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Simplex Algorithm for Optimizing Drainage Design

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ABSTRACT- A methodology and computer model is developed to determine economically optimum closed subsurface drainage systems in irrigated areas. The model maximizes net benefits, by comparing profit driven by crop yield to drain system cost and selects an optimum drain layout. The optimization methodology used, is the SIMPLEX method, Nelder and Mead,[8]. The SIMPLEX model was linked to the subsurface drainage model DRAINMOD Skaggs [10], and to the surface hydraulic model KINE, Walker and Skogerboe[14]. The selected optimum drainage system maximizes the difference between total revenue, and the total cost of installation, operation and management of a particular drainage system. The optimization sub-program provides a workable and simple procedure for optimizing water management simulation models.

1. Introduction

In order to design an effective drainage system, the determination of the functional requirements to be met by the system, is an essential step. In agriculture drainage, this step involves the establishment of the drainage requirement of the crop to be grown and the characterisation of soil properties affecting irrigation and drainage. Therefore, the aim of a drainage system is to provide a healthy environment for plant growth. This implies that a drainage system must be designed with the requirements of the plant to be grown in mind. Enormous investments in drainage of irrigated areas have already been made or are planned. In the Imperial Irrigation District of California, the irrigated area increased rapidly

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at the beginning of this century. By the 1920's waterlogging and salinity problems began to appear and by the end of the 1930's, 20,000 hectares temporarily went out of production. In 1922, construction was begun on the planned system of open drains. Using the channels of two rivers as main trunk outlets, the system was extended on a pattern of parallel drains approximately a half-mile apart. By 1966, almost 20,000 kilometers of tile drains and 2,200 kilometers of deep open outlet drains had been installed to maintain or restore the land's productivity, Moore[7].

Development of privately installed tile in the Valley has been remarkable. From 1929 through 1960 a total of 12,000 kilometers has been laid. Most of the tile has been installed during the 1950-1960 period. The average yearly installation for this period was 800 kilometers. In 1960, 1,200 kilometers of tile were installed, Molof [6]. Today almost 60% of the half-million acres of productive land in Imperial Valley has adequate tile drainage, Imperial Irrigation District [5]. Figures 1 and 2 illustrates the layout of irrigation canals and the open drain systems.

The selection of an optimum design alternative for a subsurface drainage system depends upon the interaction of two conditions. First, maximizing crop production by closely spacing laterals, and second minimizing installation cost by spacing laterals as widely as possible. In addition, these two conflicting conditions must be balanced. There are many other factors influencing crop production. In order to isolate the effect of water in corn yield, it was assumed that all other factors such as soil fertility, disease and pest control are properly managed so that any decrease in yield will be a consequence of water management alone. Therefore, given a particular soil, climate and crop condition on-farm water management and drainage design decisions can be solved as an optimization problem.

Maximum yield for corn is achieved when moisture stress is not allowed. Managing to obtain zero moisture stress, however, may involve considerable cost due to drain installation and control of the amount of water applied and the labor and energy used. The greater the cost of installation, operation of drainage and irrigation systems and restricted water limits, the higher the unit cost of production becomes. In addition, the operational cost of any particular system would be different under different water management practices, Ehteshami et al. [3]. The question then becomes if and by how much yield should be sacrificed in order to obtain maximum profit per unit of land.

The need to make an economic evaluation of agricultural drainage systems is well recognized among numerous researchers. Among them Menz (1964), has presented an incremental analysis of the benefit-cost ratio. He noted that in some cases overall benefit cost ratios for several project scales may be greater than one, but the optimum project scale is, that at which the excess of net income over net cost is greatest and this can be determined by incremental analysis.

The method used by Wiser et al [15], gives an estimation of the effect of water table changes on crop response. The criterion for final system choice is maximization of net benefits. The change in water table height was calculated using an equation developed by Van Schilfgaarde [13], which estimates the water table height at any time due to an assumed pulse input which is uniform over the period. The water table height is a function of the drain spacing, depth and input to the water table.

A water balance approach for subsurface drainage design has been proposed by Bhattacharya et al. [2]. In this approach the system installation cost and the market value of the harvested crop were compared for drainage system designs with different drainage rates. These distributions were used to find the crop losses. A drainage system was considered inadequate, and crop loss was assumed if the water table remained within 30*cm* of the surface for more than two successive days. In another study, Bhattacharya and Broughton [1] developed a procedure to compute crop loss for corn. Different depths and durations of high water table conditions, based on available data and probability concepts, were used to calculate the revenue increases from a subsurface drainage system design with different spacings in various soil types.

Durnford et al. [4] presented a procedure which can be used to identify economically optimum subsurface drainage system designs in an irrigated area. She assumed that crop growth and yield are directly related to a minimum water table depth and found a unique least cost combination. She defined an optimum drain system, which maximizes the difference between the value of increased crop yield attributing to drain installation and the cost of the drains.

2. Procedure

The following procedure was adapted for maximization of the net benefit. The objective function (Obj), for optimizing the net benefit can be formulated as follows:

Obj = maximize net benefit (1)

To practically compute the objective function, acceptable limits such as the following must be set out:

min. spacing<drain spacing<max. spacing, (2)

min. depth<drain depth<max. depth, (3)

min. diam<drain diameter<max. diam, (4)

min. Q<furrow inflow Q<max.Q, (5)

min. Lf<furrow length<max. Lf, (6)

min. Zn<depth applied at end of furrow<max.Zn, (7)

min. F<irrigation frequency F<max.F (8)

and,

Net Benefit = Total Benefit - total Costs. (9)

Where Total Benefit in this case is the income to the farmer crop production (yield), and Total Cost included drainage system costs plus irrigation system costs plus production costs.

2.1. Drainage Costs

The total cost of drainage system is a function of several variables as follows:

$$Totcd = CMN + CMA + CTU + CIN + COU + CFI \quad (10)$$

or,

$$Toted=(C5/L+(i\times C6 \times Ddepth^{C7}/L)+i \times C8 \times ddiam^{C9}/L)+(i \times CIO/MANL \times L)$$
$$+(i \times C11/L \times OUTL) + (i \times C21/L) \quad (11)$$

and,

$$C21 = C14 \times .00164 \times ddiam^{-.86}. \quad (12)$$

Where Totcd is total drainage cost per unit area, CMN is cost of drain maintenance per unit area, CMA is cost of drain installation per unit area, CTU is cost of tubing per unit area, CIN is cost of man holes per unit area, COU is cost of outlets per unit area, CFI is cost of envelope per unit area, L is drain spacing (m), D depth is drain depth (*m*), i is the annualized economic factor, MANL is distance between each manhole (*m*), OUTL is distance between each outlet (*m*), C5, C6, C7, C8, C9, C1O and Cll are cost coefficients. C21 is cost per linear meter of envelop material, C21 could be approximated by a simple power function (Equation 12), where ddiam is drain diameter (*mm*), and C14 is a cost coefficient.

2.2. Irrigation Costs

Total cost of the irrigation system is:

$$\text{Totci} = \text{Nise}\,(\text{Cotlb}+\text{Cotwt})+\text{Cothd} \quad (13)$$

or,

$$\text{Totci} = \text{Nise}\,((1/60 \times C2 \times C4 \times \text{Tirr}) + (\text{Cl} \times \text{Nf} \times \text{Tco})/\text{Effc})) + C3 \times \text{Wf} \quad (14)$$

and,

$$\text{Noset} = \text{Nf}/\text{Nfs} \quad (15)$$

$$\text{Nfs} = \text{Qmax}/\text{Qin} \quad (16)$$

$$\text{Tirr} = \text{Tco} \times \text{Noset} \quad (17)$$

$$\text{Nf} = 10{,}000/\text{Lf} \times \text{Fs} \quad (18)$$

$$\text{Wf} = \text{Nf} \times \text{Fs}. \quad (19)$$

Where Totci is total cost of the irrigation system, Nise is number of irrigations per season, Cotlb is cost of labor per unit area, Cotwt is water cost, Cothd is cost of head ditch construction per unit area, Tirr is time of irrigation, Noset is number of irrigation sets, Nf is number of furrows, Nfs is number of furrows per set, Qmax is maximum volume of available water, Qin is volume of inflow to one furrow, Tco is time of inflow cutoff to furrow, Lf is furrow length, Fs is furrow spacing, Wf is head ditch length, Effc is conveyance efficiency, Cl, C2, C3 are cost coefficients, and C4 is fraction of time. The surface irrigation hydraulic performance was simulated using the KINE model, Walker and Skogerboe [14].

2.3. Production Cost

Cp is the agronomic production cost per ha, excluding the cost of drainage and irrigation system construction and operation. A production cost of $500/ha is assumed.

2.4. Benefit or Unit Income

Total Benefit can be described as:

$$\text{Beft} = \text{Ry} \times \text{Py} \times \text{Cl}, \quad (20)$$

where Beft is the total benefit ($ per unit area or $/ha), Ry is relative yield (%). The relative yield has computed using DRAINMOD Skaggs et al.[11]. Py is potential yield (kg/ha) and Cl is price of the corn crop ($/kg).

3. Solution to the Optimization Problem

Maximization of net benefit is more comprehensive than minimization of cost in that it incorporates a decision about the desired level of system performance. In this study, benefit will be measured in terms of crop yield value, and the net benefit is defined as that income derived by the farmer from any additional crop yield attributable to installation of a drain system minus the cost of that system. Maximization of net benefits further implies that differing levels of system performance are compared. Assuming that the level of performance as a function of maximizing net benefit can be quantified satisfactorily, then for each performance level there is a consequent minimum system and operation cost at which that performance level is achieved. The relationship between benefits, cost and system performance level can be visualized as shown in Figure 3.

In this figure benefits and costs are plotted. The net benefit is the distance between the two curves. In general, it is expected that as the performance level of the system increases, the benefit or yield increases at least to a point. But the cost must also increase to obtain the additional performance. In the example curve shown, it is assumed that some benefit is derived from the land with no artificial drainage. In addition, benefits are shown as levelling off as the crop yields approach some minimum level. Finally, the derived net benefits level off as the crop yield approaches some maximum attainable level and may even decline beyond this point i.e. extra contribution of the cost which is due to additional crop protection. In the economic consideration of a particular drainage system, the level of protection should not be increased if the total cost exceeds the total benefit. Therefore, theoretically, the point where marginal cost equal marginal benefit or, in another word, where the slope of the cost function and the benefit function are equal represents an optimum point.

The problem then is, to define the best system and develop a feasible procedure for finding this system. As above, in this study, it is assumed that the best system is the one which maximizes net benefits on the farm level. The general procedure commonly used to find a solution for the best system can be classified as two types: 1. simulation and 2. optimization. Using the first approach, the simulation method, possible drain spacings and depths and surface irrigation parameters and their effects on crop yield can be determined realistically. The second approach, optimization requires more detailed analysis than the simulation model, but it is capable of including most of the interdependencies inherent in irrigation and drainage systems. A simplified optimization routine which provides most of the advantage of the optimization method, can be employed.

Spendley et al. [12] introduced a clever idea for tracking optimum function conditions by evaluating, from the output form a set of points forming a simplex in the space and called it "SIMPLEX". The procedure was modified byNelder and Mead [8]. The name simplex is derived from its shape in space. The Spendley method employs a regular sequential pattern search of points in the design space while maintaining efficiency compared to the simple direct method .The idea is to pick a base point and, rather than attempting to cover the entire range of the variables, to evaluate the design parameters in some pattern about the base point. For example, in two dimensions, a triangular pattern which the best of them (the node with the lowest value of the objective function) would be selected as the next base point around, which to locate the next pattern of points. If none of the corner points is better than the base point, the scale of the grid is reduced and the search continues.

In this method the search to optimize the objective function, trail x vectors Figure 4, can be selected at a point in space located at the vertices of the simplex. The objective function can be evaluated at each of the vertices of the simplex, and a projection made from the point yielding the highest value of the objective function (point x_1 in Figure 4) through the centroid of the simplex. Point x_1 is deleted and a new simplex is formed by reflection, expansion or contraction. The simplex is then composed of remaining old points and the one new point, and then the procedure continues until a prescribed error tolerance is met and optimization reaches final convergence.

Some definitions are as follows from Nelder and Mead, [8].

Reflection: The reflection of Ph is denoted by P* and its coordinates are defined by the relation;

$$P^* = (1+\alpha)\, P_b - Ph \tag{21}$$

where a is positive constant, the reflection coefficient, Pb is centred of simplex, and Ph is value of vertex with function in highest value (the suffix of h, l are to define high and low respectively).

If y* is less than y_l, i.e. if reflection produced a new minimum, then we expand P* to P** by the relation;

$$P^{**} = \delta P^* + (1 - \delta)P_b \tag{22}$$

where δ is expansion coefficient, which is greater than unity and finally if on reflecting P to P* it is found that y* is biggrr than y_1 for all i # h, i.e. that replacing P by P* leaves y* the maximum, (y is function value at P_i) then we define a new Ph to be either the old Ph or P*, whichever has the lower function value and form;

$$P^{**} = \beta Ph + (1 - \beta) Pb \tag{23}$$

where β is contraction coefficient which lies between 0 to 1. The final point of concern is halting the procedure .which is concerned with the variation in the y values over the simplex. The form chosen is to compare the standard error of y'_s in the form of;

$$Err = \left\{ \Sigma (y_i - y_b)^2 /n \right\}^{1/2} \tag{24}$$

where y_b is mean value of y, n is number of vertices that are compared to a preset value (Err) or to so-called error tolerances and to stop when the value falls below this value. Figure 5 shows a brief outline of the procedure used in the optimization subroutine.

9. Results

The Simplex method is a usefull technique for optimizing simulation models. The method was used to optimize interaction between irrigation and drainage requirement of the crop. The drainage system optimization model could be used for comparing a wide range of design parameter values and to produce a series of graphs that will allow practicing drainage engineers and farmers to select a subsurface drainage system optimized for a given set of conditions.

The estimated costs of drain installation and materials are shown in Table 1 and a summary of input data and the values of parameters used are shown in Table 2. The drain design computed by the drainage optimization model is the least cost system for the highest level of yield that would be achieved based on the input cost data, soil conditions, crop production, and one particular irrigation layout. The computational procedure, as described, is an iterative process. For example, for a field situation where a single corn crop is planted each year, and the costs for a closed drain system are shown in Table 1. By using these values and an initial trial drain spacing of, for example, 60 meters (Table 3), a relative yield of 82% would be determined using the drainage system design results with the yield model. The net benefit from this particular system was determined to be $170/ha/year. The optimization model then evaluates a second alternative spacing of 69 meters and determines a corresponding relative yield of 68% and net benefit of $3/ha/year. Therefore, the net benefit gradient is negative and the net benefit will decrease if the spacing is increased. Since a higher net benefit is required, the optimization sub-model decrease the spacing to 58 meters and re-evaluates the corresponding costs and benefits, and the gradient for the new results is determined. Table 3 showes the sequence of data obtained by following this iteration method of optimization. When the change in the net benefits is less than a per-defined tolerance, the optimization sub-model will end the procedure and the chosen system would be the system giving the highest annual net return, using the current input data. Convergence occurs fairly quickly in a few iterations. The numerical values of net benefit for different combinations of hydraulic conductivity and for one interest rate, one amortization period and one installation cost are shown in Figure 6 for different soil permeabilities. Of all the various hydraulic parameters considered in the conductivity has the greater on drain spacing net benefit. Figure 6 indicates the drain spacing needed to achieve the maximum annual net benefit from subsurface drainage for various values of hydraulic conductivity increases with hydraulic conductivity.

The sensitivity of model as a function of drain spacing was evaluated by varying the unit price of crop production, and varying the unit cost of installation using different interest rates and system life times (Figures 7, 8). In each case, one input cost was tested while keeping the other parameters constant. Figure 7 shows the effect of capital recovery factors on net benefit for different drain spacings. Figure 8 shows the effect of crop prices on the net benefit for different drain spacings. Figure 8 indicates that the crop prices are a major influence on the net benefit. It is obvious from Figures 7 and 8 that changes in the cost of the system components and crop price would influence the net benefit, while not significantly affecting the drain spacing.

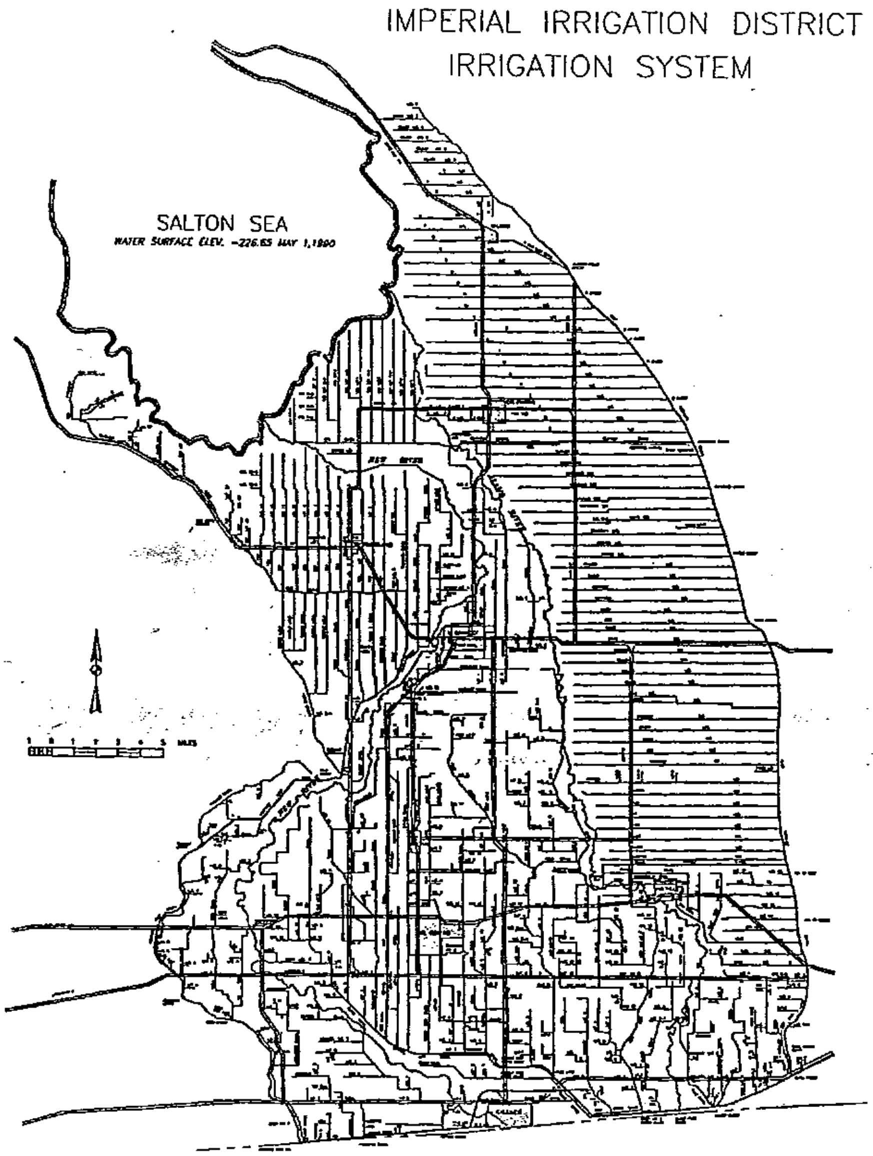

Fig. 1. Imperial irrigation destrict, map of the imperial unit irrigation system showing concrete lined and unlined laterals.

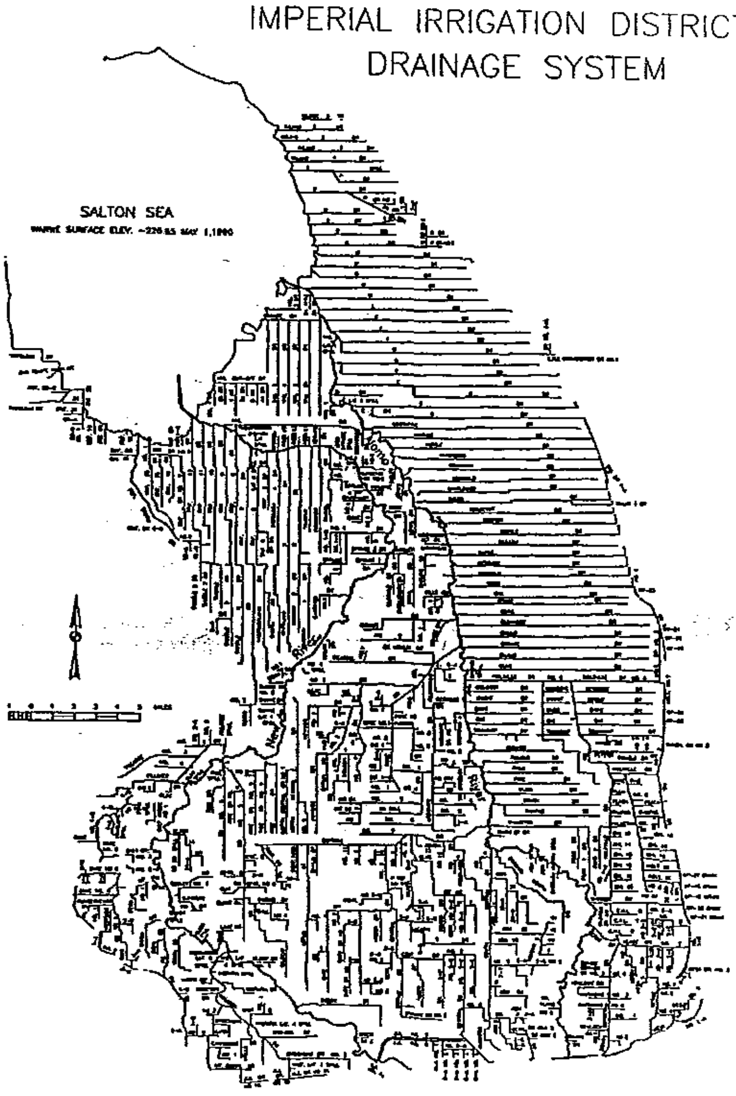

Fig. 2. Imperial irrigation district, map of the drainage system, imperial unit showing layout of the open drain ditches.

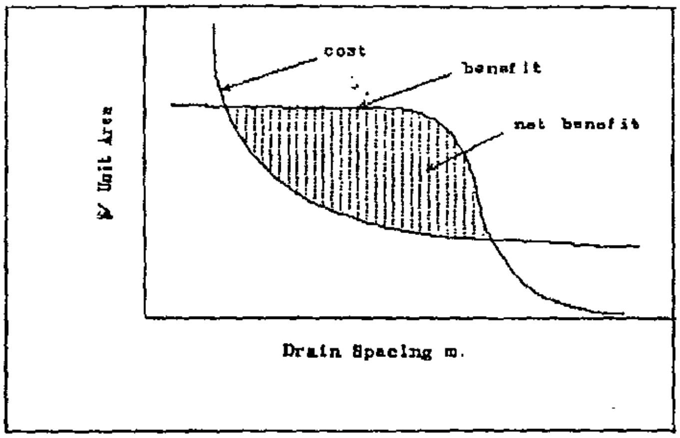

Fig. 3. Example curve showing cost relationships between cost, benefit and net benefit, for one system performance level.

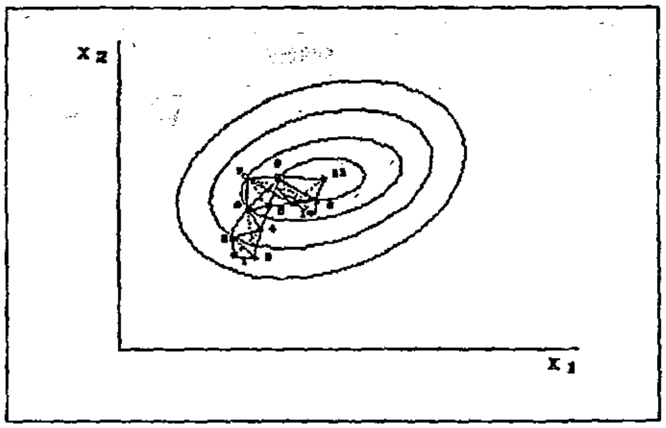

Fig. 4. An outlook of the Simplex Method with sequence of Simplexes obtained in maximization of the objective function.

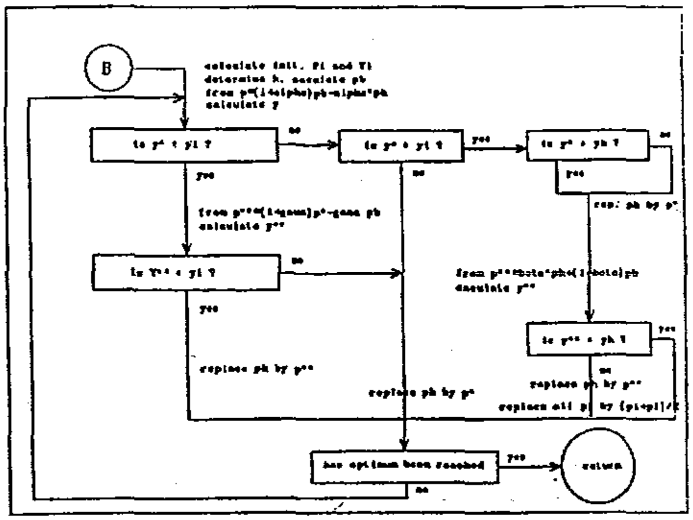

Fig. 5. Flow diagram of Simplex Method (from Nelder and Mead, 1965).

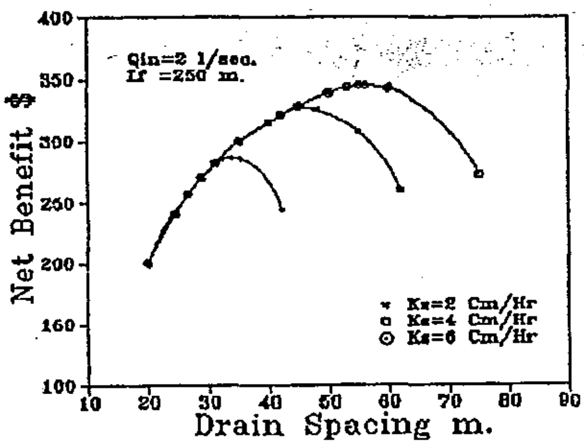

Fig. 6. Net benefit due to subsurface drainage for various soil hydraulic conductivity values.

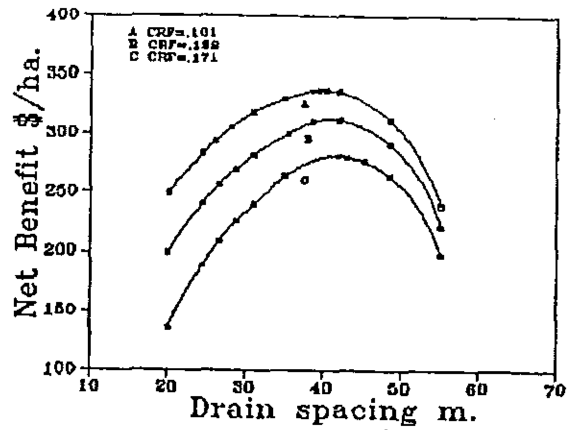

Fig. 7. Net benefit due to subsurface drainage for different capital cost recovery factor.

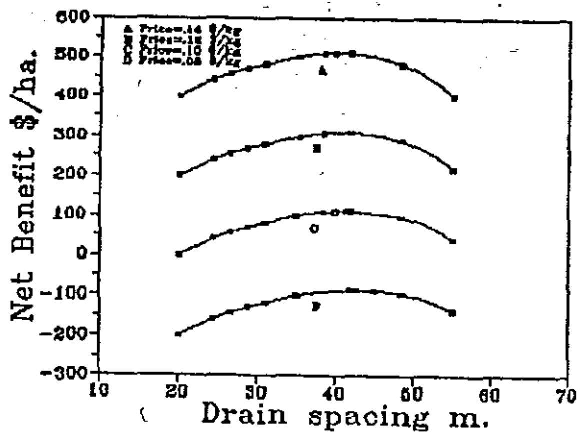

Fig. 8. Net benefit due to subsurface drainage for various prices of corn.

Table 1. Costs assumed for closed drain systems and irrigations water management practices.

Variable	Cost Assumed	Units	Explanation
C1	0.0100	$/m^3	water cost
C2	4.0000	$/hr	labor cost
C3	3.1000	$/m	annual cost of ditch constraction
C4	1.0000		fraction of time
C5	0.0311	$/m/year	maint cost
C6	0.2770	$/m	inst. cost
C7	2.1800	$/m	inst. cost
C8	0.0200	$/m	tubing cost
C9	0.7600	$/m	tubing cost
C10	175.00	$/unit	manhole cost
C11	100.00	$/unit	outlet cost
C14	8.7600	$/m^3	envelope cost
price/kg	0.1200	$/kg	price of crop
Rate	0.1320		

Table 2. Summery of the input data used in drainage and optimization model.

Input parameters	Values
Years of simulation	1982/83
Rainfall station (#)	
Temperature station (#)	
Crop type	corn
Planting date (julian day)	105
Growing season (days)	130,142

Drain depth (*cm*)	180,200,220
Drain spacing (*cm*)	4000,5000
Profile depth (*cm*)	230
drain tubing (*mm*)	104
Soil layers	2
Saturated hydraulic conduc.. (*cm*/hr)	2,3,4,5
Infiltration parameters A and B	3.3,1.0 6.0,1.0 9.2,1.0
Length of furrow (*m*)	200,300
Furrow spacing (*m*)	1.00
Roughness coefficient	0.04
Field slop (*m/m*)	0.014
Hydraulic section parameters	0.66,2.87
Furrow geometry parameter	0.96,0.604
Kostiakov-Lewis infiltration parameters	0.0088,0.212, 0.00017
Flow rate (l/s)	0.5-0.07
Water applied at end of furrow (*m*)	0.05-0.07
Maximum flow available (m^3/sec)	10.00
Potential yield (kg/ha)	10000.00
Distance between each manhole (*m*)	500.00
Distance between each outlet (*m*)	500.00
Irrigation frequencies	10-20

Table 3. Sequence for optimization trail in one particular case.

#	Spacing (m)	Relative (%)	Net benefit ($/ha)
1	6	82	170
2	69	68	3
3	58	86	213
4	53	92	270
5	47	97	325
6	34	100	314
7	43	98	334
8	53	92	270
9	38	99	333
10	33	100	315
11	41	99	335

10. Conclusions

A comprehensive procedure is presented which uses available information on weather, soil, water and plant properties and related cost parameters to establish rational guidelines to enable the investor or engineer to select an appropriate design alternatives which will result in increased maximum average annual net benefit. The procedure conducted in this study introduces the use of state-of-the-art computer simulation techniques to optimize water management models. The Simplex algorithm was linked together with the surface irrigation and subsurface drainage model to optimize water management decisions in irrigated agriculture. The optimization routine is based on net benefit maximization in which the benefits are crop yields, and the cost components are installation and maintenance of drainage system costs, plus costs associated with surface irrigation, and the seasonal production cost. The optimization routine is proven to be an effective methodology.

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