract:

$$F(t_n, T_1) \times \beta^C = P(c,\ t_n) e^{(r(t_n)-C)*(T_1-t_n)/365} \tag{34}$$

where is the C is the coupon rate. Formula (34) could be written in the following way:

$$F(t_n, T_1) \times \beta^C = P(c,\ t_n) e^{(R(t_n)-C-\Delta r(t_0))*(T_1-t_n)/365} \tag{35}$$

Rearranging and taking logarithms yields

$$SPREAD \equiv R(t_n) - \ln\frac{F(t_n, T_1) \times \beta^C}{P(c,\ t_n)} \times 365 \times (T_1 - t_n)^{-1} - C \tag{36}$$

where $SPREAD$ is the empirical equivalent of $\Delta r(t_n)$ and corresponds to Brennan's (1986) implied convenience yield.

Furthermore, Eq. (27) shows that the equilibrium price spread is positively related with the remaining time that the bond is special collateral. Because the empirical results in Section 2 and the theoretical results in Section 3 both imply that the cheapest-to-deliver bonds trade on a special financing rate, the equilibrium spread should then decrease with the remaining time that the bond is cheapest-to-deliver. The remaining time is measured by $T_2 - t_n$ in the empirical analysis, where T_2 is the last possible delivery date. An alternative specification $T_1 - t_n$ is included as well to test whether the equilibrium spread decreases with respect to the next delivery, rather than the last possible delivery indicated by the cumulative profits in Section 2.

Since the cheapest-to-deliver bond may have different liquidity than the benchmark bond, the estimated equilibrium spread $P(c, t_n) - \bar{P}(c, t_n)$ may be contaminated by the liquidity factors not accounted for in the theoretical model. Therefore, the effect of liquidity differences in the estimated spread are controlled with two well-known indicators: the bid-ask spread, $BIDASK$, and logarithm of daily trading volume, $LogVol$. The summary for all variables is provided in Table 4.

4.2 Empirical Model

In order to empirically test the how the variables specified in the previous section affect the estimated equilibrium spread, the following reduced-form equation is estimated:

$$\begin{aligned} P(c, t_n) - \bar{P}(c, t_n) \quad = \quad & \alpha + \gamma_1[UTIL] + \gamma_2[MARGIN] + \gamma_3[SPREAD] + \gamma_4[T_2 - t_n] \\ & + \gamma_6[BIDASK(c, t_n)] + \gamma_7[T_1 - t_n] + year(t_n) + bond(c) + \varepsilon(c, t_n) \end{aligned} \tag{37.1}$$

where

$$\varepsilon(c, t_n) = \gamma_8[\, e(c, t_{n-1})\,] + u(c, t_n)$$

and $year(t_n)$ and $bond(c)$ are binary variables for different years and bonds, respectively.

Table 4. Summary of the variables used in the empirical analysis.

Model variable	Empirical variable	Explanation	Predicted sign	Motivation
$P(c, t_n)$	$P(c, t_n)$	Market price at time t_n for cheapest-to-deliver		
$P(d, t_n)$	$\bar{P}(c, t_n)$	Sum of discounted cash flows at time t_n for bond		
	Independent variables			
π	$UTIL$	Percentage of cheapest-to-deliver bonds utilized in physical delivery	Positive	Marginal delivery profit, Eq. (22)
$\Delta r(t_n)$	$SPREAD$	Implied repurchase rate spread	Positive	Benefit from reduced fin. costs, Eq. (24)
$t_2 - t_n$	$T_2 - t_n$	Time to last possible delivery (years)	Positive	Cumulative benefit from reduced fin. costs, Eq. (25)
$\Delta_i\beta$	$MARGIN$	Cheapest-to-deliver margin	Positive	Marginal delivery profit, Eq. (22)
	Control variables			
	$T_1 - t_n$	Time to next delivery		Alternative for
	$BIDASK(c, t_n)$	Bid-ask spread		Liquidity effects
	$LogVol(c, t_n)$	Log of trading volume		Liquidity effects
	$year(t_n)$	Year dummy		Fixed year
	$bond(c)$	Bond dummy		Fixed bond

An alternative specification is estimated as well:

$$P(c,\, t_n) - \bar{P}(c,\, t_n) \quad = \quad \alpha + \gamma_1[UTIL] + \gamma_2[MARGIN] + \gamma_3[SPREAD] + \gamma_4[T_2 - t_n] \qquad (37.2)$$
$$+\ \gamma_6[LogVol(c,\, t_n)] + \gamma_7[T_1 - t_n] + year(t_n) + bond(c) + \varepsilon(c,\, t_n)$$

Both equations are estimated by panel least squares regression. Preliminary Lagrange multiplier test and Wooldridge (2001) test for serial correlation detect heteroscedasticity and first-order autocorrelation in the regression error term. Therefore, $\varepsilon(t_n, c)$ is modeled as an AR(1) process, and the standard errors for regression coefficients are corrected for arbitrary heteroscedasticity and correlation by using Thompson (2005) method.

In addition to Equations (37.1) and (37.2), which concern only cheapest-to-deliver bonds, another two equations are estimated comprising any ten-year German government bond $j,\ j \in \{\, 8 < M < 10.5 \,\}$:

$$P(j,\ t_n) - \bar{P}(j,\ t_n) = \alpha + \gamma_1[OTR(j,t_n)] + \gamma_2[CTD(j,t_n)] + \gamma_3[NONDEL(j,t_n)] + \gamma_4[UTIL * CTD(j,t_n)]\,\gamma_5[MARGIN * CTD(j,t_n)] + \gamma_6[SPREAD * CTD(j,t_n)] + \gamma_8[BIDASK(j,t_n)] + year(t_n) + bond(j) + \varepsilon(j,t_n) \tag{38.1}$$

$$P(j,\ t_n) - \bar{P}(j,\ t_n) = \alpha + \gamma_1[OTR(j,t_n)] + \gamma_2[CTD(j,t_n)] + \gamma_3[NONDEL(j,t_n)] + \gamma_4[UTIL * CTD(j,t_n)]\,\gamma_5[MARGIN * CTD(j,t_n)] + \gamma_6[SPREAD * CTD(j,t_n)] + \gamma_8[LogVol(j,t_n)] + year(t_n) + bond(j) + \varepsilon(j,t_n) \tag{38.1}$$

where $P(j,t_n)$ is the market price for bond j, and $OTR(j,t_n)$, $CTD(j,t_n)$, and $NONDEL(j,t_n)$ are binary variables that distinguish on-the-run, cheapest-to-deliver, and non-deliverable bonds, respectively. The time-slope variables for cheapest-to-deliver bonds, $T - t_n$, are dropped from Equations (38.1) and (38.2) in order to maintain the comparability between the level variables $OTR(j,t_n)$, $CTD(j,t_n)$, and $NONDEL(j,t_n)$.

4.3 Results

Table 5 reports the descriptive statistics for the variables used in the empirical analysis. The second column in Table 5 indicates that the estimated equilibrium spread $\bar{e}[\cdot]$ is on average positive, and ranges from 10.3 cents to 15.5 cents (15.2 to 21.4) per €100 for cheapest-to-deliver bonds (all bonds), depending on the estimation method. Out of all bond–day observations in the sample, cheapest-to-deliver bonds cover 24.5 percent, on-the-run bonds 24 percent, and non-deliverable bonds 29 percent of the observations.

Table 5. Descriptive statistics of the variables used in the empirical analysis.

	Mean	Median	Max.	Min.	S.D.	Skew.	Kurt.	Sum	Obs.
Cheapest-to-deliver bonds, Eq. (36.x)									
$\bar{e}[VRP]$	0.155	0.149	0.871	–0.367	0.147	0.415	3.498	237.507	1534
$\bar{e}[NSS]$	0.103	0.104	0.910	–0.525	0.166	0.268	4.818	158.327	1534
$\bar{e}[FB]$	0.114	0.120	0.908	–0.578	0.173	–0.052	4.616	174.415	1534
UTIL	0.159	0.164	0.476	0.039	0.083	1.764	7.751	244.156	1534
MARGIN	0.457	0.384	1.241	0.000	0.277	0.751	2.678	700.357	1534
SPREAD	–0.027	–0.025	0.812	–1.132	0.156	–0.460	10.874	–41.852	1534
$T_2 - t_n$	0.250	0.255	0.499	0.000	0.143	–0.041	1.821	384.499	1534
All bonds, Eq. (37.x)									
$\bar{e}[VRP]$	0.214	0.143	1.287	–0.563	0.251	1.092	3.706	1335.072	6227
$\bar{e}[NSS]$	0.152	0.113	1.244	–0.546	0.237	0.904	4.516	951.771	6227
$\bar{e}[FB]$	0.169	0.137	1.337	–0.710	0.247	0.728	4.373	1055.066	6227
OTR	0.240	0.000	1.000	0.000	0.427	1.215	2.477	1503.000	6227
CTD	0.245	0.000	1.000	0.000	0.430	1.183	2.399	1534.000	6227
NONDEL	0.290	0.000	1.000	0.000	0.454	0.924	1.854	1815.000	6227
Control variables									
$T_1 - t_n$	0.123	0.123	0.249	0.000	0.072	0.033	1.831	188.322	1534
VOL (M€)	1630	1450	4860	106	1030	1.052	3.691	9880000	6227
BIDASK	0.025	0.020	0.240	0.000	0.027	3.239	17.619	153.280	6227

In Table 5, $\bar{e}[\cdot]$ is the equilibrium spread, estimated with VRP, NSS, and FB methods described in Appendix I. Other variables are UTIL (%-tage of bonds utilized in delivery); MARGIN (relative cheapness in delivery); SPREAD (implied repurchase rate spread); $T_2 - t_n$ (years to last possible delivery); OTR (a dummy variable for on-the-run bonds); CTD (a dummy variable for cheapest-to-deliver bonds); NONDEL (a dummy variable for non-deliverable bonds). The control variables are $T_1 - t_n$ (years to next delivery); BIDASK (bid-ask spread); and VOL is the daily trading volume (in millions).

Table 6 categorizes the estimated equilibrium spreads by the status of the bond. The first column describes the status: on-the-run, cheapest-to-deliver, or non-deliverable bond. According to the results, the on-the-run bond trades on an average premium ranging from 32.3 cents to 41.9 cents per €100, depending on the estimation method.[8]

[8] These estimates are over twice the size of Jankowitsch et al. (2006), but may be explained by the different estimation period (Jankowitsch et al. used the period from January 1999 to March 2001) or estimation procedure (estimating $\bar{P}(t_n, j)$, Jankowitsch et al. did not exclude on-the-run bonds). Especially the latter issue is may be decisive, because including on-the-run bonds in the term structure estimation results downward-biased estimates of on-the-run premium.

The second line in Table 6 provides the average estimates for cheapest-to-deliver premium, and the third line presents the estimates for non-deliverable bonds. Note that the former ranges from 10.3 cents to 15.5 cents per €100 and differs statistically from zero, whereas the latter varies from 0.1 cents to 3.3 cents per €100 and does not generally statistically deviate from zero.

Here, the cheapest-to-deliver premium could be approximated in two different ways: by the second line in Table 6 alone, or by adjusting the values on the second line with the ones on the third line. Either way, these results provide unequivocal support for the existence of the cheapest-to-deliver premium, and validate the convergence assumption A.I in the theoretical model. Albeit the estimated cheapest-to-deliver premiums in Table 6 are on average half the size indicated by the convergence strategy in Section 2, it should be noted that the values in Table 6 are averages and presumably larger at the time the convergence strategy is initiated.

Table 6. The estimates of equilibrium spread, categorized by bond status.

Status	$\bar{e}[VRP]$	$\bar{e}[NSS]$	$\bar{e}[FB]$	N
OTR	**0.419**	**0.323**	**0.357**	
	[65.94]	[55.12]	[58.52]	1503
	(0.00)	(0.00)	(0.00)	
CTD	**0.154**	**0.103**	**0.112**	
	[24.46]	[17.82]	[18.63]	1534
	(0.00)	(0.00)	(0.00)	
NONDEL	**0.033**	0.001	0.003	
	[5.76]	[0.22]	[0.49]	1815
	(0.00)	(0.83)	(0.62)	
N	6227	6227	6227	

The categories are OTR (on-the-run bonds), CTD (cheapest-to-deliver bonds), and NONDEL (non-deliverable bonds). $\bar{e}[VRP], \bar{e}[NSS]$, and $\bar{e}[FB]$ are the estimates of the equilibrium spread. Student's t-values (p-values) in brackets (parentheses) report the statistical significance from zero mean. Bolded values indicate statistical significance on at least 5% level.

Table 7 provides the results for multivariate analysis specified by Equations (37.1) and (37.2). In general, the coefficients for the futures–market related variables are statistically significant and consistent with the theoretical model; as predicted by the model, increases in delivered amounts (*UTIL)*, relative cheapness (*MARGIN*), and implied repurchase spread (*SPREAD*) are all positively related to the price premium. In addition, the coefficient for the remaining time to last delivery $T_2 - t_n$ appears positive and statistically significant in all regressions, confirming the cyclical pattern of cheapest-to-deliver premium. In contrast, the coefficient for time to next delivery $T_1 - t_n$ is generally small and insignificant, confirming that the premium decreases towards the last, instead of the next, delivery. All relevant coefficients are somewhat unchanged over different estimation methods and the measures of liquidity, which indicates that the results are robust to model misspecification. The model fits are good as well; adjusted R^2 ranges from 38 to 73 percent.

Table 8 reports the estimation results for Equations (38.1) and (38.2). Since the estimation concerns the complete sample, the bonds having a special status are allowed for an individual constant term: "*OTR*" for on-the-run bonds, "*CTD*" for cheapest-to-deliver bonds, and "*NONDEL*" for non-deliverable bonds. This sort of classification is strongly supported by the results: according to first four rows in Table 8, the coefficients for *OTR*, *CTD*, and *NONDEL* appear statistically significant in all regressions. The differences between the status coefficients are analogous to the values reported in Table 6, indicating that the cheapest-to-deliver bonds are on average €0.25 cheaper than on-the-run bonds, but trade on a €0.10 premium to non-deliverable bonds.

The results for the futures-markets related variables *UTIL*, *MARGIN*, and *SPREAD* are little changed from Table 7. The signs of the regression coefficients are consistent with model predictions, and appear statistically significant in most regressions. According to additional tests not reported here, the log-likelihood ratio tests for restricted model $\{\gamma_4, \gamma_5, \gamma_6\} = 0$ reject the null hypothesis in all regressions with ratios ranging from 81.87 to 329.51. This indicates that the futures-market related variables have explanatory power beyond the bond status and liquidity coefficients, thus providing further support for the model developed in Section 3.

Table 7. The effect of model-specified variables on the estimated equilibrium spread, cheapest-to-deliver subsample.

Variable	Prediction	$\bar{e}[VRP]$ 1	$\bar{e}[VRP]$ 2	$\bar{e}[NSS]$ 1	$\bar{e}[NSS]$ 2	$\bar{e}[FB]$ 1	$\bar{e}[FB]$ 2
Constant		**0.173**	**0.519**	**0.152**	**–0.235**	**0.246**	**0.199**
		[4.41]	[1.73]	[2.46]	[-0.48]	[3.63]	[0.37]
		(0.00)	(0.08)	(0.01)	(0.64)	(0.00)	(0.72)
UTIL	(+)	**0.354**	**0.376**	–0.156	–0.154	**0.286**	**0.371**
		[6.64]	[6.42]	[-1.47]	[-1.38]	[2.85]	[3.57]
		(0.00)	(0.00)	(0.14)	(0.17)	(0.00)	(0.00)
MARGIN	(+)	**0.237**	**0.248**	**0.355**	**0.346**	**0.297**	**0.309**
		[7.50]	[7.49]	[8.34]	[7.80]	[6.30]	[6.23]
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
SPREAD	(+)	**0.221**	**0.209**	**0.178**	**0.162**	**0.205**	**0.185**
		[5.47]	[5.13]	[4.86]	[4.68]	[4.47]	[4.03]
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$T_2 - t_n$	(+)	**0.360**	**0.372**	**0.229**	**0.253**	**0.301**	**0.327**
		[15.47]	[15.58]	[2.79]	[5.78]	[7.12]	[7.36]
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$T_1 - t_n$		0.006	–0.006	–0.054	–0.075	**–0.037**	-0.079
		[0.10]	[0.10]	[-0.69]	[-0.90]	[4.89]	[0.89]
		(0.92)	(0.92)	(0.49)	(0.36)	(0.00)	(0.37)
BIDASK		**0.456**		**0.544**		**0.457**	
		[3.93]		[2.69]		[2.09]	
		(0.00)		(0.00)		(0.04)	
Log(Vol)			–0.016		0.017		-0.004
			[-1.18]		[0.78]		[-0.18]
			(0.24)		(0.44)		(0.85)
AR(1)		**0.273**	**0.277**	**0.322**	**0.342**	**0.315**	**0.337**
		[6.25]	[6.12]	[9.60]	[9.95]	[8.54]	[8.83]
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Year ID		*Yes*	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*
Bond ID		*Yes*	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*
Adj. R^2		0.73	0.71	0.39	0.38	0.42	0.43
N		1534		1534		1534	

The dependent variable in all regressions is $\bar{e}[\cdot]$, the daily spread between the market price and the sum of discounted cash flows. $\bar{e}[\cdot]$ is estimated by VRP, NSS, and FB methods described in Appendix I. The independent variables are UTIL (%-tage of bonds utilized in delivery); MARGIN (relative cheapness in delivery); SPREAD (implied repurchase rate spread); $T_2 - t_n$ (years to last possible delivery). The control variables are $T_1 - t_n$ (years to next delivery); BIDASK (bid-ask spread); and VOL (daily trading volume). Bond ID and year ID are dummy variables that specify individual bonds and years, respectively. The regression residual term is assumed to follow an AR(1) process. Thompson (2005) robust standard errors are used to correct for arbitrary heteroscedasticity and correlation. Adjusted t-values (p-values) are in brackets (parentheses). Bolded values indicate statistical significance on at least 5% level.

Table 8. The effect of model-specified variables on the estimated equilibrium spread, all bonds.

Variable	Prediction	$\bar{e}[VRP]$ 1	$\bar{e}[VRP]$ 2	$\bar{e}[NSS]$ 1	$\bar{e}[NSS]$ 2	$\bar{e}[FB]$ 1	$\bar{e}[FB]$ 2
Constant		**0.477**	0.531	**0.517**	–0.331	**0.608**	0.081
		[3.10]	[1.42]	[3.63]	[-0.85]	[3.50]	[0.18]
		(0.00)	(0.15)	(0.00)	(0.39)	(0.00)	(0.86)
OTR		**0.134**	**0.130**	**0.124**	**0.099**	**0.146**	**0.125**
		[3.65]	[3.60]	[-3.29]	[2.57]	[3.14]	[2.54]
		(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.01)
CTD		**–0.141**	**–0.145**	**–0.134**	**–0.138**	**–0.149**	**–0.153**
		[-4.40]	[-5.25]	[-4.80]	[-5.13]	[-4.03]	[-4.23]
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
NONDEL		**–0.254**	**–0.253**	**–0.238**	**–0.213**	**–0.273**	**–0.257**
		[2.65]	[-4.68]	[-4.52]	[-4.42]	[-4.27]	[-4.41]
		(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
*UTIL*CTD*	*(+)*	0.450	0.347	0.131	0.048	**0.438**	**0.372**
		[1.85]	[0.46]	[1.28]	[0.41]	[4.95]	[3.15]
		(0.06)	(0.65)	(0.20)	(0.51)	(0.00)	(0.00)
*MARGIN*CTD*	*(+)*	**0.147**	**0.155**	**0.147**	**0.142**	**0.163**	**0.169**
		[2.66]	[3.05]	[2.82]	[3.02]	[2.25]	[2.47]
		(0.01)	(0.00)	(0.00)	(0.00)	(0.02)	(0.01)
*SPREAD*CTD*	*(+)*	**0.238**	**0.1224**	**0.181**	**0.164**	**0.210**	**0.189**
		[4.78]	[3.13]	[3.52]	[3.30]	[3.60]	[3.21]
		(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.00)
BIDASK		**0.364**		**0.514**		**0.342**	
		[3.70]		[4.03]		[3.00]	
		(0.00)		(0.00)		(0.02)	
LogVol			–0.003		**0.040**		0.025
			[-0.18]		[1.98]		[1.07]
			(0.86)		(0.05)		(0.28)
AR(1)		**0.533**	**0.532**	**0.427**	**0.439**	**0.478**	**0.492**
		[15.70]	[13.61]	[14.33]	[14.20]	[13.90]	[14.52]
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Year ID							
Bond ID							
Adj. R^2		0.85	0.83	0.59	0.56	0.61	0.59
N		6227		6227		6227	

The dependent variable in all regressions is $\bar{e}[\cdot]$, the daily spread between the market price and the sum of discounted cash flows. $\bar{e}[\cdot]$ is estimated by VRP, NSS, and FB methods described in Appendix I. The independent variables are OTR (a dummy variable for on-the-run bonds); CTD (a dummy variable for cheapest-to-deliver bonds); NONDEL (a dummy variable for non-deliverable bonds); UTIL (%-tage of cheapest-to-deliver bonds utilized in delivery); MARGIN (relative cheapness in delivery); SPREAD (implied repurchase rate spread). The control variables are BIDASK (bid-ask spread) and VOL (daily trading volume). Bond ID and year ID are dummy variables that specify individual bonds and years, respectively. The regression residual term is assumed to follow an AR(1) process. Thompson (2005) robust standard errors are used to correct for arbitrary heteroscedasticity and correlation. Adjusted *t*-values (p-values) are in brackets (parentheses). Bolded values indicate statistical significance on at least 5% level.

5 SUMMARY AND CONCLUSIONS

The theory of portfolio choice states that the demand for bonds is governed by four factors: the wealth of the marginal investor, and the expected return, risk, and liquidity of bonds relative to alternative assets. Besides these four factors, the present study argues that investor's contractual liability regarding bond-futures settlement process increases the demand, and thus the equilibrium price, for certain types of bonds. These bonds are cheapest-to-deliver against bond futures contracts, and play a central part in bond-futures settlement process known as physical delivery. In case of the physical delivery, the futures seller is obligated to deliver an eligible bond to the futures buyer. The futures seller incurs losses if any other than the cheapest bond is delivered.

As a consequence, the futures-market initiated demand concentrates on that particular bond. Section 2 of this paper reports on a test of the effects that futures-market initiated demand causes on the prices of the cheapest-to-deliver bonds. The test comprises an arbitrage strategy that generates profits if the cheapest-to-deliver bond trades on a premium. Using end-of-day price data from German government ten-year Bund markets, systematic profits from the trading strategy imply that a premium exists, and that it decreases gradually until the cheapest-to-deliver bond becomes ineligible for delivery. Based on no-arbitrage arguments, it is also deduced that the cheapest-to-deliver bonds trade on special financing rates. Next, a general equilibrium model is developed to analyze the underlying factors governing the cheapest-to-deliver premium in Section 3. The model implies that the premium increases as a function of the delivered amount and the relative cheapness of the cheapest-to-deliver bond. The cumulative benefit from a special financing rate results in the characteristic time trend effect indicated by the arbitrage profits. The implications of the theoretical model are empirically tested in Section 4. Throughout the tests, the results consistently support and corroborate the theoretical model. The estimated equilibrium spread between the cheapest-to-deliver and a non-deliverable bond is approximately €0.10 per every €100 and robust to the various specifications of the empirical model. As hypothesized, the premium increased as a function of the amount of delivered bonds, the relative cheapness, and the (cumulative) benefit from reduced financing costs.

In conclusion, the results of this paper strongly suggest that the futures-market initiated demand inflates the price of the bond that is cheapest-to-deliver for the Bund futures contract. The results are of importance for futures exchanges, bond market researchers and investors alike. The welfare effects of the price premium, and reversed-causality effects to the price of the futures contract, are possible issues for future research.

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APPENDIX 1 TERM STRUCTURES MODELS

A term structure model refers to a statistical method to extract discount factors, spot, or forward rates from the prices of coupon bonds. In general, the term structure has to be estimated, because a continuous set of default-free zero-coupon bonds is not traded on the actual markets. The German government bond market is no exception: the continuous term structure has to be estimated from a discrete series of coupon bond prices. The purpose of this Appendix is to demonstrate the term structure models and estimation techniques used in the study.

A.1 Extracting Term Structure from Coupon Bond Prices

The fundamental theorem of asset pricing implies that the price of a default-free bond in frictionless markets is a linear function of time m cash flows C_m and unique discount factors δ_m:

$$P = \sum_{m=1}^{M} C_m \, \delta_m \qquad (A1)$$

Basically, δ_M is the current price of a zero-coupon bond that at maturity M pays one dollar with certainty. Absence of arbitrage and consumer impatience imply that the discount factor strictly positive, but decreasing with respect to time to maturity. The discount factor could be also expressed in terms of spot (s_m) or forward (f_m) interest rate using following relations:

$$\delta_m = \exp\,(-m \times s_m)$$

$$\delta_m = \exp\,(-\int_0^m f(t)\, dt)$$

These relations allow to extract discount factors from the term structure of interest rates, or correspondingly from prices of default-free zero-coupon bonds. In practice, however, frictions in real market lead to sets of parallel discount factors that hold approximately in Equation (A1). Therefore, it is more sensible to use an inexact relation:

$$P_i = \sum_{m=1}^{M} C_{i,m}\,\delta_m + e_i \qquad (A2)$$

and apply a term structure model to estimate a continuous set of unique discount factors that result in the smallest errors e_i, $i = 1, \ldots, N$, in the cross-section of coupon bond prices.

In addition to choosing appropriate pricing function as in Equation (A2), the term structure estimation involves selection of functional form to be used and statistical method to obtain the parameters of the functional form. In order to minimize the effect of misspecification of functional form on the error term, three distinctive functional forms are estimated in parallel. These models are taken from the past literature, and are widely used in central banks and the academia [e.g. BIS (2005); Jordan and Mansi (2003); Subramanian (2001); Anderson and Sleath (1999); Bliss (1997)].

A.2 Term Structure Models

Svensson's (1994) parsimonious model uses six parameters to characterize the entire term structure of spot rates:

$$s_m(\beta,\tau) = \beta_0 + \beta_1\left[\frac{1-\exp(-m/\tau_1)}{(m/\tau_1)}\right] + \beta_2\left[\frac{1-\exp(-m/\tau_1)}{(m/\tau_1)} - \exp\left(-\frac{m}{\tau_1}\right)\right] \qquad (A3)$$
$$- \beta_3\left[\frac{1-\exp(-m/\tau_2)}{(m/\tau_2)} + \exp\left(-\frac{m}{\tau_2}\right)\right]$$

where β_0 represents the asymptote of long-term interest rates, and β_1 defines the slope. The remaining parameters define the position, magnitude, and direction of humps in the term structure. The parameters are estimated using non-linear constrained optimization procedure:

$$\min_{\beta,\tau} \sum_{i=1}^{N} (\ w_i \times e_i\ )^2$$

subject to

$$\beta_0 \geq 0$$
$$\beta_0 + \ \beta_1 \geq 0$$
$$1 \leq \tau_1, \tau_2 \leq M_{max}$$

The weights w_i are computed using the inverse of bond *i* Macauley duration d_i:

$$w_i = \frac{1/d_i}{\sum_i^N 1/d_i}$$

Due to non-linearity of the function, the optimization procedure could end up in local minima rather than the global minimum. In order to find the global minimum, the procedure is repeated using various sets of starting parameters (altogether 91 sets).

The smoothed Fama-Bliss method [Bliss (1997); Fama and Bliss (1987)] requires a two-step process: first, iterative extraction of forward rates to obtain spot rates (s_m), and then fitting extended Nelson-Siegel [Nelson and Siegel (1987)] type of approximating function through them:

$$s_m(\beta, \tau) = \beta_0 + \beta_1 \left[\frac{1 - \exp\left(-\mathrm{m}/\tau_1\right)}{(\mathrm{m}/\tau_1)}\right] + \beta_2 \left[\frac{1 - \exp\left(-\mathrm{m}/\tau_2\right)}{(\mathrm{m}/\tau_2)} - \exp\left(-\frac{\mathrm{m}}{\tau_2}\right)\right] \qquad (A4)$$

by

$$\min_{\beta,\tau} \sum_{m=1}^{M} [s_m - s_m(\beta, \tau)]^2$$

where the parameters (β, τ), constraints, and the estimation procedure is are analogous to Equation (3) with few exceptions: the model contains only one “hump” parameter β_2, and the difference between iterated and approximated spot rate is minimized rather than the weighted price error.

Finally, the smoothed spline method with variable roughness penalty [Waggoner (1997)] approximates forward rates as piecewise cubic polynomials, with segments joined at knot points. This guarantees very flexible representation of the forward curve, as the segments move almost independently of each other. The downside is that the flexibility and excessive curvature makes the outcome inappropriate for smooth term structure representation. To prevent this, excessive curvature of the forward curve is measured by the square of the second derivative and penalized accordingly. The objective function becomes a trade-off between (1) the cubic spline that minimizes the error between observed prices and prices implied by the forward curve, and (2) the smoothness of the forward curve

$$\min_{f} \sum_{i=1}^{N} {e_i}^2 + \int_0^M \Pi(m)[\, f''(m)\,]^2 dm \qquad (A5)$$

$$\Pi(t) = \begin{cases} 10, & 1 < m \leq 10 \\ 100, & 10 < m \leq 30 \end{cases}$$

where $[\, f''(m)\,]^2$ is the measure of curvature, and $\Pi(m)$ is the variable roughness penalty (VRP) that controls the smoothness of the forward curve as a function of time to maturity. As in Waggoner (1997), the VRP used in this paper is a step function with increasing penalty values, but with few modifications to German data; first, due to exclusion of one-year or shorter bonds, it has only one step occurring at ten years to maturity. Second, the original VRP values were chosen to maximize the out-of-sample fit in the U.S. Treasury market; these values, however, generate poor out-of-sample fit in the German market. Based on Waggoner's (1997) original criterion, the out-of-sample weighted mean absolute error, extensive testing suggest that much smaller values as in Equation (A5) are more appropriate. This result may stem from the differences in maturity spectrum between German and more diverse U.S. Treasury government bond markets; the number of knot points that separate maturity segments are usually placed at the maturity of every third bond, and, *ceteris paribus*, more knot points increase the flexibility of the forward curve and thus require larger penalty values.

A.3 Methodology and Data

Following Bliss (1997) and Waggoner (1997), the available set of bonds is sorted by maturity and divided into two subsamples: the estimation subsample is used to extract the term structure on a daily basis, and compute fitted prices and errors for bonds in the hold-out subsample. The bond with the longest time to maturity is always included in the estimation sample. Then, the goodness-of-fit of estimated curve is analyzed using two distinctive measures: the weighted mean absolute error ("WMAE"), and the hit rate. WMAE indicates how much the fitted price deviates from the midpoint of bid and ask prices. Each price error is weighted by the inverse of Macauley duration; this feature enables to combine the price errors across maturities, irrespective of the asymmetric relationship between interest rate and price changes. The second performance measure, hit rate, is the percentage of "perfectly" priced bonds; in other words, it describes how many bonds have the estimated price inside the bid and ask quotes of the total number of available bonds. Only the hold-out sample is used to calculate the goodness-of-fit statistics; optimization procedure guarantees that any method will produce a good in-sample fit.

The data consist of ICMA end-of-day price quotes for German government long-term Bundesanleihen ("Bund") and medium-term Bundesobligationen ("*Bobl*") bonds, and Bundesschatzanweisungen ("*Schatz*") notes. The data extends from January 2001 to March 2007, for a total of 1578 trading days. Based on the past studies [Bliss (1997); Schich (1997); Fisher (1996)], the following types of filters are imposed to ensure that the securities used in term structure extraction are largely homogenous:

- Bonds with special terms or option features
- Bonds or notes that have could have abnormal liquidity:
  - less than one year to maturity
  - outstanding amount less than 5 billion
  - bonds issued by government special funds
  - depending on the number of bonds in each maturity segment, one or two most recently issued securities
- Bonds or notes that are cheapest-to-deliver against the near-month futures contract
  - In order to avoid any conceptual problems in the empirical analysis of the study

A.4 Results

The goodness-of-fit results for tested models are presented in Table A1. Overall, both performance measures indicate that all three methods and the associated estimation procedures do well in pricing hold-out sample bonds. Although the estimation method proposed by Svensson performs best and the Smoothed Fama-Bliss does worst by both measures, the differences are very small. For example, Table A1 shows that the WMAE of the best method is 6.1 cents per €100, but the marginal to the worst method is only 1.3 cents per €100. The hit rate confirms that the good fits: approximately 38 percent of the price estimates hit inside the actual bid-ask prices. The unit root tests indicate that the residual terms of all models follow stationary processes.

Table A1. The goodness-of-fit results for estimated term structure models.

Criterion	NSS	FB	VPR
Weighted Mean Absolute Error ("WMAE")			
Mean	0.06	0.07	0.07
Median	0.06	0.07	0.07
Standard deviation	0.03	0.04	0.04
Hit Rate			
Mean	0.39	0.36	0.39
Median	0.35	0.35	0.37
Standard deviation	0.20	0.19	0.18
Tests for unit root			
Common (LLL)†	−2.92***	−9.31***	−8.77***
Individual (IPS)†	−10.06***	−16.46***	−15.18***
Average number of daily observations (in/out)		18 / 17	
Number of business days		1578	

† Under the null hypothesis, the residual term follows unit root process. Lag selection is based on Schwarz criterion. "***" denotes rejection of the null hypothesis at 1% level. "LLL" and "IPS" denote panel unit root test by Levin, Lin, and Chu (2002) and Im, Pesaran, and Shin (2003), respectively. Panels are allowed to have individual intercepts.