SPACECRAFT FORMATION FLYING VIA DYNAMIC COMPENSATION

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Abstract

In this paper we present a passivity based control for spacecraft formation flying. We derive both attitude and position control. The controls do not require velocity information. They only require information about the position and attitude of the given robot and two of its neighbors. Furthermore, our approach has the advantage that we provide convergence results and establish bounds on the formation error.

Nomenclature

r;	The inertial position of the i th spacecraft
Vi	The inertial velocity of the i th spacecraft
\mathbf{q}_i	The unit quaternion representing
-	attitude of the i th spacecraft with
	respect to the inertial frame.
đ i	The vector part of \mathbf{q}_i .
- ą	The scalar part of q_i .
ω_i	The angular velocity of the i th
-	spacecraft with respect
	to the inertial frame.
f?	The control force applied to the i th
•	spacecraft.
T:	The control torque applied to the i th
•	spacecraft.
M,	The mass of the i th spacecraft.
J,	The inertia of the i th spacecraft.
IF	The inertial final position of the
•	formation.
Q <i>F</i>	The unit quaternion representing
**	desired attitude of the

	formation with respect to the inertial frame.
\tilde{r}_{iF}^d	The desired position of the i th
	spacecraft in the formation frame.
Ĩ;	The distance to go for each spacecraft
	i.e. $\tilde{\mathbf{r}}_i = \mathbf{r}_i - \mathbf{r}_F - \mathbf{r}_{iF}^d$
Ĩ _{ij}	The formation error between the i th
	and j th spacecraft,
	i.e. $\tilde{\mathbf{r}}_i - \tilde{\mathbf{r}}_j$
$ ilde{q}_i$	The Orientation of the i th spacecraft
	with respect to the F frame.
<i>Ĩ</i> ij	The relative orientation of the i th
-	spacecraft with respect to the j th spacecraft.

The indices are defined mod N i.e. $\mathbf{r}_{N+1} = \mathbf{r}_1$ and $\mathbf{r}_0 = \mathbf{r}_N$.

1 Introduction

The multiple agent formation problem has application to areas in both robotics and aerospace. In [Decou, 1991a] and [Decou, 1991b] a free-flying multiple spacecraft interferometer is proposed. For this application a group of spacecraft fly in a rigid formation within very fine tolerances. In aviation [Blake and Multhopp, 1998] flying multiple aircraft in formation can reduce the induced drag on each airplane. This can result in less power expended and thus fuel savings.

In addition there are many applications of formation control to robotics. Many robots working together can push a box [Kube and Zhang, 1996], move large awkward objects [Dickson et al., 1996], move a large number of small objects effectively [Vidal et al., 1996], or sweep out a given area to obtain information about the terrain [Rao et al., 1996] or the environment [Kurabayashi et al., 1996, Anderson et al., 1996].

Specifically we will address the problem of maintaining multiple spacecraft in rigid formation while maintaining alignment among the spacecraft to within a fine tolerance. Many of the previous approaches to formation control implement leader-follower hierarchal control [Wang, 1991, Wang and Hadaegh, 1996]. In these approaches some agents are designated as leaders. The leaders establish the motion for the formation and the remaining agents (followers) track the leader's motion to a fixed separation distance. The advantage of this control is that it implements feedback control on each agent and there exists convergence results for these control algorithms [Wang, 1991, Wang and Hadaegh, 1996]. The difficulty with implementing these approaches is the need to have position, velocity and acceleration information of one or more of the the neighboring agents. Furthermore the leader-follower approach provides for many points of failure. If a given agent is impaired or fails completely there exists no feedback between the impaired agent and its leaders in the hierarchal chain. The impaired agent will simply be left behind.

One approach that adds feedback between all of the agents in the formation is the virtual structure approach [Tan and Lewis, 1997]. In this approach each agent checks its position within the given formation, then it runs an open loop control to correct for any formation errors. This process is then iteratively repeated.

Other alternatives hierarchal to control biological-model based are control Sekiyama and Fukuda, 1996, Mitsumoto et al., 1996], behavior based control [Balch and Arkin, 1998] template based control[Beard and Hadaegh, 1998]. These controls all require velocity and acceleration information about neighboring agents or they do not guarantee that strict bounds on the relative distance between agents are meet.

The approach that we present here is to implement a passivity based control [Lizarralde and Wen, 1996, Tsiotras, 1998, Ortega et al., 1995] on each spacecraft. The control only requires information about its own position and attitude and the position and attitude of two neighboring spacecraft. No velocity or acceleration information is required.

The paper is organized as follows:

 Section 2 reviews the equations of motion of the problem and introduces the notion of formation control.

- Section 3 presents the position and attitude controls for each spacecraft.
- Section 4 gives some simulations.
- Section 5 presents the conclusions of the paper.

2 Problem Statement

Consider a fleet of N spacecraft. The motion of each spacecraft is governed by a translational and a rotational set of equations. The translational equations are given by double integrator dynamics

$$\mathbf{M}_i \bar{\mathbf{r}}_i = \mathbf{f}_i^c. \tag{1}$$

For the rotational equations we will use a unit quaternion attitude representation. A useful review of properties of the unit quaternion are given in the Appendix. In terms of the unit quaternion the rotational equations are given by

$$\begin{aligned}
\mathbf{J}_{i}\dot{\boldsymbol{\omega}}_{i} &= -\boldsymbol{\omega}_{i}^{\times}\mathbf{J}_{i}\boldsymbol{\omega}_{i} + \boldsymbol{\tau}_{i}^{c} \\
\dot{\mathbf{q}}_{i} &= \frac{1}{2}E(\mathbf{q}_{i})\boldsymbol{\omega}_{i},
\end{aligned} \tag{2}$$

where the cross product operator is defined by

$$\boldsymbol{\omega}^{\times} = \begin{bmatrix} 0 & -\boldsymbol{\omega}_3 & \boldsymbol{\omega}_2 \\ \boldsymbol{\omega}_3 & 0 & -\boldsymbol{\omega}_1 \\ -\boldsymbol{\omega}_2 & \boldsymbol{\omega}_1 & 0 \end{bmatrix}$$

and $E(\mathbf{q})$ is given in the appendix. The formation structure is defined by

- \mathbf{r}_F , the desired inertial position of the formation center.
- **q**_F, the desired inertial attitude of each spacecraft.
- \mathbf{r}_{iF}^d , the desired position of each spacecraft with respect to the formation center.

The formation control problem is to derive controls τ_i^c and \mathbf{f}_i^c such that

- 1. $\|\tilde{\mathbf{r}}_i\| \to 0$
- 2. $\rho(\mathbf{q}_i, \mathbf{q}_F) \rightarrow 0$

while

- 3. $\sum_{i=1}^{N} ||\tilde{r}_{i,i+1}(t)|| < \epsilon_1$
- 4. $\sum_{i=1}^{N} \rho(q_i(t), q_{i+1}(t)) < \epsilon_2$,

where the metric $\rho(q_i, q_j)$ is defined in the Appendix.

Furthermore we wish to implement this control in such a way that the coordination between spacecraft is closed loop. In other words if the i^{th} spacecraft is moving too far away from the other spacecraft it will slow down. Second we wish to implement this control with as little communication as possible. Specifically, we will only require that each spacecraft have information about its own position and that of two other spacecraft. To implement this control we will use a passive control law.

3 The Main Result

Do to difficulty in communicating velocity information from one spacecraft to another, we will implement a passive control. Theorem 3.1 and Theorem 3.2 will present a control that is motivated by that given in [Lizarralde and Wen, 1996, Tsiotras, 1998] First we present results for the translational problem.

Theorem 3.1 Given that

1. A Hurwitz,

2. B full rank,

3.
$$P > 0$$
 such that $A^T P + PA = -Q$.

4. K > 0

5. c > 0

6. $\alpha > 0$

then the control

$$\mathbf{f}_{i}^{c} = -\alpha K \tilde{\mathbf{r}}_{i} - K \tilde{\mathbf{r}}_{i,i-1} - K \tilde{\mathbf{r}}_{i,i+1} - c y_{i} (3)$$

$$\mathbf{r}_i(0) = \mathbf{r}_{io} \tag{4}$$

$$\mathbf{v}_i(0) = 0 \tag{5}$$

$$x_i = Ax_i + B\bar{\mathbf{r}}_i \tag{6}$$

$$y_i = B^2 P A x_i + B^2 P B \tilde{\mathbf{r}}_i \tag{7}$$

$$x_i(0) = -A^{-1}B\bar{r}_i(0),$$

asymptotically stabilizes $\tilde{\mathbf{r}}_i$ and

$$\sum_{i=1}^{N} \tilde{\mathbf{r}}_{i,i+1}^{T}(t) K \mathbf{r}_{i,i+1}(t) \leq \sum_{i=1}^{N} \left(\tilde{\mathbf{r}}_{i,i+1}(0) K \mathbf{r}_{i,i+1}(0) + \alpha \tilde{\mathbf{r}}_{i}(0)^{T} K \tilde{\mathbf{r}}_{i}(0) \right) \quad (8)$$

Proof: Consider the function

$$V = \frac{1}{2} \sum_{i=1}^{N} (\alpha \bar{\mathbf{r}}_{i}^{T} K \tilde{\mathbf{r}}_{i} + \tilde{\mathbf{r}}_{i,i+1}^{T} K \mathbf{r}_{i,i+1} + \mathbf{v}_{i}^{T} \mathbf{M}_{i} \mathbf{v}_{i} + c \dot{\mathbf{x}}_{i}^{T} P_{i} \dot{\mathbf{x}}_{i}). \quad (9)$$

Differentiating we find that

$$\begin{split} \dot{V} &= \sum_{i=1}^{N} \mathbf{v}_{i}^{T} (\alpha K \tilde{\mathbf{r}}_{i} + K \tilde{\mathbf{r}}_{i,i+1} + K \tilde{\mathbf{r}}_{i,i-1} + f_{i} \\ &+ c B^{T} P \dot{x}_{i}) - \frac{c_{i}}{2} \dot{x}_{i}^{T} Q \dot{x}_{i} \\ &= \sum_{i=1}^{N} \mathbf{v}_{i}^{T} (K \tilde{\mathbf{r}}_{i} + K \tilde{\mathbf{r}}_{i,i+1} + K \tilde{\mathbf{r}}_{i,i-1} + f_{i} \\ &+ c_{i} y_{i}) - \frac{c_{i}}{2} \dot{x}_{i}^{T} Q \dot{x}_{i} \\ &= -\frac{c_{i}}{2} \dot{x}_{i}^{T} Q \dot{x}_{i} \\ &\leq 0. \end{split}$$

Consider the set $\Omega = \{\tilde{\mathbf{r}}_{i,i+1}, \bar{\mathbf{r}}_i, \mathbf{v}_i, \dot{x}_i | V = 0\}$. Let $\bar{\Omega}$ be the largest invariant set in Ω . We have that $\dot{x}_i = 0$. It follows that $\forall t \geq 0, x_i = x_{io}$ a constant. From equation (6) and the fact that B is full rank we know that $\forall t \geq 0$ $\mathbf{r}_i = \mathbf{r}_{io}$ a constant. Therefore we have that $\mathbf{v}_i = 0$. Since $y_i = B^T P \dot{x}_i, y_i = 0$. Since $\mathbf{v}_i = 0$ it follows that in $\bar{\Omega}$, $\mathbf{f}_i^c = 0$. On this set we get from (3) that

$$\tilde{\mathbf{r}}_{i,i-1} + \tilde{\mathbf{r}}_{i,i+1} + \alpha \tilde{\mathbf{r}}_i = \mathbf{0}. \tag{10}$$

Since $\tilde{\mathbf{r}}_{i,j} = \tilde{\mathbf{r}}_i - \bar{\mathbf{r}}_j$ then we may write equation (10) as a matrix equation

The matrix in equation (11) is nonsingular $\forall \alpha > 0$. Therefore $\tilde{\mathbf{r}}_i = 0$ in $\bar{\Omega}$. Hence by LaSalle's Invariant set theorem $\bar{\mathbf{r}}_i = 0$. The bound (8) follows from the fact that $\dot{V} \leq 0$ and $\mathbf{v}_i(0) = 0$ and $\dot{\mathbf{x}}_i(0) = 0$.

Note from equation (9), that by choosing α small places low emphasis on \tilde{r}_i or little importance on getting to the final destination relative to maintaining formation. By choosing a small α most of the control effort goes toward maintaining formation.

Theorem 3.2 presents an analogous control law for maintaining formation alignment. Theorem 3.2 Given that

1. A Hurwitz,

- 2. B full rank,
- 3. P > 0 such that $A^T P + P A = -Q$,
- 4. k > 0
- 5. c > 0

6.
$$\alpha > 0$$

then the control

$$\tau_{i}^{c} = -\alpha k Vec(\mathbf{q}_{i}\mathbf{q}_{F}^{*}) - k Vec(\mathbf{q}_{i}\mathbf{q}_{i+1}^{*}) -k Vec(\mathbf{q}_{i}\mathbf{q}_{i-1}^{*}) - cy_{i}$$
(12)
$$\omega_{i}(0) = 0 \mathbf{q}_{i}(0) = \mathbf{q}_{io}$$
$$\dot{\tau} = A\tau + B Vec(\tau, \tau^{*})$$
(12)

$$\begin{aligned} x_i &= A x_i + B V ec(\mathbf{q}_i \mathbf{q}_F) \end{aligned} \tag{13} \\ y_i &= B^T P A x_i + B^T P B V ec(\mathbf{q}_i \mathbf{q}_F^*), \end{aligned} \tag{14}$$

asymptotically stablizes $q_i q_F^*$ and

$$\sum_{i=1}^{N} \rho(q_i(t), q_{i+1}(t)) \leq \sum_{i=1}^{N} \rho(q_i(0), q_{i+1}(0)) + \alpha \sum_{i=1}^{N} \rho(q_i(0), q_F(0)) \quad (15)$$

Proof: Given the Lyapunov function candidate

$$V = \frac{1}{2} \sum_{i=1}^{N} (\alpha k \rho(\mathbf{q}_i, \mathbf{q}_F) + k \rho(\mathbf{q}_i^T, \mathbf{q}_{i+1}) + \omega_i^T \mathbf{J}_i \omega_i + c \dot{\mathbf{x}}_i^T P_i \dot{\mathbf{x}}_i).$$

the proof is analogous to the proof of Theorem 3.1.

The next section applies this control to the three spacecraft formation problem. We provide simulations for the translational control problem.

4 Simulations

Consider the three spacecraft problem with • $\mathbf{r}_{1F}^d = \mathbf{r}_1(0) = [5, 0, 0]^T$

- $\mathbf{r}_{2F}^d = \mathbf{r}_2(0) = [0, 5, 0]^T$
- $\mathbf{r}_{3F}^d = \mathbf{r}_3(0) = [0, 0, 5]^T$
- $\mathbf{r}_F^d = [10, 10, 10]^T$



Figure 1: Plot of $\|\tilde{\mathbf{r}}_{12}\|$, $\|\tilde{\mathbf{r}}_{23}\|$ and $\|\tilde{\mathbf{r}}_{1}\|$

Furthermore, suppose that each spacecraft is initially in perfect formation i.e. $\tilde{r}_{12} = \tilde{r}_{23} = 0$. Given the control parameters $K = B = P = -A = I_3$, c = 1 and $\alpha = .1$. Figure 1 plots the quantities $\tilde{r}_1, \tilde{r}_{12}$, and \tilde{r}_{13} versus time. Observe that the quatity \tilde{r}_1 slowly decays to zero. Thus each spacecraft slowly converges to its final position. Furthermore, since the quanties \tilde{r}_1 and \tilde{r}_{13} remain small, we know that the formation is begin keep during the entire maneuver.

Now consider the same problem, but with a small random position error added on to each spacecraft. Thus $\tilde{\mathbf{r}}_{12}(0) \neq \tilde{\mathbf{r}}_{23}(0) \neq 0$. Given the control parameters $-A = B = I_3$, $K = 10I_3$, $P = 5I_3$, c = 1 and $\alpha = .01$ Figure 2 plots $\tilde{\mathbf{r}}_1$, $\tilde{\mathbf{r}}_{12}$ and $\tilde{\mathbf{r}}_{23}$ versus time for the case where we start with some initial position error. Again note that the error $\tilde{\mathbf{r}}_1$ slowly decays to zero. This indicates that spacecraft one is converging to it proper final formation position. We also see that the formation keeping is improving with time since $\tilde{\mathbf{r}}_{12}, \tilde{\mathbf{r}}_{13} \rightarrow 0$. For both of these examples similar plots for the second and third spacecraft are obtained.

5 Conclusion

The main result of this paper was the development of a passivity based formation control. This control rigidly moves a group of N spacecraft from one position to another while maintaining spacecraft formation and alignment. This control guarantees both formation keeping and spacecraft



Figure 2: Plot of $\|\tilde{\mathbf{r}}_{12}\|$, $\|\tilde{\mathbf{r}}_{23}\|$ and $\|\tilde{\mathbf{r}}_{1}\|$

alignment to within a given bound. The advantage of using a passivity based control is that the control does not depend on velocity information. In formation problems it is difficult enough to communicate positions between agents. It is nearly impossible to communicate accurate velocity information. Thus this approach has the advantage of being more easily implemented than previous controls.

The set of unit quaternions is defined by $\mathbf{q} \in {\mathbf{R}}^4 | \mathbf{p}^T \mathbf{p} = 1$. The quaternion is generally particulation into a vector and a scalar component i.e.

$$\mathbf{q} = \begin{bmatrix} \vec{\mathbf{q}} \\ ar{m{q}} \end{bmatrix},$$

where $\vec{q} \in \mathbb{R}^3$ and $\bar{q} \in \mathbb{R}$. The conjugate of q is

$$\mathbf{q^{*}} = \begin{bmatrix} -\vec{\mathbf{q}} \\ \bar{q} \end{bmatrix},$$

We define the following quaternion matrix operators

$$E(\mathbf{q}_{i}) = \begin{bmatrix} \bar{q}_{i}I_{3} - (\mathbf{\vec{q}}_{i}^{\times}) \\ -\mathbf{\vec{q}}_{i}^{T} \end{bmatrix}$$

$$H_{1}(\mathbf{q}_{i}) = \bar{q}_{i}I_{4} + \left(\frac{(\mathbf{\vec{q}}_{i}^{\times}) \mid \mathbf{\vec{q}}_{i}}{-\mathbf{\vec{q}}_{i}^{T} \mid 0}\right) \qquad (16)$$

$$H_{2}(\mathbf{q}_{i}) = \tilde{q}_{i}I_{4} + \left(\frac{-(\mathbf{\vec{q}}_{i}^{\times}) \mid \mathbf{\vec{q}}_{i}}{-\mathbf{\vec{q}}_{i}^{T} \mid 0}\right).$$

The composition of two unit quaternions is closed

under the operation

$$\mathbf{q}_1 \mathbf{q}_2 = H_1(\mathbf{q}_1) \mathbf{q}_2
 = H_2(\mathbf{q}_2) \mathbf{q}_1.$$
 (17)

For this paper we will define the following metric on the set of unit quaternions

$$\rho(\mathbf{q}_1, \mathbf{q}_2) = 2(1 - \mathbf{q}_1^T \mathbf{q}_2).$$
(18)

Since \mathbf{q}_1 , and \mathbf{q}_2 are both unit vectors, $\rho(\mathbf{q}_1, \mathbf{q}_2) > 0$ for $\mathbf{q}_1 \neq \mathbf{q}_2$. For $\mathbf{q}_1 = \mathbf{q}_2$ $\rho(\mathbf{q}_1, \mathbf{q}_2) = 0$.

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