Staggered Interplex

Mariano Vergara and Felix Antreich
Institute for Communications and Navigation, German Aerospace Center (DLR), Germany
Email: {mariano.vergara, felix.antreich}@dlr.de

Abstract—**In this paper we present a modification of the interplex scheme, which allows to increase the power efficiency. The proposed method consists in introducing a constant time offset for each signal component, in order to maximise the power at the output of the receive matched filter. With the chosen high power amplifier (HPA) setup, this technique improves the receive power efficiency by 2-3% without any change in the transmitter/receiver hardware.**

The interplex scheme is a phase-shift-keyed/phase modulation (PSK/PM) that combines multiple signal components into a phase modulated composite signal [1]. The interplex offers a higher power efficiency than a conventional PSK/PM signal for a low number of signal components (less or equal than five [1]). In the following we assume that the signal components consist of direct-sequence-code-division-multiple-access (DS-CDMA) signals. Like the PSK/PM technique, the interplex mapping scheme is a constant envelope modulation, which means that the constellation points lie on a circle in the complex plain. This contributes to the reduction of the distortions due to the non-linearities of the high power amplifier (HPA). In order to establish a constant envelop modulation, some inter-modulation product terms are introduced by the interplex mapping scheme. If the power of these terms is not used in the demodulation process, the transmit power efficiency is jeopardised [1], [2].

The *transmit (Tx) power efficiency* is the ratio of the useful radiated signal power to the total radiated signal power. Nevertheless, this does not describe all the power inefficiency of the interplex scheme. Indeed, not all the useful signal power that is transmitted can be used at receiver side. To begin with, a realistic interplex signal is only approximately constant envelope due to state changes in the signal constellation. As a result of that, the interplex signal will experience some distortion, when amplified by the HPA, which results in a correlation loss at receiver side. On top of that, since DS-CDMA spreading sequences are not perfectly uncorrelated, and the cross-correlation among different signal causes in average correlation loss. This correlation loss is also called Multiple Access Interference (MAI). The intermodulation product terms not only constitute a form of power inefficiency but they also contribute in increasing the noise floor caused by introducing additional MAI. We define the *receiver (Rx) power efficiency* as the ratio between the useful power at the output of the receiver's matched filters and the total transmit power. The Rx power efficiency is a more accurate and useful measure than the Tx power efficiency to make a power budget analysis for a mapping scheme.

In this paper we propose a method for increasing the Rx power efficiency, which we call *staggerd interplex*. This method consists in the introduction of delays on the signal components of the interplex scheme, so that the sum of the Rx power at the correlator outputs at the receiver is maximised. This results in a non-linear optimization problem, which is solved by an evolutionary algorithm [3]. The signal is distorted by an HPA modelled after the well-known Saleh model [4].

We show that for the chosen HPA configuration an increase of Rx power efficiency of 2-3% can be achieved without requiring any hardware modification either at the transmitter or the receiver side.

I. STANDARD INTERPLEX

An N-channel interplex [1] signal is a phase-shift-keyed/phase modulated (PSK/PM) signal

$$x_N(t) = \sqrt{2A}\sin(2\pi f_c t + \Theta(t)), \tag{1}$$

in which the phase modulation is

$$\Theta(t) = \left[\beta_1 + \sum_{n=2}^{N} \beta_n s_n(t)\right] s_1(t), \tag{2}$$

where A is the effective value of the real-valued carrier, f_c denotes the carrier frequency, β_n are the modulation angles, and $s_n(t)$ is the n-th binary phase shift keying (BPSK) signal component. We use the convention of calling $s_1(t)$ *primary* or *main component* and $s_n(t)$,$\forall\ n \neq 1$ *secondary components*. In the following the quantity β_1 is chosen equal to $\frac{\pi}{2}$ for any N, as this suppresses the carrier and good part of the intermodulation product.

The signal (1) should theoretically be implemented by varying the phase of an oscillator according to the information symbols, but this is not practical and an I-Q modulator is preferred [5, chap.6]. Thus, practically the above signal for $N = 3$ is generated as:

$$\begin{aligned} x_3(t) = &\big[s_1(t)\cos\beta_2\cos\beta_3 \\ &- s_1(t)s_2(t)s_3(t)\sin\beta_2\sin\beta_3\big]\sqrt{2A}\cos(2\pi f_c t) \\ &- \mathrm{j}\big[s_2(t)\sin\beta_2\cos\beta_3 \\ &+ s_3(t)\cos\beta_2\sin\beta_3\big]\sqrt{2A}\sin(2\pi f_c t). \end{aligned} \tag{3}$$

We observe that (1) and (3) are exactly equal if the bandwidth is infinite, and as a consequence of that the signal components are *purely* binary, i.e. $s_n(t) = \mp 1$. Same applies for the signal (1) and (28) for the case N=4. Nevertheless, signals with infinite bandwidth do not exist in real life and the signal components of the signal (1) can only be binary in their constellation, because band-limited BPSK signals are not *purely* binary. The novelty of this paper consists in analysing the interplex scheme presented in [1] in a band-limited case and in proposing a very simple way to improve the power efficiency.

Each signal component is of the kind:

$$s_n(t) = \sum_{k=-\infty}^{\infty} b^{(n)}_{k\,(mod\,N_{sf})} c^{(n)}_k p_n(t-kT_c) \qquad (4)$$

where $b^{(n)}_{k\,(mod\,N_{sf})}$ and $c^{(n)}_k$ are the data symbols and the spreading sequence of the n-th signal component, respectively. The subscript $_{k\,(mod\,N_{sf})}$ indicates the modulo N_{sf} operation, where N_{sf} is the spreading factor and $\frac{1}{T_c}$ is the chip rate, equal for all signal components. $p_n(t)$ is the pulse shape of the n-th signal component. The energy of the pulse is

$$\int_{-\infty}^{\infty} |p_n(t)|^2 \mathrm{d}t = T_c \qquad \forall n, \qquad (5)$$

and the energy of the spreading sequence is

$$\sum_{k=0}^{N_{sf}-1} |c^{(n)}_k|^2 = N_{sf} \qquad \forall n. \qquad (6)$$

Thus, the power of each signal component is

$$P_n = \frac{1}{N_{sf}\,T_c} \int_{N_{sf}\,T_c} |s_n(t)|^2 \mathrm{d}t = 1 \qquad \forall n. \qquad (7)$$

Both $b^{(n)}_{k\,(mod\,N_{sf})}$ and $c^{(n)}_k$ are assumed binary. We indicate the chip pulse train by

$$c^{(n)}(t) = \sum_{k=0}^{N_{sf}-1} c^{(n)}_k p_n(t-kT_c) \qquad \forall n. \qquad (8)$$

The chip pulse train in (8) has a duration of

$$T = N_{sf}\,T_c. \qquad (9)$$

The power of the signal components and intermodulation products within the three-channel interplex are:

$$\begin{aligned} P_1 &= A\ \cos^2\beta_2\cos^2\beta_3 \\ P_2 &= A\ \sin^2\beta_2\cos^2\beta_3 \\ P_3 &= A\ \cos^2\beta_2\sin^2\beta_3 \\ P_{im} &= A\ \sin^2(\beta_2)\sin^2(\beta_3) \\ &\times \tfrac{1}{N_{sf}T_c}\textstyle\int_{-\infty}^{\infty} |s_1(t)s_2(t)s_3(t)|^2 \mathrm{d}t \end{aligned} \qquad (10)$$

We report in the Appendix the expressions for $x_N(t)$ with $N=4$. In the following we will discuss the case $N=3$ as an example case.

II. POWER DISTRIBUTION

The power of the main component $s_1(t)$ is indicated by P_{main}, the power of the secondary components by P_{sec}. As in [1], we assume that all secondary components have the same power. Introducing this condition to the three-channels interplex equations (10) and in the four-channel interplex equations (cf. Appendix) leads to:

$$P_{main}(\beta) \;=\; A[\cos\beta]^{2(N-1)} \qquad (11)$$

$$P_{sec}(\beta) \;=\; A[\cos\beta]^{2(N-2)}[\sin\beta]^2 \qquad (12)$$

where β indicates the interplex angle that is the same for all the secondary components. We will drop the dependency on β because we will focus on the ratio $\frac{P_{main}}{P_{sec}}$. With this notation the three-channel interplex can be written as:

$$\begin{aligned} x_3(t) = &\big[\sqrt{P_{main}}\ s_1(t) \\ &- \mu\ s_1(t)s_2(t)s_3(t)\big]\sqrt{2}\cos(2\pi f_c t) \\ &- \mathrm{j}\big[\sqrt{P_{sec}}\ s_2(t) \\ &+ \sqrt{P_{sec}}\ s_3(t)\big]\sqrt{2}\sin(2\pi f_c t) \end{aligned} \qquad (13)$$

where

$$\mu = \sqrt{A}\,\sin^2\beta \qquad (14)$$

and

$$P_{im} = \mu^2\,\frac{1}{N_{sf}T_c}\int_{-\infty}^{\infty} |s_1(t)s_2(t)s_3(t)|^2 \mathrm{d}t \quad . \qquad (15)$$

III. STAGGERED INTERPLEX

The staggered interplex is a novel modification of the original interplex scheme [1] in which the signal components have relative time offsets. We use the convention of taking the offset of the first component as reference, and as such it is set equal to zero ($\tau_1 = 0s$). The time offsets of the other components are indicated by τ_n, $n = 2\ldots,N$ with $\tau_n \in [-T_c/2, T_c/2]$. The staggered interplex for $N=3$ is

$$\begin{aligned} x_3^{stagg}(t) = &\big[\sqrt{P_{main}}\ s_1(t) \\ &- \mu\ s_1(t)s_2(t-\tau_2)s_3(t-\tau_3)\big]\sqrt{2}\cos(2\pi f_c t) \\ &- \mathrm{j}\big[\sqrt{P_{sec}}\ s_2(t-\tau_2) \\ &+ \sqrt{P_{sec}}\ s_3(t-\tau_3)\big]\sqrt{2}\sin(2\pi f_c t) \end{aligned} \qquad (16)$$

We note that the intermodulation product power of the three-channel staggered interplex is dependent on the time offsets τ_2 and τ_3:

$$P_{im}^{stagg}(\tau_2,\tau_3) = \mu^2\frac{1}{N_{sf}T_c}\int_{-\infty}^{\infty} |s_1(t)s_2(t-\tau_2)s_3(t-\tau_3)|^2 \mathrm{d}t \qquad (17)$$

IV. OPTIMISATION OF THE RX POWER EFFICIENCY

In this section we will define the Rx power efficiency, the HPA model, and we will formulate the non-linear optimisation problem in order to minimise the Rx power efficiency by means of the delays τ_n with $n = 2,\ldots,N$.

A. High power amplifier (HPA)

The HPA is modelled after the Saleh model [4], which is an acknowledged way of modelling a travelling-wave tube amplifier (TWTA). The AM-AM and the AM-PM curves of the Saleh model are respectively:

$$\begin{cases} A(r) = & \frac{\alpha_a\, r}{1+\beta_a\, r^2} \\ \\ \Phi(r) = & \frac{\alpha_\Phi\, r^2}{1+\beta_\Phi\, r^2} \end{cases} \qquad (18)$$

where r indicates the instantaneous envelope of the input signal. The HPA parameters we chose for our simulations are:

$$\begin{aligned} \alpha_a &= 1 \\ \beta_a &= 2 \\ \alpha_\Phi &= 0 \\ \beta_\Phi &= 0 \end{aligned} \tag{19}$$

In order to drive the HPA at the maximum efficiency, we set the input power backoff (IBO) equal to 0 dB. In other words, the working point of the amplifier is at the saturation point of the AM-AM characteristic, where the non-linear effects are maximal.

B. Interplex Rx power efficiency

The output of the HPA is

$$y(t) = \mathcal{T}\left[x(t)\right] \tag{20}$$

where $x(t)$ indicates the input signal, i.e. the signal in (1) or the signal in (16), while $\mathcal{T}\left[\cdot\right]$ indicated the HPA transfer function indicated in (18). The signal (20) propagates through an ideal channel without noise. At the receiver it is down-converted and it is processed by a bank of matched filters. The receiver structure for $N = 3$ is depicted in Fig.1. The correlator outputs z_n with $n = 1, 2, 3$ are:

$$\begin{aligned} z_1 &= \frac{1}{T}\int_T \mathrm{Re}\left\{y(t)\right\} c^{(1)}(t)\mathrm{d}t \\ z_2 &= \frac{1}{T}\int_T \mathrm{Im}\left\{y(t)\right\} c^{(2)}(t)\mathrm{d}t \\ z_3 &= \frac{1}{T}\int_T \mathrm{Im}\left\{y(t)\right\} c^{(3)}(t)\mathrm{d}t \end{aligned} \tag{21}$$

while $s_2(t)$ and $s_3(t)$ cause interference among each other, the first component $s_1(t)$ suffers from the interference coming to the intermodulation product. If the receiver does not have an ideal estimate of the carrier phase and/or the HPA has a non-ideal AM-PM characteristic, then the intermodulation product term causes additional interference to the signal components $s_2(t)$ and $s_3(t)$. In case the inter-chip interference is negligible, the intermodulation product can be seen as an additional DS-CDMA signal with a spreading sequence given by

$$\mathbf{c}^{(im)} = \mathbf{c}^{(1)} \odot \mathbf{c}^{(2)} \odot \mathbf{c}^{(3)}, \tag{22}$$

where $\mathbf{c}^{(n)} = \left[c_0^{(n)}, c_1^{(n)}, \ldots, c_{N_{sf}-1}^{(n)}\right]^{\mathrm{T}}$, $n = 1, 2, 3$ are the vectors containing the spreading sequences of the signal components, and the symbol $\odot$ indicates the Hadamard product. In the standard interplex, the intermodulation product has an *equivalent* pulse shape

$$p_{im}(t) = p_1(t)p_2(t)p_3(t), \tag{23}$$

while in case of the staggered interplex we get

$$p_{im}^{stagg}(t) = p_1(t)p_2(t-\tau_2)p_3(t-\tau_3). \tag{24}$$

The staggered interplex introduces the new degree of freedom given by the variables τ_2 and τ_3. This degree of freedom can be used to shape the equivalent pulse in order to reduce its power, and to decorrelate it from $p_1(t)$, so that the MAI affecting the first component $s_1(t)$ is minimised. At the same time, we want the signal is affected as less as possible by the non-linear distortion produced by the HPA on a signal that it is not exactly constant-envelope. To this end, we define the Rx power efficiency as:

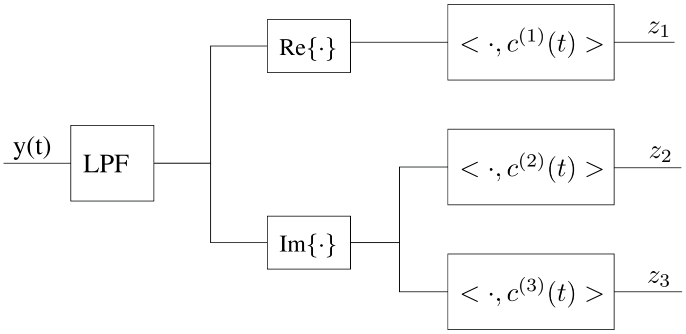

Fig. 1. Receiver structure for the interplex signal with N=3.

$$\eta^{Rx} = \frac{\sum_{n=1}^{N} |z_n|^2}{P} \tag{25}$$

where P is the overall average power of the standard or staggered interplex signal. The Rx power efficiency measures the useful power that at the receiver and it accounts for

1) the non-useful power of the intermodulation product terms,
2) correlation losses due to HPA distortion.
3) MAI between signal components as well as MAI between signal components and intermodulation product terms,

C. Optimisation

The optimisation problem at hand is:

$$\max_{\tau_2, \tau_3, \ldots, \tau_N} \eta^{Rx}(\tau_2, \tau_3, \ldots, \tau_N) \tag{26}$$

subject to:

$$|\tau_n| \leq \frac{T_c}{2} \qquad n = 2, 3, \ldots, N \tag{27}$$

The problem is non-linear and it is solved numerically. The algorithm used is an evolutionary algorithm described in [3]. We restrict our search to a semi-interval $\left[0; \frac{T_c}{2}\right]$, having used a symmetrical pulse.

V. RESULTS

In the following simulations we assume a band-limited rectangular pulse shape for all signal components. The one-sided bandwidth has been set to $B = 10.23$ MHz. The spreading sequences are Gold codes as used for the GPS C/A (*coarse acquisition*) code [6], with code period $T = 1$ ms, $N_{sf} = 1023$ chips per code period, each with a time duration $T_c = 977.52$ ns.

In Fig.2 and Fig. 3 we report the Rx efficiency for $N = 3$ and $N = 4$ for the standard interplex and for the staggered interplex, with optimised delays. The gain is up to roughly 3% for the three-channels interplex and up to 2.5% for the four-channels interplex. This gain comes from a reduction of the intermodulation product terms, a reduction of the peak-to-average-power-ration (PAPR) of the staggered interplex signal and a reduction of the MAI.

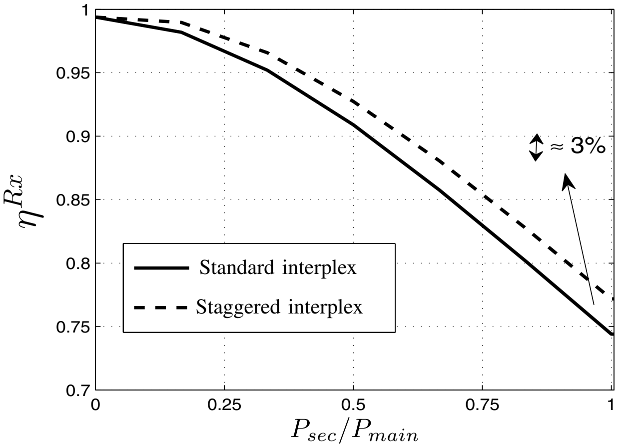

Fig. 2. Rx Efficiency of standard and staggered three-channels interplex.

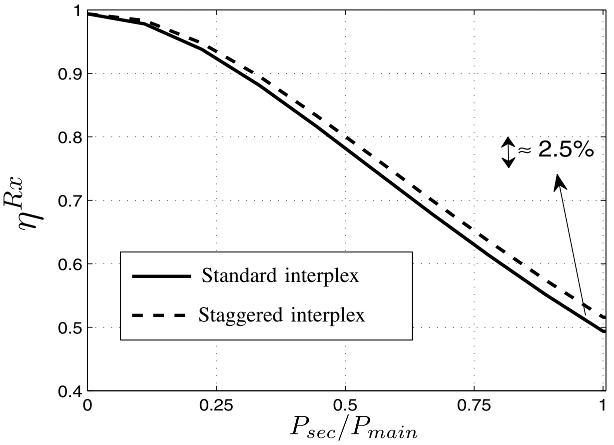

Fig. 3. Rx Efficiency of standard and staggered four-channels interplex.

The optimal delays for $N = 3$ and $N = 4$ are plotted in Fig. 4 and Fig. 5, respectively.

The signal constellation diagrams for the $N = 3$ and $N = 4$ channel interplex are depicted in Fig.6 and Fig. 7, both for the standard and the staggered interplex with optimised delays. These figures were calculated in the case in which all useful signal components have the same power, i.e. $P_{main} = P_{sec}$. The constellation points are marked in green and the state transitions are plotted in blue. The corresponding PAPRs of the interplex and the staggered interplex signal are written in the bottom right corner of the figures. The PAPR is reduced, because the optimised delays do not allow all signal components to change polarity at the same time. This is the same principle underlying the offset-quadrature phase shift keying (OQPSK) [7, p. 361]. The staggered interplex scheme that we propose may be seen as a generalization of the OQPSK modulation to a phase shift keying (PSK) modulation with more than 2 signal components.

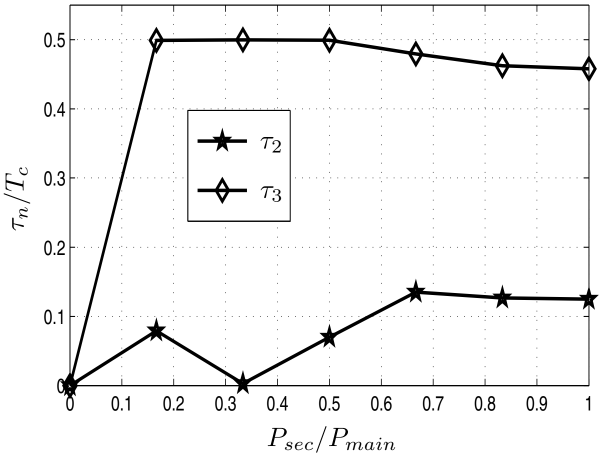

Fig. 4. Optimised time offsets of the three-channels staggered interplex.

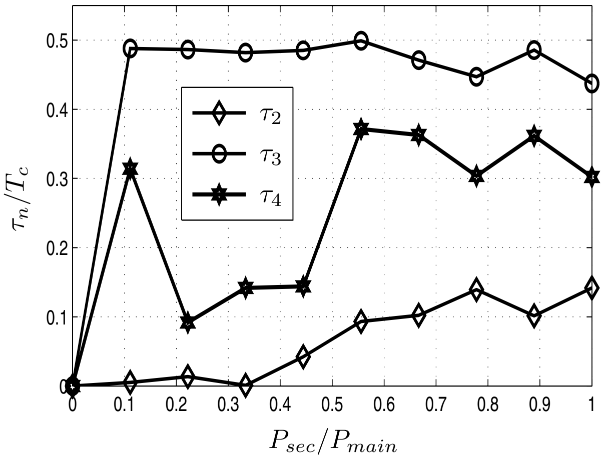

Fig. 5. Optimised time offsets of the four-channels staggered interplex.

VI. CONCLUSIONS

In this paper we proposed a method for increasing the useful power at the transmitter side of a band-limited interplex signal. This new technique is called staggered interplex and consists in the introduction of sub-chip delays on the signal components of the interplex scheme. These delays are so that the sum of the Rx power at the correlator outputs at the receiver is maximised. This optimisation has taken into account the effects of the HPA distortion, the MAI between signal components, and the the power inefficiency due to the intermodulation product terms. The proposed staggered interplex makes use of a degree of freedom that had not been used in the standard interplex. We have shown that the staggered interplex achieves an increase of the Rx power efficiency of 2-3% for $N = 3, 4$, considering

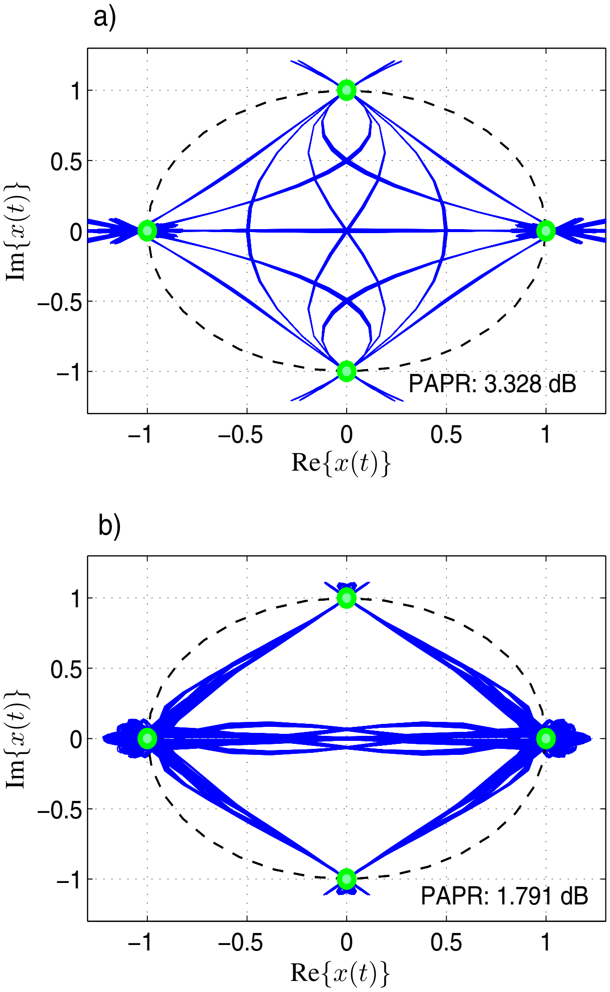

Fig. 6. Signal envelope of a three-channels interplex with uniform power distribution. a) Standard interplex, b) Staggered interplex.

the chosen values of the HPA parameters. This gain is likely to be higher as the non-linearity of the HPA becomes more prominent. To this regard, we remind that we chose a HPA model without phase noise, i.e. an ideal AM-PM transfer function. Moreover, if one chooses distinct pulse shapes for each signal component, especially if their spectra overlap only partially, a further increase of the Rx efficiency can be expected.

The technical relevance of this scheme is that the resulting gain in Rx power efficiency, with respect to the standard interplex scheme, can be achieved without any change in the transmitter and receiver architecture.

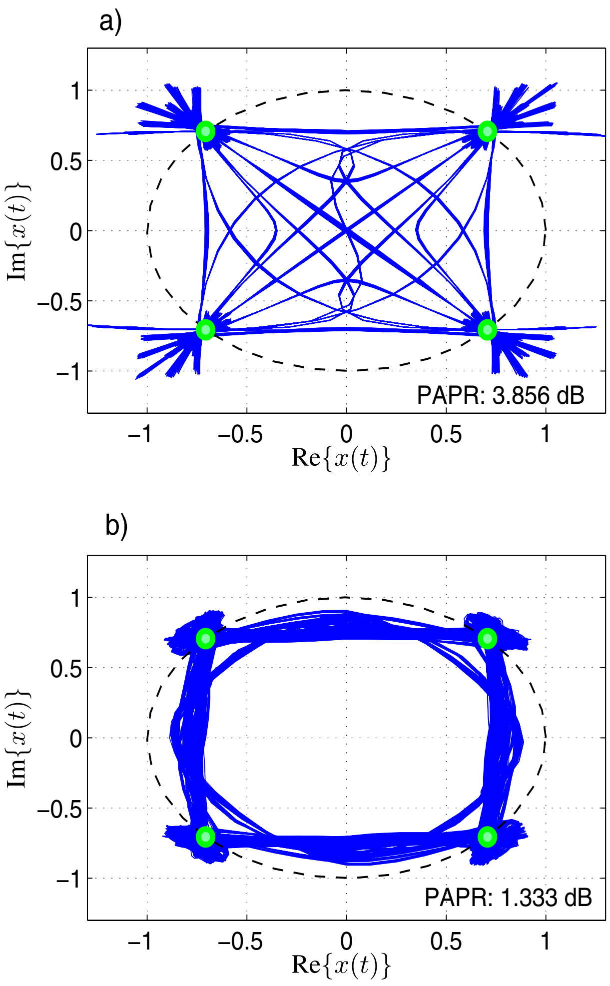

Fig. 7. Signal envelope of a four-channels interplex with uniform power distribution. a) Standard interplex, b) Staggered interplex.

APPENDIX

A. *Interplex with four signal components* ($N = 4$)

$$\begin{aligned} x_4(t) = \big[&s_1(t)\cos\beta_2\cos\beta_3\cos\beta_4 \\ &- s_1(t)s_3(t)s_4(t)\cos\beta_2\sin\beta_3\sin\beta_4 \\ &- s_1(t)s_2(t)s_3(t)\sin\beta_2\sin\beta_3\cos\beta_4 \\ &- s_1(t)s_2(t)s_4(t)\sin\beta_2\cos\beta_3\sin\beta_4\big]\sqrt{2A}\cos(2\pi f_c t) \\ &- j\big[s_2(t)\sin\beta_2\cos\beta_3\cos\beta_4 \\ &- s_2(t)s_3(t)s_4(t)\sin\beta_2\sin\beta_3\sin\beta_4 \\ &+ s_3(t)\cos\beta_2\sin\beta_3\cos\beta_4 \\ &+ s_4(t)\cos\beta_2\cos\beta_3\sin\beta_4\big]\sqrt{2A}\sin(2\pi f_c t) \end{aligned} \tag{28}$$

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