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Optimising the periodic distribution of gas cylinders with customers priority

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Abstract: This paper deals with a real-life application related to the distribution of gas cylinders to customers over a one-week horizon. Every customer should be served a pre-specified number of times during the week horizon. Customers are also characterised by a priority factor that expresses the importance of that customer to the company. Three heuristic methods have been develop based on a two-stage decomposition approach. The first stage assigns the customers to the service days and then, the second stage solves several VRPs, one for each working day. Several real-life experiments have been conducted to show that the heuristic that considers explicitly the customers priority performs well with respect to the other heuristics and also to the actual solution adopted by the company.

Keywords: Transportation problems, gas cylinders distribution, customers priority, periodic constraints.

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1 Introduction

Many companies in the sector of oil and gas industry continue to search for tools that make the distribution more efficient in a way that minimises their costs and maximises their profits. Currently in Oman, many of the oil and gas companies perform the distribution planning manually because they do not have any systematic approach that allows them to achieve this cumbersome task automatically. Implementing, thus, a scientific method based on the use of mathematical models will help in reducing the distribution costs and in the same time improving the customers' satisfaction. This is the main goal of this paper.

This paper focuses on a real-life application related to a company established in Oman that deals with gas cylinder distribution covering hotels, restaurants, companies and households. This gas company has been in business since 1992 and to this present day uses manual methods to plan the distribution of their gas cylinders on a daily basis. Our contribution in this paper is twofold:

1. to propose a scientific solution of the problem based on the use of optimisation tools
2. to extend the horizon for planning the distribution activities to cover a one-week horizon.

This will allow to perform more efficiently since our approach seeks to achieve a time-oriented consolidation of the customers belonging to the same region to be served in the same days.

For this purpose, our approach considers that every customer is characterised by a weekly demand that should follow a pre-defined pattern over the weekly horizon. Some of the customers with very high demand should be served daily, those with low demand will be served only once a week and all the others need either twice or three times service per week. In these last two cases several periodicity patterns are usually possible and our approach will select the most appropriate one. For example, for customers requiring twice a week service, the heuristics define the following three feasible combinations $\{$(Saturday, Tuesday), (Sunday, Wednesday); (Monday, Thursday)$\}$ and selects one of them. In the same way, the three times a week combinations are $\{$(Saturday, Monday, Wednesday); (Sunday, Tuesday, Thursday)$\}$. Friday is a non-working weekend day in Oman.

The paper is organised as follows: Section 2 will summarise the literature references related to our topic. Section 3 will describe our optimisation solution approaches and Section 4 will be devoted to the computational experiments carried out to compare the developed heuristics. Finally, Section 5 will conclude the paper.

2 Literature review

The distribution problems in the gas industry are generally split into two main classes. The first one is related to the gas when distributed in cylinders that are considered as the unit of the commodity to be distributed. The second class deals with the bulk gas distribution by using reservoirs. The unit of commodity in this case is the litre and a flow meter is used to measure the quantity to be transferred from the tank-truck to the customer's reservoir. With this respect, the second class turns to be very similar to the well known Petrol Station replenishment problem (PSRP). In the sequel, a survey on the most important contributions appeared to solve both the classes will be conducted.

Starting from the distribution of gas as cylinders, we are aware of one single work due to Fölsz et al. (1998). The authors discuss a step-by-step method that involves first the assignment of the capacities of the filling customers, followed by a planning step of the daily routes of the trucks and finally developing a database for the application. However, contrarily to what this paper is proposing, this contribution covers a single day horizon and does not take into account the periodic nature of the customers demand.

Considering the second class, the first work to be cited is due to Hulshof (2008) who presents a distribution model in the context of bulk gas distribution that involves inventories. The author takes into account the fact that the tanks at each customer have a starting inventory and that whenever the tank is nearly empty a refilling request is sent to the depot. This implemented system embraces what is known as a central-inventory system that has the aim of reducing the overall inventory and distribution costs since the gas company can perform important consolidation of delivery over both the space and the time.

In the same context, You et al. (2011) propose an approach based on a continuous adjustment method of the inventories. Their main idea is to plan the tank-trucks deliveries that satisfy the customers demand and maintain an acceptable level of their inventories.

The last paper we are aware of is due to Bell et al. (1983) who suggest an optimisation module based on a mixed-integer programming formulation that can span over two- to five-day horizon. During this period, any customer is classified as either receiving one or many deliveries. The problem was solved by using a Langrangian relaxation technique as a central concept for the route selection module.

Given the scarcity of the references in the context of gas distribution, one can get some insights from the field of petrol replenishment. The two problems have several similarities even though the difference in the products to be transported should be taken into account. Our focus here will be devoted to the multi-period variant of the PSRP that has been subject to three different works.

Allah et al. (2000) consider an unlimited homogeneous fleet of vehicles and a single depot problem. The authors propose several construction and improvement heuristics to solve first the one-trip-one-customer variant then the general variant of the problem over a multi-period horizon.

Cornillier et al. (2008a, 2008b) propose a multi-phase heuristic involving route construction, vehicle loading procedures, route packing procedure, and look-back as well as look-ahead of deliveries procedures. They limit the number of stations to be visited at each route to two stations only over a multi-day period horizon.

Finally, more recently Triki (2013) takes into account the periodic nature of the replenishment planning and selects the minimisation of the total distance travelled by the tank-trucks as objective function. He proposes several schedule-first-route-second heuristics followed by an improvement technique to solve a real-life application for a company established in Italy.

The main characteristics related to the modelling issues arising in the extended horizon PSRPs have been also summarised in the recent overview by Triki and Al-Hinai (2015). The authors distinguish between the periodic PSRPs and the general multi-period PSRPs, i.e., whose demand does not have any specific pattern over the multi-day horizon, and present an optimisation formulation for both the variants. Moreover, they also survey the exact and heuristic techniques developed for their solution.

Finally, it is worth noting that several other contribution have focussed on the one-day version of the PSRP without taking into account the multi-period nature of the problem (Abdelaziz et al., 2002; Avella et al., 2004; Boctor et al., 2011; Brown et al., 1987; Cornillier et al., 2009, 2012; Hanczar, 2012; Ng et al., 2008; Popvić et al., 2011; Rizzoli et al., 2003; Surjandari et al., 2011).

The analysis of the scientific literature reported above clearly shows that the gas cylinder distribution problem that covers an extended horizon while taking into account the priority of customers has never been addressed before in an integrated way. The present paper will try to fill this gap by proposing and solving a novel solution approach.

3 Optimisation solution methods

The main objective of our work is to extend the existent models in order to cover a one-week horizon while taking into account the priority of customers and minimising the distance to be travelled by all the vehicles. The expansion of time horizon will include additional variables and constraints that need to be introduced to describe the periodicity patterns occurring within the company distribution network. The extended horizon introduces several computational challenges related to the huge size of the resulting optimisation models. For this reason, solving the problem exactly till optimality seems to be intractable computationally and also impractical from the service level point of view. Consequently, we will avoid to report here the mathematical formulation corresponding to our problem in order to focus in the sequel on the development of heuristic methods based on the customers' priority.

Specifically, each customer will be assigned a priority value that expresses the importance of that customer for the company business. A starting value of 1 means a low priority and the maximum level of ten represents the highest care to be given to the customer. Moreover, every customer is also characterised by his weekly demand of gas cylinders that can be translated in terms of equally split quantity to be delivered over the periodic days of service.

Three solution methods have been developed to solve the problem under exam. All the heuristics are based on the following two-stage general decomposition framework:

Stage 1 Assign the customers to the days of the week while respecting their weekly frequency of service and other operational constraints.

Stage 2 Solve the resulting VRPs (one for each day) in order to minimise the travelled distance and define the vehicles routes.

In Stage 1 of the above framework, the assignment of customers to the working days will be performed by using three alternative methods:

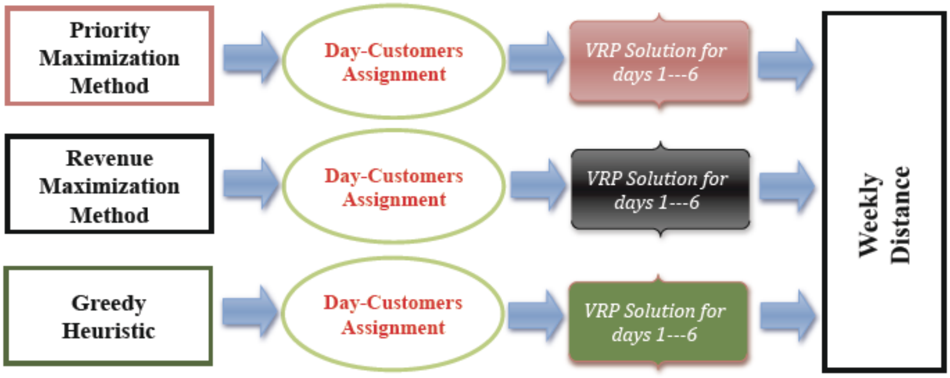
1 an IP approach based on maximising the total priorities over all the customers

2 another IP approach that maximises the revenues deriving from selling gas to all the customers

3 a greedy approach.

In the sequel, the first two approaches will be denoted as the *priority maximisation method* and the *revenue maximisation method*, respectively (see Figure 1). Section 4 will be devoted to conducting an experimental study in order to compare how these three heuristics perform from the computational viewpoint and to identify the most efficient one. The one-day VRPs resulting from Stage 2 are usually of moderated size with respect to the original time-extended formulation and can, thus, be solved even exactly in an acceptable amount of time for the company.

Figure 1 Solution methods for solving the periodic distribution problem (see online version for colours)



All the three heuristics should take into account the following restrictions as suggested by the company:

- the average working time for each driver must be taken into consideration
- the capacity of each truck must not be exceeded
- each truck must perform only 1 route every day (given the extend of the geographical area of distribution)
- the service frequency of each customer should be respected
- the demand of all high-priority customers should be satisfied as much as possible (in order to reduce the chance that they run out of gas cylinders)
- the demand of the other customers will be ensured only if serving them will not violate any of the operational constraints.

Some of these requirements will be fulfilled by incorporating them within the mathematical model for solving exactly the daily VRPs, while others will be satisfied, in Step 1, during the day-customers assignment. Before describing our heuristic approaches the following parameters should be introduced:

N	number of customers to be served (including the depot)
t	time horizon, indexed by $l = 1, \dots, t$
K	number of vehicles, indexed by $k = 1, \dots, K$
$\lvert(i, j)\rvert$	distance between customers i and j (or customer i and the depot)
c_k	cost of using vehicle k per unit of distance
Q_k	capacity of vehicle k
Tk	working time limit for the driver operating vehicle k
v_k	average speed of vehicle k
Pi	priority assigned to customer i (scale 1 – 10)
C_i	feasible periodicity combinations of customer i, indexed by r
d_{ilr}	demand of customer i on day l corresponding to combination r
R	revenue deriving from selling one gas cylinder
a_{rl}	input constant that equals to 1 if day l belongs to combination r of C_i, and 0 otherwise.

3.1 *Priority maximisation approach*

The description of this approach will be based on the use of the notation defined above and, moreover, the use of two sets of binary variables associated with the IP assignment model:

y_{ir}	equals 1 if customer i is served following periodicity combination r, 0 otherwise
$x_{(i,j)k}^{l}$	is 1 if truck k travels from customer i to customer j (or to the depot) on day l, and 0 otherwise.

The optimisation model related to our first heuristic can be represented as follows:

$$\text{Maximise} \quad \sum_{i=1}^{N} P_i \sum_{l=1}^{t} \sum_{r \in C_i} a_{rl} y_{ir} \tag{1}$$

Subject to:

$$\sum_{r \in C_i} y_{ir} = 1 \qquad i = 1, \ldots, N \tag{2}$$

$$\sum_{j=1}^{N} x_{(i,j)k}^{l} = \sum_{r \in C_i} a_{rl} y_{ir} \qquad i = 1, \ldots, N;\ l = 1, \ldots, t \tag{3}$$

$$\sum_{r \in C_i} \sum_{i,j=1}^{N} d_{ilr} x_{(i,j)k}^{l} \leq Q_k \qquad i = 1, \ldots, t;\ k = 1, \ldots, K \tag{4}$$

$$\sum_{i,j=1}^{N} \left(\frac{|(i,j)|}{v_k} \right) x_{(i,j)k}^{l} \leq T_k \qquad i = 1, \ldots, t;\ k = 1, \ldots, K \tag{5}$$

$$x_{(i,j)k}^{l} \in \{0,1\} \qquad i, j = 1, \ldots, N; t = 1, \ldots, T;\ k = 1, \ldots, K \tag{6}$$

$$y_{ir} \in \{0,1\} \qquad i = 1, \ldots, N;\ r \in C_i \tag{7}$$

Expression (1) represents the objective function that maximises the priority of all the served customers over all the days in which they are served. Constraints (2) ensure that only one periodicity combination is assigned to each customer. Constraints (3) represent the periodicity condition that forces each customer to be visited only on the days corresponding to the assigned combination. Constraints (4) impose the condition that the total demand served in one trip should not exceed the capacity of the vehicle. Constraint (5) is the time bound condition for the working hours of the drivers. Finally, the restriction on the variables to be binary are expressed by (6) and (7).

3.2 Revenue maximisation approach

This heuristic is very similar to the previous one and differs only by its objective function that is based on a pure economic criterion. It attempts, indeed, to maximise the gas revenue rather than the priority:

$$\text{Maximise} \quad \sum_{i=1}^{N} \sum_{l=1}^{t} \sum_{r \in C_i} Rd_{ilr}$$

Subject to: constraints (2) to (7).

Both the assignment models reported in this and the previous section result to be pure integer formulations whose size increases with the number of customers, service days, feasible combinations and trucks. However, they will still be easier to solve with respect to the whole assignment-routing problem.

It is also worthwhile noting that both the above models do not include the obligation of serving the demand of all the customers. The decision of serving or not any of the customers will be based on the selection criteria used and restricted by constraints (4) and (5) related to the vehicles capacity and drivers time limit, respectively. While solving the VRPs in the second stage of the decomposition framework, there will be a set of demand satisfaction constraints that will ensure the service of the all the customers that have been selected by the assignment model in stage 1.

3.3 Greedy approach

Our last approach for the day-customers assignment is based on the use the fundamental characteristics related to each customer concerning his priority and frequency of service. It consists in performing the following steps:

1. Sort all the customers in a non-increasing order of their priority P_i.
2. Consider, in the above order, only customers with priority level bigger than 6 and assign randomly to each of them one of his periodicity combinations in C_i.
3. Check the satisfaction of the capacity Q_k for each vehicle and of the maximum working hours T_k for each driver over all the working days. In case of violation on, say, day l:
    a. select one of the customers assigned to day l having the minimum service frequency and assign him a different periodicity combination
    b. if after a given number of trials the feasibility is not regained, then involve an additional vehicle on day l, if available
    c. otherwise, select one of the customers that are scheduling in day l and declare him as unserved and eliminate him from the other service days related to his combination
    d. repeat step c till regaining the trucks capacity.
4. Sort the yet unserved customers in the decreasing order of priority and, in case of equal priority in the increasing order of frequency of service.
5. Try to accommodate the unserved customers, in the order identified in Step 4, within the available daily vehicles while satisfying the operational constraints.

The idea behind the above heuristic is to guarantee the service of customers characterised by high priority (more than 6, in this case) even by pushing the distribution system to its maximum operational limits. The other customers are served only if they can be accommodated on the available resources by giving priority to lower frequency of service. Obviously, other variants of the same heuristic may be developed by allowing, for example, to hire additional vehicles or overtime work even to serve low-priority customers if required or by trying to accommodate first the customers with highest frequency of service. These variant are not investigated here but left for possible future work.

4 Experimental results

Computational experiments were carried out with the aim of showing the validity of our model and also to compare our heuristics even with respect to the solution adopted manually by the company. The experiments have been based on the following data collected from the weekly operation of the company:

- data concerning the demand of each customer in terms of the number of gas cylinders required every week equally distributed among the service days
- data related to the distance between the depot and the customers and between any two customers
- data related to the trucks such as: speed, capacity, availability of the fleet of trucks and the number working time.

Moreover, our experiments have focused on two specific districts of the capital city *Muscat*, namely *Ameerat* and *Barka,* where the company has important portion of its business. The first district involves 16 customers and the second one 25. The two districts are located too far away to be merged together within the same region (one is located in the north of the Capital and the other is in the south). The two districts will represent, thus, in the sequel two different test problems. All our models, including the one that solves the VRPs exactly, have been developed by using the modelling language and solving package Lingo 13.

All the results, collected while solving realistic instances of our application, are summarised in Table 1. Besides the total distances over the six working days, corresponding to our three methods, the table reports also the practical solution as adopted by the company in that specific week. It is worth noting that all the solutions refer to the minimum distance since it is the standard objective function of most of the VRP. The criteria of priority/revenue maximisation has been used only in Stage 1 of our general framework in order to assign the service days to the customers. The ultimate objective of the company remains minimising the distribution distance while respecting the periodicity and operational constraints.

Table 1 Comparison of the solution obtained by the different methods (in km)

	Priority method	*Revenue method*	*Greedy method*	*Company solution*
Ameerat	1,809	1,839	2,345	*2,862*
Barka	9,168	9,174	9,862	*10,710*
Weekly distance	10,977	11,013	12,207	13,572

The results of Table 1 show how the priority method yields the minimum weekly distance in terms of total travel distance. The heuristic based on the priority maximisation outperforms the revenue maximisation method by 3.2%, the greedy heuristic by 11.2% and the solution adopted by the company by 23.6%. Moreover, the detailed daily schedules (not reported here for brevity) have shown that on one day, over the six working days, there was no need for any distribution service to be executed. This positive side-effect is due to the temporal consolidation of the distribution when the time horizon is extended to one week. During that specific idle day the company can focus on performing some other useful tasks that can improve its level of service to customers. This would never be possible whenever the cylinder distribution to customers is planned on a day-by-day basis as achieved now by the company.

As far as the execution time is concerned, it should be noted how the most time consuming part of the methods (few hours) is solving exactly the VRPs over the six working days. As explained above, this stage is common to all the three heuristic methods. The assignment stage is of the order of few minutes in the case of the first two heuristics and just few seconds for the greedy method. The overall time to solve both the stages have been declared to be acceptable by the company logistics manger even in the case of the biggest among the two test problems (*Barka*). However, it is clear that in case of distribution network expansion or number of customers increase, then larger scale instances should be solved and, thus, heuristic approaches should be explored even for the solution of the daily VRPs.

5 Concluding remarks

A detailed analysis was performed in order to solve a real-life application that refers to the optimisation of the cylinder gas distribution. Our main contribution consists in considering an extended horizon during which the demand of each customer is characterised by a particular pattern having a periodic nature. Our best approach has considered the priority assigned to customers as a key criteria for deciding on how to consolidate the distribution service over the six working days. The daily distribution routes have been then optimally defined by solving several VRPs, one for each day. Computational experiments that refer to two districts of Muscat city were also carried out in order to show the validity of our method and its superiority with respect to other heuristics and also to the solution adopted by the company.

Now we expect from the company to apply our method for a relatively long period of time in order to gain confirmation on its real benefits in minimising the company distribution costs and in the same time in increasing the customers satisfaction. Moreover, the company may also suggest modifications that can further improve the performance of our time-extended periodic approach. Further investigations may consider explicitly in the optimisation models additional environmental impact factors rather than minimising only the travelled distance.

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