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CONTROL AND OPTIMIZATION OF SEMI-PASSIVELY ACTUATED MULTIBODY SYSTEMS*)

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Abstract

The controlled multibody systems are under the consideration. At the lecture special emphasis is put on the study of underactuated and overactuated systems having different type of actuators (external powered drives, unpowered spring-damper like drives, etc.). Several questions are addressed about the role of inherent dynamics, and how much multibody system should be governed by external powered drives and how much by the system's inherent dynamics. The lecture consists of the following parts: introduction to the subject in question; mathematical statement of the optimal control problems that are suitable for modelling of controlled motion and optimization of semi-passively controlled multibody systems with different degrees of actuation; description of the methodology and the numerical algorithms for solution of control and optimization problems for semi-passively actuated multibody systems. The solutions of several optimal control problems for different kind of semi-passively actuated multibody systems are presented. Namely, the energy-optimal control of planar semi-passively controlled three-link manipulator robot, the energy-optimal control of closed-loop chain semi-passively actuated SCARA-like robot; optimization of the hydraulic and pneumatic drives of the multibody system modelled the human locomotor apparatus with above-knee prostheses, and others. Future perspectives in area of control and optimization problems of the semi-passively actuated multibody systems are discussed.

1. Introduction

Today our knowledge in mechanics, control engineering, electronics and computer sciences is actively integrated into a new interdisciplinary science – *mechatronics* [1]. One of the primary goals of mechatronics is to gain as many advantages as possible from the optimal interaction between the mechanical, control, electronic and computer subsystems. This requires more fundamental research on a number of topics of controlled multibody systems, e.g. parameter identification and optimal design, contact and impact problems, large deformation problems, control-structure interaction, etc., [2-3]. The research in above areas can help to improve performance characteristics of modern mechatronic products.

The most important and relevant characteristics of interaction between inherent dynamics and control of any mechanical system are its degree and type of actuation. Most technical systems, e.g. industrial robots, have been designed based on the commonsense rule of minimum complexity of the structure. The industrial robots have usually the same number of actuators as of the degrees of freedom of their mechanical subsystems, i.e. they belong to the class of fullyactuated mechanical systems. A lot of research has already been done in area of control and optimization of fullyactuated robotic systems that successfully supported industrial robotics. If multibody system (MBS) has less actuators than joints or more precisely if the dimension of the configuration space exceeds that of the control input space, the system is called *underactuated.* Examples of underactuated MBS are a car with n trailers having spring-damper-like joints, manipulator robots with failed actuators, free-flying space manipulators without jets or momentum wheels, manipulator robots with flexible links, legged robots with passive joints, etc. The general advantages of using underactuated mechanical systems reside in the fact that their weight is lower, and they consume less energy than their fullyactuated counterparts. For hyper-redundant robots or multi-legged mobile robots, where large kinematic redundancy is available for dexterity and specific task completion, underactuation allows a more compact design and simpler control schemes. The analysis of dynamics and control of underactuated MBS is significantly more complex than that for fullyactuated ones. A survey of papers in the above area has shown that the dynamics and control problems of underactuated mechanical systems have actively been studied for the last decade [4-7].

The next generation of robots must be autonomous and dexterous [8]. Dexterity implies the mechanical ability to carry out various kinds of tasks in various situations. Robots must have many sensors and more actuators than degrees of freedom, i.e. being the controlled mechanical system with sensing and actuation redundancies. To carry out optimally the complex tasks in various situations it can be desirable to change a number of actively controlled degrees of freedom of robotic system. It can easy be done, for instance, by locking or unlocking some of the actuators of robot during its performance of the specific subtask of a given complex task. From the point of view of control it means that robot can be considered as over, -fully, or underactuated

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mechanical system during its performance of the complex task. Obviously, the type of actuators used can also be different depending on the task of robot [9-14].

The analysis of the literature and the above-mentioned fundamental aspects shows the importance of studying dynamics, control and optimization problems of MBS with different degrees of actuation and the robotic systems, in particular. This research is of a great challenge.

In the paper a controlled MBS of rigid bodies is under study. External controlling forces and moments can be applied directly to arbitrary points in the system. These controlling stimuli are generated by external (powered) drives. It is assumed that displacement and velocity dependent internal controlling forces and torques can also be applied to the system. These controlling stimuli are generated by internal (unpowered) drives, e.g. spring-damper actuators located between arbitrary points and described by linear and angular stiffness and damping parameters. MBS including both external (powered) and internal (unpowered) drives we shall term a semi-passively actuated MBS. In the paper we tackle optimization problems for MBS having unpowered actuators. The reasons of this study are as follows. To incorporate spring-damper actuators into the structure of MBS and to design optimally their parameters can give several advantages, e.g. to decrease a number of external drives and, as a consequence, to decrease the weight of moving links and the energy consumption of the system. It can give great advantages to use different passive compliance elements to control some degrees of freedom of manipulator robots and legged mechanisms for their performance of working tasks with periodic laws of motion [9-13]. We study a fundamental question about optimal interaction between the controlling stimuli generated by the external drives and the proportional-differential internal forces described by linear and angular stiffness and damping parameters. A range of questions is also addressed about the role of inherent dynamics in controlled motion, and how much MBS should be governed by the external drives and how much by the system's inherent dynamics. We are in particular investigating semi-passively actuated manipulator robots and bipedal walking mechanisms having spring-damper actuators.

In this paper we outline the results that will be presented during the lecture.

2. Statement of the problem

Consider a MBS the controlled motion of which can be described by the following equations:

$$x^{\bullet} = f(x, u, w(t,\xi)), \qquad g(x, w(t,\xi)) = 0, \qquad t \in [0, T] \tag{1}$$

Here $x = (x_1, x_2, ..., x_n)$ is a state vector, $u = (u_1, u_2, ..., u_m)$ is a vector of controlling stimuli (forces, torques) generated by the external (powered) drives of the MBS, $w = (w_1, w_2, ..., w_r)$ is a vector of the controlling stimuli of the internal (unpowered) drives of the MBS, and T is the duration of the controlled motion of MBS. Vector functions f and g are determined by the structures of the MBS and unpowered drives, respectively, ξ is a vector of design parameters of the unpowered drives.

Constraints and restrictions are imposed on the state vector $x(t)$, the controlling stimuli of the unpowered drives $w(t,\xi)$, and the external control laws $u(t)$ of the system. These restrictions can be written in the following way:

$$\{x(t)\} \in Q, \qquad t \in [0, T] \tag{2}$$

$$w(t,\xi) \in W, \qquad t \in [0, T] \tag{3}$$

$$u(t) \in U, \qquad t \in [0, T] \tag{4}$$

In formulas (2) - (4), Q and U are given domains in the state and control spaces of the system, respectively; W is a set of addmissible controlling stimuli determined by the structure of the unpowered drives.

The differential equations (1) together with the restrictions (2)-(4) are called the mathematical model of the semi-passively actuated MBS. This model can be used for many applications, e.g. to solve the design problems of lower limb prostheses and to study control strategies for the stable motion of bipedal locomotion systems with compliance elements at the joints [6, 10], for computer simulation of the energy-optimal motion of closed-loop chain manipulator robots with unpowered drives [12], etc.

Assume that there exists a non-empty set of vector-functions $\{x(t), u(t), w(t,\xi), \ t \in [0, T]\}$ which satisfy the equations (1) and the constraints (2)-(4). The following optimal control problem can be formulated.

Problem A. Given a MBS the controlled motion of which is described by equations (1). It is required to determine the vector-function of unpowered drives $w_*(t,\xi)$, the motion of the system $x_*(t)$ and the external controlling stimuli $u_*(t, x_*, w_*)$ which alltogether satisfy the equations (1), the restrictions (2)-(4), and which minimize the given objective functional $\Phi[u]$.

As a result of the solution of *Problem A* the optimal structure of MBS having both powered and unpowered drives is designed. The external controlling stimuli for the system are also found which minimize the given objective functional.

One of the primary goals for the incorporation of unpowered drives into the structure of MBS is an improvement of their control processes. It means that the validity of the following inequality is expected: $\Phi[\, u_*(t, x_*, w_*)\,] < \Phi[\, u_{0*}(t, x_{0*})\,]$, where $x_{0*}(t)$, $u_{0*}(t)$ are the optimal motion and the controlling stimuli of the MBS without the unpowered drives obtained under the restrictions (2), (4). In this sense the solution of *Problem A* could help to estimate the limiting possibility of improvement of the external control strategies for MBS due to incorporation into their structure different unpowered drives determined by the constraints (3).

3. Methodology

We have formulated the optimal control problem for the semi-passively actuated MBS. The key feature of the proposed mathematical statement of the problems is the direct utilization of the differential equations describing the inherent dynamics of internal unpowered drives together with all other constraints that are imposed on the state vector and the controlling stimuli of the system. It leads to the non-uniqueness of the solution of the direct and inverse dynamics problems and makes it possible to design optimal unpowered actuators for MBS.

In general case for MBS with many degrees-of-freedom powerful numerical algorithms are needed to solve *Problem A*. Futhermore, during the calculation of optimal control laws for MBS it is necessary to design at the same time the optimal structure of the unpowered drives taking into account the restriction (3). This can significantly increase the complexity of the computation.

The numerical method has been developed [6, 10, 12] for the solution of *Problem A* for MBS, which model semi-passively actuated manipulator robots and bipedal locomotion systems with unpowered drives at their joints. The method is based on a special procedure to convert the initial optimal control problem into a standard nonlinear programming problem. This is made by an approximation of the independent variable functions using a combination of polynomial and Fourier series and by the solution of semi-inverse dynamics problem. The key features of the method are its high numerical efficiency and the possibility to satisfy a lot of restrictions imposed on the phase coordinates of the system automatically and accurately. The efficiency of the developed method has been illustrated by solution of several problems, e.g. by computer simulations of the energy-optimal motion of closed-loop chain semi-passively actuated manipulator robot, the bipedal walking robot, by the solution of design problems of the energy-optimal above-knee prostheses with several types of unpowered knee mechanisms [6, 10, 12].

4. Optimal Passive Drives for Given Motion of MBS

Consider a MBS having *n* degrees-of-freedom. Let the equation of its controlled motion be as follows:

$$A(q)\ddot{q} + B(q,\dot{q}) = u(t), \qquad t \in [0,T] \tag{5}$$

Here $q = (q_1, q_2, ..., q_n)$ is a vector of the generelized coordinates, $u = (u_1, u_2, ..., u_m)$ is a vector of the controlling stimuli (forces, torques) generated by powered drives of the MBS, $w = (w_1, w_2, ..., w_m)$ is a vector of the controlling stimuli of the internal (unpowered) drives of MBS, $A(q)$, $B(q,\dot{q})$ are given matrices.

At the same time, assume that MBS has additional passive drives, namely non-linear visco-elastic spring-damper-like actuators in its structure. The mathematical model of the semi-passively actuated MBS can be written as follows:

$$A(q)\ddot{q} + B(q,\dot{q}) = u(t) + w(q,\dot{q}), \; w(q,\dot{q}) + kf(q,\dot{q}) = 0, \qquad t \in [0,T] \tag{6}$$

where the function $f(q,\dot{q})$ determines the inherent dynamics of the passive drives under the restriction (3) and k is a "damper coefficient".

To estimate the quality of the control processes the following objective functional is exploied

$$\Phi[u(t)] = \int_0^T \|u(t)\|^2 dt \,, \; \|u(t)\| = (u_1^2(t) + ... + u_n^2(t))^{1/2} \tag{7}$$

Let $\{q_0(t),\ u_0(t),\ t\in[0,T]\}$ be any pair of functions that satisfy equation (5).
It is assumed that the motion $\{q_0(t),\ t\in[0,T]\}$ can also be realised by the considered semi-passively actuated MBS. Using the equations (6), the external controlling stimuli that need for the motion are written as follows

$$u_{w0}(t)=u_0(t)+kf(q_0,\dot q_0)\text{, where } u_0(t)=A(q_0)\ddot q_0(t)+B(q_0,\dot q_0) \tag{8}$$

For the control law (8) the objective functional (7) will be equal to

$$\Phi[u_{w0}(t)]=\int_0^T\left\|u_0(t)+kf[q_0(t),\dot q_0(t)]\right\|^2dt=\Phi[u_0(t)]+ak^2+2bk\ , \tag{9}$$

$$a=\int_0^T\left\|f(q_0,\dot q_0)\right\|^2dt\ ,\quad b=\int_0^T\langle u_0(t),f(q_0,\dot q_0)\rangle dt\ ,\quad \langle u_0(t),f(q_0,\dot q_0)\rangle=(u_{01}f_1+\ldots+u_{0n}f_n) \tag{10}$$

It can be shown that the function $\Phi[u_{w0}(t)]$ has a global minimum with respect to the damper coefficient *k*. The value of this minimum is equal to $\Phi_{\min}=\Phi[u_0(t)]-b^2/a$ for the following optimal value of the damper coefficient $k_*=-b/a$.

The above mentioned makes it possible to conclude that for a MBS with *n* degrees-of-freedom and for any admissible motion $\{q_0(t),\ t\in[0,T]\}$ the energy-optimal non-linear visco-elastic spring-damper-like actuators are determined by the formulae $w_*(q_0,\dot q_0)+k_*f(q_0,\dot q_0)=0, t\in[0,T]$.

As follows from above due to the incorporation of the optimal spring-damper-like actuators into the MBS structure the decrease in energy consumption is equal to

$$\Phi[u_0(t)]-\Phi_{\min}=\left\{\int_0^T\langle u_0(t),f(q_0,\dot q_0)\rangle dt\right\}^2\Big/\int_0^T\left\|f(q_0,\dot q_0)\right\|^2dt\ .$$

This value depends only on the given motion $\{q_0(t),\ t\in[0,T]\}$ and the function $f(q,\dot q)$ determining the inherent dynamics of the passive drives.

Usually some restrictions are imposed on the external controlling stimuli $u(t)$. In this case the function $f(q,\dot q)$ can not be choosen arbitrarily. Indeed, let us assume that the external controlling stimuli $u(t)$ are restricted by the constraint $\|u(t)\|\le u_{\max}, t\in[0,T]$ with given positive number $u_{\max}$. Then the function $f(q,\dot q)$ must satisfy not only the restriction (3) but also the inequality $\|u_0(t)-bf(q_0,\dot q_0)/a\|\le u_{\max},\ t\in[0,T]$, where $u_0(t),\ a$ and b are determined by the expressions (8) and (10).

5. Optimization of Controlled Motion of Semi-Passively Actuated Manipulator Robots

Here we present several results of optimization of controlled motion of semi-passively actuated three-link manipulator robot (Figure 1) and closed-loop chain semi-passively actuated SCARA-like robot (Figure 2).

5.1. THREE-LINK MANIPULATOR ROBOTS

In the Figure 1 the sketch of semi-passive actuated three-link manipulator robot is shown. The robot can have powered drives U_i and unpowered (passive) drives P_i (spring-damper-like actuators) located at the joints. The torques of the unpowered drives are modelled by formulae

$$p_i=-k_i(\varphi_i-\varphi_{i0k})-c_i\dot\varphi_i\quad i=1,2,3 \tag{11}$$

where k_i are the stiffness coefficients, c_i are the damping coefficients, φ_{i0k} are the no-load angles of the torsional spring.

The controlled motion of the considered semi-passive actuated three-link manipulator robot is described by equations (6). For the robot in question several variants of the *Problem A* have been solved by using the developing methodology. The functional (7) has been used to estimate energy consumption of the controlled motion of the robot. The set of pick-and-place operations of the robot are under the particular study. For comparison in the Figure 3-4 the polar paths of the end-effector of the robot are depicted that correspond to the solution of energy-optimal control problems obtained for fullyactuated robot without unpowered drives and for underactuated robot with powered drive located at the joints *O* and *A* and with unpowered spring-damper-like drive located at joint *B,* respectively.

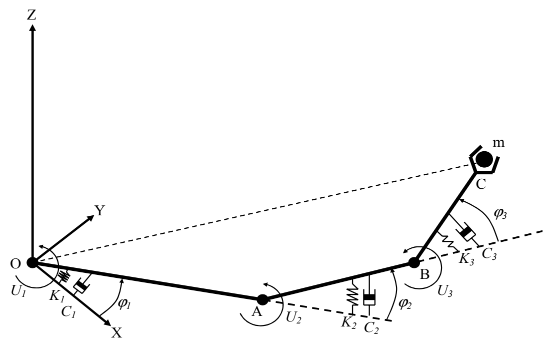


Figure 1.

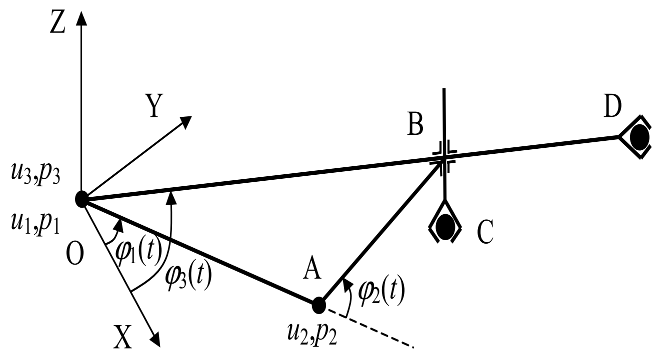


Figure 2.

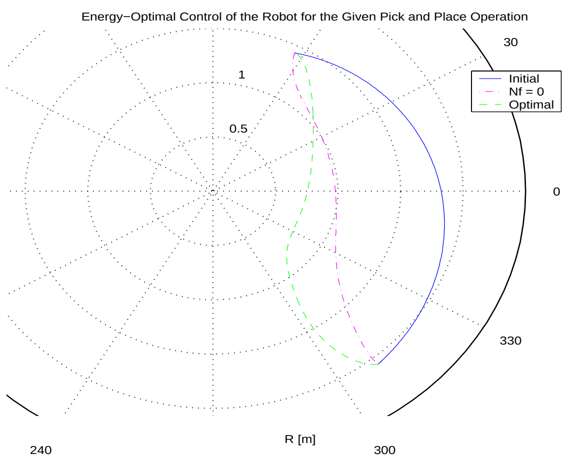


Figure 3.

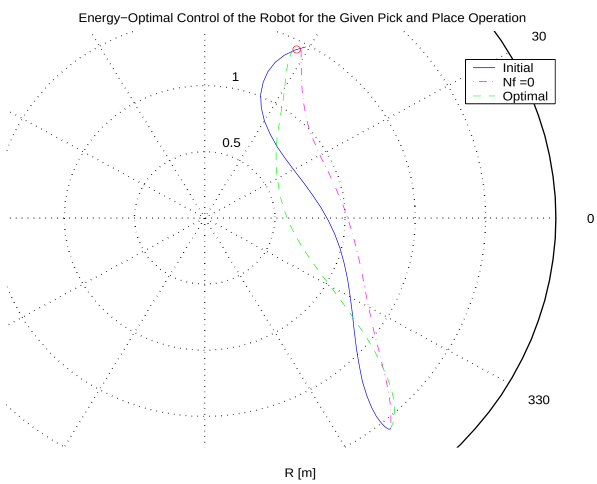


Figure 4.

5.2. SEMI-PASSIVELY ACTUATED SCARA-LIKE ROBOT

In the paper the optimal control problems of a new structure of manipulator robot (Figure 2) is under study. Proposed robotic system has the following new features in comparison with the well-known SCARA robot. In addition to powered drives it comprises several unpowered (passive) spring-damper-like drives. An additional link has also been incorporated into the structure that gives the possibility to obtain a semi-passively actuated closed-loop chain robot. Emphasis is put on a study of the interaction between the controlling stimuli of the powered drives and the torques exerted by the unpowered drives needed to provide the optimal motion of the robots with different degrees of actuation.

The robot depicted in Figure 2 comprises four links that are modeled by the rigid bodies OA, AB, OD and EC. There are one degree-of-freedom rotational joints at the points O and A, and translational joints at the point B. All joints are considered frictionless.

It is assumed that the robot's links OA, AB and OD move in the horizontal plane OXY under the action of the torques $u_1(t)$, $u_2(t)$ and $u_3(t)$ applied to the links OA, AB, and OD, respectively. Under the action of the force $F(t)$ the link EC moves along the direction of the axes OZ. The controlling stimuli $u_i(t)$, $i = 1, 2, 3$ and $F(t)$ are exerted by the powered drives of the robot. The robotic system also comprises spring-damper actuators at joints O and A. The torques exerted by these actuators p_1 , p_2 and p_3 act on the links OA, AB and OD, respectively. They will be treated as the controlling stimuli of unpowered (passive) drives of the robot. Using ϕ_1, ϕ_2 and z as the generalized coordinates the equations of motion of the considered system can be derived by using the Lagrange formalism [12]. Here we study motions of the robot in the horizontal plane OXY only. The inherent dynamics of the passive drives of the robot can be modeled in different ways, e.g. by the differential constraints (11). The equations of the plane motion of the robot can be written as follows:

$$f_1(\phi_i,\dot{\phi}_i,\ddot{\phi}_i) = u_1 + p_1 + u_3 + p_3, \quad f_2(\phi_i,\dot{\phi}_i,\ddot{\phi}_i) = u_2 + p_2 + b(\phi_i)(u_3 + p_3) \tag{12}$$

The functions f_1 and f_2 are determined by means of the Lagrange operator [12].

The considered robot is an overactuated mechanical system. This makes it possible to optimize the controlling stimuli of powered drives for an arbitrary given motion of the robot.

Problem A.1. Assume that an arbitrary motion of the robot and control torques of unpowered drives are given, i.e. the functions $\phi_i(t), p_i(t)$ are specified. It is required to find the control stimuli $u = (u_1, u_2, u_3)$ which minimize the functional

$$E[u(t)] = \int_0^T (u_1^2(t) + u_2^2(t) + u_3^2(t))dt \tag{13}$$

subject to the differential constraints (12).

It can be shown that the solution of *Problem A.1.* is

$$u_3^*(t) = (g_1 + bg_2)/(2 + b^2), \quad u_1^*(t) = g_1 - u_3^*(t), \quad u_2^*(t) = g_2 - bu_3^*(t) \tag{14}$$

Here the functions g_1 and g_2 have the expressions:

$$g_1 = f_1 - p_1 - p_3, \quad g_2 = f_2 - p_2 - bp_3 \tag{15}$$

The obtained controlling stimuli (14) provide execution of an arbitrary given motion of the overactuated robot with minimal energy consumption E^*.

The simplest way to reduce the overactuation of the considered robot is to exclude one of the powered drives. For instance, assuming that

$$u_3(t) = 0, \quad t \in [0, T] \tag{16}$$

the unique solution for the functions $u_1(t)$, $u_2(t)$ can be obtained from the equations (12). In this case the functional (13) is

$$E^0 = \int_0^T (g_1^2(t) + g_2^2(t))dt \tag{17}$$

where the function $g_1(t)$, $g_2(t)$ are given by the formulas (15). Comparing the value E^0 with the value of the functional (13) for the obtained optimal controlling stimuli $u_i^*(t)$ it is easy to show the validity of the following expression

$$E^0 - E^* = \int_0^T (g_1(t) + bg_2(t))^2 /(2+b^2)dt \tag{18}$$

The formula (18) shows that the energy consumption needed to execute an arbitrary given motion by the considered overactuated robot with obtained optimal controlling stimuli (14), (15) is less than the energy consumption of the same robot but without powered drive acting on the link OD.

During the lecture other results of optimization of motion of closed-loop chain semi-passively actuated SCARA-like robot will be presented.

6. Optimization of Controlled Motion of Semi-Passively Actuated Bipedal Locomotion Systems

Here we will demonstrate the application of methodology of optimization of semi-passively actuated MBS to solve the design problems of lower limb prostheses.

There is an important difference between the dynamics of an intact limb and a prosthetic limb of an amputee. In the paper the mathematical modeling of a human gait of an amputee with the above-knee prosthesis is considered based on a supposition that the force moments at the knee and at the ankle joints of the prosthetic leg are passive ones. The values of these moments depend not only on the gait pattern, but also on the prosthesis construction.

The model of the amputee locomotor system (ALS) with above-knee prosthesis is depicted in Figure 5. It is assumed that the above-knee prosthesis comprises the linear-viscoelastic ankle mechanism and the hydraulic or the pneumatic knee mechanism.

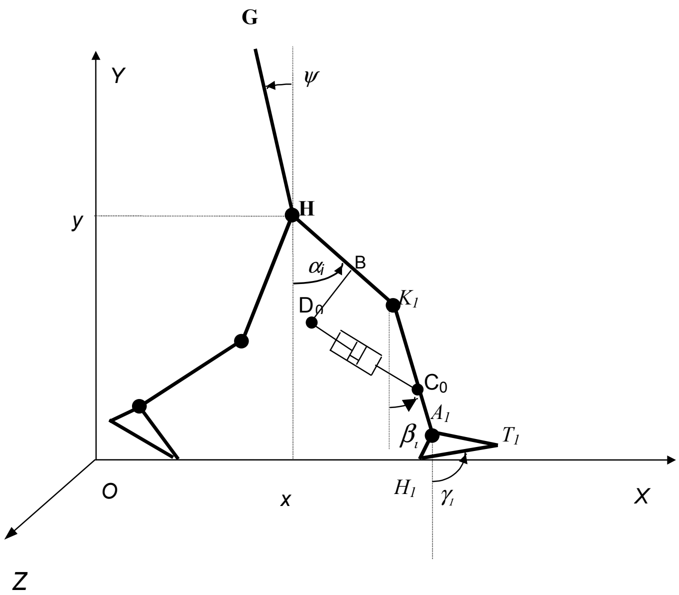


Figure 5.

During locomotion of ALS with the above-knee prosthesis the control torques

$$p_1(t) = C(\beta_1 - \gamma_1 + \pi/2) + K(\dot{\beta}_1 - \dot{\gamma}_1) + D\,, \tag{19}$$

$$u_1(t) = (P_2 - P_1)S_p d_2 (d_1^2 + l_0^2)^{1/2} \sin(\alpha_1 - \beta_1 + \eta)/l_1$$

are generated at the ankle and at the knee joints of the prosthetic leg, respectively.

Here *C, K* are the torsion spring and the damping coefficients of the ankle mechanism; *D* is determined by the free angle of the spring and torsion spring coefficients; P_1, P_2 are the chamber pressures of the hydraulic or the pneumatic actuator that can be calculated by using the equations of dynamics of the knee mechanism of the prosthesis [15], S_p is the cylinder piston cross-area,

$$l_1 = (d_1^2 + d_2^2 + l_0^2 + 2d_2(d_1^2 + l_0^2)^{1/2}\cos(\alpha_1 - \beta_1 + \eta))^{1/2}, \ \eta = a\tan(l_0 / d_1), \quad (20)$$

$$d_1 = |BK_1|, \quad d_2 = |K_1C_0|, \quad l_0 = |BD_0|.$$

The controlled motions of ALS with above-knee prosthesis are described by the equations (6) and the formulae (19), (20).

The design problem of the above-knee prosthesis can be formulated as the optimal control problem with parameters (*Problem A).* The considered semi-passively actuated MBS that models ALS has the state vector $\left\{x, \dot{x}, y, \dot{y}, \psi, \dot{\psi}, \alpha_i, \dot{\alpha}_i, \beta_i, \dot{\beta}_i, \gamma_i, \dot{\gamma}_i, i = 1,2\right\}$, the vector of the powered drives controlling stimuli $u(t) = \{q_1, q_2, u_2, p_2\}$, and the vector of the constructive parameters of the unpowered drives $C_p = (C, K, D, d_1, d_2, l_0, S_p, S_0)$. The following functional

$$E_p = \frac{1}{2L}\int_0^T \{\sum_{i=1}^{2} |q_i(\dot{\psi} - \dot{\alpha}_i)| + |u_2(\dot{\alpha}_2 - \dot{\beta}_2)| + |p_2(\dot{\beta}_2 - \dot{\gamma}_2)|\}dt \quad (21)$$

is used during the solution of *Problem A.* The objective functional (21) estimates the energy expenditure per unit of distance traveling of ALS. The same approach as described in paragraph 3 has been used to solve the problem of design energy-optimal above-knee prostheses. Due to the dynamic constraints (19) the procedure of converting the *Problem A* into the standard nonlinear programming problem includes the solution of the semi-inverse dynamics problems for the mechanical system that models ALS with the above-knee prosthesis. It sufficiently increases the time consumption of the numerical algorithm for designing the energy-optimal above-knee prosthesis. The Problem A has been solved numerically for two types of the prostheses: the above-knee prosthesis with the hydraulic actuator at the knee, and the prosthesis with the pneumatic knee mechanism. Some kinematic and dynamic characteristics of the energy-optimal motion of ALS with optimal structure of the above-knee prosthesis obtained by the numerical solution of the Problem A for the gait with natural cadence are shown in Figures 6 - 9 (solid thin curves correspond to the prosthesis with the hydraulic actuator at the knee, dashed curves - to the prosthesis with the pneumatic knee mechanism). Knee angle ($\alpha_1 - \beta_1$) and hip angle ($\alpha_1 - \psi$) of the prosthetic leg are depicted in Figures 6-7, respectively, (in degrees). Hip torque of the prosthetic leg, ($q_1(t)/M$), and knee torque of the healthy leg, ($u_2(t)/M$) are presented in Figure 8-9, respectively, (in Nm/kg). For the comparison purposes in Figures 6-9 the domains of the values of the respective kinematic and dynamic characteristics obtained by the biomechanical experiments for a human normal gait [16, 17] are depicted by heavy solid curves.

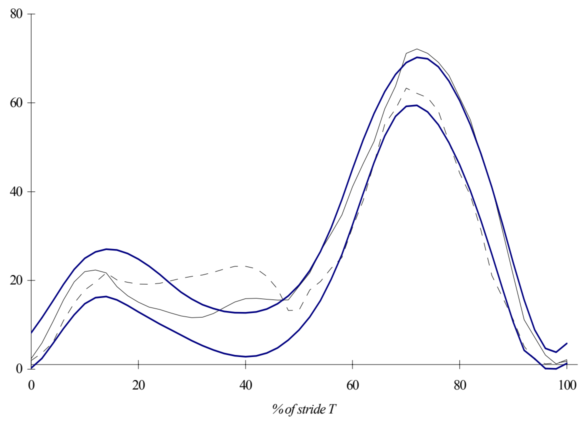
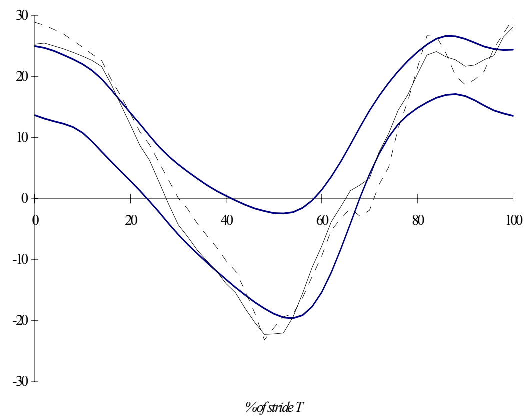


Figure 6.



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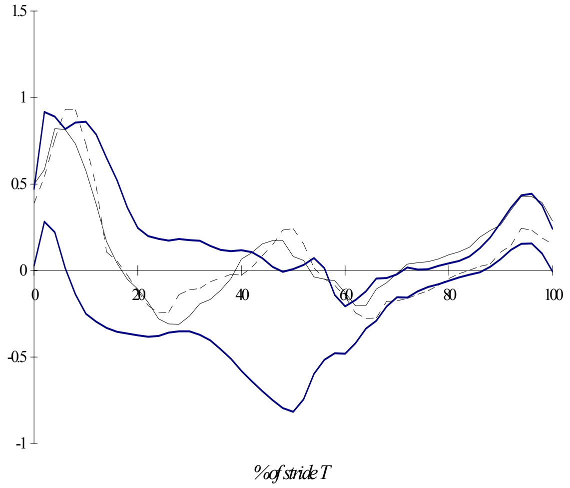

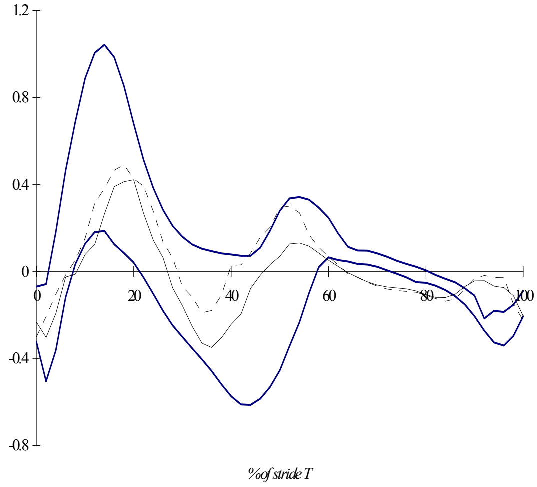

Figure 8.

Figure 9.

The obtained kinematic and dynamic characteristics of the motion of ALS with optimal above-knee prosthesis structures are within reasonable proximity to the respective characteristics of a human normal gait [16, 17]. The analysis of a number of numerical simulations shows that the natural cadence of ALS gait gives a minimum to the energy expended per unit of distance traveled comparing to the amount of energy needed for the slow or fast gaits [10].

7. Conclusion

We have formulated the optimal control problem for the semi-passively actuated MBS (*Problem A*). The key feature of the proposed mathematical statement of the problems is the direct utilization of the differential equations describing the inherent dynamics of the passive actuators (internal unpowered drives) together with all other constraints that are imposed on the state vector and the controlling stimuli of the system. It leads to the non-uniqueness of the solution of the direct and inverse dynamics problems and makes it possible to design optimal passive actuators for MBS.

In the general case to solve *Problem A* for MBS with many degrees-of-freedom powerful numerical algorithms are needed. Futhermore, during the calculation of optimal control laws for MBS it is necessary to design at the same time the optimal structure of the passive drives taking into account the restriction (3). This can significantly increase the complexity of the computation.

Closed-form solution of *Problem A* has been obtained for arbitrary given motion of MBS having n degrees-of-freedom MBS and passive drives. The analysis of the obtained solutions shows that in several cases the incorporation of passive drives into the structure of MBS can decrease the energy consumption needed for the given motion of the system.

The numerical method has been developed for the solution of *Problem A* for MBS, which model several types of semi-passively actuated robotic and biorobotic systems. Efficiency of the proposed method is illustrated by computer simulations of the energy-optimal motion of closed-loop semi-passively controlled manipulator robot, the bipedal walking robot, the solution of design problems of the energy-optimal above-knee prostheses with several types of passively controlled knee mechanisms, etc. Analysis of the numerical results obtained has shown that during the optimal motion of the considered MBS there is a strong interaction between the gravity force, the external control torque exerted by the actively controlled drives and the internal torque exerted by the passive linear spring-damper actuators. Moreover, the incorporation of the optimal passive linear spring-damper actuators into the structure of the closed-loop robot leads to a significant reduction of the energy consumption of the robot for cyclic pick and place operations [12, 18]. The kinematic, dynamic, and energetic characteristics of controlled motion of MBS that model human locomotor system with above-knee prosthesis are strongly sensitive to the essential parameters of the passive actuators of the prosthesis. For a given individual and cadence of a gait there exist optimal values of the spring and damper parameters of the prosthesis's ankle and knee mechanisms. These parameters give minimum energy expended per unit of distance travelled [10].

Results obtained can help to design simpler control systems of manipulator robots and autonomous legged mechanisms having less weight and energy consumption. They will also be use to design energy efficient passively controlled mechanisms of the lower limb prostheses.

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