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Optimization of Semi-Passively Controlled Multibody Systems

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Introduction

Some challenging applications of multibody dynamics require more fundamental research on a number of topics [1-2], e.g. contact and impact problems, control-structure interaction, large deformation problems, etc. In this paper, we tackle optimization problems for multibody systems having spring-damper actuators. The reasons of this study are as follows. To incorporate spring-damper actuators into the structure of multibody systems and to design optimally their parameters can give several advantages, e.g. to decrease the number of external drives and, as a consequence, to decrease the weight of the moving links and the energy consumption of the system. It can give great advantages to use different passive compliance elements to control some degrees of freedom of manipulator robots and legged mechanisms for their performance at working tasks with periodic laws of motion [3-8].

The object under study is a controlled mechanical system of rigid bodies. Revolute, prismatic, free, and fixed joints can interconnect the bodies. External controlling forces and moments can be applied directly to arbitrary points in the system. These controlling stimuli are generated by external drives. It is assumed that displacement and velocity dependent internal controlling forces and torques can be also applied to the system. These controlling stimuli are generated by spring-damper actuators (internal drives) that located between arbitrary points and described by linear and angular stiffness and damping parameters. A multibody system (MBS) including both external and internal drives we shall term a semi-passively controlled (actuated) multibody system. Many mechanical systems fit into this category. Examples of these systems are a car with *n* trailers having spring-damper-like joints, manipulator robots with elastic hinges, legged mechanisms with compliant elements at joints, etc. We study a fundamental question about optimal interaction between the controlling stimuli generated by the external drives and the proportional-differential forces described by linear and angular stiffness and damping parameters. A range of questions is also addressed about the role of inherent dynamics in controlled motion, and how much the MBS should be governed by the external drives and how much by the system's inherent dynamics. We are in particular investigating semi-passively actuated manipulator robots and bipedal walking mechanisms having spring-damper actuators.

Statement of the problem

Consider a MBS the controlled motion of which can be described by the following equations:

$$x^{\bullet} = f[x(t), u(t), w(t,\xi)], \qquad t \in [0,\tau] \tag{1}$$

$$g[x(t), x^{\bullet}(t), w(t,\xi)] = 0, \quad t \in [0,\tau]$$

Here $x(t)$ is a state vector, $u(t)$ is a vector of controlling stimuli (forces, torques) generated by the active (powered) drives of the MBS, $w(t,\xi)$ is a vector of the controlling stimuli of the passive (unpowered) drives of the MBS, and τ is the duration of the controlled motion of MBS. Vector functions f and g are determined by the structures of the MBS and passive drives, respectively, ξ is a vector of design parameters of the passive drives.

Usually some constraints and restrictions are imposed on the state vector $x(t)$, the controlling stimuli of the passive drives $w(t,\xi)$, and the external control laws $u(t)$ of the system. These restrictions can be written in the following way:

$$\{x(t)\} \in Q, \qquad t \in [0,\tau] \tag{2}$$

$$w(t,\xi) \in W, \qquad t \in [0,\tau] \tag{3}$$

$$u(t) \in U, \qquad t \in [0,\tau] \tag{4}$$

In formulas (2) - (4), Q and U are given domains in the state and control spaces of the system, respectively; W is a set of addmissible controlling stimuli determined by the structure of the passive drives.

The differential equations (1) together with the restrictions (2)-(4) is called the mathematical model of the semi-passively actuated MBS. This model can be used for many applications, e.g. to solve the design problems of lower limb prostheses [3, 4, 7], to study of control strategies for the stable motion of bipedal locomotion systems with compliance elements at the joints [5], for computer simulation of the energy-optimal motion of closed-loop chain manipulator robots with passive drives [8], etc.

Assume that there exists a non-empty set of vector-functions

$$\{x(t), u(t), w(t,\xi), \;\; t \in [0,\tau]\}$$

which satisfy the equations (1) and the constraints (2)-(4). The following optimal control problem can be formulated.

Problem A. Given a MBS the controlled motion of which is described by equations (1). It is required to determine the vector-function of passive drives $w_*(t,\xi)$, , the motion of the system $x_*(t)$ and the external controlling stimuli $u_*(t,x_*,w_*)$ which alltogether satisfy the equations (1), the restrictions (2)-(4), and which minimize the given objective functional $\Phi[u]$.

As a result of the solution of *Problem A* the optimal structure of the MBS having both powered and unpowered drives is designed. The external controlling stimuli for the system are also found which minimize the given objective functional.

One of the primary goals for the incorporation of passive drives into the structure of MBS is an improvement of their control processes. It means that the validity of the following inequality is expected:

$$\Phi[u_*(t,x_*,w_*)] < \Phi[u_{0*}(t,x_{0*})],$$

where $x_{0*}(t)$, $u_{0*}(t)$ are the optimal motion and the controlling stimuli of the MBS without the unpowered drives obtained under the restrictions (2), (4). In this sense the solution of *Problem A* could help to estimate the limiting possibility of improvement of the external control strategies for MBS due to incorporation into their structure of different passive drives determined by the constraints (3).

Results and discussion

We have formulated the optimal control problem for the semi-passively actuated MBS. The key feature of the proposed mathematical statement of the problems is the direct utilization of the differential equations describing the inherent dynamics of the passive actuators (internal unpowered drives) together with all other constraints that are imposed on the state vector and the controlling stimuli of the system. It leads to the non-uniqueness of the solution of the direct and inverse dynamics problems and makes it possible to design optimal passive actuators for a MBS.

In the general case to solve *Problem A* for MBS with many degrees-of-freedom powerful numerical algorithms are needed. Futhermore, during the calculation of optimal control laws for a MBS it is necessary to design at the same time the optimal structure of the passive drives taking into account the restriction (3). This can significantly increase the complexity of the computation.

Within the frame of the above-mentioned we will present a closed-form solution of *Problem A* that obtained for n degrees-of-freedom MBS with passive drives [9]. The solution of *Problem A* has been obtained for two cases. First, for a MBS without any restrictions that can be imposed on the controlling stimuli of its passive drives. Second, for a MBS with n degrees-of-freedom having non-linear visco-elastic spring-damper actuators. In both cases the motion of the MBS is specified in advance. The analysis of the obtained solutions shows that in several cases the incorporation of passive drives into the structure of a MBS can decrease the energy consumption needed for the given motion of the system.

We will also present the numerical method that has been developed for the solution of *Problem A* for the MBS, which model the semi-passively actuated manipulator robots and the bipedal locomotion systems with unpowered drives at their joints. The method is based on a special procedure to convert the initial optimal control problem into a standard nonlinear programming problem. This is made by an approximation of the independent variable functions using a combination of splines and Fourier series and by the solution of a semi-inverse dynamics problem. The key features of the method are its high numerical effectiveness and the possibility to automatically and accurately satisfy a lot of restrictions imposed on the phase coordinates of the system.

The proposed method is illustrated by computer simulations of the energy-optimal motion of closed-loop semi-passively controlled manipulator robot, the bipedal walking robot, and by the solution of design problems of the energy-optimal above-knee prostheses with several types of passively controlled knee mechanisms. Analysis of the numerical results obtained has shown that during the optimal motion of the considered MBS there is a strong interaction between the gravity force, the external control torque exerted by the actively controlled drives and the internal torque exerted by the passive linear spring-damper actuators. Moreover, the incorporation of the optimal passive linear spring-damper actuators into the structure of the closed-loop robot leads to a significant reduction of the energy consumption of the robot for cyclic pick and place operations. The kinematic, dynamic, and energetic characteristics of controlled motion of the MBS that model human locomotor system with an above-knee prosthesis are strongly sensitive to the essential parameters of the passive actuators of the prosthesis. For a given individual and cadence of a gait there exist optimal values of the spring and damper parameters of the prosthesis's ankle and knee mechanisms. These parameters give minimum energy expended per unit of distance travelled.

Results obtained can help to design simpler control systems of manipulator robots and autonomous legged mechanisms having less weight and energy consumption. They will also be use to design energy efficient passively controlled mechanisms of the lower limb prostheses.

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