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Effect of Three Nucleon Forces in $p - {}^3$He Scattering

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Abstract The effect of the inclusion of different models of three nucleon (3N) forces in $p - {}^3$He elastic scattering at low energies is studied. Two models have been considered: one derived from effective field theory at next-to-next-to-leading order and one derived from a more phenomenological point of view—the so-called Illinois model. The four nucleon scattering observables are calculated using the Kohn variational principle and the hyperspherical harmonic technique and the results are compared with available experimental data. We have found that with the inclusion of both 3N force models the agreement with the experimental data is improved, in particular for the proton vector analyzing power A_y.

1 Introduction

Realistic nucleon-nucleon (NN) potentials reproduce the experimental NN scattering data up to energies of 350 MeV with a χ^2 per datum close to 1. However, the use of these potentials in the description of three-nucleon (3N) and four-nucleon (4N) bound and scattering states gives a χ^2 per datum much larger than 1 (see, e.g., Ref. [1,2]). To improve that situation, different 3N forces have been introduced [3].

Recently, the development of 3N forces has been brought forth following mainly two lines. First, there are 3N force models derived from a chiral effective field theory [4]. At present, these models have been derived at next-to-next-to-leading order (N2LO) of the so-called chiral expansion. At this particular order, the 3N force contains two unknown constants usually determined either by fitting the 3N and 4N binding energies or, alternatively, the 3N binding energy and the tritium β-decay Gamow-Teller matrix element [5]. The 3N force depends also on a cutoff function, which in general includes a cutoff parameter Λ. With a particular choice

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of the cutoff function, a local version of the N2LO 3N interaction (hereafter referred as N-N2LO) has been derived [6]. The parameter Λ is chosen to be for physical reason of the order of 500 MeV. The derivation of chiral 3N force at successive orders is now in progress [7,8].

Alternatively, within a more phenomenological approach, the so-called Illinois model for the 3N force model has been derived [9]. This model has been constructed to include specific two- and three-pion exchange mechanisms among the three nucleons. The model contains a few unknown parameters, which have been determined by fitting the spectra of $A = 4 - 12$ nuclei.

It is clearly very important to test these new models in order to understand how they can describe the nuclear dynamics. The $A = 3$ and 4 scattering observables are among the best testing grounds to understand their validity. It has been proven that most of the $A = 3$ scattering observables are quite insensitive to the effect of the 3N force [10]. It is therefore of relevance to study their effect in $A = 4$ scattering observables.

In recent years, there has been a rapid advance in solving the four nucleon scattering problem with realistic Hamiltonians. Accurate calculations of four-body scattering observables have been achieved in the framework of the Alt-Grassberger-Sandhas (AGS) equations [11,12], solved in momentum space, where the long-range Coulomb interaction is treated using the screening-renormalization method [13,14]. Also solutions of the Faddeev-Yakubovsky (FY) equations in configuration space [15,16] and several calculations using the resonating group model [17,18] were reported. In this contribution, the four-body scattering problem is solved using the Kohn variational method and expanding the internal part of the wave function in terms of the hyperspherical harmonic (HH) functions (for a review, see Ref. [19]).

Very recently, the efforts of the various groups have culminated in a benchmark paper [20], where it was shown that $p - {}^3$He and $n - {}^3$H phase-shifts calculated using the AGS, FY, and HH techniques and using several types of NN potentials are in very close agreement with each other (at the level of less than 1 %).

Since now the 4N scattering observables can be calculated with a good accuracy, it is therefore timely to start to investigate the effect of the 3N force in these systems. It is important to note that the 4N studies performed so far have emphasized the presence of several discrepancies between the theoretical predictions and experimental data. Let us consider $p - {}^3$He elastic scattering, where exist several accurate measurements of both the unpolarized cross section [21–23], the proton analyzing power A_y [23–25], and other polarization observables [26]. The calculations performed so far with a variety of NN interactions have shown a glaring discrepancy between theory and experiment for A_y [11,17,23,25,27]. This discrepancy is very similar to the well known "A_y Puzzle" in $N - d$ scattering. This is a fairly old problem, already reported about 20 years ago [28,29] in the case of $n - d$ and later confirmed also in the $p - d$ case [30]. Also for other $p - {}^3$He observables (the ^{3}He analyzing power A_{0y} and some spin correlation observables as A_{yy}, A_{xx}, etc.) discrepancies have been observed.

In this paper we report a preliminary study of the effect of including 3N forces in $p - {}^3$He elastic scattering calculations in order to see if this inclusion helps in reducing these discrepancies. Clearly, it is important to specify which NN potential has been used together with a particular model of 3N interaction. The 3N force derived from the effective field theory at N2LO has been used together with a NN potential constructed within the same approach, in particular the next-to-next-to-next-to-leading order (N3LO) interaction by Entem and Machleidt [31], with cutoff $\Lambda = 500$ MeV (I-N3LO interaction model). The two free parameters of the N-N2LO 3N potential have been chosen from the combination that reproduces the $A = 3, 4$ binding energies [6]. The Illinois 3N models has been used in conjunction with the Argonne v_{18} (AV18) NN potential [32]. Among the different Illinois models, we have considered the most recent one, the so called Illinois-7 model (IL7) [33].

This paper is organized as follows. In Sect. 2, we discuss the preliminary results obtained with four interaction models, two (I-N3LO and AV18) including only NN interaction and two (I-N3LO/N-N2LO and AV18/IL7) including also the 3N forces under consideration. The conclusions will be given in Sect. 3.

2 Study of the 3N Force

In this Section, we present the results of the inclusion of the considered 3N force models in elastic $p - {}^3$He observables. This work is still preliminary, final results with complete tests of numerical stability will be reported in a forthcoming paper [34]. We present in particular the results obtained using I-N3LO, AV18, I-N3LO/N-N2LO, and AV18/IL7 models.

The HH method and the Kohn variational principle used to perform the calculations are described in Ref. [19]. The accuracy reached by this technique is rather good, as evidenced in Ref. [20], where $p - {}^3$He and

Table 1 Phase-shifts and mixing angle parameters for $p - {}^3$He elastic scattering at incident proton energy $E_p = 5.54$ MeV calculated using the I-N3LO potential. The values reported in the columns labeled HH have been obtained using the HH expansion and the Kohn variational principle, those reported in the columns labeled AGS by solving the AGS equations [12], and those reported in the columns labeled FY by solving the FY equations [16]

Phase-shift	HH	AGS	FY	Phase-shift	HH	AGS	FY
${}^1S_0$	−68.5	−68.3	−69.0	${}^3P_0$	25.1	25.4	25.8
${}^3S_1$	−60.1	−60.0	−60.0	${}^1P_1$	23.0	23.0	23.2
${}^3D_1$	−1.51	−1.45	−1.40	${}^3P_1$	44.3	44.5	44.1
$\epsilon(1^+)$	−1.07	−1.08	−1.18	$\epsilon(1^-)$	9.36	9.28	9.28

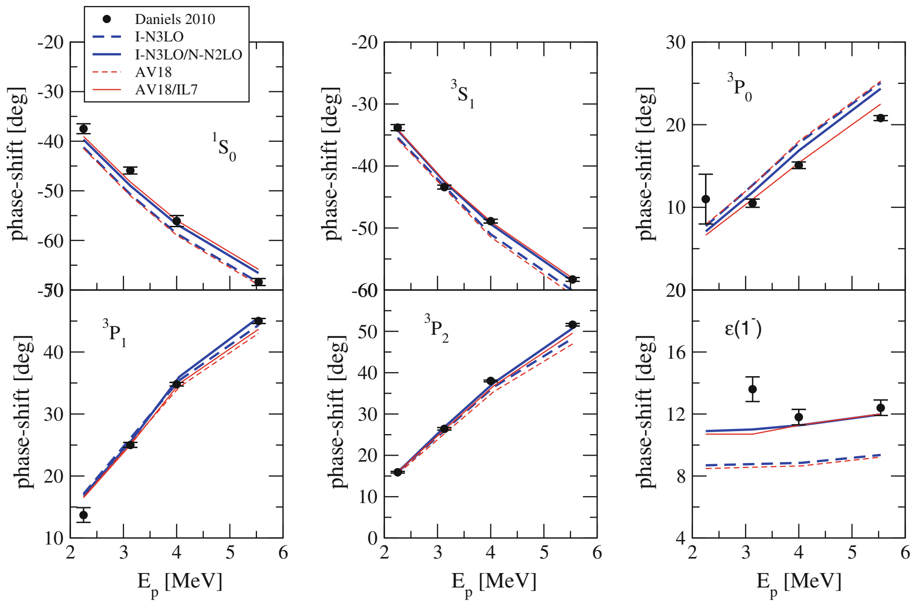

Fig. 1 $p - {}^3$He phase shifts calculated with I-N3LO (*blue dashed line*), I-N3LO/N-N2LO (*blue solid line*), AV18 (*thin red dashed line*), and AV18/IL7 (*thin red solid line*) interaction models. The results of the PSA performed at TUNL have been also reported [26]

$n - {}^3$H phase shifts calculated with the present method were found in good agreement with the results obtained by other groups using the AGS [12] and FY [16] techniques. As an example, a comparison of a selected set of phase-shifts and mixing angle parameters for $p - {}^3$He elastic scattering at $E_p = 5.54$ MeV calculated by means of the HH, AGS, and FY techniques is reported in Table 1. As can be seen, there is a good overall agreement among the results of the three calculations.

In the energy range considered here ($E_p \leq 6$ MeV), the various $p - {}^3$He observables are dominated by S-wave and P-wave phase shifts (D-wave phase shifts give only a marginal contribution, and more peripheral phase shifts are negligible). A comparison of a selected set of calculated phase-shifts and mixing parameters with those obtained by the recent phase-shift analysis (PSA) [26] is reported in Fig. 1. For the ${}^1S_0$ and ${}^3S_1$ phase shifts, the results obtained including NN interactions slightly overpredict (in absolute value) the PSA values. Including the 3N force, the calculated phase shifts agree very well with the PSA values (for both AV18/IL7 and I-N3LO/N-N2LO models). In fact, the $p - {}^3$He interaction in S-wave is repulsive, being dominated by the Pauli repulsion, and the corresponding phase shifts are generally well reproduced by an interaction model giving the correct value of the ^{3}He binding energy (and radius).

Let us consider now P-wave phase shifts. For the ${}^3P_0$ phase shift we observe that the models including NN interaction only overpredict the PSA values. With the inclusion of the 3N forces, the results come close to the PSA values. Note that the ${}^3P_0$ phase shift calculated with the I-N3LO/N-N2LO model still overpredict the PSA values, while the values obtained with AV18/IL7 are in better agreement with data. The ${}^3P_2$ and ${}^3P_1$ phase shifts are underpredicted by the models including NN interaction only, and again including the 3N force the agreement with the PSA values is improved. For these two phase shifts, the values calculated with the I-N3LO/N-N2LO models are in slightly better agreement with the PSA values. Finally, for the 1^- mixing

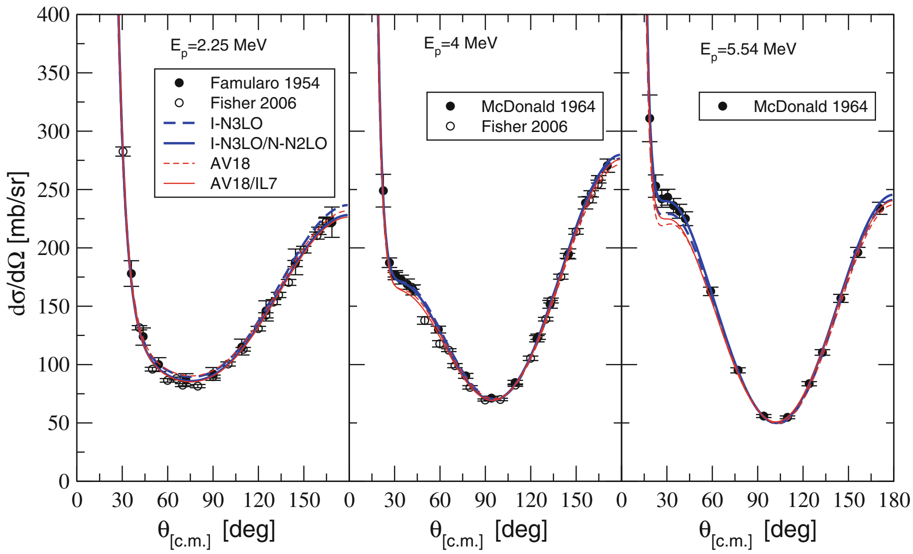

Fig. 2 $p - {}^3$He differential cross sections for three different incident proton energies (notation as in Fig. 1). The experimental data are from Refs. [21–23]

parameter we observe that the theoretical results obtained with the I-N3LO and AV18 models disagree with the PSA values. On the contrary, the results obtained with AV18/IL7 and I-N3LO/N-N2LO are very close to the data. In conclusion, including a 3N force, we observe a general improvement of the description of the S- and P-wave phase shifts and mixing parameters.

Let us now compare the theoretical results directly with a selected set of available experimental data (the complete comparison will be presented in the forthcoming paper [34]). The calculated $p - {}^3$He differential cross sections a energies $E_p = 2.25, 4$, and 5.54 MeV are reported in Fig. 2 and compared with the experimental data of Refs. [21–23]. As can be seen, there is a good agreement between the theoretical calculations and experimental data. This observable is sensitive to small changes of the phase shifts only in the "interference" region around $\theta = 30$ deg. In fact, we note that only the I-N3LO/N-N2LO reproduces the differential cross section there, as is more evident at $E_p = 5.54$ MeV.

More interesting is the situation for the proton vector analyzing power A_y, shown in Fig. 3. Here, we observe a larger sensitivity to the employed interaction model. The calculations performed using I-N3LO and AV18 largely underpredict the experimental points, a fact already observed before [20,23,25]. A sizable improvement is found by adopting both I-N3LO/N-N2LO and AV18/IL7 models, as it was expected from the discussion regarding the comparison with the PSA phase shifts. The analyzing powers calculated including the two 3N force models are very close with each other.

Finally, in Fig. 4, we show a further polarization observable, the ^{3}He spin correlation coefficient A_{yy}. This observable (and other spin correlation coefficients) is found to be not very sensitive to the interaction model. We observe, however, that the I-N3LO/N-N2LO results are in slightly better agreement with data.

3 Conclusions

In this paper preliminary results for $p - {}^3$He elastic scattering including the effect of different of 3N force models have been reported. We have considered two 3N interaction models. The IL7 model is based on a phenomenological approach, being constructed from a select set of two- and three-pion exchange processes among the three nucleons. The N-N2LO model is derived from a chiral effective theory up to N2LO in the chiral expansion. Both models used in conjunction with a "consistent" NN potential (AV18 with IL7 and I-N3LO with N-N2LO) reproduce the $A = 3, 4$ binding energies.

The results obtained for the considered scattering observables have been compared with the available experimental data and a very recent PSA performed at TUNL [26]. We have found that the phase shifts obtained with both the I-N3LO/N-N2LO and AV18/IL7 models are very close with those derived from the PSA. The direct comparison of the theoretical results with the experimental data has shown that there are still some

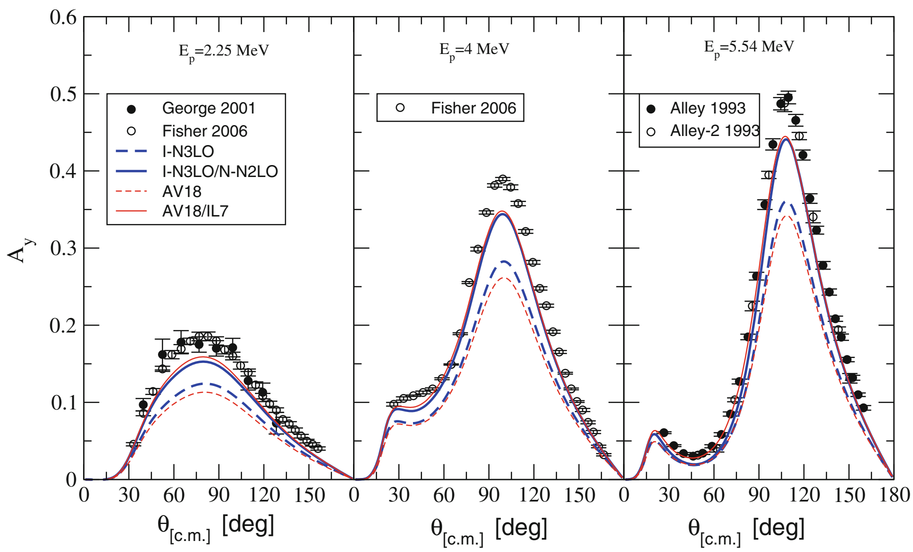

Fig. 3 Same as in Fig. 2 but for the spin-correlation A_y observable. The experimental data are from Refs. [23–25]

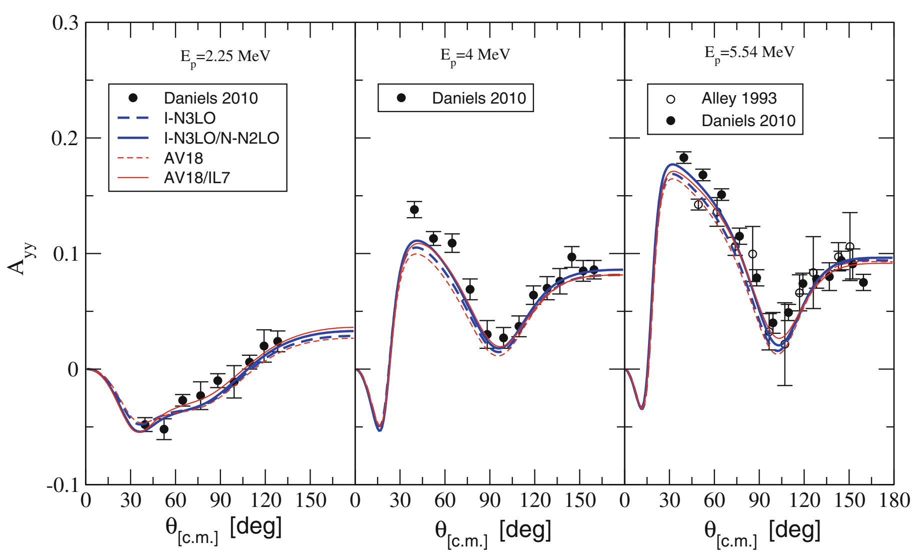

Fig. 4 Same as in Fig. 2 but for the spin-correlation A_{yy} observable. The experimental data are from Refs. [24,26]

discrepancies, but the A_y problem is noticeably reduced. In fact, we observe that now the discrepancy is reduced to be of the order of 10 % at the peak, much less than before. We have also found that the results obtained with the I-N3LO/N-N2LO and AV18/IL7 are always very close with each other (see Fig. 3). Since the frameworks used to derive these 3N force models are rather different, this outcome is somewhat surprising.

Finally, we would like to remark that the previously observed large underprediction of the $p - {}^3\mathrm{He}A_y$ observable was considered to be due to some deficiencies of the interaction in P-waves [27,25], as, for example, due to the appearance of a unconventional "spin-orbit" interaction in $A > 2$ systems [35]. In fact, some of the parameters of the IL7 model have been fitted to reproduce the P-shell nuclei spectra [33]. Therefore, the AV18/IL7 model is constructed to take into account (at least effectively) this unconventional "spin-orbit" interaction and this can explain the improvement in the description of the $p - {}^3\mathrm{He}A_y$ observable. Regarding the N-N2LO 3N force model, its two parameters have been fitted to the $A = 3$ and $A = 4$ binding energies. Therefore, its capability to improve the description of the $p - {}^3\mathrm{He}A_y$ observable is not imposed but it is somewhat built-in. Investigations to understand these issues are in progress. It is interesting to note that in the $N - d$ case, the use of the I-N3LO/N-N2LO model does not give a significant improvement in the solution of the "A_y puzzle" [2]. A detailed analysis of effect of the AV18/IL7 interaction in $A = 3, 4$ systems is currently underway [34]. Finally, it will be certainly very interesting to test the effect of the inclusion of the next order 3N forces derived from the effective field theory.

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