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DOES DEMAND PRESSURE ON OPTIONS EXPLAIN MOVEMENTS IN IMPLIED VOLATILITY?

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TABLE OF CONTENTS **PAGE**

LIST OF FIGURES

LIST OF TABLES

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ABSTRACT

The purpose of this study is to examine how options demand explains movements in implied volatility. The study takes a stock option approach and uses Barclays Plc. stock options to determine how stock options demand affects to corresponding implied volatility. The Barclays Plc. stock options behaviour can be seen as a reflection of stock options markets in the London International Futures and Options Exchange (LIFFE). The option demands ability to explain implied volatility changes is investigated in five different moneyness categories.

The empirical part of this study contains the use of Cox, Ross and Rubinstein binomial tree option pricing model and bisection method to calculate option implied volatilities. The hypotheses used in the study are based on the option pricing theory of flat option supply curves and the effects of option demand pressure on implied volatility changes are tested with specified regressions and ordinary least squares (OLS) estimation method. The data set of this study contains tick- and end-of-day Barclays Plc. stock options data from 4 January, 2005 to 30 December, 2005. These options are traded in the London International Financial Futures and Options Exchange (LIFFE).

The empirical results show that changes in stock option implied volatility are directly related to demand pressure from public order flow and especially changes in implied volatility are dominated by call option demand. As a result – the demand pressure moves stock option prices. The trading is also partly motivated by changes in expected future volatility, but price reversals of implied volatilities are an average as much as 47 percent.

KEYWORDS: Options, implied volatility, demand pressure.

1. INTRODUCTION

Derivative markets are expanding continuously. The growth of the markets started in the 1970s and 1980s, when contracts written on financial contracts were introduced and the modern-day option valuation theory was developed. The major breakthrough in the theory was the development of the Black-Scholes option valuation model, derived by Fischer Black and Myron Scholes (1973) and expanded by Robert Merton (1973). The key implication of their model is that contract valuation in general is possible under the assumption of risk-neutrality. (Whaley 2003.)

The Black-Scholes model has the known deficiency of often inconsistently pricing deep in-the-money and deep out-of-the-money options. Option professionals refer to this well-known phenomenon as a volatility "smile" or "skew". A volatility smile is the pattern that results from calculating implied volatilities across the range of exercise prices spanning a given option class. The name smile comes from the fact that, prior to the October 1987 market crash, the relation between the Black-Scholes implied volatility of equity options and exercise price gave the appearance of a smile. Since October 1987, however, the implied volatility decreases as the exercise price increases and performs a skew. Still, under the assumptions of the Black-Scholes model, the smile should be flat and constant through time. There are two major strands of studies trying to explain the smile pattern. The first strand of literature derives modified versions of options pricing models using different volatility assumptions (deterministic local volatility, stochastic volatility and explicitly model volatility). The second strand of the literature emphasizes that the outcomes of implied volatility smiles come from the options market microstructure. Throughout the study when speaking of volatility smile I refer to the pattern that implied volatilities differ across exercise prices. (Bollen & Whaley 2004; Chan, Cheng & Lung 2004: 1167; Corrado & Su 1997.)

Investigating option market microstructure and particularly supply and demand of options leads to a better understanding of volatility smile phenomenon. Theoretically speaking, under dynamic replication, the supply curve for each option series[1] is a horizontal line. No matter how large the demand for

[1] An option series is defined by three attributes – call or put, exercise price, and expiration date.

buying a particular option, its price and implied volatility are unaffected. As pointed out later, in reality, prices are affected by supply and demand considerations. (Bollen at al. 2004.)

This study examines how efficiently options supply and demand explains the implied volatility smile pattern by assessing the relation between demand pressure and implied volatility movement. The hypotheses of this study are based on the demand pressure[2] hypothesis described by Bollen et al. (2004) and each of them are tested in the empirical part. The hypotheses are defined in more detail in Chapter 5.2. Hypotheses are:

H_0: No relation exists between demand for options and related implied volatilities.

H_1: With supply curves upward sloping, an excess of buyer-motivated trades will cause price and implied volatility to rise, and an excess of seller-motivated trades will cause implied volatility to fall.

H_2: A positive relation between demand for options and related implied volatilities would be observed if the order imbalance merely reflects a change in investor expectations about future volatility.

The demand pressure hypothesis states that, although there are several possible reasons for the implied volatility smile, the demand pressure from supply and demand imbalance explains the smile pattern. Next, in Chapter 1.1., the previous studies of trading pressure effects on implied volatility smile are reviewed. The problem statement and the structure of this Thesis will be demonstrated in Chapter 1.2.

[2] Bollen et al. (2004) used ”net buying pressure” phrase as I use more convenient “demand pressure” phrase.

1.1. Previous studies

The trading pressure effects on implied volatility smile are previously studied by Dennis and Mayhew (2002), Bollen et al. (2004), Chan et al. (2004), and Chan, Cheng and Lung (2006).

Dennis et al. (2002) investigated the volatility skew observed in the prices of stock options. Their data covers quotes and trades of individual stock options listed on Chicago Board Options Exchange (CBOE) from April 1986 through December 1996. They tested whether leverage, firm size, beta, trading volume, and/or the put/call volume ratio can explain cross-sectional variation in risk-neutral skew. They find that risk-neutral density implied by individual stock option is negatively skewed and notes that skewness is more negative for stocks with large betas, in periods of high volatility and times when risk-neutral density for index options is more negatively skewed. Also firm size and trading volume explains the risk-neutral density skewness. They also argued that one possible explanation for implied volatility skew is that demand for out-of-the-money puts drives up the prices of low strike price options. However, they did not find a robust cross-sectional relationship between the risk-neutral skew and the put/call volume ratio[3]. In other words they did not find any relation between trading pressure and implied volatility smile.

Bollen et al. (2004) studied the trading pressure effects on implied volatility smile in both index options and individual stock options markets. They find that demand pressure is related to the daily changes in the implied volatility. They document that particularly out-of-the-money (OTM) put options implied volatility is higher because of the demand pressure.

Bollen et al. (2004) also find that average stock option volatility curve differs remarkably from the index volatility curve. The index volatility curve is monotonically declining whereas the stock option volatility curve forms a smile. In generally index option implied volatilities are higher than stock option implied volatilities. Their regression analysis show that there is strong statistical relation between the change in implied volatility and demand pressure furthermore its evident that for index options the demand pressure for index puts dominates

[3] Dennis et al. (2002) used put/call volume ratio as a proxy for trading pressure.

whereas for stock options the demand pressure for call dominates. Also the analysis show that option's own demand pressure is the dominant trading pressure variable in explaining implied volatility changes.

Chan et al. (2004) examined the demand pressure hypothesis of Bollen et al. (2004) on rather new Hong Kong Hang Seng Index options. They produced five different moneyness categories for options at various time frames and calculated implied volatilities, options premiums, and options trading profits. Their results indicate that the demand pressure hypothesis exists also in the Hang Seng index options markets due to a reverse relation between exercise prices and options trading profits. They also found that delta neutral strategy involving trading with out-of-the-money put options can generate abnormal returns. In more recent research Chan et al. (2006) investigated demand pressure in the Hong Kong Hang Seng index options market during the Asian financial crisis from July 1997 to August 1998. They find that over the entire crisis period, the changes in market expectations, rather than changes in demand pressure, drive changes in option implied volatility.

1.2. Problem statement and structure of the Thesis

The purpose of this study is to examine how well the options' demand and supply explains the movements in options' implied volatilities. As mentioned, if the implied volatility change results from demand for options, the option demand generates price pressure and implied volatility changes. When the demand affects are studied the stock options' implied volatilities and demand pressure variables need to be characterised and estimated.

In global markets the common commodity prices are formed from supply and demand equilibrium. However, option prices are valuated differently; theoretically options price and implied volatility are unaffected of options supply and demand. Therefore it can be hypothesized that no relation exists between demand for options and corresponding implied volatilities. In this study stock option implied volatilities are estimated using the binomial option pricing model developed by Cox, Ross and Rubinstein (1979).

The demand pressure hypothesis of Nicolas Bollen and Robert Whaley (2004) is used in the study. The demand pressure is defined as the difference between buyer- and seller-motivated contracts traded per day. The study hypotheses are investigated by using Barclays Plc. stock options (BBL). The BBL stock options are traded in London International Financial Futures and Options Exchange (LIFFE) and the sample data includes options traded on LIFFE between January 2005 and December 2005. Barclays Plc. is a global financial services provider and the BBL stock options were the fifth most traded stock option in LIFFE during sample period. Thou, Barclays Plc. stock options describe well the stock option markets in LIFFE.

The thesis is divided into seven chapters. In the first chapter the topic and research problem were introduced and also the previous research, related to this study, was covered. The option theory, option pricing and volatility framework are discussed in the chapters two, three and four respectively. In chapter five the data, hypotheses and methodology are introduced before the regression analysis is introduced. Chapter six reveals the empirical results and chapter seven summarises and concludes the study.

2. OPTION THEORY

In economics the concept of market is understood as a organizational device which brings together buyers and sellers. In financial markets different kind of financial assets and -instruments are traded (i.e. Deposits, Bills, Bonds, Currencies, Equities, Assurances, Pensions and Derivatives). In the beginning of this chapter derivative instruments and -markets are introduced, but later on this chapter concentrates on options and their features. (Howells & Bain 2005: 19.)

Derivatives are securities whose prices are determined by the prices of other securities. These assets are also called contingent claims because their payoffs are contingent on the price of other securities. Derivative securities include futures, forwards and options as basic instruments. Swaps and some complicated instruments are hybrid securities, which can eventually be decomposed into sets of basic forwards and options. As derivatives' underlying asset almost everything can be used. Traditionally, the variables underlying options and other derivatives have been stock prices, stock indices, interest rates, exchange rates, and commodity prices. In this study stock options are under consideration and they are derivatives whose value is dependent on the price of a stock. Because the value of derivatives depends on the value of other securities, they can be powerful tools for both hedging and speculation. (Bodie, Kane & Marcus 2005: 697; Neftci 2000: 2-3; Hull 2003: 15.)

2.1. Derivative markets

Although, the origin of derivatives use dates back thousands of years, still in the last 35 years derivatives has grown its importance and the most important innovations occurred. Not coincidently the most important theoretical developments in the derivative literature are done in the 1970s and 1980s. Nowadays all kinds of derivatives are traded actively on exchanges throughout the world. A derivatives exchange is a market where individual's trade standardized contracts that have been defined by the exchange. Traditionally the trading of derivatives has occurred on the floor of exchanges via shouting and hand signalling between traders. Today most of the trading is completed via electronic trading while floor trading is dying. However not all trading is done on exchanges. The over-the-counter (OTC) market is an important alternative to ex-

changes and, measured in terms of the total volume of trading, has become much larger than the exchange-traded market. It is a telephone- and computer-linked network of dealers, who do not physically meet. Trades are done over the phone and are usually between two financial institutions or between a financial institution and one of its corporate clients. (Hull 2003: 1-2; Whaley 2003: 1132.)

A forward contract is a particularly simple derivative. It is an agreement to buy or sell an asset at a certain future time for a certain price. A forward contract is traded in the over-the-counter market—usually between two financial institutions or between a financial institution and one of its clients. One of the parties to a forward contract assumes a long position and agrees to buy the underlying asset on a certain specified future date for a certain specified price. The other party assumes a short position and agrees to sell the asset on the same date for the same price.

Like a forward contract, a futures contract is agreement between two parties to buy or sell an asset at a certain time in the future for a certain price. Unlike forward contracts, futures contracts are normally traded on an exchange. To make trading possible, the exchange specifies certain standardized features of the contract. As the two parties to the contract do not necessarily know each other, the exchange also provides a mechanism that gives the two parties a guarantee that the contract will be honoured. One way in which a futures contract is different from a forward contract is that an exact delivery date is usually not specified. The contract is referred to by its delivery month, and the exchange specifies the period during the month when delivery must be made. For commodities, the delivery period is often the entire month. (Hull 2003: 2-6; Kolb 1999: 3.)

It is defined that swap is the simultaneous selling and purchasing of cash flows involving various currencies, interest rates, and a number of other financial assets. Usually a swap is an agreement between two companies to exchange cash flows in the future. The agreement defines the dates when the cash flows are to be paid and the way in which they are to be calculated. Usually the calculation of the cash flows involves the future values of one or more market variables. (Hull 2003: 125; Neftci 2000: 10.)

As the name implies, an option is the right to buy or sell, for a limited time, a particular good at a specified price. There are two basic types of options. A call option gives the holder right to buy the underlying asset by a certain date for a certain price. A put option gives the holder the right to sell the underlying asset by a certain date for a certain price. The price in the contract is known as the exercise price or strike price; the date in the contract is known as the expiration date or maturity. Options are also divided into two groups weather they are American or European options. American options can be exercised at any time up to the expiration date. European options can be exercised only on the expiration date itself. It should be emphasized that an option gives the holder the right to do something. The holder does not have to exercise this right. This is what distinguishes options from forwards and futures, where the holder is obligated to buy or sell the underlying asset. Note that whereas it costs nothing to enter into a forward or futures contract, there is a cost to acquiring an option. Options are traded both on exchanges and in the over-the-counter market. Prior to 1973, options of various kinds were traded over-the-counter. An over-the-counter market is a market without a centralized exchange or trading floor. In 1973, Chicago Board Options Exchange (CBOE) began trading options on individual stocks. Since that time, the options market has experienced rapid growth. (Hull 2003: 6; Kolb 1993: 5-6; Neftci 2000: 7.)

2.2. Market participants

The markets are composed from many participants. Market makers are ready to sell and purchase financial instruments and provide the traders with two-way quotes. They provide liquidity and smoothens market fluctuations. At every security at which they are making the market, the market maker must quote a bid and an ask price. Market makers vital task is to buy and sell at their quoted prices. Since the market maker generally takes a position in the security (if only for a short time while waiting for an offsetting order to arrive), the market maker also has a dealer function. Dealers quote two-way prices and hold large inventories of a particular instrument. They are institutions that act in some sense as market makers. Traders buy and sell securities. Trader does not make the markets, on the contrary they execute clients´ orders and trade also for the company's behalf. Customers submit orders to buy or sell. These orders may be contingent on various outcomes, or they may be direct orders to transact imme-

diately. Brokers transmit orders for customers. Brokers provide a platform where the buyers and sellers can get together. Brokers do not hold inventories, but take care of client's orders but do not trade to his/her own account. There are also risk managers who check trades and positions taken by trader and approve them if they are within the preselected boundaries on various risks. (Neftci 2004: 17; O´Hara 1995: 8.)

The main reason, why derivatives markets have been outstandingly successful, is that they have attracted many different types of traders and have a great deal of liquidity. When an investor wants to take one side of a contract, long position (i.e., buy the option) or short position (i.e., sell or write the option), there is usually no problem in finding someone that is prepared to take the other side. Positions are usually taken for hedging, arbitrage, and speculation purposes. Hedgers use futures, forwards, and options to reduce the risk that they face from potential future movements in a market variable. Speculators use them to bet on the future direction of a market variable. Arbitrageurs take offsetting positions in two or more instruments to lock in a profit. Therefore arbitrage involves the simultaneous purchase and sale of equivalent securities in order to profit from discrepancies in their prices. According to capital market theory the equilibrium market prices are rational and they rule out arbitrage opportunities. If security prices are misspriced the markets immediately restore the equilibrium of the markets. In a sense, arbitrage free prices represent the fair market value of the underlying instruments. Gains without taking some risk and without some initial investment should not exist. In market practice "arbitrage" represents a position that has risks, a position that may lose money but is still highly likely to yield a high profit. (Bodie et al. 2005: 343; Hull 2003: 8-10; Neftci 2004: 27-31.)

The law of one price states that if two assets are equivalent in all economically relevant respects, then they should have the same market price. If arbitrageurs observe a violation of the law, they will engage in arbitrage activity—simultaneously buying the asset where it is cheap and selling where it is expensive. In the process, they will bid up the price where it is low and force it down where it is high until the arbitrage opportunity is eliminated. All investors will want to take an infinite position in arbitrage opportunity and because those large positions will quickly force prices up or down until the opportunity vanishes, security prices should satisfy a no-arbitrage condition. No-arbitrage condition rules out the existence of arbitrage opportunities. (Bodie et al. 2005: 349.)

2.3. Option payoffs

At expiration option value is relatively easy to determine. At expiration the owner of the option either exercises the option or allows it to expire as worthless. The value of an option at expiration depends only on the stock price and the exercise price. To focus on the principle of option pricing, commissions and other transaction costs are ignored. As at expiration, the payoff from a call option is usually given as:

(2.1) $$\max(S_T - K, 0).$$

And the function indicates that the call option will be exercised if $S_T > K$ and will not be exercised if $S_T \leq K$. The payoff to the holder of a long position in a put is

(2.2) $$\max(K - S_T, 0),$$

where $S_T =$ Spot price of stock at maturity, and $K =$ Strike price of an option.

The value of an option is divided into two parts: the intrinsic value; and the time value of an option. The intrinsic value of an option is defined as the maximum of zero and the value the option would have if it were exercised immediately. In the case of in-the-money American option the value is worth at least as much as its intrinsic value because the holder can realize a positive intrinsic value by exercising immediately. Often it is optimal for the holder of an in-the-money American option to wait rather than exercise immediately. Then the option is said to have time value. The time value of the option reflects the amount buyers are willing to pay for the possibility that, at some time prior to expiration, the option may become profitable to exercise. In general, the value of an option equals the intrinsic value of the option plus the time value of the option. The time value of the option is zero when (a) the option has reached maturity or (b) it is optimal to exercise the option immediately. (Das 1997b: 221-225; Hull 2003.)

2.4. Factors affecting option prices

There are six factors affecting the price of a stock option, see for example, Cox and Rubinstein (1985: 33-39) and Hull (2003: 167-170):

1. The current stock price, S_0
2. The strike price, K
3. The time to expiration, T
4. The volatility of the stock price, σ
5. The risk-free interest rate, r
6. The dividends expected during the life of the option.

In the Table 1, it is shown what happens to option prices when one of these factors changes with all of the others remaining fixed. Only effects to the American option prices are pointed out in this case. Capital C is a notation for American call option and capital P is a notation for American put option, while c and p are their European counterparts.

Table 1. The effect on the price of an American stock option of increasing one variable while keeping all others fixed.[4]

Variable	American call, C	American put, P
Current stock price	+	-
Strike price	-	+
Time to expiration	+	+
Volatility	+	+
Risk-free rate	+	-
Dividends	-	+

American call options become more valuable as the stock price increases and are less valuable as the strike price increases. This is because the payoff from a call option will be the amount by which the stock price exceeds the strike price. Controversially, American put options become less valuable as the stock price increases and are more valuable as the strike price increases. This is because the

[4] +indicates that an increase in the variable causes the option price to increase; -indicates that an increase in the variable causes the option price to decrease.

payoff from put option will be the amount by which the strike price exceeds the stock price. Cox and Rubinstein (1985: 215-216) pointed out how changes in the factors affect option values in extreme level. The extreme changes are as follows:

Stock price (S_0):

as $S_0 \to 0$, then $C \to 0$ and $P \to K$
as $S_0 \to \infty$, then $C \to \infty$ and $P \to 0$

Strike price (K):

as $K \to 0$, then $C \to S_0$ and $P \to 0$
as $K \to \infty$, then $C \to 0$ and $P \to \infty$

In the case of time to expiration, both put and call American options become more valuable as the time to expiration increases. This is because the owner of the long-life option has more exercise opportunities open than the owner of the short-life option. The long-life option must therefore always be worth at least as much as the short-life option.

Time to expiration (t):

given $S_0 < K$: as $t \to 0$, then $C \to 0$ and $P \to K - S_0$
given $S_0 > K$: as $t \to 0$, then $C \to S_0 - K$ and $P \to 0$
as $t \to \infty$, then $C \to S_0$ and $P \to K$

The volatility of a stock price is a measure of how uncertain we are about future stock price movements. When volatility increases the extreme price movements are more likely. The owner of a call benefits from price increases but as price decreases the most the owner can lose is the price of the option. The owner of a put benefits from price decreases, but has limited downside risk in the event of price increases. The value of both calls and puts therefore increase as volatility increases.

Volatility (σ):

given $S_0 < Kr^{-t}$: as $\sigma \to 0$, then $C \to 0$ and $P \to K - S_0$
given $S_0 > Kr^{-t}$: as $\sigma \to 0$, then $C \to S_0 - Kr^{-t}$ and $P \to 0$
as $\sigma \to \infty$, then $C \to S_0$ and $P \to K$

The risk-free rate affects the price of an option in a less clear-cut way. As interest rates in the economy increases, the expected return required by investors from the stock tends to increase. Also, the present value of any future cash flow received by the holder of the option decreases. The combined impact of these two effects is to decrease the value of put options and increase the value of call options. The effect on change in risk-free interest rate is as follows:

Risk-free rate (r):

as $r \to \infty$, then $C \to S_0$ and $P \to 0$

Dividends have the effect of reducing the stock price on the ex-dividend date. Therefore the value of a call option is negatively related to the size of any anticipated dividends, and the value of a put option is positively related to the size of any anticipated dividends. (Hull 2003: 167-170.)

2.4.1. Bounds on option prices

Option prices have theoretical boundaries which they can not past. If option prices go either above or under these boundaries, there are profitable opportunities for arbitrageurs.

Upper bounds

A call option, which gives the holder the right to buy one share of a stock for a certain price, can never be worth more than the stock. Hence, the stock price is an upper bound to the option price: $c \le S_0$ and $C \le S_0$. If these relationships were not true, an arbitrageur could easily make a riskless profit by buying the stock and selling the call option. A put option, which gives the holder the right to sell one share of a stock for K, can never be worth more than K. Hence, $p \le K$ and $P \le K$. For European options, we know that at maturity the option cannot be worth than K. It follows that it cannot be worth more than the present value of K today:

(2.3) $$p \le Ke^{-rT}.$$

Lower bounds

A lower bound for the price of a European call option on a non-dividend – paying stock is

(2.4) $$S_0 - Ke^{-rT}.$$

Because the worst that can happen to a call option is that it expires worthless, its value cannot be negative. This means that $c \geq 0$, and therefore that

(2.5) $$c \geq \max(S_0 - Ke^{-rT}, 0).$$

For a European put option on a non-dividend-paying stock, a lower bound for the price is

(2.6) $$Ke^{-rT} - S_0.$$

Because the worst can happen to a put option is that it expires worthless, its value cannot be negative. This means that (Hull 2003; Das 1997b.)

(2.7) $$p \geq \max(Ke^{-rT} - S_0, 0).$$

A lower bound for the price of an American call option is its exercise value

(2.8) $$S_0 - K.$$

Again because the worst that can happen to a call option is that it expires worthless, its value cannot be negative. This means that $C \geq 0$, and therefore that

(2.9) $$C \geq \max(S_0 - K, 0).$$

An American call option can never be worth less than a European call option: $C \geq c$. Given no dividends on the underlying stock and positive interest rates an American call option will never be prematurely exercised, implying that an American option will be priced as European option: $C = c$. If two American call options have the same exercise price and are written on the same stock, the op-

tion with the longer maturity date cannot be worth less than the other option: if $T_1 > T_2$ then $C(T_1) \geq C(T_2)$.

The minimum value of an American put option is either zero or $K - S_0$. This means that

(2.10) $$P \geq \max(K - S_0, 0).$$

An American put option is worth at least as much as a European put option: $P \geq p$. Unlike the situation for an American call option, even in the absence of dividends, it may be optimal to prematurely exercise an American put option. This happens when the stock price falls low enough so that any potential benefit received from the likelihood that it falls more is less than the interest gained on the cash received from immediately exercising the option. The difference

(2.11) $$e_t \equiv P_t - p_t > 0$$

is called the early-exercise premium. This is the extra amount one pays for an American put to have the right to exercise it early. (Elliot & Hoek 2006: 31-32; Jarrow & Turnbull 2000: 68-78.)

2.4.2. Early exercise of an American option

American call on a non-dividend-paying stock should never be exercised early. For that there are two reasons. One relates to the insurance that it provides. A call option, when held instead of stock itself, in effect insures the holder against the stock price falling below the exercise price. Once the option has been exercised and the exercise price has been exchanged for the stock price, this insurance vanishes. The other reason concerns the time value of money. From the perspective of the option holder, the later the strike price is paid out the better. The call option has unlimited upside potential, so there is always some additional benefit of waiting to exercise, namely, more profits are possible. When dividends are expected, we can no longer assert that an American call option will not be exercised early. Sometimes it is optimal to exercise an American call immediately prior to an ex-dividend date. It is never optimal to exercise a call option at other times.

In the case of American put option on a non-dividend-paying stock, the option can be optimal to exercise early. Indeed, at any given time during its life, a put option should always be exercised early if it is sufficiently deep in-the-money. Like a call option, a put option can be viewed as providing insurance. A put option, when held in conjunction with the stock, insures the holder against the stock price falling below a certain level. However, a put option is different from a call option in that it may be optimal for an investor to forgo this insurance and exercise early in order to realize the strike price immediately. In other words the upside potential of American put option is limited by the strike price K. Hence, if the put option has reached its maximum, it is better to exercise and earn interest on the proceeds than to wait. In general, the early exercise of a put option becomes more attractive as S_0 decreases, as r increases, and as the volatility decreases. This difference between the premature exercise of an American call option versus an American put option exists because of the differences in their payoff diagrams. (Hull 2003: 175-179; Jarrow et al. 2000: 79.)

2.5. Stock price behaviour

The underlying stock price process is important issue in the valuation of stock options. It is usually assumed that the stochastic process behind a stock price is geometric Brownian motion. In this section, the basic stochastic price processes are introduced.

Any variable whose value changes over time in an uncertain way is said to follow a stochastic process. Stochastic processes can be classified as discrete time or continuous time. A discrete-time stochastic process is one where the value of the variable can change only at certain fixed points in time, whereas a continuous-time stochastic process is one where changes can take place at any time. Stochastic processes can also be classified as continuous variable or discrete variable. In a continuous-variable process the underlying variable can take any value within a certain range, whereas in a discrete-variable process, only certain discrete values are possible. (Hull 2003: 216.)

2.5.1. Stochastic processes

The Markov process

A Markov process is a particular type of stochastic process where only the present value of a variable is relevant for predicting the future. The past history of the variable and the way that the present has emerged from the past are irrelevant. In other words, given the history of the process, the past values can be ignored as long as you know the present state. Predictions for the future are uncertain and must be expressed in terms of probability distributions. The Markov property implies that the probability distribution of the price at any particular future time is not dependent on the particular path followed by the price in the past. The Markov property of stock prices is consistent with the weak form of market efficiency.[5] It is stated that the present price of a stock impounds all the information contained in a record of past prices. If the weak form of market efficiency is not true, technical analysis could make above-average returns by interpreting charts of the past history of stock prices. There is very little evidence that they are in fact able to do this. It is competition in the marketplace that tends to ensure that weak-form market efficiency holds. When the variable follows a Markov stochastic process, the change in the value of the variable during any time period of length T is $\phi(0,\sqrt{T})$, $\phi(\mu,\sigma)$ denotes a probability distribution that is normally distributed with mean μ and standard deviation σ. (Hull 2003: 216-217; Pliska 1997: 107.)

The Wiener process

Wiener processes are usually used to describe stock price process. Wiener process, also referred to as Brownian motion, is a particular type of Markov stochastic process with a mean change of zero and a variance rate of 1,0 per year. Variable z which follows a Wiener process has the following two properties:

1. The change Δz during a small period of time Δt is

(2.12) $$\Delta z = \varepsilon\sqrt{\Delta t}$$

[5] The efficient market theory is described in Fama (1970, 1991).

where ε is a random drawing from a standardized normal distribution, $\phi(0,1)$.

2. The values of Δz for any two different short intervals of time Δt are independent.

The basic Wiener process, dz (the limit as $\Delta t \to 0$), has a drift rate of zero and a variance rate of 1,0. The drift rate of zero means that the expected value of z at any future time is equal to its current value. The variance rate of 1,0 means that the variance of the change in z in a time interval of length T equals T. A generalized Wiener process for a variable x is defined in terms of dz as follows:

(2.13) $$dx = a\,dt + b\,dz.$$

The $a\,dt$ term implies that x has an expected drift rate of a per unit of time. The $b\,dz$ term can be regarded as adding noise or variability to the path followed by x. In a small time interval Δt, the change Δz in the value of x is given by equations (2.12) and (2.13) as (Hull 2003: 218-221.)

(2.14) $$\Delta x = a\,\Delta t + b\varepsilon\sqrt{\Delta t}.$$

Itô Process

A further type of stochastic process can be defined. This is known as an Itô process. This is a generalized Wiener process in which the parameters a and b are functions of the value of the underlying variable x and time t. Algebraically, an Itô process can be written as

(2.15) $$dx = a(x,t)dt + b(x,t)dz.$$

2.5.2. The process for stock prices

It is tempting to suggest that a stock price follows a generalized Wiener process. However, this model fails to capture the key aspect of stock prices. The key aspect is that the expected percentage return required by investors from a stock is not dependent of the stock's price. The constant expected drift-rate assumption is inappropriate and needs to be replaced by the assumption that the expected return (i.e., expected drift divided by the stock price) is constant. If S is the

stock price at time t, the expected drift rate in S should be assumed to be μS for some constant parameter μ. This means that in short interval of time, Δt, the expected increase in S is $\mu S \Delta t$. The parameter μ is the expected rate of return on the stock, expressed in decimal form. If the volatility of the stock price is always zero, this model implies that

(2.16) $$\Delta S = \mu S \Delta t.$$

In the limit as $\Delta t \to 0$,

$$dS = \mu S \, dt$$

or

$$\frac{dS}{S} = \mu \, dt.$$

The stock price at time T is then derived by integrating between time zero and time T:

(2.17) $$S_T = S_0 e^{\mu T}$$

where S_0 and S_T are the stock price at time zero and time T. Equation (2.17) shows that, when the variance rate is zero, the stock price grows at a continuously compounded rate of μ per unit of time. In practice, of course, a stock price does exhibit volatility. A reasonable assumption is that the variability of the percentage return in a short period of time, Δt, is the same regardless of the stock price. This suggests that the standard deviation of the change in a short period of time Δt should be proportional to the stock price and leads to the model

(2.18) $$dS = \mu S \, dt + \sigma S \, dz$$

or

(2.19) $$\frac{dS}{S} = \mu \, dt + \sigma \, dz.$$

Equation (2.19) is the most widely used model of stock price behaviour. The variable σ is the volatility of the stock price. The variable μ is its expected rate of return. (Hull 2003: 222-223.)

The model of stock price behaviour developed above is known as geometric Brownian motion. Based on the model, the change in the stock price during a short time period, Δt, is

$$\frac{\Delta S}{S} = \mu \Delta t + \sigma \varepsilon \sqrt{\Delta t} \tag{2.20}$$

or

$$\Delta S = \mu S \Delta t + \sigma S \varepsilon \sqrt{\Delta t}. \tag{2.21}$$

The variable ΔS is the change in the stock price S in a small time interval Δt, and ε is a random drawing from normal distribution, $\phi(0,1)$. The parameter μ is the expected rate of return per unit of time from the stock, and the parameter σ is the volatility of the stock price. Both of these parameters are assumed constant. The left-hand side of equation (2.20) is the return provided by the stock in a short period of time Δt. The term $\mu \Delta t$ is the expected value of this return, and the term $\sigma \varepsilon \sqrt{\Delta t}$ is the stochastic component of the return. The variance of the stochastic component (and therefore the whole return) is $\sigma^2 \Delta t$. This is consistent with the definition of the volatility σ, so that $\sigma \sqrt{\Delta t}$ is the standard deviation of the return in a short time period Δt. Equation (2.20) shows that $\Delta S / S$ is normally distributed with mean $\mu \Delta t$ and standard deviation $\sigma \sqrt{\Delta t}$. In other words, (Hull 2003: 223-224.)

$$\frac{\Delta S}{S} \sim \phi(\mu \Delta t, \sigma \sqrt{\Delta t}). \tag{2.22}$$

Itô's lemma

The price of a stock option is a function of the underlying stock's price and time. More generally, the price of any derivative is a function of the stochastic variables underlying the derivative and time. The behaviour of functions of stochastic variables is very important in the pricing of derivatives. A mathematical rule from stochastic calculus called Itô's lemma is used for computing differentials of functions of stochastic random variables.

When it is taken that the value of a stock S follows the Itô process

(2.23) $$dS = a(S,t)dt + b(S,t)dz$$

where dz is a Wiener process and a and b are the functions of S and t. The stock has a drift rate of a and a variance rate of b^2. Itô's lemma shows that a derivative f of S and t follows the process

(2.24) $$df = \left(\frac{\partial f}{\partial S} a + \frac{\partial f}{\partial t} + \tfrac{1}{2} \frac{\partial^2 f}{\partial f^2} b^2 \right) dt + \frac{\partial f}{\partial S} b \, dz$$

where the dz is the same Wiener process as in equation (2.23). Thus, f also follows an Itô process. It has a drift rate of

(2.25) $$\frac{\partial f}{\partial S} a + \frac{\partial f}{\partial t} + \tfrac{1}{2} \frac{\partial^2 f}{\partial S^2} b^2$$

and a variance rate of

(2.26) $$\left(\frac{\partial f}{\partial S} \right)^2 b^2.$$

The earlier equation (2.18)

(2.27) $$dS = \mu S \, dt + \sigma S \, dz$$

with μ and σ constant, is a reasonable model of stock price movements. From Itô's lemma, it follows that the process followed by a derivative f of S and t is

(2.28) $$df = \left(\frac{\partial f}{\partial S} \mu S + \frac{\partial f}{\partial t} + \tfrac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) dt + \frac{\partial f}{\partial S} \sigma S \, dz.$$

Note that the both S and f are affected by the same underlying source of uncertainty, dz.

The lognormal property

Now Itô's lemma can be used to derive the process followed by $\ln S$. When $f = \ln S$ and after calculations of

$$\frac{\partial f}{\partial S} = \frac{1}{S}, \qquad \frac{\partial^2 f}{\partial S^2} = -\frac{1}{S^2}, \quad \frac{\partial f}{\partial t} = 0$$

it follows from equation (2.28) that the process followed by f is

$$df = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma\, dz. \tag{2.29}$$

The equation indicates that f follows a generalized Wiener process, with constant drift rate $\mu - \sigma^2/2$ and constant variance rate σ^2. Therefore the change in f between time zero and T is normally distributed with mean

$$\left(\mu - \frac{\sigma^2}{2}\right)T \tag{2.30}$$

and variance

$$\sigma^2 T. \tag{2.31}$$

Therefore

$$\ln S_T \sim \phi\left[\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right] \tag{2.32}$$

where ϕ is a normal distribution. The equation above shows that $\ln S_T$ is normally distributed. This implies that stock price is log normally distributed, because a variable has a lognormal distribution if the natural logarithm of the variable is normally distributed. A variable that has a lognormal distribution can take any value between zero and infinity. (Hull 2003: 226-235; Jarrow et al. 2000: 213-214; Wilmott, Howison & Dewynne 1995: 42.)

3. OPTION PRICING

In this chapter, a useful and very popular binomial tree model, the Cox, Ross and Rubinstein (1979) model and Black-Scholes model for pricing stock options are introduced.

Binomial tree is a diagram that represents different possible paths that might be followed by the stock price over the life of the option. The simple one-step binomial model can determine the rational price today for a call option. The following approach to a simple discrete-time option pricing formula was introduced in seminar paper by Cox, Ross and Rubinstein in 1979.

3.1. The Binomial model for stock options

In binomial tree model it is assumed that the stock price follows a multiplicative binomial process over discrete periods. The rate of return on the stock over each period can have two possible values: $u-1$ with probability q, or $d-1$ with probability $1-q$. Thus, if the current stock price is S, the stock price at the end of the period will be either uS or dS. This movement is represented in the following diagram:

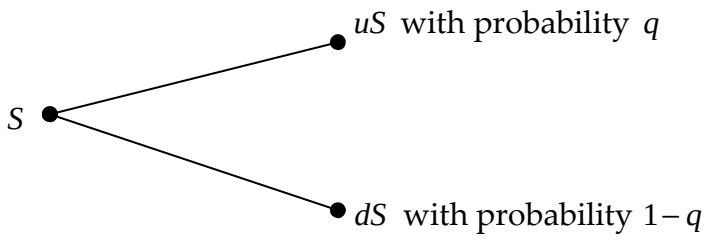

Figure 1. Binomial movement in stock price.

Next it is assumed that the interest rate is constant. Individuals may borrow or lend as much as they wish at this rate. To focus on the basic issues, it is also assumed that there are no taxes, transaction costs, or margin requirements. Hence, individuals are allowed to sell short any security and receive full use of the proceeds.

Letting r denote one plus the riskless interest rate over one period, $u > r > d$ is required. If these inequalities do not hold, there would be profitable riskless arbitrage opportunities involving only the stock and riskless borrowing and lending.

Denote that C is the current value of the call, C_u is its value at the end of the period if the stock price goes to uS and C_d is its value at the end of the period if the stock price goes to dS. Since there is only one period remaining in the life of the call, the terms of its contract and a rational exercise policy imply that $C_u = \max[0, uS - K]$ and $C_d = \max[0, dS - K]$. Therefore,

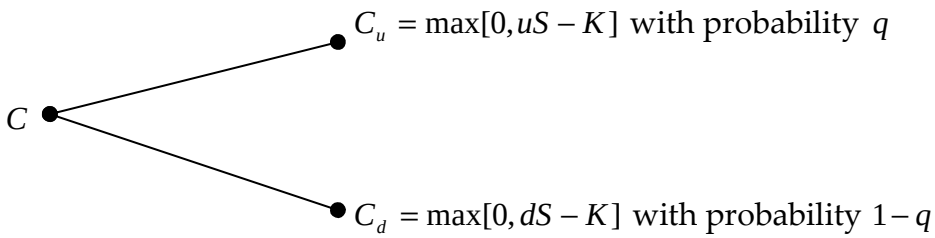

Figure 2. Stock price movement in one step binomial model.

Then a portfolio containing Δ shares of stock and the Euro amount B in risk-less bonds is formed. This will cost $\Delta S + B$. At the end of the period, the value of this portfolio will be

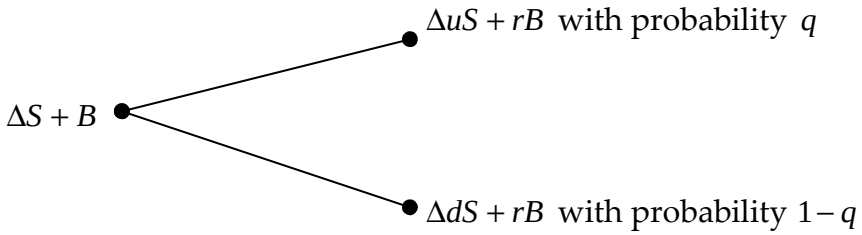

Figure 3. The movement of portfolio value in one step binomial model.

Since the Δ and B can be selected in any way, they are selected to equate the end-of-period values of the portfolio and the call for each possible outcome. This requires that

(3.1) $$\Delta uS + rB = C_u,$$

(3.2) $$\Delta dS + rB = C_d .$$

From these equations, it is found

(3.3) $$\Delta = \frac{C_u - C_d}{(u-d)S}$$

and

(3.4) $$B = \frac{uC_d - dC_u}{(u-d)r}.$$

Portfolio selected with Δ and B is called the hedging portfolio.

If there are no riskless arbitrage opportunities, the current value of the call, C, cannot be less than the current value of the hedging portfolio, $\Delta S + B$. Thus, if there are no riskless arbitrage opportunities, it must be true that

(3.5) $$C = \Delta S + B = \frac{C_u - C_d}{u-d} + \frac{uC_d - dC_u}{(u-d)r} = \left[\left(\frac{r-d}{u-d}\right)C_u + \left(\frac{u-r}{u-d}\right)C_d\right]/r$$

if this value is greater than $S - K$, and if not, $C = S - K$.

Equation (3.5) can be simplified by defining

$$p \equiv \frac{r-d}{u-d}$$

and

$$1 - p \equiv \frac{u-r}{u-d}$$

so the value of call option is

(3.6) $$C = [\,pC_u + (1-p)C_d\,]/r.$$

It is easy to see that in the present case, with no dividends, this will always be greater than $S - K$ as long as the interest rate is positive. When assumed that r is always greater than one, the equation (3.6) is the exact formula for the value of a call one period prior to the expiration in terms of $S, K, u, d,$ and r.

It is observe that $p \equiv (r-d)/(u-d)$ is always greater than zero and less than one, so it has the properties of a probability. In fact, p is the value q would have in equilibrium if investors were risk-neutral[6]. To see this, the expected rate of return on the stock would then be the riskless interest rate, so

(3.7) $$q(uS) + (1-q)(dS) = rS$$

and

(3.8) $$q = (r-d)/(u-d) = p.$$

Hence, the value of the call can be interpreted as the expectation of its discounted future value in a risk-neutral world. The risk-neutral valuation principle is correct not just in a risk-neutral world but in the real world as well. The risk-neutral valuation principle states that an option can be valued under the assumption that the world is risk neutral. This means that for valuation purposes the following is assumed:

1. The expected return from all traded securities is the risk free interest rate.
2. Future cash flows can be valued by discounting their expected values at the risk-free interest rate.

The pricing formula in equation (3.6) does not involve the probabilities of the stock price moving up or down. The key reason, why probabilities are not needed, is that the option is not valued in absolute terms. The options value is calculated in terms of the price of the underlying stock. The probabilities of future up or down movements are already incorporated into the price of the stock. Hence, there is no need to take them into account again when valuing the option in terms of stock price. (Hull 2003.)

Next a call with two periods remaining before its expiration date is considered. In keeping with the binomial process, the stock can take on three possible values after two periods,

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[6] In a risk-neutral world all individuals are indifferent to risk. In such a world investors require no compensation for risk, and the expected return on all securities is the risk-free interest rate.

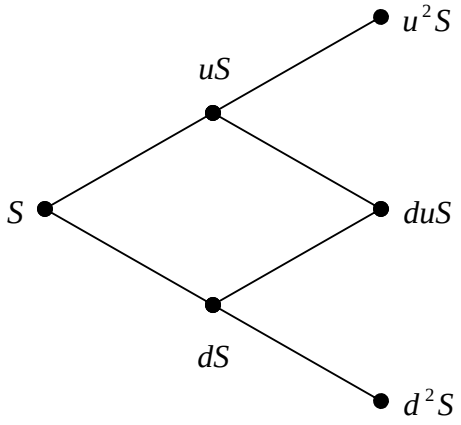

Figure 4. Stock price movement in two step binomial model.

Similarly, for the call,

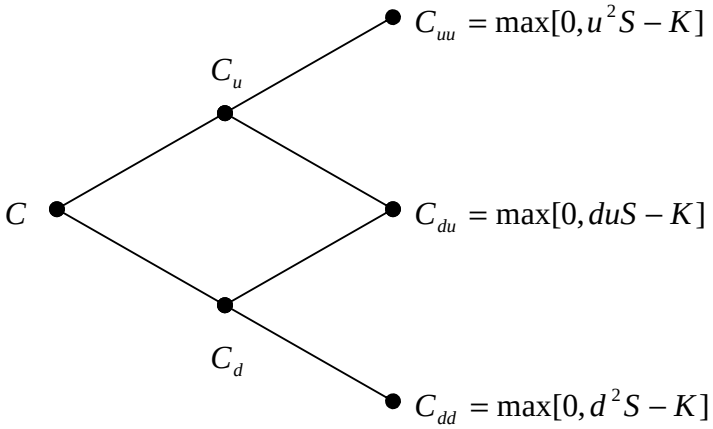

Figure 5. The movement of portfolio value in two step binomial model.

C_{uu} stands for the value of a call two periods from the current time if the stock price moves upward each period; C_{du} and C_{dd} have analogous definitions. At the end of the current period there will be one period left in the life of the call, so the problem is identical to the previously solved one. Thus, the values are

(3.9) $$C_u = [pC_{uu} + (1-p)C_{ud}]/r$$

and

(3.10) $$C_d = [pC_{du} + (1-p)C_{dd}]/r.$$

Again, a portfolio is selected with ΔS in the stock and B in bonds whose end-of-period value will be C_u if the stock price goes to uS and C_d if the stock price goes to dS. Indeed, the functional form of Δ and B remains unchanged. To get the new values of Δ and B, we simply use equations (3.3) and (3.4) with the new values of C_u and C_d.

Since Δ and B have the same functional form in each period, the current value of the call in terms of C_u and C_d will again be $C = [pC_u + (1-p)C_d]/r$ if this is greater than $S-K$, and $C = S-K$ otherwise. By substituting from equations (3.9) and (3.10) into the former expression, and noting that $C_{du} = C_{ud}$, the value for a call option is obtained

(3.11) $$C = [p^2 C_{uu} + 2p(1-p)C_{ud} + (1-p)^2 C_{dd}/r^2$$

$$= [p^2 \max[0, u^2 S - K] + 2p(1-p)\max[0, duS - K] + (1-p)^2 \max[0, d^2 S - K]]/r^2.$$

For equation (3.11), $n = 2$. Now the value of a call with any number of periods to go can be classified. By starting at the expiration date and working backwards, the general valuation formula for any n is written as:

(3.12) $$C = \left[\sum_{j=0}^{n}\left(\frac{n!}{j!(n-j)!}\right)p^j(1-p)^{n-j}, \max[0, u^j d^{n-j} S - K]\right]/r^n.$$

Where

(3.13) $$\frac{n!}{j!(n-j)!}p^j(1-p)^{n-j},\ j = 0,1,\ldots,n.$$

is called the binomial distribution, and

(3.14) $$\frac{n!}{j!(n-j)!} = \binom{n}{j}$$

is called a binomial coefficient.

Binomial tree methods can be used to value derivatives when exact option pricing formulas are not available. Binomial tree method is particularly useful when the holder has early exercise decisions to make prior to maturity. One- and two-

step binomial trees for non-dividend-paying stocks, introduced above are not very precise models of reality. A more realistic model is one that assumes stock price movements are composed of a much larger number of small binomial movements. (Hull 2003: 392-393.)

The binomial tree for a non-dividend paying stock represents the stock price movements in a risk-neutral world. The parameters $p, u,$ and d must give correct values for the mean and variance of stock price changes during a time interval of length Δt. In a risk-neutral world, the expected return from a stock is the risk-free interest rate, r. Hence the expected value of the stock price at the end of a time interval of length Δt is $Se^{r\Delta t}$, where S is the stock price at the beginning of the time interval. It follows that

(3.15) $$Se^{r\Delta t} = pSu + (1-p)Sd$$

(3.16) $$e^{r\Delta t} = pu + (1-p)d.$$

The stochastic process for stock prices implies that the variance of the percentage change in the stock price in a small time interval of length Δt is $\sigma^2 \Delta t$. Because the variance of a variable Q is defined as $E(Q^2) - [E(Q)]^2$, it follows that

(3.17) $$pu^2 + (1-p)d^2 - [pu + (1-p)d]^2 = \sigma^2 \Delta t.$$

Substituting for p from equation (3.16) reduces this to

(3.18) $$e^{r\Delta t}(u+d) - ud - e^{2r\Delta t} = \sigma^2 \Delta t$$

equation (3.16) and (3.18) impose two conditions on $p, u,$ and d. A third condition used by Cox, Ross, and Rubinstein is

(3.19) $$u = \frac{1}{d}.$$

These three conditions imply

(3.20) $$p = \frac{a-d}{u-d}$$

(3.21) $$u = e^{\sigma\sqrt{\delta t}}$$

(3.22) $$d = e^{-\sigma\sqrt{\delta t}}$$

where

(3.23) $$a = e^{r\delta t}$$

and terms of order higher than Δt are ignored. The variable a is sometimes referred to as the growth factor. (Hull 2003: 393-394.)

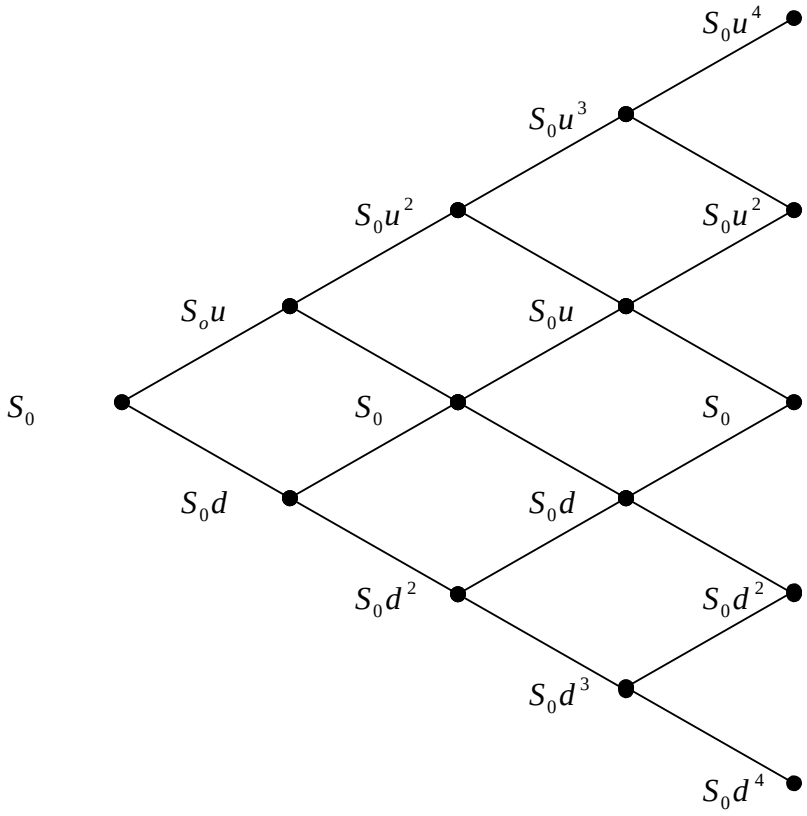

Figure 6. Binomial tree used to value a stock option.

Figure 6. illustrates the complete tree of stock prices that is considered when the binomial model is used. At time zero, the stock price, S_0, is known. At time Δt, there are two possible stock prices, S_0u and S_0d; at time $2\Delta t$, there are three possible stock prices, S_0u^2, S_0, and S_0d^2; and so on. In general, at time $i\Delta t$, $i+1$ stock prices are considered. These are

(3.24) $$S_0u^jd^{i-j}, \quad j = 0,1,...,i.$$

The relationship $u = 1/d$ is used in computing the stock price at each node of the tree in figure 6. For example, $S_0u^2d = S_0u$. Also the tree recombines in the sense that an up movement followed by a down movement leads to the same stock price as a down movement followed by an up movement. Since the tree of stock prices is constructed, options are evaluated by starting at the end of the tree (time T) and working backward. The value of the option is known at time T. For example, a put option is worth $\max(K - S_T, 0)$ and a call option is worth $\max(S_T - K, 0)$, where S_T is the stock price at time T and K is the exercise price. Because a risk-neutral world is being assumed, the value at each node at time $T - \Delta t$ can be calculated as the expected value at time T discounted at rate r for a time period Δt. Similarly, the value at each node at time $T - 2\Delta t$ can be calculated as the expected value at time $T - \Delta t$ discounted for a time period Δt at rate r, and so on. Eventually, by working back through all the nodes, the value of the option at time zero is obtained. Also American options can be valued using a binomial tree model. The procedure is to work back through the tree from the end to the beginning, testing at each node to see whether early exercise is preferable to holding the option for a further time period Δt. The value of the option at the final nodes is the same as for the European option. (Hull 2003.)

More generally, The life of an American put option on a non-dividend-paying stock is divided into N subintervals of length Δt. It is referred to the j th node at time $i\Delta t$ as the (i, j) node, where $0 \le i \le N$ and $0 \le j \le i$. The $f_{i,j}$ is the value of the option at the (i, j) node. The stock price at the (i, j) is $S_0u^jd^{i-j}$. Because the value of an American put at its expiration date is $\max(K - S_T, 0)$, we know that

(3.25) $$f_{N,j} = \max(K - S_0u^jd^{N-j}, 0), \quad j = 0,1,..., N.$$

There is a probability p of moving from the (i, j) node at time $i\Delta t$ to the $(i+1, j+1)$ node at time $(i+1)\Delta t$, and a probability $1-p$ of moving from the (i, j) node at time $i\Delta t$ to the $(i+1, j)$ node at time $(i+1)\Delta t$. When there is no possibility of early exercise, risk neutral valuation gives

(3.26) $$f_{i,j} = e^{-r\Delta t}[pf_{i+1,j+1} + (1-p)f_{i+1,j}]$$

for $0 \le i \le N-1$ and $0 \le j \le i$. When early exercise is taken into account, this value for $f_{i,j}$ must be compared with the option's intrinsic value, and obtain

(3.27) $$f_{i,j} = \max\{K - S_0 u^j d^{i-j}, e^{-r\Delta t}[pf_{i+1,j+1} + (1-p) f_{i+1,j}]\}.$$

Because the calculations start at time T and work backward, the value at time $i\Delta t$ captures not only the effect of early exercise possibilities at time $i\Delta t$ but also the effect of early exercise at subsequent times. In the limit as Δt tends to zero, an exact value for American put is obtained. In practice, $N = 30$ usually gives reasonable results. (Hull 2003: 397.)

3.2. The Black-Scholes model

The Black-Scholes model, derived by Black and Scholes (1973) expanded by Merton (1973), made a major breakthrough in the pricing of stock options. The model has had a huge influence on the way traders price and hedge options. It has also been pivotal to the growth and success of financial engineering in the 1980s and 1990s. (Hull 2003: 234.)

When describing the Black-Scholes model few simplified assumptions are made.

1. The stock price follows a lognormal distribution described earlier with μ and σ constant.
2. The short selling of securities is permitted.
3. There are no transactions costs, taxes or bid-ask spreads. All securities are perfectly divisible.
4. There are no dividends, stock splits or other corporate actions during the life of the option.
5. There are no riskless arbitrage opportunities.
6. Security trading is continuous.
7. The interest rate is constant and the same for all maturities.

Some of these assumptions can be relaxed and even interest rates can be allowed to be stochastic providing that the stock price distribution at maturity of the option is still lognormal. (Hull 2003: 242; Wilmott et al.1995: 41-42.)

Black-Scholes model assumes that stock prices behave as lognormal property. It assumes that percentage changes in the stock price in a short period of time are normally distributed. Define:

μ : Expected return on stock
σ : Volatility of the stock price

The mean of the percentage change in time Δt is $\mu \Delta t$ and the standard deviation of this percentage change is $\sigma\sqrt{\Delta t}$, so that

(3.28) $$\frac{\Delta S}{S} \sim \phi(\mu \Delta t, \sigma\sqrt{\Delta t})$$

where ΔS is the change in the stock price S in time Δt, and $\phi(m,s)$ denotes a normal distribution with mean m and standard deviation s. The expected value, $E(S_T)$, of S_T is given by $E(S_T) = S_0 e^{\mu T}$. This fits in with the definition of μ as the expected rate of return. The variance, var(S_T), of S_T can be shown to be given by $\text{var}(S_T) = S_0^2 e^{2\mu T}(e^{\sigma^2 T} - 1)$. (Hull 2003: 234-236; Neftci 2004: 213-214.)

The lognormal property of stock prices can be used to provide information on the probability distribution of the continuously compounded rate of return earned on a stock between times zero and T. The continuously compounded rate of return per annum realized between times zero and T is defined as η. It follows that

(3.29) $$S_T = S_0 e^{\eta T}$$

so that

(3.30) $$\eta = \frac{1}{T} \ln \frac{S_T}{S_0}.$$

It follows from equation (2.32) that

(3.31) $$\eta \sim \phi\left(\mu - \frac{\sigma^2}{2}, \frac{\sigma}{\sqrt{T}}\right).$$

Thus, the continuously compounded rate of return per annum is normally distributed with mean $\mu - \sigma^2/2$ and standard deviation $\sigma/\sqrt{T}$. (Hull 2003: 236.)

3.2.1. Derivation of the Black-Scholes differential equation

It is assumed that the stock price process follows the geometric Brownian motion and as derived earlier the change in the stock price during a short time period is:

(3.32) $$\Delta S = \mu S \Delta t + \sigma S \Delta z.$$

When f is the price of a call option contingent on S. The variable f must be some function of S and t. Hence, from equation (2.28),

(3.33) $$\Delta f = \left(\frac{\partial f}{\partial S} \mu S + \frac{\partial f}{\partial t} + \tfrac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) \Delta t + \frac{\partial f}{\partial S} \sigma S \Delta z$$

where ΔS and Δf are the changes in f and S in a small time interval Δt. Based on the discussion of Itô´s lemma the Wiener processes underlying f and S are the same. Therefore constructing a portfolio of the stock and the derivative, the Wiener process can be eliminated. The appropriate portfolio is: one derivative short and an amount $\partial f / \partial S$ of shares long. Thus, the value of the portfolio, Π, is:

(3.34) $$\Pi = -f + \frac{\partial f}{\partial S} S$$

and the change in the value of the portfolio in the small time interval Δt is

(3.35) $$\Delta \Pi = -\Delta f + \frac{\partial f}{\partial S} \Delta S.$$

Substituting Δf and ΔS from equations (3.31) and (3.33) into equation (3.35) the $\Delta \Pi$ is:

(3.36) $$\Delta \Pi = \left(-\frac{\partial f}{\partial t} - ½ \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) \Delta t.$$

Because this equation does not involve Δz, the portfolio must be riskless during time Δt. The assumptions listed earlier imply that the portfolio must instantaneously earn the same rate of return as other short-term risk-free securities. If it earned more than this return, arbitrageurs could make a riskless profit by borrowing money to buy the portfolio; if it earned less, they could make a riskless profit by shorting the portfolio and buying risk-free securities. Therefore

(3.37) $$\Delta\Pi = r\Pi\,\Delta t$$

where r is the risk-free interest rate. Substituting from equations (3.34) and (3.36), it is obtained

(3.38) $$\left(\frac{\partial f}{\partial t} + ½\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t = r\left(f - \frac{\partial f}{\partial S}S\right)\Delta t$$

so that

(3.39) $$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \tfrac{1}{2}\sigma^2 S^2\frac{\partial^2 f}{\partial S^2} = rf\,.$$

This is the Black-Scholes partial differential equation. It has many solutions, corresponding to all the different derivatives that can be defined with S as the underlying variable. The particular derivative that is obtained when the equation is solved depends on the boundary conditions that are used. These specify the values of the derivative at the boundaries of possible values of S and t.

The Black-Scholes differential equation (3.39) does not involve any variable that is affected by the risk preferences of investors. The variables that do appear in the equation are the current stock price, time, stock price volatility, and the risk-free rate of interest. Which all are independent of risk preferences. Though, because risk preferences do not enter the differential equation, they cannot affect its solution. Therefore, any set of risk preferences can be used when evaluating f. However it is noteworthy that the portfolio used in the derivation of equation (3.39) is not permanently riskless. It is riskless only an infinitesimally short period of time. As S and t changes, $\partial f / \partial S$ [7] also changes. To keep the portfolio riskless, it is therefore necessary to frequently change the relative proportions of the derivative and the stock in the portfolio. (Black 1975: 37; Hull 2003: 242-245; Wilmott et al. 1995: 42-44.)

[7] The delta, $\Delta = \partial f / \partial S$, of an option is the rate of change of its price with respect to a change in the underlying asset price and it is the amount of shares respect to derivative needed to hedge the portfolio.

3.2.2. Black-Scholes pricing formulas

The Black-Scholes formulas are derived by solving the differential equation (3.39) subject to the boundary conditions. A boundary condition specifies the behaviour of the required solution at some part of the solution domain. The Black-Scholes formulas for the prices at time zero of a European call option on a non-dividend-paying stock and a European put option on a non-dividend-paying stock are

(3.40) $$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

and

(3.41) $$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

where

(3.42) $$d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

(3.43) $$d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

The function $N(x)$ denotes the cumulative standard normal probability:

(3.44) $$N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du.$$

In other words, it is the probability that a variable with a standard normal distribution, $\phi(0,1)$ will be less than x. (Black et al. 1973: 643-644; Hull 2003: 246; Neftci 2004: 213-214; Wilmott et al. 1995: 44-48.)

4. VOLATILITY

The concept of volatility of asset prices and returns is central to financial markets. Volatility provides essential data about the probability of achieving certain outcomes in terms of price levels which is intrinsic to key decisions in financial markets. In the context of option pricing, an estimate of volatility is essential to the valuation of the instrument. Volatility estimation, in the context of option pricing, must be considered in the broader context of asset price and return volatility generally. The framework for volatility estimation, in reality, must recognise the causes of volatility in asset prices and the inter-relationship between volatility and option pricing models. Price volatility in asset markets is caused by a variety of factors, the most important of which is information release. A second cause of volatility is the process of trading and market-making in financial instruments. The study of market micro-structure seeks to isolate the impact of trading on volatility. (Das 1997a: 307-308.)

Usually volatility refers to the standard deviation of the change in the value of a financial instrument with a specific time horizon. The volatility of a stock, σ , is a measure of our uncertainty about the returns provided by the stock. Stocks typically have volatility between 20% and 50%. From equation (3.31), the volatility of a stock price can be defined as the standard deviation of the return provided by the stock in one year when the return is expressed using continuous compounding. When T is small, equation (3.28) shows that $\sigma\sqrt{T}$ is approximately equal to the standard deviation of the percentage change in the stock price in time T . Equation (3.28) shows that our uncertainty about a future stock price, as measured by its standard deviation, increases—at least approximately—with the square root of how far ahead we are looking. For example, the standard deviation of the stock price in four weeks is approximately twice the standard deviation in one week. (Hull 2003: 238-239.)

4.1. Implied volatility

The one parameter in the Black-Scholes pricing formulas that cannot be directly observed is the volatility of the stock price. The volatility plugged in to the option pricing formula to obtain the market value of the option is called the implied volatility. Implied volatilities are important because they are embedded in

option prices, and the prices of options reflect future expectations of market participants. This means that implied volatilities constitute a forward-looking estimate of the volatility of the underlying asset. In practice, implied volatilities are used to monitor the markets opinion about the volatility of a particular stock. Traders like to calculate implied volatilities from actively traded options on a certain asset and interpolate between them to calculate the appropriate volatility for pricing a less actively traded option on the same stock. It is important to note that the prices of deep-in-the-money and deep-out-of-the-money options are relatively insensitive to volatility. Implied volatilities calculated from these options tend, therefore, to be unreliable. (Hull 2003: 250-251; Neftci 2004: 432; Rouah & Vainberg 2007: 304-305.)

Option pricing formulas usually cannot be inverted analytically, so implied volatility must be calculated numerically. Usually implied volatilities are calculated using either the Black-Scholes formula or the Cox-Ross-Rubinstein binomial model. Based on the assumptions of the Black-Scholes model, implied volatility is interpreted as the market's estimate of the constant volatility parameter. If the underlying asset's volatility is allowed to vary deterministically over time, implied volatility is interpreted to be the market's assessment of the average volatility over the remaining life of the option. Implied volatility calculation is accomplished by feeding the value-price difference,

$$C(\sigma) - C_M, \tag{4.1}$$

into a root-finding program, where $C(\ )$ is an option pricing formula, σ is the volatility parameter, and C_M is the observed market price of the option. Various algorithms can be used to find the value of σ that makes this expression equal to zero. Methods like bisection method and Newton-Raphson algorithm are used to calculate implied volatilities. (Mayhew 1995: 8-9.)

The binomial search algorithm; bisection method, also used in this research, is well suited to problems for which the function is continuous on an interval $[a,b]$ and for which the function is known to take a positive value on one endpoint and a negative value on the other endpoint. By the Intermediate Value Theorem, the interval will necessarily contain a root. A first guess for the root is the midpoint of the interval. The bisection algorithm proceeds by repeatedly

dividing the subintervals of $[a,b]$ in two, and at each step locating the half that contains the root. (Rouah et al. 2007: 9.)

It is known that a closed-form solution for an implied volatility is not possible. Although few approximation formulas for implied volatility have been proposed, see for example, Brenner and Subrahmanyam (1988), Chance (1996), and Chambers & Nawalkha (2001). (Li 2005.)

4.2. Implied volatility research

It is argued that option's implied volatility is a good measure when forecasting the underlying assets future volatility. Academic literature has researched the information content and effectiveness of implied volatility in numerous studies. The lion's share of the studies claims that implied volatility is far better predictor of future volatility than other forecasting models.

Already Latané and Rendleman (1976), in their early study, found that the implied volatility contained relevant information regarding future volatility. Jorion (1995) studied the information content and predictive power of volatility implied in options on foreign currencies and found that implied volatility outperformed the statistical time-series models. However, implied volatility also appeared to be biased volatility forecast. Christensen and Prabhala (1998), find that implied volatility outperforms past volatility in forecasting future volatility and even subsumes the information content of past volatility. They also pointed out that implied volatility is interpreted as an efficient volatility forecast in a wide range of settings (e.g., Day and Lewis, 1988; Harvey and Whaley, 1992; Poterba and Summers, 1986; Sheikh, 1989).

However Canina and Figlewski (1993), find that for S&P 100 index options implied volatility to be a poor forecast of subsequent realized volatility. In aggregate and across subsamples separated by maturity and exercise price, implied volatility has virtually no correlation with future volatility, and it does not incorporate the information contained in recent observed volatility. Also Fleming (1998) found that implied volatility is a biased forecast which significantly overstates future volatility. However, he found that the implied volatility's forecast power dominates that of the historical volatility rate.

Poon and Granger (2003) summarized the research area and surveyed 52 studies and report that in 43 studies implied volatility yields better forecast of future volatility than forecasts based on past volatility while only 9 find the reverse. By expanding the earlier research Poon and Granger (2005) surveyed 93 studies that conducted tests of volatility-forecasting methods on a wide range of financial asset returns. They found that option-implied volatility provides more accurate forecasts than time-series models.

4.3. The implied volatility smile

In reality, options that are identical in every respect, expect for their exercise price, have different implied volatilities. The implied volatility curve, a plot of implied volatilities versus exercise price, gives shape to this phenomenon. Overall, the more out-of-the-money a call (put) option is, the higher is the corresponding implied volatility. This well-established empirical fact is known as the volatility smile, or volatility skew, and has major implications for trading, hedging, and pricing financial instruments. (Neftci 2004: 435; Rouah et al. 2007: 304-305.)

The volatility smile reflects a variety of factors, including:

1. adjustments for the distributional assumptions underlying standard option pricing models;
2. directional assumptions regarding the movement in the underlying asset prices which are incorporated into the option volatility and price;
3. clientele effects and the demand for out-of-the-money options;
4. the management of option hedging risks by traders; and
5. liquidity effects.

The volatility smile is capable of explaining the deviation of observed asset price movements from the assumed log normal distribution. In practice, asset price movements seem to be characterised by the following:

- The market distribution of the changes of asset prices appears to demonstrate fat tails (statistically described as the kurtosis of the distribution).

This type of distribution is characterised by larger changes of value price than is consistent with a normal distribution.

- The fat tails are consistent with the presence of "jump" risk, that is, non-stochastic (or discontinuous) changes or movements in the price of the asset which cause deviation from the assumption of a normal distribution.

The actual observed pattern of price changes would systematically underestimate the value of deep in and out-of-the-money options because of the above characteristics. This reflects the fact that the log normal distribution systematically underestimates the expected values that the option may take at maturity in either tail of the distribution. The volatility smile is consistent with trader behaviour which seeks to adjust the option premium for these deficiencies in an option pricing model such as Black-Scholes. This adjustment is effected through an increase in the volatility for both deep in and out-of-the-money options to equate the premium received to the expected payouts under the option incorporating the true asset price change distribution. (Das 1997a: 328-329.)

The volatility smile, particularly the skew in the structure of the smile, may reflect expectations regarding the expected direction of price movements which are incorporated in the option price and by implication the implied volatility. Directional view may be reflected in option price which will be higher or lower than in the absence of new expectations and reflected in the implied volatilities. (Das 1997a: 329; Hull 2003: 335.)

The market for options with different strike prices appears to exhibit significant biases in demand and supply (a clientele effect). Out-of-the-money options are attractive vehicles for speculative investment demand, reflecting the following factors:

1. the gearing or leverage of the out-of-the-money options (expressed as the asset price divided by the option premium) is higher; and
2. the low absolute cash investment entailed in the purchase of the option.

The presence of these factors dictates significant demand for these options. The option traders are reluctant to supply out-of-the-money options because of the

difficulty of hedging or replicating these options in the event of a jump in the asset price. In contrast, the position for in-the-money options is influenced by different factors. The dominating characteristic of these types of options is that they have a high delta and move closely with movements with the underlying asset price. This allows in-the-money options to be used as a direct substitute for the asset itself.

The supply of these options is limited. This reflects the reluctance to write a deep in-the-money option because in the absence of a large or extreme price movement the option will be exercised, requiring the seller to buy or sell the asset at a price which is disadvantageous to them. In addition, such options do not have significant time or volatility value, further reducing their attractiveness to the seller. The interaction of supply and demand for these deep in-the-money options results in the option price and implied volatilities being bid up above comparable volatilities for at-the-money options of the same maturity. Also the volatility smile appears to incorporate the impact of traders seeking to manage the risk of option transactions.

The combination of the above factors results in differential liquidity of options with different exercise prices for a given maturity. The volatility of at-the-money options is lower reflecting the higher liquidity of these options from the greater balance between supply and demand for these options. In and out-of-the-money options are less frequently traded and the imbalance of demand relative to supply is reflected in the higher implied volatility compared to the at-the-money options. (Das 1997a: 329-331.)

5. DATA, HYPOTHESIS AND METHODOLOGY

In the previous chapters, the introduction and the theoretical basics to option theory and the most common option pricing methods were discussed. The purpose of this chapter is to describe the data, study hypothesis and methods used to generate implied volatilities.

At the beginning of the empirical part of this study, the used data is presented in Chapter 5.1. Also the data description and the screening criteria will be introduced. Chapter 5.2 will introduce the hypotheses and Chapter 5.3 introduces the analysis methods used in this study.

5.1. Description of the data

To test the demand pressure effects on implied volatility the tick- and end-of-day data on Barclays Plc. stock options (BBL) are used. The data used in the study is gathered from the time period 4 January, 2005 to 30 December, 2005. The underlying instrument is the Barclays Plc. stock, traded in the London Stock Exchange. Barclays Plc. is a global financial services provider operating around the world (Barclays 2007). Data availability issues and the fact that the underlying stock and the option are liquid mostly motivated the choice of this specific stock option. In the sample period 2005; 304067 BBL options were traded and they were fifth most traded stock options in the London International Financial Futures and Options Exchange (LIFFE). Overall, BBL options should describe the behaviour of LIFFE stock options markets rather well.

The options are American style, i.e. an option can be exercised any time during option's maturity. The expiration months for these options are the three nearest calendar months, and the three following months within the quarterly cycle March, June, September and December. The detailed contract specifications for BBL stock option are reviewed in the Table 2.

Table 2. The contract specifications for Barclays Plc. stock option (American-Style Exercise). (Euronext 2007.)

Ticker	BBL
Exercise type	American
Unit of trading	One option equals rights over 1000 shares
Quotation	Pence per share
Minimum price movement (tick size and value)	0,5 pence per share / £5,00
Exercise day	Exercise by 17:20 on any business day, extended to 18:30 for all series on a Last Trading Day
Last trading day	Third Friday in expiry month
Settlement day	Settlement Day is four business days following the day of exercise / Last Trading Day
Trading hours	08:00 – 16:30

5.1.1. Data availability

The data used is from the London International Financial Futures and Options Exchange's Market Data System and it was obtained from the database of the University of Vaasa. The end-of-day data used reports timestamp, the characteristics of the option (call or put, expiration date, strike price, open and close prices, high and low prices), number of contracts traded, open interest, volatility, ATM volatility, and the price of the underlying stock at its last trade. The options tick data contained trading time (year, month, hour, minute, and second), expiration date, call or put, strike price, price and trading volume.

For implied volatility calculations the estimate risk-free interest rate has to be defined. The risk-free rate (r) is in theory the rate at which money is borrowed or lent in a default free market. Usually studies done in the United States use Treasury Bill –interest and this Treasury rate is the rate at which a national government borrows in its own currency. However banks and other large financial institutions usually set r equal to the London Interbank Offer Rate (LIBOR) instead of Treasury rate when they evaluate derivatives transactions. LIBOR is a rate at which a financial institution is willing to lend its surplus funds to other financial institutions in the interbank markets. LIBOR is published by the British Bankers Association (BBA). Theoretically the accurate interest would be ex-

actly the interest, which equals to the option maturity. Since there is not that kind of interest available in UK from the observed years, the three-months LIBOR is chosen as the risk-free rate. When calculating the implied volatility, the logarithmic adjustment for the interest rate is done by using the formula: $\ln(1+r_3)$. Following figure illustrates the changes in the three-month LIBOR from 4 January, 2005 to 30 December, 2005.

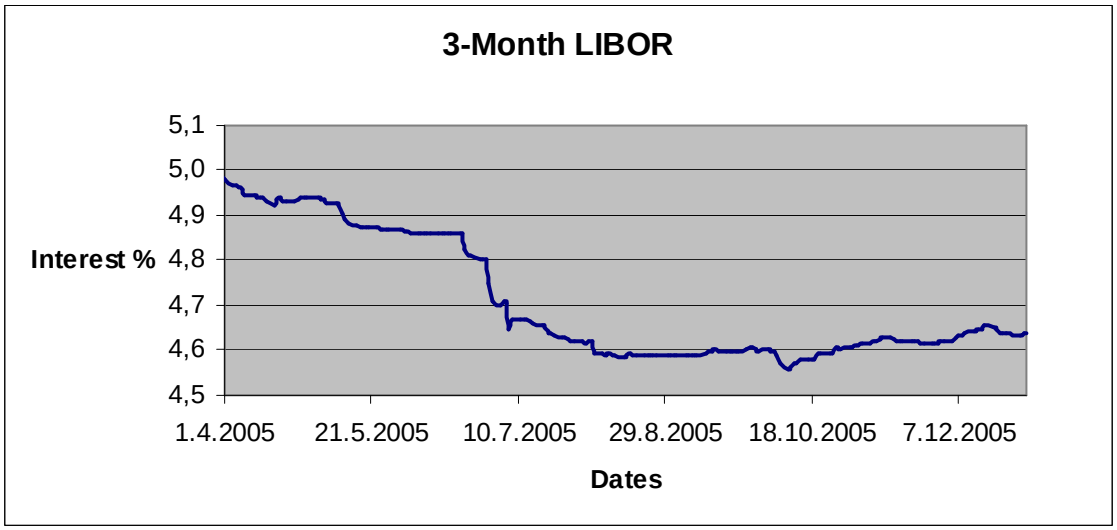

Figure 7. The three-month LIBOR from 4 January to 30 December, 2005.

5.1.2. Descriptive statistics

To construct the sample of the study, several exclusion filters were used to make sure that the data would be as accurate and reliable as possible. First, options with more than 111 days and less than 22 days until expiration were excluded. These options were excluded because of possible liquidity biases. Second, options with dividend payments during the maturity were also excluded. For some contracts it is not possible to obtain an implied volatility, and also these contracts are, therefore, removed from the sample data. The options with less than 22 trading days left are ignored because options markets are very volatile during this period, which could lead to unreliable estimates of implied volatility. 22 days equals approximately the number of trading days in one moth. On the other hand, options with more than 111 trading days left are ignored because of thin trading. For example Ederington and Guan (2005) used options with at least 20 trading days to expiration to avoid unstable implied volatilities, Fleming (1998) used options with at least 15 days, to expiration, and

Canina and Figlewski (1993) used options with more than 7 and fever than 127 days to expiration to calculate implied volatilities.

The Cox et al. (1979) binomial tree model with 100 time steps and bisection method is used to calculate individual stock options implied volatilities for both put and call options in five different moneyness categories. One implied volatility is computed for each option series each day and the volatility is based on the midpoint of the high/low prices of trading day. Option maturities are between 22 to 111 days and dividends are not allowed during option's life. Since any dividends are allowed during the option maturity, the calls should not optimally be exercised early and their prices should not include an early exercise premium.

To characterize the shape of implied volatility, the options are divided into five different moneyness categories and then the average implied volatility for each of the categories is computed. The moneyness is derived approximately as in Bollen and Whaley (2004) and it reflects the option's likelihood of being in the money at expiration. Moneyness for put and call options are measured using the option's delta:

$$\Delta_C = N\left[\frac{\ln(Se^{rT} / X) + 0{,}5\sigma^2 T}{\sigma\sqrt{T}}\right] \tag{5.2}$$

$$\Delta_P = \left(N\left[\frac{\ln(Se^{rT} / X) + 0{,}5\sigma^2 T}{\sigma\sqrt{T}}\right]\right) - 1. \tag{5.3}$$

The notations are as follows: Δ_C and Δ_P are the deltas for call and put options respectively, S is the current stock price, X is the option's strike price, T is the option's time to expiration, r is the risk-free rate of interest, σ is the stock return volatility, and $N(.)$ is the normal cumulative density function. Deltas are computed for each option series each day during the sample period. The estimate for the volatility rate is the realized return volatility of the underlying stock in year 2005 multiplied by the square root of options maturity. Based on the deltas, options are divided into five moneyness categories. The upper and lower bounds of the categories are listed in Table 3. Listed are category numbers, labels, and corresponding delta ranges of options used in the study. Options with absolute deltas below 0,02 or above 0,98 are excluded due to the distortions caused by price discreteness.

Table 3. Moneyness category definitions.

Categories	Labels	Range
1	Deep in-the-money (DITM) call	$0{,}875 < \Delta_C \leq 0{,}98$
	Deep out-of-the-money (DOTM) put	$-0{,}125 < \Delta_P \leq -0{,}02$
2	In-the-money (ITM) call	$0{,}625 < \Delta_C \leq 0{,}875$
	Out-of-the-money (OTM) put	$-0{,}375 < \Delta_P \leq -0{,}125$
3	At-the-money (ATM) call	$0{,}375 < \Delta_C \leq 0{,}625$
	At-the-money (ATM) put	$-0{,}625 < \Delta_P \leq -0{,}375$
4	Out-of-the-money (OTM) call	$0{,}125 < \Delta_C \leq 0{,}375$
	In-the-money (ITM) put	$-0{,}875 < \Delta_P \leq -0{,}625$
5	Deep out-of-the-money (DOTM) call	$0{,}02 < \Delta_C \leq 0{,}125$
	Deep in-the-money (DITM) put	$-0{,}98 < \Delta_P \leq -0{,}875$

The trading activity of included BBL stock options over the 2005 sample period used in the study is summarized. The total numbers of contracts traded in each moneyness category are reported in Table 4. First, the summary shows that call option volume is greater than put option volume, 73,8 percent of all contracts traded were call options, with only 27,2 percent being puts. Second, comparing across moneyness categories, trading volume for calls is heaviest for OTM options. On the other hand, for puts, ITM options have the heaviest trading volume.

Table 4. Summary of qualified Barclays Plc. options traded on the London International Financial Futures and Options Exchange during the sample period 2005.
The delta value of each option series is computed using the closing stock price, the three month LIBOR, and the realized volatility in sample period matching the options maturity.

	Stock Options			
	Calls		Puts	
Delta Value Category	No. of Contracts	Prop. of Total	No. of Contracts	Prop. of Total
1	139	0,0018	28	0,0004
2	2240	0,0286	1518	0,0194
3	20762	0,2647	5147	0,0656
4	28565	0,3642	12635	0,1611
5	5358	0,0683	2040	0,0260
Total	57064	0,7276	21368	0,2724

Figure 8 illustrates the BBL option's implied volatility and Barclays Plc. stock level. Implied volatility is defined as the average implied volatility of ATM or category 3 options. The implied volatility and stock level are plotted over the full sample. The stock level has its highest peak around 614,5 and lowest around 519,5. The implied volatility is highest at 27,4% (31.10.2005) and lowest at 8,8% (19.5.2005). Figure 9 depicts Barclays Plc. daily returns, which are rather stable moving approximately between minus four percent to plus three percent. Table 5 contains the average implied volatilities of the BBL stock options over the period January 2005 to December 2005. The average BBL option implied volatility plotted versus moneyness categories performs a smile, as is shown in figure 10. The implied volatility of the ATM options is lowest and increasing with movement in either direction. Implied volatility is highest in moneyness category 1, 29,35 percent.

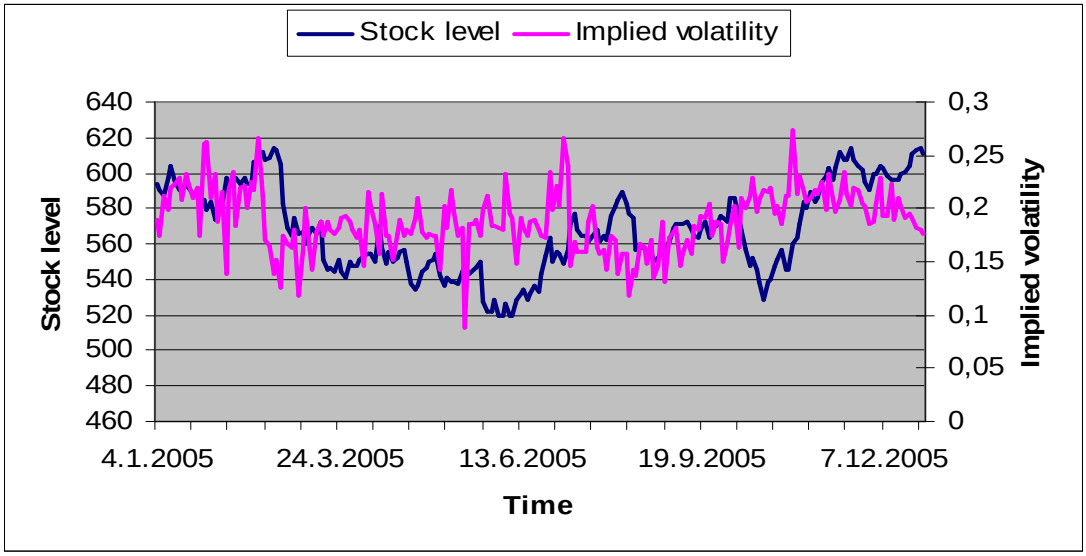

Figure 8. BBL implied volatility level and Barclays Plc. level from January 2005 through December 2005.
The Implied volatility is the average implied volatility of ATM options and Stock level is the closing Barclays Plc. stock level on the dates the implied volatilities are estimated.

Table 5. Average implied volatilities by option delta for Barclays Plc. stock options traded on the London International Financial Futures and Options Exchange during the period January 2005 through December 2005.

	Average implied volatility				
Category	1	2	3	4	5
BBL	0,2935	0,2068	0,1916	0,2015	0,2190

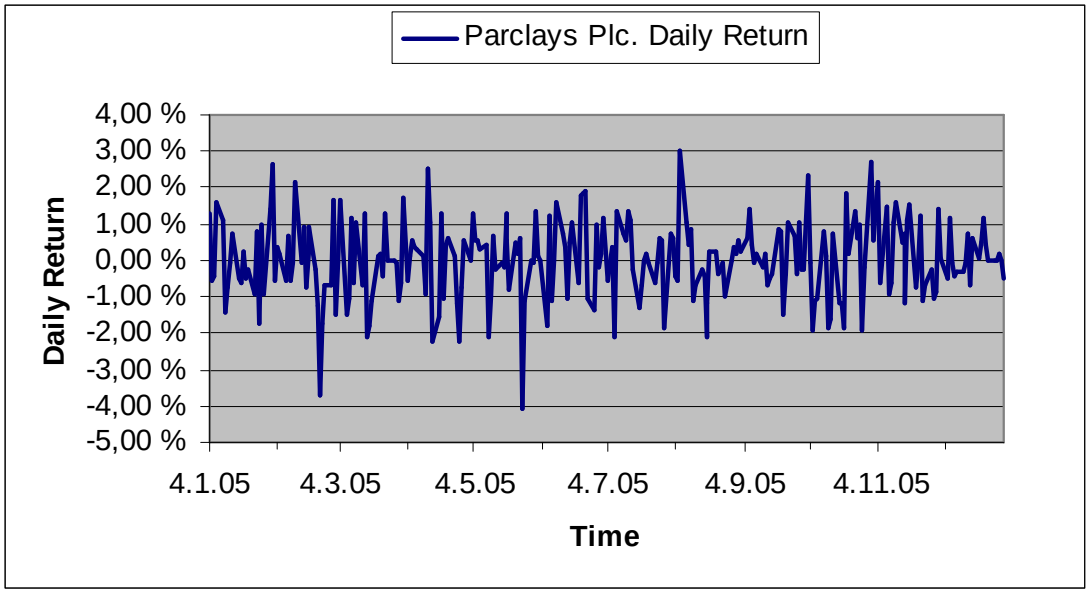

Figure 9. Barclays Plc. daily return. The daily return is calculated as log(stock level$_t$/stock level$_{t-1}$).

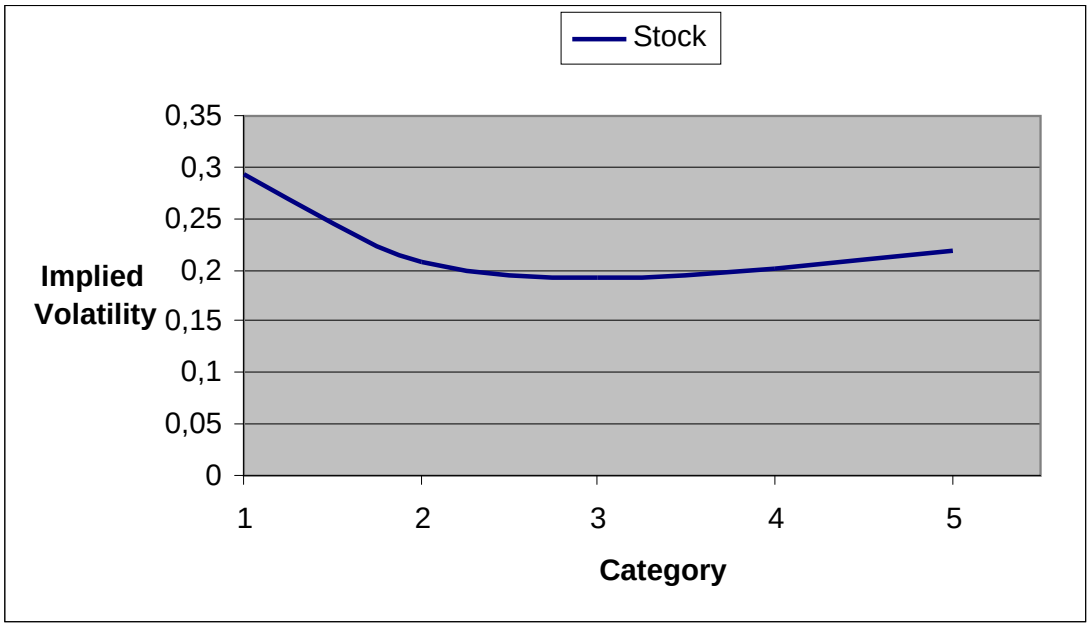

Figure 10. Estimated implied volatility smile of Barclays Plc. stock option from January 2005 to December 2005.

5.2. Hypotheses

In empirical studies the main aim is to study the relation between the implied volatility movement and demand pressure by regression analysis. As pointed out by Bollen and Whaley (2004), under the assumption of frictionless markets, suppliers of option market liquidity can perfectly and costlessly hedge their inventories, so supply curves will be flat. Also neither time variation in the demands to buy or sell options nor public imbalances for particular option series will affect option price and, hence, implied volatility. The null hypothesis, therefore, is:

H_0: No relation exists between demand for options and related implied volatilities.

Two alternative hypotheses support a positive relation between demand for options and related implied volatilities. The first alternative hypothesis is based on the reality that, when suppliers of liquidity are required to take larger positions in particular options series, their hedging costs and/or desired compensation for risk increase. This leads to a position where option price and implied volatility increase as well. The hypothesis is:

H_1: With supply curves upward sloping, an excess of buyer-motivated trades will cause price and implied volatility to rise, and an excess of seller-motivated trades will cause implied volatility to fall.

The second alternative hypothesis is based on the view that the trading activity of investors provides information to the market maker, who continually learns about the underlying asset dynamics and updates prices as a result.

H_2: A positive relation between demand for options and related implied volatilities would be observed if the order imbalance merely reflects a change in investor expectations about future volatility.

The regression model (5.3) includes the lagged change in implied volatility as an independent and explanatory variable to assess the relation between changes in implied volatility and option demand. The null hypothesis predicts that its coefficient is not different from zero.

Based on Bollen et al. (2004) the hypothesis one and two generates different predictions. Hypothesis one predicts that coefficient of the lagged change in implied volatility is negative and significant. Though, if the implied volatility change results from the demand for options, the option demand changes should generate price pressure, which causes option prices and implied volatilities to change. However, the changes in implied volatility will reverse; at least in part, as the market maker has the opportunity to rebalance his portfolio. Also, the hypothesis predicts that implied volatilities of different option series do not need to move together as they are primarily affected by option series' own demand.

Based on hypothesis two, if changes in implied volatility are driven by shifts in investor expectations regarding volatility, changes in implied volatility should be permanent and uncorrelated through time; hence it also predicts an insignificant coefficient on the lagged change in implied volatility. Also, demand for ATM options should be the dominant factor determining the implied volatility of all options, since ATM options are most informative about future volatility. Thus, the implied volatility of all option series in a class should move in concert.

5.3. Methodology

The empirical methodology is designed to uncover the role demand pressure plays in determining changes in the level of implied volatility for options with different exercise prices. To analyze the time-series dynamics of implied volatility, the levels of implied volatility in the five different moneyness categories are considered separately. To asses the relation between implied volatility and demand pressure, the daily change in the average implied volatility of options in a particular moneyness category on contemporaneous measure of security return, security trading volume, and demand pressure is regressed. Based on earlier studies, the security return and trading volume are known determinants of volatility. Though, the contemporaneous return of the underlying security and its trading volume are included as control variables for holding constant the leverage and information flow effects. The lagged chance in implied volatility is included as an independent variable in a regression that assesses the relation between changes in implied volatility and option demand. The null hypothesis

predicts that its coefficient is not different from zero. Also the learning hypothesis (H_2) predicts an insignificant coefficient on the lagged change in implied volatility. In contrast to the null hypothesis and learning hypothesis, the limits to arbitrage hypothesis (H_1) predict that the coefficient of the lagged change in implied volatility is negative.

The regression model specification, defined by Bollen et al. (2004), is

$$\Delta IV_t = \beta_0 + \beta_1 RS_t + \beta_2 VS_t + \beta_3 D_{1,t} + \beta_4 D_{2,t} + \beta_5 \Delta IV_{t-1} + \varepsilon_t, \tag{5.3}$$

where ΔIV_t is the change in the average implied volatility in a moneyness category from the day $t-1$ to the day t, RS_t is the underlying security return from the close on day $t-1$ to the close on day t, VS_t is the trading volume of the underlying security on day t, and $D_{1,t}$ and $D_{2,t}$ are the demand pressure variables (whose definitions vary in the regression tests that follow). ΔIV_{t-1} is the lagged change in implied volatility.

Daily demand pressures in the first six regressions are estimated based on the difference between the volume of buyer motivated option contracts and the volume of seller motivated option contracts within a trading day. Based on the tick data, a traded option contract is a buyer motivated (seller motivated) option contract if the option is traded higher (lower) price than the high/low midpoint of option series traded within a trading day. Demand pressure variables used in the first six regressions are computed as

$$D_t = \frac{\sum_i^n B_i \Delta_i - \sum_i^n S_i \Delta_i}{\sum_i^n B_i + \sum_i^n S_i}, \tag{5.4}$$

where B_i is the contract size of buyer motivated trade, S_i is the contract size of seller motivated trade, Δ_i is the options delta and n is the number of specific trades done at day t. In the last two regressions demand pressure variables $D_{1,t}$ and $D_{2,t}$ are replaced by $Put/Call$ volume ratio and it is computed as

$$Put/Call_t = \frac{\sum_i^n P_i - \sum_i^n C_i}{\sum_i^n P_i + \sum_i^n C_i}, \tag{5.5}$$

where P_i is the volume of a put option contract traded in a specified moneyness group of a particular option series, and C_i is the volume of a call option contract traded in a specified moneyness group of a particular option series. Dennis et al. (2002) used the put/call volume ratio as a proxy for demand pressure.

6. EMPIRICAL RESULTS

This chapter presents and discusses the empirical results. The hypotheses are tested based on the equation (5.3) presented earlier. The regression results are obtained by using EViews5 econometric software programme. The estimated beta (β_i) coefficients show the magnitude and direction of different variables effectiveness on implied volatility changes. A positive beta coefficient indicates that the increase of a factor increases the implied volatility changes and a negative beta coefficient indicates that the increase of a factor decreases implied volatility changes. The regression results are estimated based on ordinary least squares (OLS) method, and if there has been autocorrelation or heteroscedasticity, they are removed with EViews. Basic starting point was to do the regression with OLS settings when ever possible.

If the demand pressure does explain the change in implied volatility, the coefficients: β_3, β_4, or both should be significantly greater than zero. On the other hand if options' trading is driven by market expectations about volatility change, β_3 and β_4 should not be significantly different from each other because overall call options and put options should respond similarly to the change of volatilities. The hypothesis that $\beta_3 = \beta_4$ is tested with Wald coefficient test. If the demand pressure is related to market expectations about future volatility, then the volatility should be more heavily affected by the demand pressure in the same moneyness category. In all equations, the lagged change of implied volatility, ΔIV_{t-1}, is used to investigate whether the impact of demand pressure on change in implied volatility is caused by limits to arbitrage. For market makers who steadily rebalance their options positions, the options implied volatility changes should move towards their previous levels. A negative estimated coefficient β_5 would suggest a transitory impact of options demand pressure and the impact due to limits to arbitrage. On the other hand, if β_5 is not different from zero, it would suggest that implied volatility changes are affected by the changes of market expectations about volatility. Because the market makers are informed by options trading activities they adjust option positions continuously. In addition, in all equations RS and VS are used to control for leverage and information flow effects. Volatility changes may be negatively related to price changes, because of a firm's leverage effect, and positively related to information flow. The asterisks (***,**,*) after coefficients denotes that

the particular coefficient is significantly different from zero at the 1, 5, or 10 percent probability level.

6.1. Changes in ATM implied volatility

In all, four pairs of regression tests are performed. In the first pair, the degree to which the variables in (5.3) explain changes in the volatility of ATM options (category 3) is assessed. The regression is estimated for calls and puts separately, and its specification is

$$(6.1)\quad \Delta IV_t = \beta_0 + \beta_1 RS_t + \beta_2 VS_t + \beta_3 ATMC_t + \beta_4 ATMP_t + \beta_5 \Delta IV_{t-1} + \varepsilon_t,$$

where $ATMC_t (ATMP_t)$ is the demand pressure for ATM calls (puts). The coefficients β_3 and β_4 should be informative regarding investor trading motivation. If trading is motivated by changes in expected future volatility, the coefficient values should be indistinguishable from one another. ATM calls and ATM puts are equally responsive to shifts in volatility, so there is no reason for traders to prefer one type of option over the other. On the other hand, if the demand pressure moves prices as a result, the coefficients will differ.

Table 6 contains a summary of the regression results of the (6.1) for Barclays Plc. options (BBL). Panel A shows the results for changes in the implied volatility of call options. The coefficients of $ATMC$ and $ATMP$ demand pressure variables are negative and insignificant. In addition, tested with Wald coefficient test, the β_3 and β_4 are not significantly different. Panel B shows the results for changes in the implied volatility of ATM put options. The coefficients on $ATMP$ is insignificantly positive and $ATMC$ is positively significant at less than 10% level. Again, the β_3 and β_4 are not significantly different.

The results regarding the lagged implied volatility variable in Table 6 are interesting. Under the null hypothesis and the learning hypothesis, the coefficient should not be different from zero. Although, the coefficient for ATM call is approximately -0,40 and the coefficient for ATM put is approximately -0,48. Apparently, prices reverse. Thus, about 44 percent of the BBL option-implied volatility change observed today gets reversed tomorrow.

The results in Table 6 indicate that demand pressure moves prices as a result but trading is also motivated by changes in expected future volatility. In addition, the results in panel B indicates that demand pressure influences implied volatility changes, and that the demand pressure of call options has a stronger effect on the implied volatility changes than put options demand pressure.

Table 6. Summary of regression results of change in at-the-money implied volatility for Barclays Plc. stock options traded on the London International Financial Futures and Options Exchange during the period January 2005 through December 2005.

Panel A. Change in ATM Call Volatility as a function of $ATMC$ and $ATMP$

				Parameter Estimates					
Ticker	No. of Obs.	R^2	Adj. R^2	β_0	β_1	β_2	β_3	β_4	β_5
BBL	179	0,2048	0,1818	0,0011	-0,5640***	$-4{,}05\cdot10^{-6}$	-0,010	-0,0073	-0,3993***

Panel B. Change in ATM Put Volatility as a function of $ATMC$ and $ATMP$

				Parameter Estimates					
Ticker	No. of Obs.	R^2	Adj. R^2	β_0	β_1	β_2	β_3	β_4	β_5
BBL	153	0,2504	0,2249	0,0025	0,4863*	$-2{,}21\cdot10^{-5}$	0,0214*	0,0049	-0,4807***

6.2. Changes in OTM call implied volatility

The second test pair examines changes in implied volatility of OTM calls. The regression specification is

$$\Delta IV_t = \beta_0 + \beta_1 RS_t + \beta_2 VS_t + \beta_3 OTMC_t + \beta_4 ATMC_t + \beta_5 \Delta IV_{t-1} + \varepsilon_t \quad (6.2)$$

for the results reported in panel C of Table 7. For panel D, the demand pressure $ATMP$ replaces $ATMC$ in (6.2). The regression attempts to asses whether demand pressure of OTM calls affects the implied volatility of OTM calls after controlling for the effects of demand pressure of ATM options. If the learning story is correct and demand pressure arises from a revision to investor expectations regarding future volatility, the demand pressure of ATM options is more likely to drive changes in OTM implied volatility than OTM demand pressure. The reason for this is that ATM options have the highest sensitivity to volatility,

hence they are the natural vehicle to exploit new information. On the other hand, if the limit to arbitrage story is correct, the OTM demand pressure should be more important to that of other series. Thus, if learning hypothesis is correct, the coefficients for *ATMC* and *ATMP* should be greater that of *OTMC*. If the limits to arbitrage hypothesis is correct and volatility changes are primarily affected by options' own demand, the coefficient for *OTMC* should be greater that of *ATMC* and *ATMP*.

The regression results are viewed in panels C and D of Table 7. In panel C the coefficient on *OTMC* is insignificantly positive and coefficient on *ATMC* is insignificantly negative. In panel D, the coefficient on *ATMP* is insignificantly positive. It shows that ATM trading does not carry much information about implied volatility changes of OTM call options. Although, in both panels the β_3 and β_4 not significantly different.

Table 7. Summary of regression results of change in out-of-the-money Call implied volatility for Barclays Plc. stock options traded on the London International Financial Futures and Options Exchange during the period January 2005 through December 2005.

Panel C. Change in OTM Call Volatility as a function of $OTMC$ and $ATMC$

				Parameter Estimates					
Ticker	No. of Obs.	R^2	Adj. R^2	β_0	β_1	β_2	β_3	β_4	β_5
BBL	119	0,3154	0,2851	0,0033	-0,5029**	$-2{,}12\cdot10^{-5}$	0,0262	-0,0078	-0,5009***

Panel D. Change in OTM Call Volatility as a function of $OTMC$ and $ATMP$

				Parameter Estimates					
Ticker	No. of Obs.	R^2	Adj. R^2	β_0	β_1	β_2	β_3	β_4	β_5
BBL	119	0,3186	0,2885	0,0025	-0,5473**	$-1{,}85\cdot10^{-5}$	0,0304	0,0107	-0,4957***

Finally, the coefficient of the lagged implied volatility variable in the results of Table 7 is again consistently negative and significant and about the same order of magnitude as in Table 6. It suggests that market makers rebalance their option positions gradually and these rebalancing activities cause the implied volatilities to revert partly back to their previous levels. Approximately 50 percent of the change in the OTM call option volatility gets reversed on the following

day. These evidences suggest that the changes of implied volatility are not merely caused by market expectations.

6.3. Changes in ITM put implied volatility

Table 8 shows the results for changes in the implied volatility of ITM put options. In panel E, the regression specification is

$$\Delta IV_t = \beta_0 + \beta_1 RS_t + \beta_2 VS_t + \beta_3 ITMP_t + \beta_4 ATMP_t + \beta_5 \Delta IV_{t-1} + \varepsilon_t . \quad (6.3)$$

If buying pressure is mainly caused by market expectations about future volatility change, the coefficients for *ATMC* and *ATMP* should be greater that of *ITMP*. ATM options have the highest sensitivity to volatility change, and therefore they are more likely to be used if market expectations for future volatility changes are the major reason for trading options. Thus, the demand pressure based on ATM options should have a stronger effect on the implied volatility change. On the other hand, if the volatility changes are primarily affected by option demand in a specific category, β_3 should be greater than β_4.

Table 8 shows the results for changes in the implied volatility of ITM put options. The coefficients of *ITMP* and *ATMP* demand pressure variables, in panel E, are negative and insignificant. In panel F, where *ATMC* replaces *ATMP* in the regression, the coefficient on *ITMP* is insignificantly positive while the coefficient on *ATMC* is positively significant at less than 5% level. Again, in both cases, the β_3 and β_4 are not significantly different.

The results indicate that the demand pressure has influence on implied volatility changes. Also it indicates that, unlike the demand pressure of ATM calls, the demand pressure of ITM puts does not affect changes in the implied volatility of ITM puts. Therefore, market expectations may affect the changes of implied volatility. This result indicates that demand pressure of call options has a stronger effect on the implied volatility changes than that of put options.

It is also worth noting that the coefficients on lagged change in volatility are again significantly negative, indicating price reversals. Again, this evidence

suggests that implied volatility changes are not caused merely by market expectations.

Table 8. Summary of regression results of change in in-the-money Put implied volatility for Barclays Plc. stock options traded on the London International Financial Futures and Options Exchange during the period January 2005 through December 2005.

Panel E. Change in ITM Put Volatility as a function of $ITMP$ and $ATMP$

Ticker	No. of Obs.	R^2	Adj. R^2	Parameter Estimates β_0	β_1	β_2	β_3	β_4	β_5
BBL	164	0,2416	0,2176	-0,0001	0,2484	$6{,}48 \cdot 10^{-6}$	-0,0060	-0,0047	-0,4653***

Panel F. Change in ITM Put Volatility as a function of $ITMP$ and $ATMC$

Ticker	No. of Obs.	R^2	Adj. R^2	Parameter Estimates β_0	β_1	β_2	β_3	β_4	β_5
BBL	97	0,3497	0,3140	0,0028	0,2976	$-1{,}54 \cdot 10^{-5}$	0,0096	0,0161**	-0,5866***

6.4. The affect of put/call volume ratio on ATM implied volatility changes

Table 9 shows the results for changes in the implied volatility of ATM options, when $Put/Call$ volume ratio is the demand pressure variable. In both panels G and H, the regression specification is

$$\Delta IV_t = \beta_0 + \beta_1 RS_t + \beta_2 VS_t + \beta_3 Put/Call_t + \beta_5 \Delta IV_{t-1} + \varepsilon_t, \quad (6.4)$$

where $Put/Call_t$ is the approximated demand pressure of ATM options. In panel G the coefficient on $Put/Call$ is insignificantly positive while the coefficient in panel H on $Put/Call$ is insignificantly negative. Based on these results it is undoubtedly clear that $Put/Call$ volume ratio do not have remarkable affect on implied volatility changes.

Again the coefficients on lagged change in volatility are significantly negative, indicating price reversals. Again, this evidence suggests that implied volatility changes are not caused merely by market expectations.

According to Tables 6, 7, 8, and 9, the β_1 (stock return) estimates are significantly negative in four cases out of eight, significantly positive in two cases and, insignificantly positive in the rest of the cases. The coefficient of β_2 (stock volume) is insignificantly negative in six cases out of eight and insignificantly positive in two cases. The results of β_1 estimates are partly consistent with previous literature that documents an inverse relation between volatility changes and return. On the other hand the results of β_2 estimates indicate, unlike the previous literature, that there is no relation between volatility and information flow.

Table 9. Summary of regression results of change in at-the-money implied volatility relative to put/call volume ratio for Barclays Plc. stock options traded on the London International Financial Futures and Options Exchange during the period January 2005 through December 2005.

Panel G. Change in ATM Call Volatility as a function of category 3 Put/Call volume ratio

				Parameter Estimates				
Ticker	No. of Obs.	R^2	Adj. R^2	β_0	β_1	β_2	β_3	β_5
BBL	179	0,2054	0,1871	0,0019	-0,5243***	$4{,}50 \cdot 10^{-6}$	0,0046	-0,4006***

Panel H. Change in ATM Put Volatility as a function of category 3 Put/Call volume ratio

				Parameter Estimates				
Ticker	No. of Obs.	R^2	Adj. R^2	β_0	β_1	β_2	β_3	β_5
BBL	153	0,2353	0,2147	0,0045	0,4704*	$-3{,}15 \cdot 10^{-5}$	-0,0015	-0,4706***

In all equations the coefficient of variable ΔIV_{t-1} is significantly negative in 1 percent level. This result shows that market makers rebalance their option positions gradually and that implied volatilities are partially correlated to their previous levels. Therefore, the impact of options demand pressure is transitory and the impact can be caused by limits to arbitrage. The evidence supports the hypothesis that limits to arbitrage permit a relation between the demand for options and corresponding implied volatility. The price reversals of implied volatilities are an average about 47 percent. Overall, this evidence suggests that the implied volatilities changes are better explained by demand pressure than by market expectations. According to the results, the implied volatility change in the Barclays Plc. options market shows a clear reversion pattern. These findings support the demand pressure hypothesis in that option demand affects implied volatilities.

7. SUMMARY AND CONCLUSIONS

The first purpose of this study was to examine how well the trading pressure explains the shape of option implied volatility smile. The second purpose was to estimate how option demand affects to option prices and associated implied volatilities. Based on the tick-by-tick Barclays Plc. stock options, the call and put options were analyzed across five different moneyness categories. The regression equations were tested using ordinary least squares method.

Chapter 1 introduced a brief review of earlier research done in the area of explaining the shape of implied volatility smile with option demand and the purpose of the study were presented. In the theoretical part of this study (i.e. Chapters 2., 3. and 4.), first the derivative markets and then the options particularly were introduced. After this, stock price behaviour and stochastic price processes were introduced. The famous option pricing models, binomial tree model and Black-Scholes model were presented in Chapter 3. Then, in Chapter 4, an important factor in the option pricing, volatility, were comprehensively introduced. Also this chapter took a close look on implied volatility. The empirical part of this study (i.e. Chapter 5. and 6.) contained first the introduction and description of the data. After that, the study hypothesis and analysis methods were presented. Finally the Chapter 6 presented the empirical test results.

To investigate the relation between demand pressure and implied volatility, the study examines the null hypothesis and two alternative hypotheses. The null hypothesis predicts that there is no relation between demand for options and corresponding implied volatilities. The two alternative hypotheses support a positive relation between demand for options and related implied volatilities.

To test the hypotheses the tick- and end-of-day data on Barclays Plc. stock options are used. These options are traded in the London International Financial Futures and Options Exchange (LIFFE) and the data is gathered from the time period 4 January, 2005 to 30 December, 2005. The Barclays Plc. stock options were the fifth most traded options in LIFFE during the sample period and they can be seen as generalized example of all stock options traded in LIFFE.

The empirical results of the study are somewhat in accordance with the theoretical hypothesis. The results document a relation between the change in im-

plied volatility and demand pressure, holding constant the effects of known determinants of volatility. For Barclays Plc. stock options, demand pressure for calls has a more dominant role than puts. Based on the results, volatility changes are not permanent and the demand pressure on stock call options appears to drive shape of the stock options implied volatility smile. As a result – demand pressure moves stock option prices. Also, implied volatility changes on day one are shown to revert in part on the following day, as market makers are gradually able to rebalance their portfolios. The empirical results are consistent with the limits to arbitrage hypothesis.

The results show that there exists relation between option demand and related implied volatility. It is clear that trading is partly motivated by changes in expected future volatility, but price reversals of implied volatilities are an average as much as 47 percent. Also, it seems that call demand and especially ATM call demand dominates explaining implied volatility movement. This is in line with earlier studies that changes in implied volatility of stock options are most affected by demand pressure for calls.

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APPENDIX 1. The Microsoft Visual Basic Codes for calculating the binomial option price and the implied volatility.

This calculates the binomial option price.

```
Public Function OptionValue( _
  OptType As String, _
  AmEur As String, _
  StockPrice As Double, _
  Strike As Double, _
  Volatility As Double, _
  IntRate As Double, _
  EndDate, _
  StartDate, _
  NoSteps As Double _
  )
  ReDim V(0 To NoSteps)
  'Calculate parameters used in binomial tree
  TimeStep = Years(EndDate, StartDate) / NoSteps
  DiscountFactor = Exp(-IntRate * TimeStep)
  u = Exp(Volatility * Sqr(TimeStep))
  d = 1 / u
  p = (Exp(IntRate * TimeStep) - d) / (u - d)

  'Binary variables to identify the type of option
  b = BinCP(OptType)
  BAE = BinAE(AmEur)

  'Calculate option value at maturity
  For j = 0 To NoSteps
    s = AssetPrice(StockPrice, u, NoSteps, j)
    V(j) = IntVal(s, Strike, b)
  Next j

  'Move back down the tree
  For n = (NoSteps - 1) To 0 Step -1
    For j = 0 To n
      V(j) = (p * V(j) + (1 - p) * V(j + 1)) * DiscountFactor
      s = AssetPrice(StockPrice, u, n, j)
      IntValue = IntVal(s, Strike, b)
      If IntValue * BAE > V(j) Then
        V(j) = IntValue
      End If
    Next j
  Next n
  OptionValue = V(0)
End Function
```

```
'Calculate maturity in years
Function Years(EndDate, StartDate)
   NumDays = (EndDate - StartDate)
   FullWeeks = Int(NumDays / 7)
   RemainderDays = ((NumDays / 7) - FullWeeks) * 7
   WeekendDays = (2 * FullWeeks) + Application.WorksheetFunction.Max(0, Application. Work-
   sheetFunction.Min((Weekday(StartDate) + RemainderDays - 6), IIf(Weekday(StartDate) = 7, 1,
   2)))
   Years = Application.WorksheetFunction.Max(0, (NumDays - WeekendDays)) / 252
End Function

'Calculate the intrinsic value
Function IntVal(s, Strike, b)
   IntVal = b * (s - Strike)
   If IntVal < 0 Then
      IntVal = 0
   End If
End Function

'Calculate asset price at period n after nd downs
Function AssetPrice(S0, u, n, nd)
      d = 1 / u                                          'd is the down factor
      nu = n – nd                                        'nu is the number of ups
      AssetPrice = S0 * u ^ nu * d ^ nd
End Function

'Check whether the option is a call or a put
'Check first letter to determine the type of option
Function BinCP(OptType As String)
      If Left(OptType, 1) = "C" Or Left(OptType, 1) = "c" Then BinCP = 1
      If Left(OptType, 1) = "P" Or Left(OptType, 1) = "p" Then BinCP = -1
End Function

'Check whether the option is American or European
'Check first letter to determine the type of option
Function BinAE(AmEur As String)
      If Left(AmEur, 1) = "E" Or Left(AmEur, 1) = "e" Then BinAE = 0
      If Left(AmEur, 1) = "A" Or Left(AmEur, 1) = "a" Then BinAE = 1
End Function

Public Function OptMat01( _
   OptType As String, _
   StockPrice, _
   Strike, _
   Volatility, _
   EndDate, _
   StartDate, _
   NoSteps, _
   nd _
```

```
  )
    TimeStep = Years(EndDate, StartDate) / NoSteps      'Calculate time step
    u = Exp(Volatility * Sqr(TimeStep))                 'Calculate up factor
    s = AssetPrice(StockPrice, u, NoSteps, nd)          'Calculate stock price at maturity
    b = BinCP(OptType)                                  'Determine type of option
    OptMat01 = IntVal(s, Strike, b)                     'Calculate value at maturity
End Function

Function OptMat02( _
  OptType As String, _
  StockPrice, _
  Strike, _
  Volatility, _
  EndDate, _
  StartDate, _
  NoSteps, _
  nd _
  )
      ReDim V(0 To NoSteps)
    'Specify size of vector V
      TimeStep = Years(EndDate, StartDate) / NoSteps    'Calculate time step
      u = Exp(Volatility * Sqr(TimeStep))               'Calculate up factor
      b = BinCP(OptType)                                'Determine type of option
      For j = 0 To NoSteps
    'Start loop
        s = AssetPrice(StockPrice, u, NoSteps, j)       'Calculate stock price at maturity
        V(j) = IntVal(s, Strike, b)                     'Calculate option value
      Next j
    'End loop
      OptMat02 = V(nd)                                  'Return element asked in argument
End Function
```

This calculates the implied volatility with bisection method.

```
Public Function ImpliedVolatility( _
  StockPrice As Double, _
  Strike As Double, _
  IntRate As Double, _
  EndDate, _
  StartDate, _
  OptionPrice As Double, _
  OptType As String, _
  AmEur As String, _
  NoSteps As Double _
  ) As Double
    'Check for arbitrage violations: if price at almost zero volatility greater than price, return 0.
      Dim sigma_low As Double
      Dim Price As Double
      Dim IV As Double
```

```
sigma_low = 0.0001
IV = -1
'Calculate the price of the volatility was almost zero, to see what would be the minimum Price.

Price = OptionValue(OptType, AmEur, StockPrice, Strike, sigma_low, IntRate, EndDate, StartDate, NoSteps)


If Price > OptionPrice Then
IV = 0
Else
'Simple binomial search for the implied volatility. Relies on the value of the option increasing in volatility.
Const ACCURACY = 0.00001
Const MAX_ITERATIONS = 100
Const HIGH_VALUE = 10000000000#
Const ERROR = -1E+40
'Want to bracket sigma. First find a maximum sigma by finding a sigma with a estimated price higher than the actual price.
Dim sigma_high As Double
sigma_high = 0.3

Price = OptionValue(OptType, AmEur, StockPrice, Strike, sigma_high, IntRate, EndDate, StartDate, NoSteps)

Do While (Price < OptionPrice)
sigma_high = 2# * sigma_high            'keep doubling.

Price = OptionValue(OptType, AmEur, StockPrice, Strike, sigma_high, IntRate, EndDate, StartDate, NoSteps)

If (sigma_high > HIGH_VALUE) Then
GoTo ReturnValue                        'return ERROR; // something is wrong.
End If
Loop
Dim i As Integer
For i = 0 To MAX_ITERATIONS
Dim sigma As Double
sigma = (sigma_low + sigma_high) * 0.5

Price = OptionValue(OptType, AmEur, StockPrice, Strike, sigma, IntRate, EndDate, StartDate, NoSteps)

Dim test As Double
test = (Price - OptionPrice)
If (Abs(test) < ACCURACY) Then
IV = sigma
Exit For
```

```
        ElseIf test < 0# Then
        sigma_low = sigma
        Else
        sigma_high = sigma
        End If
        Next i
        End If
        ReturnValue:
        If IV >= 0 Then
        ImpliedVolatility = IV
        Else
        'return ERROR;
        'Err.Raise 702
        ImpliedVolatility = -1
        End If
End Function
```