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Recognition of intrinsic quality properties for automatic geometric inspection

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Abstract In the last few years the need for methodologies capable of performing an automated geometric inspection has increased. These methodologies often use 3D high-resolution optical digitisers to acquire points from the surface of the object to be inspected. It is expected that, in the near future, geometric inspection will be requiring more and more the use of these instruments. At present geometric inspection is not profiting from all the opportunities attainable by 3D high-resolution optical scanners or from the numerous tools which can be used for processing the point cloud acquired from the inspected product. For some years now, these authors have been working on a new methodology for automatic tolerance inspection working from a 3D model acquired by optical digitisers. In this paper all the information recognisable in a scanned object is organised into a new data structure, called *Recognised Geometric Model* (*RGM*). The final aim is to define a representation of the inspected object for the automatic evaluation of the non-idealities pertaining to the form, orientation and location of the *non-ideal features* of the acquired object. The key concept of the proposed approach is the capability to recognise some *intrinsic nominal properties* of the acquired model. These properties are assumed as references to evaluate the non-idealities of the inspected object. With this approach the references of geometric inspection are searched for in the inspected object independently of a tolerance specification and of the availability of a 3D nominal representation. The high-level geometric information within RGM depends on the *rules* used for its identification. The capability to recognise specific categories of nominal references offers the possibility of introducing new tolerances to be specified. The proposed approach has been implemented in original software by means of which a specific test case has been analysed.

Keywords ISO tolerancing · Three-dimensional metrology · Automated inspection

1 Introduction

Geometric inspection of industrial products is an important phase of the manufacturing process; it serves to verify whether the manufactured parts comply with the geometric requirements identified during the designing process. In the last few years, the push toward the manufacture of products having an ever more complex and varied geometry, as well as the ever higher demands for geometric accuracy have made geometric inspection crucial and labour-intensive. As a consequence, the need for methodologies capable of performing automated geometric inspection has increased.

In order to specify the design geometric requirements of industrial products, dimensional and geometric tolerances must be prescribed. Any language intended for tolerance specification is based on:

- the measuring instruments available for inspection;
- the way to represent the product geometry.

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Several properties of measuring instruments affect the way to specify tolerances. For example, the resolution of the device identifies the minimum permissible deviation which the instrument is capable of detecting and which is therefore possible to specify. Another important aspect is the way to inspect the object geometry: based on the measuring method, specific geometric properties can be evaluated and then specified. The nature of the geometric properties that can be evaluated results from the way the measure is given as output and how this output can be processed. In CMM the object surface is usually touched using mechanical probes at the end of an arm so that point data can be captured in a set of few selected discrete points. By using high-resolution optical digitisers the surfaces of the object can be digitised in the form of a 3D high-density point cloud. Having all these data at our disposal allows us to detect the geometric properties of the acquired object and to estimate even some differential geometric properties. Consequently, it is possible to inspect not-usually-adopted categories of geometric properties and to specify new tolerance categories. The extraction of high-level geometric information from high-density point cloud requires proper data processing methodologies. Nowadays optical instruments for 3D high-density scanning offer an excellent performance with high accuracy and short measurement times. It is expectable that in the near future the use of these instruments will spread widely.

As regards the way to represent the product geometry, currently 3D CAD models are extensively used. They supply a representation of the object geometry which is coherent with the way to inspect an object by 3D high-resolution optical digitisers. 3D models are semantically richer than the traditional 2D representations on which tolerance specifications are at present based.

ISO has proposed a language, acronymed Geometric Product Specification (GPS), to express the design geometric requirements. This language focuses on the measuring instruments available for inspection at the time of the writing of the standards (i.e. CMM, gauges, dial gauges, etc.). At the moment, GPS standards do not adequately take into account the opportunities brought about by high-resolution optical digitisers. The GPS language is mainly confined to a 2D representation of the product geometry, which is obtained by projecting the object orthogonally onto a plane.

For some years now, the present authors have been working on a new methodology for automatic tolerance inspection which takes advantage from the capability to acquire a high-density 3D model of the object to be inspected and from a 3D representation of the specified object. In this paper all the information recognisable in a scanned object is, at the current state of our research project, organised into a new data structure, called *Recognised Geometric Model* (*RGM*). The final aim is to define a representation of the inspected object for the automatic evaluation of non-idealities in the form, orientation and location of the *non-ideal features* of the acquired object. The key concept of the proposed approach is the capability to recognise some *intrinsic nominal properties* of the acquired model. These *intrinsic nominal properties* are assumed as references to evaluate the non-idealities of the inspected object. Following this approach, the references of geometric inspection are searched for in the inspected object independently of a tolerance specification and of the availability of a 3D nominal representation. RGM represents the geometric entities and the mutual geometric properties that are automatically recognised in the inspected object. The high-level geometric information within RGM depends on the *rules* used for its identification. The capability to recognise specific categories of nominal references offers the possibility of introducing new tolerances to be specified. The RGM data structure can be *queried* in order to gather some information pertaining to intrinsic geometric features and/or their related non-idealities.

The proposed approach has been implemented in original software, coded in C++, by using a library dedicated to the processing of tessellated geometric models, which has been developed at the University of L'Aquila. In order to verify the reliability of the methodology, a specific test case has been analysed, which refers to a real object whose acquisition has been carried out by means of an optical scanner (http://www.scansystems.it).

2 Literature review

For the last few years a great number of methodologies have been proposed with a view to performing an automated geometric inspection of the manufactured parts [1–3]. Generally speaking, these methodologies require as input the CAD model of the workpiece to be inspected and use the high-resolution optical digitisers to acquire points from the surface of the object. The CAD model provides a nominal geometric description of the object in which the *nominal references* are in the form of parameterised equations describing the surfaces of the geometric model [1–4]. This model describes explicitly the mutual geometric relationships between the surfaces; these relationships include adjacency, orientation and localisation.

Tolerance specifications usually refer to some of the object's features and, for each of them, it is necessary to identify the corresponding scanned point sub-cloud. These specifications can be either interactively defined by the user or included in the CAD model. At this time there exists no standard language aimed at tolerance specification in 3D models which is suited to the automatic verification of industrial products. In [1] the authors propose and test a methodology for automatic inspection of dimensional and geometric tolerances. Firstly the 3D acquired point cloud is registered with the CAD model of the workpiece by an *Iterative Closest Point* (*ICP*) algorithm. Then the point cloud is segmented by identifying the points matching the nearest features of the CAD surface.

Gao et al. in [3] build up a geometric inspection system within commercial software for reverse engineering. In order to specify interactively dimensional and geometric tolerances, Gao et al. define a *Nominal Inspection Frame* (*NIF*). This *NIF* is associated to a reference model which can be a CAD model or a Master Model in the form of a tessellated surface. Once the *NIF* is created, the scanned data are then registered with the reference model.

In the aforementioned approaches form inspection is carried out verifying that each point of the sub-cloud to be inspected is within the tolerance zone of amplitude t, which is limited by two virtual surfaces placed inside and outside the reference surface at a distance t/2 from it. The approaches presented in literature do not extract high-level information from the acquired geometric data and are therefore constrained to use specification "languages" which are poorer than those traditionally defined by ISO or ANSI/ASME. In ISO 1101 [5] and ASME 14.5Y [6], for example, form tolerances, such as straightness or circularity, can be applied to derived or extracted features (axis, planar section of the surface, etc.), but these tolerances are not used in the approaches by Prieto [1] and Gao [3]. Moreover, these methods do not profit from all the opportunities which high-resolution optical digitisers offer or from the numerous tools which can be used for processing the acquired point cloud. If these opportunities were grasped, new categories of tolerance could be introduced in accordance with the *duality principle* [7], according to which measurement should mirror specification.

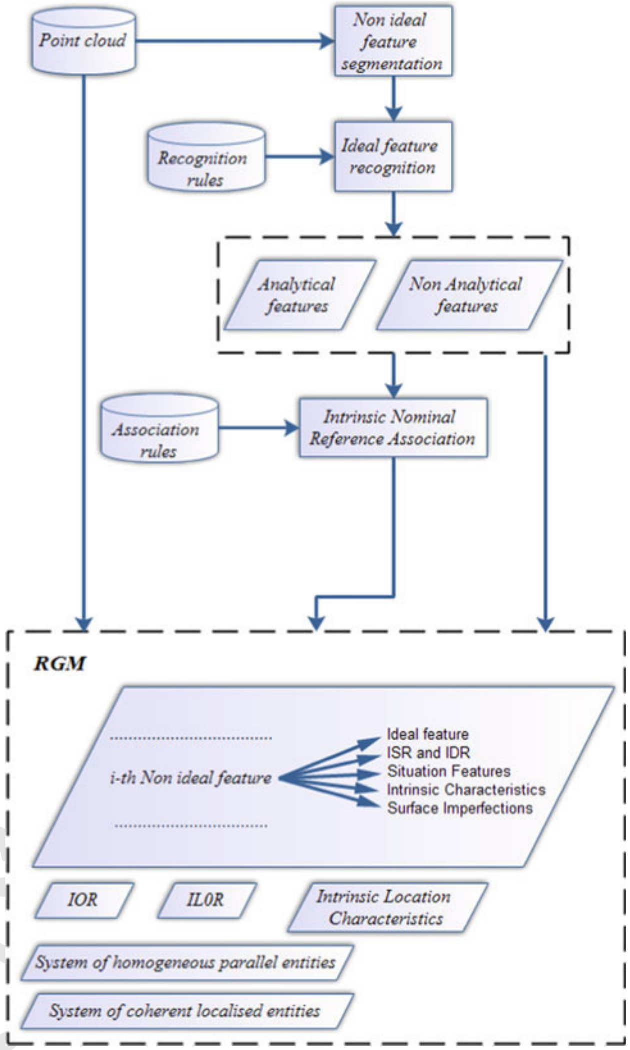


Fig. 1 The several phases of the methodology here proposed for ideal geometric property recognition

3 The concept of non-ideal feature

Some ideal geometric properties can be recognised from an acquired high-density object. Generally speaking, these properties, which refer to specific features of the inspected object, pertain to form, orientation and location and can be automatically recognised by means of specific *rules*. These rules intrinsically include the criteria to identify well-defined ideal geometric categories.

The recognition of the nominal geometric properties of a discretised surface can be performed if the acquired surface has been previously recognised to be regular. In a continuous surface, a point is *regular* if two conditions are satisfied: derivatives of all orders exist (the surface at the point is a differentiable surface or C^∞) and a tangent plane exists [8]. The vertices of a non-planar tessellated surface are naturally *non-regular points* since just C^0-continuity is verified. Therefore, the point regularity property must be recognised by using some criteria and cannot be assumed as an objective property of the surface. When the tessellated surface is obtained by scanning a real object, non-regular points can be associated with surface singularities such as imperfections or edges. In what follows the *regular points* of the tessellated surface are those where C^2-continuity is recognised so that differential geometric properties (surface normals and curvatures), necessary to geometry analysis, can be defined.

Figure 1 shows the several phases of the methodology here proposed to recognise the ideal geometric properties of the object under inspection. Firstly a complex *segmentation process* is carried out with the aim to identify the *non-ideal features* and the associated category of recognisable ideal properties. This segmentation process is performed by a hybrid approach based on fuzzy logic, which is more detailed in [9].

Several surface imperfections, such as scratches, dents or grooves, can be found within a *non-ideal feature*. These defective zones appear as islets (usually non-regular)

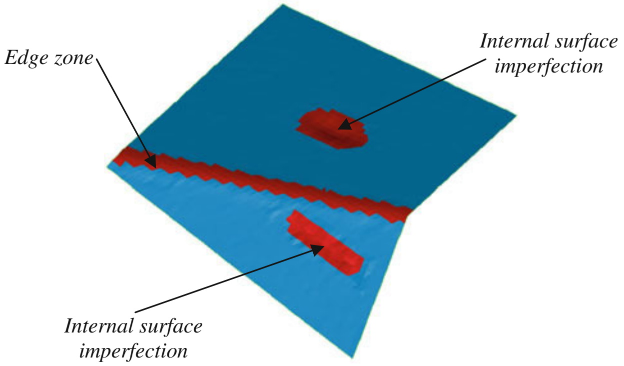


Fig. 2 Internal surface imperfections and edge zone segmentation

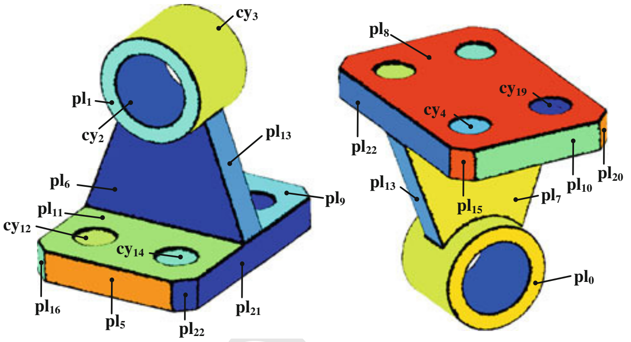


Fig. 3 Application of the first two steps of the methodology to an acquired high-density workpiece

embedded in the regular portion of the *non-ideal feature* (Fig. 2). In the acquired model the edge usually appears as a non-regular zone located between the *non-ideal features* sharing the edge. Due to the acquisition process the edge is sampled with points which pertain to the *edge zone* (i.e. points belonging to the defective surface of the edge) or which lie extended across the edge. How accurately the edge is acquired depends on the sampling rate and on the width of the surface associated with the *edge zone*.

A *non-ideal feature* is a portion of the external surface of the inspected object which is substantially characterised by specific geometric properties and consists of a regular connected zone inside which islets of the non-regular surface can be found. The *regular portion* of a non-ideal feature consists of points of the same type (flat, umbilical, ruled and generic). The type of point is deduced by investigating some differential geometric properties of the tessellated model.

The recognition of the ideal geometric properties of the tessellated model applies to the *regular portion* of each *non-ideal feature*. Non-regular parts of the *non-ideal feature* are considered as *surface imperfections* associated with the *non-ideal feature*.

According to the ISO standards, an *ideal feature* is associated with each *non-ideal feature*. The *ideal feature* denotes the geometric type of the feature. It does not identify quantitative elements, although some measurable (dimensionable) intrinsic characteristics (diameter, apex angle, etc.) and situation features (centre, axis, etc.) can be identified. The *ideal feature recognition* is carried out by investigating some intrinsic local and global differential properties of the surface portion of the *non-ideal feature*. The *ideal features* can be classified into two main categories: *analytical* and *non-analytical features*. *Analytical features* are those recognised to pertain to an analytical geometric surface (plane, sphere, cylinder, cone, etc.). An analytical type of geometric surface is associated with these *ideal features*. The other *ideal features* are defined to be *non-analytical features*. They also include surfaces characterised by some specific and recognisable geometric properties. Based on the recurrence of specific differential geometric properties among the points pertaining to the *non-ideal feature*, they can be classified as: generic extruded, generic cone, generic axially symmetric. The *non-analytical features*, which cannot be included within any of the previous categories, are named *free-form* features. In any case, for any non-analytical feature a parameterised equation can be associated with a CAD model by a *registration process*.

In Fig. 3 these first two steps of the methodology are applied to an acquired high point density workpiece. The figure shows as coloured areas the *non-ideal features* deriving from the segmentation process. The edges are also highlighted in the same figure; these parts of the inspected object do not pertain to any *non-ideal feature*. A label expressing the *ideal feature* is associated with each *non-ideal feature*.

4 The intrinsic nominal references

An *intrinsic nominal reference* is an ideal analytical geometric entity which characterises the *ideal feature*; it is located by the quantitative data retained in the *non-ideal feature*. The identifiable *intrinsic nominal references* can be of different types:

- *Intrinsic Shape Reference (ISR);*
- *Intrinsic Derived Reference (IDR);*
- *Intrinsic Local Reference (ILR);*
- *Intrinsic Orientation Reference (IOR);*
- *Intrinsic Position Reference (IPR).*

The first three types are associated with ideal shape properties, which are globally and/or locally evaluated. The last two pertain to mutual geometric properties between *intrinsic references* associated with *non-ideal features*.

The ideal geometric properties recognisable in the inspected object are organised into a data structure, called

Recognised Geometric Model (*RGM*). This is an idealised geometric representation which can be described by a hyper-graph structure denoted by $RGM(V, \varepsilon)$. In RGM(V, ε) V is the finite set of *nodes* v_i of the hyper-graph and ε is a family of sub-sets of V, known as *hyper-edges*. Each node v_i is associated with a *non-ideal feature* identified by the above-mentioned segmentation process. A few labels, which describe some attributes of the *non-ideal feature* (such as, for example, the *ideal feature*) are assigned to each node of the hyper-graph. ε involves two sub-sets of V, $\varepsilon = \{V_A, V_{GR}\}$, which represent the *non-ideal features* for which adjacency relationships and mutual geometric properties are recognised. Associated with an adjacency relationship between two *non-ideal features* in V_A there is an edge and/or an edge zone in the point cloud. V_{GR} identifies the *non-ideal features* which are oriented and located with respect to each other in a specific way; it includes two sub-sets of V ($V_{GR} = \{V_{OR}, V_{PR}\}$) which represent, respectively, the *non-ideal features* for which orientation and location properties are recognised. In the *Mutual Orientation Relationship Graph* $MORG = (V_{OR}, E_{OR})$ the arcs (E_{OR}) identify mutual orientation properties between connected nodes. Analogously, the *mutual location relationship graph* $MLRG = (V_{LR}, E_{LR})$ is a graph where each arc identifies a mutual location property between connected nodes.

4.1 The Intrinsic Shape Reference (ISR)

An *ISR* is recognised when the points of the regular portion of a *non-ideal feature* can be considered as lying on an analytical surface (for example, plane, sphere, cylinder, cone, torus, etc.). In order to estimate this reference, an analytical surface of the type of the *ideal feature* and approximating the points of the regular portion must be computed. Depending on the association rule used to approximate the feature points, different *ISRs* can be obtained. Typical association rules are L_2 and L_∞-norm rules. Within *RGM* the *ISR* is stored as a label of the node representing the *non-ideal feature* with which this reference is associated. Based on the type of *IS*R, some dimensionable intrinsic geometric parameters, referred to as *Intrinsic Characteristics*, can be identified. Table 1 lists the intrinsic characteristics of the *ISR*s which can be recognised by the methods implemented in the present work.

Table 1 The intrinsic characteristics of the *ISR*s

Type of ISRs	Intrinsic characteristics
Plane	None
Sphere	The diameter d
Cylinder	The diameter d
Cone	The apex angle α

4.2 The Intrinsic Derived Reference (*IDR*)

An *IDR* is a *derived feature* [10] since it cannot be directly identified in the acquired object, but can nevertheless be derived from the measured data. *IDR* is an analytically-known geometric entity which is identified when some geometric properties of a feature are previously evaluated. Examples of *IDR*s are the axis and the circular sections of axially symmetric surfaces or the extrusion direction of extruded surfaces. In many cases the *IDR*s are the situation features of the *non-ideal features,* so they orient and/or locate the features. Other *IDR*s could be defined based on functional or manufacturing properties of the acquired object, such as, for example, the symmetry plane of a *free-form* mirrored surface or the base cylinder of a cylindrical gear. The implementation of these *IDRs* in the *RGM* requires the introduction of specific *recognition rules*.

IDRs are estimated by approximating the point cloud, which identifies a non-ideal feature, by one or more *association operations*. Depending on the *association rule* which is used, different *intrinsic references* can be estimated. The *association rule*, therefore, affects significantly the subsequent evaluation of non-ideality. In particular, the axis is an *IDR* whose identification is important for the recognition of other *IDRs*, such as the circular sections of axially symmetric surfaces. In this work an *association rule*, which is based on the *curvature centres method* [11], is implemented for its estimation. The axis thus evaluated is used to identify the set of cutting planes (perpendicular to the axis), from which the circular sections of axially symmetric surfaces can be identified. Each of them is a reference ideal circle obtained by applying the L_2-norm rule to the points derived from the projection of the point data in the neighbourhood of the cutting plane onto it.

Within *RGM* the *IDR* is stored as a label of the node representing the non-ideal feature from which this reference is derived.

Table 2 The situation features for each type of *non-ideal feature* considered

Type of non-ideal feature	Situation features
Plane	The plane Π
Sphere	The centre $\wp$
Cylinder	The axis r
Cone	The axis r
	The apex $\wp$
Generic axially symmetric	The axis r
Generic extruded[a]	The extrusion direction r
Generic cone	The apex $\wp$

[a] Unlike the other situation features the extrusion direction is a non-localised situation feature

4.3 The Intrinsic Local Reference (*ILR*)

Intrinsic Local References identify a new category of nominal references, which is not considered in the current tolerancing standards. An *ILR* is a geometric entity, defined in a point of the regular portion of a *non-ideal feature*, which identifies local geometric properties of a surface. Typical *intrinsic local references* are the tangent plane in a point **p** and the differentiable surface which approximates the object in the neighbourhood of **p**. These references serve to evaluate locally the non-idealities of the surface or to verify some specific geometric properties which are identified in the recurrence of intrinsic local properties all over the surface or along a straight line, such as, for example, *ruledness* or *rigosity*. The main aim of *ILR* is analysing the differential geometric properties which characterise the shape of the feature. Due to the discrete nature of the inspected model, these properties require that a finite-sized region should be investigated by analysing a set of points in the neighbourhood of **p**.

4.4 The Intrinsic Orientation Reference (*IOR*)

An *IOR* is detected whenever a mutual geometric property of parallelism and perpendicularity or frequently recurring orientations (such as 30°, 45° and 60°) are recognised between two *non-ideal features*. These properties are recognisable between *non-ideal features* whose situation features include a spatial direction, such as plane, cylinder, cone, generic axially symmetric and generic extruded. Table 2 lists the situation features for each type of *non-ideal feature* being considered.

In order to recognise the *intrinsic orientation properties* it is suitable to classify the features into two types: planar features (henceforth referred to as *P-feature*) and features whose situation feature is an axis or an extrusion direction (henceforth referred to as *R-feature*). The orientation is recognised by specific rules. In this work, these rules are based on the evaluation of the dot product of the angle between the spatial directions orienting the features. For a given intrinsic orientation property the dot product varies depending on whether the features are of the same type (that is to say, both features are *P-features* or *R-features*) or of a different type (Table 3). Owing to the non-ideality of real objects, the dot product never exactly matches the ideal value. Consequently, an IOR is recognised if the dot product falls within a properly given tolerated range around the expected value. Each mutual orientation property detected is represented in RGM by connecting with an arc the *non-ideal features* involved.

The mutual parallelism properties between features of the same type (*R-features* or *P-features*) should be the first to be recognised. They are represented in the *Mutual Parallelism Relationship Graph* (*MPRG*) as two distinctive components $G_{//R} = (V_{//R}, E_{//R})$ for *R-features* and $G_{//P} = (V_{//P}, E_{//P})$ for *P-features*. Figure 4a, b furnish these two graphs for the test case shown in Fig. 3).

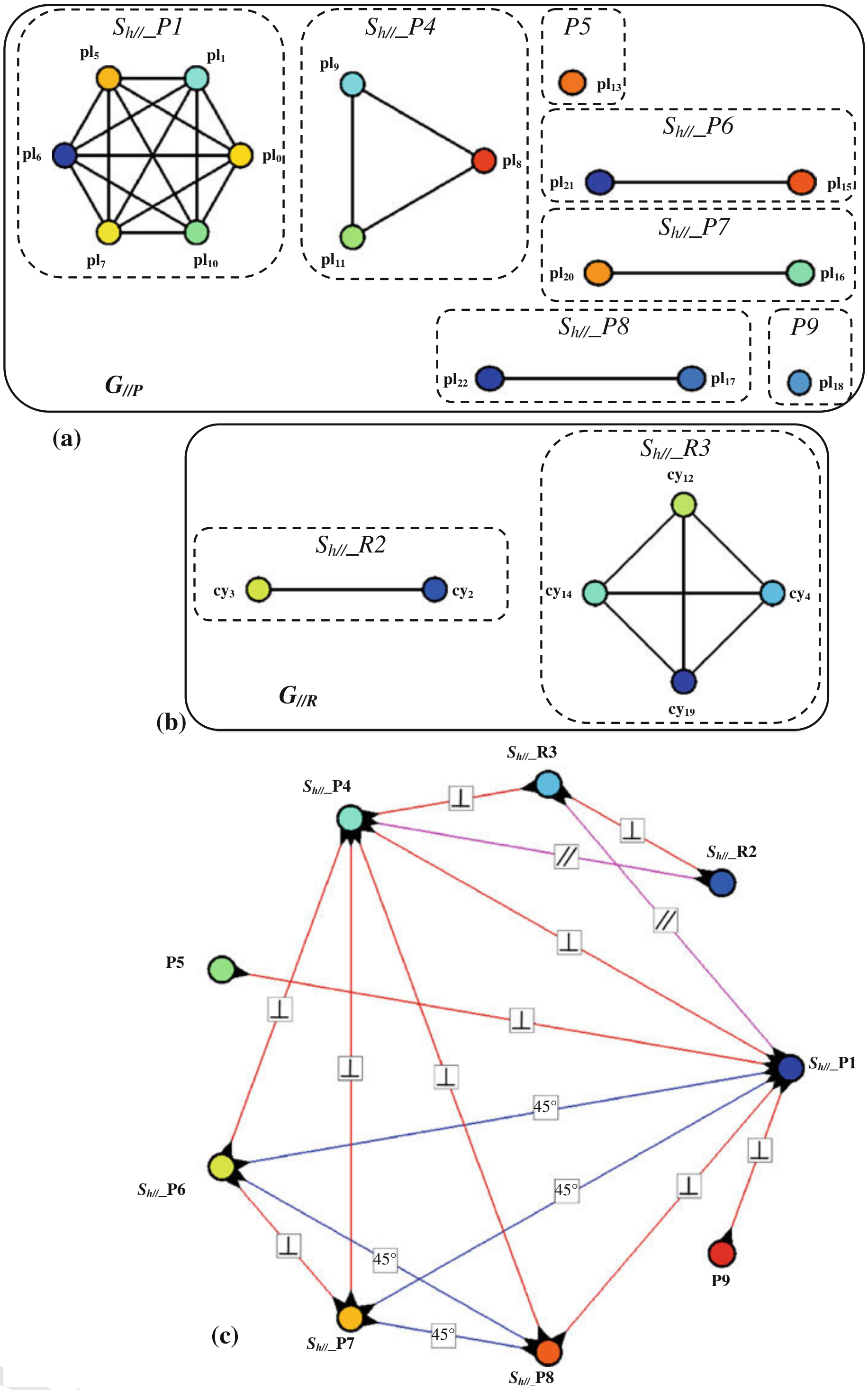
All the features oriented toward the same direction are highlighted in *MPRG* as connected components. In *MPRG* there are as many connected components as spatial directions for which a parallelism property has been recognised. If a connected component involves more than two features (or nodes) a further control should be carried out. This control consists in verifying the transitive property, which imposes the coherence of the parallelism properties between all the features involved. This is the same as verifying that the graph is completely connected. A set of N parallel features of the same type, which satisfy the transitive property, is referred to as *system of homogeneous parallel entities* ($S_{h//}$). In particular two types of systems can be identified: the system including *R-features* ($S_{//R}$) and that including *P-features* ($S_{//P}$). These systems can be automatically detected by analysing the graphs $G_{//R}$ and $G_{//P}$. A component of $G_{//R}(G_{//P})$, which is characterised by N nodes, is a *system of homogeneous parallel entities* $S_{//R}$ ($S_{//P}$) if the degree of each node is equal to N − 1. For example, for the object shown in Fig. 3, seven $S_{h//}$ systems are recognised (Fig. 4a, b). Each $S_{h//}$ in $G_{//P}$ or $G_{//R}$ is associated with a reference spatial direction.

In practical cases the transitive property cannot be verified, so a non-coherent set of parallel features is generated. In order to solve this incoherence two approaches can be used. The

Table 3 The rules for intrinsic orientation property recognition

Dot product value		Intrinsic orientation property	Symbol
Features of the same type	Features of a different type		
0	1	Perpendicularity	$\boxed{\perp}$
±0.5	$\pm\sqrt{3}/2$	Angular orientation of 60°, −60°, 120° and 240°	$\boxed{60^\circ}$
$\pm\sqrt{2}/2$	$\pm\sqrt{2}/2$	Angular orientation of 45°, −45°, 135° and 225°	$\boxed{45^\circ}$
$\pm\sqrt{3}/2$	±0.5	Angular orientation of 30°, −30°, 150° and 210°	$\boxed{30^\circ}$
±1	0	Parallelism	$\boxed{//}$

Fig. 4 **a** The sub-graph $G_{//R}$ and the $S_{h//R}$ recognition for the inspected object. **b** The sub-graph $G_{//P}$ and the $S_{h//P}$ recognition for the inspected object. **c** The *MORG* for the inspected object



first consists in imposing the missing parallelism conditions so that the transitive property is verified. The second approach splits the incoherent set of parallel entities into two or more *systems of homogeneous parallel entities*. Which of these approaches needs to be adopted is specified by the parallelism recognition rule. Whatever the approach adopted, uncertainties are produced. In particular, the first approach may imply the recognition of non-true parallelism properties between the features. In the second approach, the recognition of true parallelism properties may fail.

The recognition of the *systems of homogeneous parallel entities* allows us to build efficiently the *Mutual Orientation Relationship Graph* $MORG = (V_{OR}, E_{OR})$. In this graph the nodes (V_{OR}) are *systems of homogeneous parallel* *entities* $S_{h//}$ or single features which do not belong to any $S_{h//}$. The arcs (E_{OR}) identify mutual orientation properties between connected nodes. An orientation property recognised between two entities is extended to the *systems of homogeneous parallel entities* to which the entities pertain. In order to specify the type of orientation property recognised (i.e. parallelism, perpendicularity, 30°, 45° and 60°), a label is assigned to each arc. The *MORG* in Fig. 4c refers to the object shown in Fig. 3.

Table 5 The geometric conditions to be verified between the situation features in order to recognisecoincidence

Situation feature	Point $\wp$	Axis r	Plane Π
Point $\wp$	$\equiv$	$\in$	$\in$
Axis r	$\in$	$\cap$	$\in$
Plane Π	$\in$	$\in$	–

$\equiv$, Coincidence between points; $\in$, belonging; $\cap$, intersection

4.5 The Intrinsic Position Reference (IPR)

An *IPR* is detected whenever a mutual geometric property of *coaxiality, concentricity* or *coincidence* is recognised between *non-ideal features*. In any case *IPR* recognition requires the identification of a *localised situation feature* (i.e. a situation feature whose location can be unequivocally identified within an arbitrarily given reference frame) for each *non-ideal feature*. In Table 4 the intrinsic position properties for each combination of *ideal feature* types are listed.

Two axially symmetric features are coaxial to each other if they share the same axis. In order to recognise coaxiality, first the parallelism is detected and then the related distance between the located features is evaluated. If the distance value falls within a properly given tolerated range around 0, then coaxiality is recognised.

Two types of coincidence are here considered. The first type, simply named *coincidence* in Table 4, is an intrinsic position property, which refers to the mutual intersection between the situation features locating the *non-ideal features*. Table 5 shows the geometric conditions which must be verified between the situation features in order for this property to be recognised.

The ISO 1101 [5] introduces the concept of *concentricity* in order to control the coincidence existing between the centres of two circular sections. This is a property confined to the necessity of using a 2D representation of the geometry. In this paper, the concept of *coincidence* is semantically richer since not only does it include the *concentricity* property defined by the ISO but it is also suited to be used in a 3D representation of the product geometry.

The second type of coincidence, which is here referred to as *S-coincidence* (and whose symbol here is **=-=**), is a property detected between different *non-ideal features* that can be approximated by the same analytical function. The recognition rules for *S-coincidence* vary depending on the *ideal feature* type associated with the *non-ideal feature*. The rules are briefly described in Table 6.

When an intrinsic location property of a specific type (i.e. coaxiality, coincidence or S-coincidence) is recognised for all the possible pairs between N features (N $\geq$ 3), the transitive property must be verified. The transitive property is verified when the recognised type of location property involving N features gives rise to $m = \sum_{i=1}^{N} (N - i)$ location relationships. This set of features is referred to as *system of coherent localised entities* (S_{Lo}).

In order to represent intrinsic location properties, a specific graph is defined: the *mutual location relationship graph* $MLRG = (V_{LR}, E_{LR})$. It is a graph where each arc identifies a mutual location property between connected nodes. A label is assigned to each arc; it indicates the type of location property recognised (*coaxiality, coincidence* and *S-coincidence*). The *systems of coherent localised entities* can be automatically detected working from this graph. In *MLRG* there are as many components as *non-ideal features* for which an intrinsic location property has been recognised. Figure 5 shows the *MLRG* of the test case in Fig. 3.

Table 4 The intrinsic location properties recognised for the *ideal feature* types

Feature type	Plane	Sphere	Cylinder	Cone	Generic axially symmetric
Plane	S-coincidence	Coincidence	Coincidence	Coincidence	Coincidence
Sphere	Coincidence	Coincidence S-coincidence	Coincidence	Coincidence	Coincidence
Cylinder	Coincidence	Coincidence	Coaxiality Coincidence S-coincidence	Coaxiality Coincidence Coaxiality	Coaxiality Coincidence
Cone	Coincidence	Coincidence	Coaxiality Coincidence	Coincidence S-coincidence	Coaxiality Coincidence
Generic axially symmetric	Coincidence	Coincidence	Coaxiality Coincidence	Coaxiality Coincidence	Coincidence Coaxiality

Table 6 The rules for the *S-coincidence* property recognition

Type of analytical features A and B	Required mutual geometric relationship	Further condition to be verified
Plane	Parallelism of the planes $(\Pi_A // \Pi_B)$	Distance between Π_A and $\Pi_B < \varepsilon_0$ $(dist(\Pi_A, \Pi_B) < \varepsilon_0)$
Sphere	Coincidence of the centre $(\wp_A \equiv \wp_B)$	Difference between diameters $< \varepsilon_1$ $(\lvert d_A - d_B \rvert < \varepsilon_1)$
Cylinder	Coaxiality between the axes r_A and r_B $(r_A \circledcirc r_B)$	Difference between diameters $< \varepsilon_1$ $(\lvert d_A - d_B \rvert < \varepsilon_1)$
Cone	Coaxiality between the axes r_A and r_B $(r_A \circledcirc r_B)$	*Difference between apex angles* $< \varepsilon_2$ $(\lvert \alpha_A - \alpha_B \rvert < \varepsilon_2)$ *Coincidence of the apexes* $(\wp_A \equiv \wp_B)$

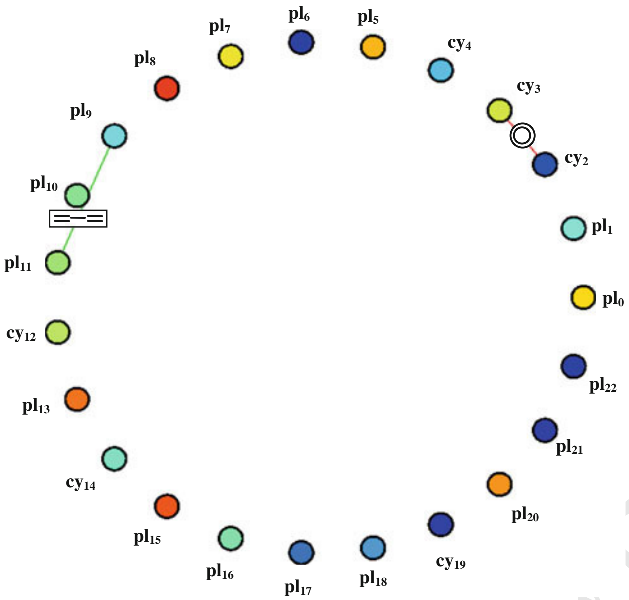


Fig. 5 The resulting *MLRG* for the test case analysed

5 Non-ideality evaluation

Recognised Geometric Model is an idealised geometric representation of an acquired high-density object aimed at the automatic non-ideality evaluation. This operation always requires the identification of a nominal (or ideal) reference. In a previous work [12] the nominal references have been classified into two main categories: *explicit* and *intrinsic*. The *explicit* references can be provided by a specification, also by using a CAD model through a registration process [1,3]. The *intrinsic references* can be detected in the inspected object by the recognition of nominal (or ideal) geometric characteristics. The *intrinsic references* here considered are those previously defined and stored in the *RGM* data structure. Working from the comparison of the object with the *intrinsic references,* an evaluation of its quality characteristics can be performed independently of the given explicit references.

The measure of non-ideality for a feature or for a geometric element derived from it is carried out by estimating the deviations of the feature's points from the associated *intrinsic references*. The non-ideality measurement can be both the maximum distance (*Max*) of the points from the reference or the *Root Mean Square Error* (*RMSE*) of these distances. If the non-ideality is expressed in terms of the *RMSE*, a probabilistic estimation of the location of the acquired points, with respect to the *intrinsic reference*, can be performed. *Max* is affected by singular errors that can be ascribed to the measuring process, while *RMSE* performs a mean evaluation of the surface deviation. The probability to detect points which are outliers can be very high when optical scanners are used, so the *RMSE* approach can be said to be more robust than the *Max* approach. The rule which is to be used to evaluate the reference (*ISR* and *IDR*) is preferably the L_∞-rule when the non-ideality is measured as *Max*, and the L_2-rule when the non-ideality is measured as *RMSE*.

5.1 Form non-ideality evaluation

In Table 7 the various categories of *form non-idealities*, which can be estimated for each type of *non-ideal feature*, are listed together with the related symbols. They include some new form non-idealities, such as *conicity, sphericity, total roundness, surface and profile regularity*, which are not considered in the current tolerancing standards. The definition of these new categories of form non-idealities is provided by the specific nature of the measured data, by the way to process them and by the possibility of simulating 3D complex virtual analytical references.

The form non-idealities of *conicity* and *sphericity* refer to *analytical features* and concern the deviation of the *non-ideal feature* from the *ISR*.

5.1.1 Total roundness non-ideality

Total roundness non-ideality can be automatically evaluated for a feature that is recognised to be characterised by

Table 7 Types of non-ideal feature and related form non-idealities

		Type of feature							
		Plane	Sphere	Cylinder	Cone	Generic axially-symmetric	Generic extruded	Generic cone	Free form
Global form non-ideality									
Straightness	⏤	–	–	Extracted median line	Extracted median line	Extracted median line	–	–	–
Flatness	⏥	X	–	–	–	–	–	–	–
Roundness	○	–	–	Any extracted cross-sectional circumferential line	Any extracted cross-sectional circumferential line	Any extracted cross-sectional circumferential line	–	–	–
Total roundness	⊗	–	–	X	X	X	–	–	–
Cylindricity	⌭	–	–	X	–	–	–	–	–
Conicity	\ /	–	–	–	X	–	–	–	–
Sphericity	⊕	–	X	–	–	–	–	–	–
Regularity non-ideality									
Profile	∿	X	X	X	X	X	X	–	–
Surface	⌇	X	X	X	X	X	X	X	X
Ruledness non-ideality									
	⌓	–	–	X	X	–	X	X	–

the property of axial symmetry (such as cylinder, cone, or generic axially symmetric). This non-ideality is particularly important for a generic axially symmetric feature, which is a non-analytical feature for which two types of *IDRs* can be recognised. These references are: the axis and the circular shape of the cross-section perpendicular to the axis.

Total roundness non-ideality concerns the deviation of the feature from a perfect axially symmetric geometry. The evaluation of this non-ideality does not require the knowledge of the nominal longitudinal profile. It measures how far the axially symmetric *non-ideal feature* deviates from a set of circles, all having their centres on the estimated ideal axis. Total roundness non-ideality differs both from traditional *roundness*, which performs single evaluations of the surface deviation section by section, and from traditional *run-out*, which requires a reference axis external to the feature. In fact, in the total roundness, the surface is virtually rotated around its intrinsic datum. Figure 6 shows an example of application for which it is important to evaluate the total roundness.

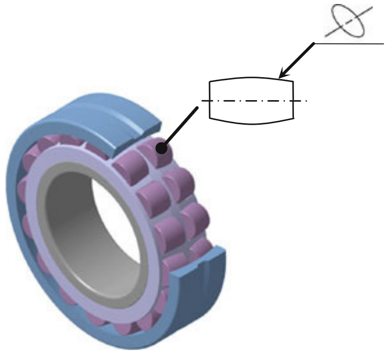


Fig. 6 Example of application of the *total roundness*

5.1.2 *Ruledness non-ideality*

Ruledness is an *intrinsic property* which can be automatically evaluated for any feature which is recognised to have the property of “being ruled” (such as cylinder, cone, generic ruled and generic extruded). *Ruledness* non-ideality is an effective substitute for the generatrix straightness non-ideality that uses an IDR consisting of a straight line which represents a surface generatrix. This traditional form non-ideality is used to evaluate the conformity of the generatrix to a straight line [13]. Although adequate to the nominal concept of ruled surface, this non-ideality is difficult to verify practically since the acquired points are not necessarily aligned with the surface generatrices. The *ruledness* measurement, which is here proposed, allows a more robust evaluation than the traditional one which is done for generatrix straightness. It aims at evaluating the deviation of a ruled surface from an *analytical ruled paraboloid*, whose vertex generatrix is the surface generatrix identified by a *growing algorithm* from a *seed point*. This paraboloid, which approximates the ruled surface in the generatrix neighbourhood, is calculated by the *weighted* L_2-norm rule, where the weight factors are the values of a Gaussian function having the mean located in the vertex generatrix as well as a value for the standard deviation chosen according to the mesh dimensions (σ = max mesh dimension). Since the generatrix direction is unknown, it is sought by the growing algorithm whose main steps are shown in Fig. 7.

The process starts with the identification of the *seed point* $\mathbf{p_s}(x, y, z)$ belonging to the ruled surface under examination. A local coordinate system (ξ, ψ, ζ) is suitably defined in such a way that the axis ζ overlaps the normal $\mathbf{n}$ at $\mathbf{p_s}$; the axis ξ is parallel to the estimated principal direction $(\mathbf{x_2})$ related to the null curvature at $\mathbf{p_s}$ (Fig. 7a). In this local coordinate system the equation of the ruled paraboloid $\boldsymbol{\Gamma}_{\boldsymbol{ruled}}$ can be conveniently expressed. In the first step of the algorithm the surface approximation is performed by analysing the *1-ring neighbourhood* around the seed point (Fig. 7b). In order to identify the generatrix direction which best fits the surface, the growing algorithm explores the nearest point (denoted as $\mathbf{p_1}$) along the $\mathbf{x_2}$ direction (Fig. 7c). A new surface approximation is performed by considering the set of points in the 1-ring neighbourhood of $\mathbf{p_s}$ and $\mathbf{p_1}$ and a new principal direction $\mathbf{x_{2,2}}$ is re-evaluated. This process continues exploring step-by-step the nearest points in this direction until a specified limited length (L_0) is reached (Fig. 7d–e). The value of L_0 must be then included in the tolerance specification. *Seed points* are chosen so as to lie on the unique directrix curve of the surface. The *ruledness* error is evaluated as the distance between the last estimation of $\boldsymbol{\Gamma}_{\boldsymbol{ruled}}$ and $\mathbf{p_s}$ and the set of nearest points $\mathbf{p_j}$.

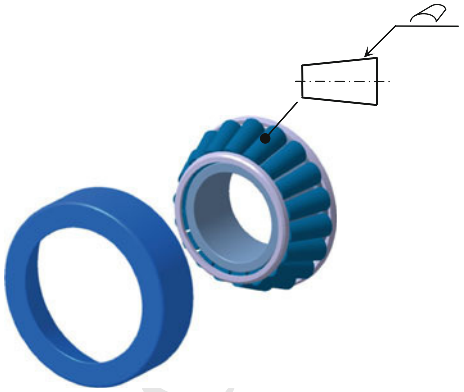


Fig. 8 Example of application of the ruledness

Figure 8 shows an exa]mple where the *ruledness* evaluation is appropriate.

5.1.3 The regularity non-ideality

Regularity is a local property of a surface (or a curve) related to its geometric *regularity* which, roughly speaking, means how far the surface (or the curve) deviates from an ideal smooth surface (or curve).

This property can be recognised for all the feature types (including the free-form features). From an operative point of view, the neighbourhood of each *regular* vertex of a *non-*

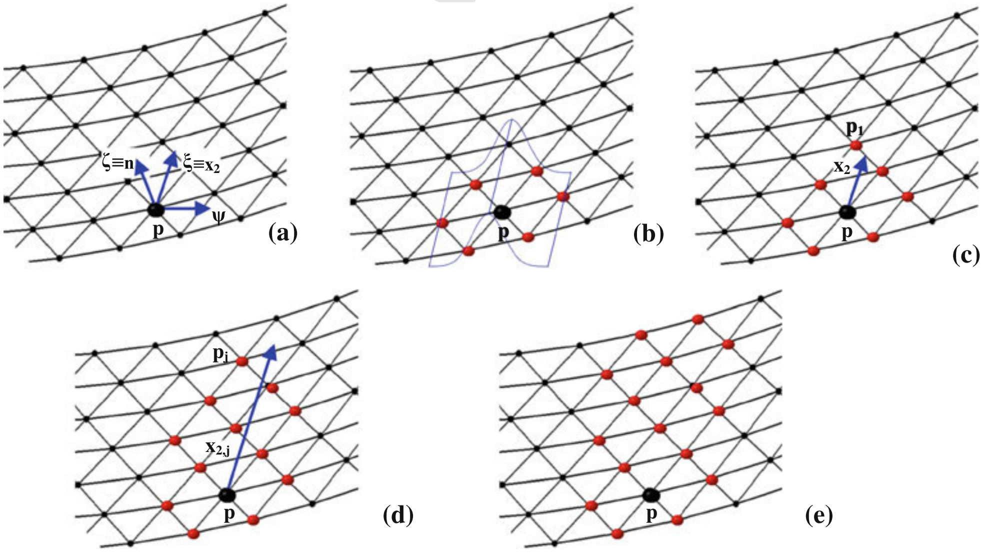


Fig. 7 Explanation of the steps in the *ruledness* evaluation process

ideal feature is compared with an *ILR* which consists in an analytical function which locally approximates the vertex neighbourhood. The measurement of the deviation from this reference represents locally the value of non-regularity. The rule to identify this *ILR* includes: the type of regular function and the method to locally approximate the neighbourhood.

In this paper two types of *regularity* are considered: the *profile*
regularity and the *surface regularity*. For *profile regularity* the *ILR* is a quadric or cubic polynomial calculated by the *weighted* L_2-norm rule. The weights of this *approximation rule* are assumed to be the values of a Gaussian function having the mean located at the analysed point and a properly selected value for the standard deviation σ. This weight defines the width (σ_0) of the inspection window of the object surface. The measurement of the profile non-ideality allows us to evaluate the profile imperfections, such as, for example, the undesired circumferential grooves caused by the tool vibration during the turning process. These imperfections escape thetraditional control of axis straightness or circularity.

For *surface regularity* the *ILR* is a quadric paraboloid; this is the typical analytical regular surface adopted to evaluate the differential geometric properties. For each *non-ideal feature* recognised, the *ILR* is evaluated at each vertex as the approximating surface resulting from the *weighted* L_2-norm rule. The weights of the approximation rule are assumed to be the values of a two-dimensional symmetric Gaussian function having the mean located at the analysed point and a properly selected value for the standard deviation σ. This weight defines the width (σ_0) of the inspection window of the object surface.

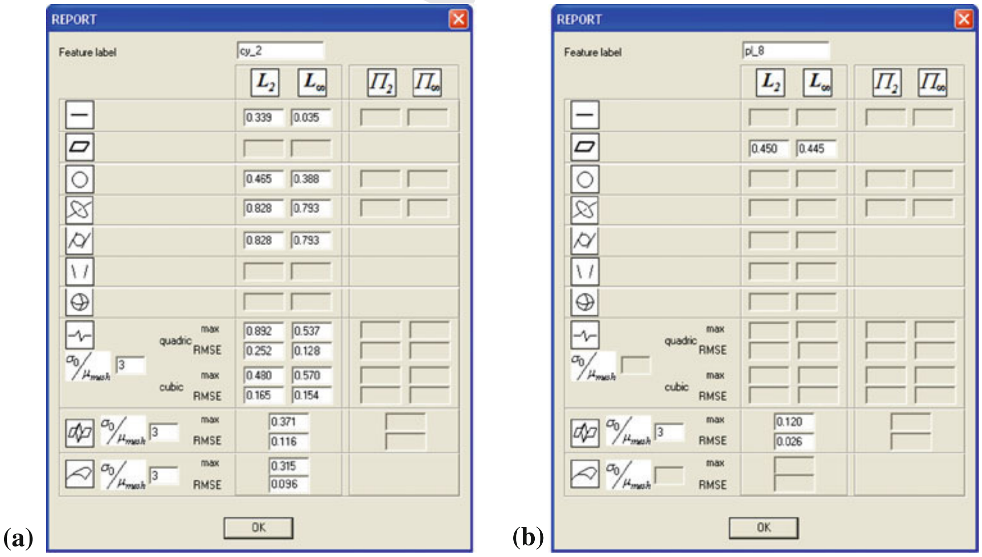
Surface regularity is potentially capable of giving a local evaluation of the combination of *roughness* and*waviness*. The possibility of evaluating such kind of surface characteristics depends on the sampling rate used to acquire the surface. The state-of-the-art optical scanners do not make it possible to detect surface *roughness* whereas other devices, such as confocal laser scanning microscopes or interferometric profilometers, can detect it. The ISO standard [14] defines the metrological characteristics of the *phase correct filters* to separate, by means of appropriate *cut-off wavelengths*, the long-wave (*waviness*) and short-wave (*roughness*) components from the acquisition of a surface profile.

Non-ideality in *surface regularity* is evaluated as the *Root Mean Square (RMS)* of the distances between the acquired points and the corresponding *ILR*. The non-ideality of the *non-ideal feature* is evaluated by averaging the local *regularity* indices.

5.1.4 Form non-ideality results

The methodology has been verified in the evaluation of form non-idealities for the test-case under examination. The reports of the developed software concerning the form non-idealities of the *non-ideal features* cy_2 and *pl_8* are displayed in Fig. 9a, b, respectively. The non-idealities are evaluated with both the L_2 and L_∞-rules. The results reported confirm the important role that the *rules* used to estimate the intrinsic references play in non-ideality evaluation.

The results of the non-idealities analysis perform a quality evaluation completely based on the *intrinsic properties* recognised in the object. These results can be useful for the geometric verification of the inspected object if the relative permissible errors are specified.



(a)

Feature label: cy_2

		L_2	L_∞	Π_2	Π_∞
—		0.339	0.035		
▱					
○		0.465	0.388		
⊗		0.828	0.793		
⌭		0.828	0.793		
\ /					
⊕					
∿ σ_0/μ_{mesh} 3	quadric max	0.892	0.537		
	quadric RMSE	0.252	0.128		
	cubic max	0.480	0.570		
	cubic RMSE	0.165	0.154		
σ_0/μ_{mesh} 3	max	0.371			
	RMSE	0.116			
σ_0/μ_{mesh} 3	max	0.315			
	RMSE	0.096			

(b)

Feature label: pl_8

		L_2	L_∞	Π_2	Π_∞
—					
▱		0.450	0.445		
○					
⊗					
⌭					
\ /					
⊕					
∿ σ_0/μ_{mesh}	quadric max				
	quadric RMSE				
	cubic max				
	cubic RMSE				
σ_0/μ_{mesh} 3	max	0.120			
	RMSE	0.026			
σ_0/μ_{mesh}	max				
	RMSE				

Fig. 9 Examples of applications of the form non-ideality evaluation

6 Conclusions

For some years now the present authors have been developing a new approach for automatic tolerance inspection working from an acquired high-density model. This approach involves the automatic construction of a geometric reference model of the scanned workpiece, called (*RGM*). Its final aim is the evaluation of the form, orientation and location non-idealities of the acquired workpiece. This approach is founded on the concepts of *non-ideal feature* and *intrinsic nominal reference*. For each *non-ideal feature* which is derived from the segmentation of the inspected object one or more *intrinsic nominal references* are identified.

The high-level geometric information, extracted from a high-density point cloud, concerns the geometric entities and the related mutual geometric properties which can be recognised from *non-ideal features*. Their domain depends on the *recognition* and *association rules* which are implemented within the RGM. This ideal geometric representation makes it possible to evaluate new and old categories of non-idealities. New procedures are also proposed which allow a more robust process of evaluation of traditional non-idealities (such as the straightness of a cylinder generatrix).

When using the RGM, tolerances can be specified according to the set of available and recognisable *intrinsic nominal references*. This allows for the automatic geometric inspection of the workpiece.

References

1. Prieto, F., Redarce, T., Lepage, R., Boulanger, P.: An automated inspection system. Int. J. Adv. Manuf. Technol. **19**(12), 917–925 (2002)
2. Li, Y., Gu, P.: Inspection of free-form shaped parts. Robotics Computer Integr. Manuf. **21**(4–5), 421–430 (2005)
3. Gao, J., Gindy, N., Chen, X.: An automated GD&T inspection system based on non-contact 3D digitization. Int. J. Prod. Res. **44**(1), 117–134 (2006)
4. Li, Y., Gu, P.: Free-form surface inspection techniques—state of the art review. Computer-Aided Des. **36**(13), 1395–1417 (2004)
5. ISO 1101:2004 Geometrical Product Specifications (GPS)—Tolerances of form, orientation, location and run out (2004)
6. ASME 14.5Y: Dimensioning and Tolerancing (2009)
7. Nielsen, H.S.: New concepts in specifications, operators and uncertainties and their impact on measurement and instrumentation. Meas. Sci. Technol. **17**(3), 541–544 (2006)
8. Do Carmo, M.: Differential geometry of curves and surfaces. Prentice-Hall, Englewood Cliffs (1974)
9. Di Angelo, L., Di Stefano, P., Morabito, A.: Fuzzy sets for geometric shape recognition in triangular meshes. In: Proceedings of the 6th International Conference on Intelligent Processing and Manufacturing of Materials (IPMM), Salerno, Italy, 25–29 June (2007)
10. ISO 14660–1: Geometrical Product Specifications (GPS)—Geometrical features—Part 1: General terms and definitions (1999)
11. Cao, Y., Mumford, D.: Geometric structure estimation of axially symmetric pots from small fragments. In: Proceedings of SHAPE Lab. @SPPRA'02 signal processing, pattern recognition and applications iasted international conference, 25–28 June. Crete, Greece (2002)
12. Di Angelo, L., Di Stefano, L., Morabito, A.E.: Automatic evaluation of form errors in high-density acquired surfaces. Int. J. Prod. Res. **49**(7), 2061–2082 (2011). First published on: 26 April 2010 (iFirst)
13. ISO 12780–1: Geometrical product specifications (GPS)—Straightness—Part 1: vocabulary and parameters of straightness (2011)
14. ISO 11562: Geometrical Product Specifications (GPS)—Surface texture: profile method—metrological characteristics of phase correct filters (1996)