PARTICLE PHYSICS FROM THE NONCOMMUTATIVE GEOMETRY POINT OF VIEW*,**

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Recent development in noncommutative geometry generalization of gauge theory is reviewed. The mathematical apparatus is reduced to minimum in order to allow the non-mathematically oriented physicists to follow the development in this interesting field of research.

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1. Introduction

The unification of electromagnetic and weak interactions is one of the biggest achievements of theoretical physics (the GWS model). This model successfully describes all known experiments involving electroweak interactions, although the gauge sector is not yet directly accessible in experiment [1]. We believe that the existence of the Higgs particle and the missing members of the third family will be soon confirmed. The situation is far less satisfactory from the theoretical point of view because the GWS model contains too many free parameters and the symmetry breaking mechanism is not yet understood. Much research have been made into the structure of string theories hoping to find answers to those questions [2]. Recently, new ideas that make use of the Connes' noncommutative geometry have been put forward [3]. Connes managed to reformulate the standard objects of differential geometry in a purely algebraic way. This allows for generalization of differential geometry to the cases of sets more exotic than manifolds. This

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new formalism has been immediately applied in gauge theory because it allows for generalization of the Kaluza–Klein program to the discrete internal space case.

2. Main ideas of noncommutative geometry

Mathematicians have proved that a given topological space X can be equivalently described by the (commutative) algebra $C(X)$ of real (complex in the complex case) valued continuous function on X. It is also possible to describe the standard notions of differential geometry in terms of algebraic structures on $C(X)$. We have the following correspondence (as this review is aimed at nonmathematically-oriented readers we will not give the precise definitions that we will not need; they can be found in [3, 4]):

topological space	$\equiv$	$C(X)$
manifold M	$\equiv$	$C^\infty(M)$
vector bundle on M	$\equiv$	projective module over $C^\infty(M)$
connection	$\equiv$	"universal" connection

The positive answer to the question: *can one go further and get rid of the adjective commutative in front of the algebra in question?* was given by Connes [3]. The result of this generalization, referred to as noncommutative geometry, allows us to do differential geometry on a more sophisticated level. As differential geometry is widely used in theoretical physics, it is not surprising that the newly invented noncommutative geometry became a very promising tool in physicists' hands. Here we will restrict ourselves to the particle physics. To do "the noncommutative particle physics", one has to specify the fermionic content of the theory and the gauge group. One introduces fermions by defining an appropriate Dirac operator. The gauge group can be a priori arbitrary but for technical reasons only unitary groups of the algebra A that generalizes $C(X)$

$$U_n = \{u \varepsilon M_n(A) : uu^\dagger = u^\dagger u = 1\}, \tag{1}$$

where $M_n(A)$ is the $n \times n$ matrix with entries from A fit naturally to the formalism. The gauge group is defined by giving "an extension" of the algebra of function on the (approximate?) spacetime. To be more precise, let us define:

Definition 1

Given an arbitrary algebra A, we can construct an algebra ΩA as follows. To every element $a \, \varepsilon \, A$ we associate a new element da. As a vector space, ΩA is the linear space of words built out of the "letters" a and da.

Multiplication of two such words is performed by concatenation and one imposes the associativity and distributivity over the action "+". Further, we will require that

$$d1 = 0, \quad d(a_0 a_1) - da_0 a_1 - a_0 da_1 = 0, \quad \mathit{and}\ d^2 = 0\,. \tag{2}$$

This is a very abstract notion. To make it more mundane, let us represent it in a (physical) Hilbert space H by setting (we neglect the very mathematical subtleties such as existence, correctness and so on) *via*

$$\pi(a_0 da_1 \ldots da_n) \equiv i^n a_0 [D, a_1] \ldots [D, a_n]\,, \tag{3}$$

where D is the free Dirac operator. In the physically motivated cases, H is a Z_2-graded space, equipped with a grading operator Γ (γ_5-matrix) such that $\Gamma^2 = 1$, A acts on H by even operators and D is an odd operator, *i.e.*:

$$a\Gamma = \Gamma a \text{ for } a \varepsilon A \text{ and } D\Gamma = -\Gamma D$$

Below, we will ignore the precise structure of the spacetime and focus our attention on the appropriate algebraic structures. To simplify our task, however, we will loose the geometrical interpretation.

Definition 2

A gauge field (connection) is any (skew) form $\alpha \varepsilon \Omega^1 A$, $\alpha = \sum a^i db^i$ such that $\sum a^i b^i = 1$. It determines the covariant derivative $\nabla = d + \alpha$. The curvature (stress tensor) is given by $\Theta = d\alpha + \alpha^2$.

Now, we have [4, 5, 6]

$$\mathcal{L}_{YM} = \frac{1}{8} \int_M \mathrm{Tr}\left(\pi(\Theta)^2\right)\,, \tag{4}$$

$$\mathcal{L}_F = \int_M \langle \psi | D + \pi(\alpha) | \psi \rangle\,, \tag{5}$$

where $\mathcal{L}_{YM}$ and $\mathcal{L}_F$ denote the bosonic and fermionic parts of the Lagrangian, respectively.

3. Models

Let S be a Riemannian (spin) 4-manifold, N_G denote the number of generations, M_{IJ} be the $N_G \times N_G$ ($I, J = 1, 2, 3, 4$) "mass matrices" and

$$D = \begin{pmatrix} \partial\!\!\!/ \otimes Id & \gamma_5 \otimes M_{12} & \gamma_5 \otimes M_{13} & \gamma_5 \otimes M_{14} \\ \gamma_5 \otimes M_{21} & \partial\!\!\!/ \otimes Id & \gamma_5 \otimes M_{23} & \gamma_5 \otimes M_{24} \\ \gamma_5 \otimes M_{31} & \gamma_5 \otimes M_{32} & \partial\!\!\!/ \otimes Id & \gamma_5 \otimes M_{34} \\ \gamma_5 \otimes M_{41} & \gamma_5 \otimes M_{42} & \gamma_5 \otimes M_{43} & \partial\!\!\!/ \otimes Id \end{pmatrix}. \tag{6}$$

Here, the matrices M_{IJ} describe the fermionic mass sector including mixing [7, 8]. Let $A = C(S) \otimes \bar{A}$, where $\bar{A}$ is the algebra

$$\bar{A} = M_{n_1} \oplus M_{n_2} \oplus M_{n_3} \oplus M_{n_4} \tag{7}$$

of direct sum of complex $n_i \times n_i$ matrices. An element $a \varepsilon A$ can be written as

$$a = \operatorname{diag}(a_1, a_2, a_3, a_4) \ , \tag{8}$$

where $a_i \varepsilon M_{n_i}(C(S))$, where the matrices are "built out" of complex function on spacetime. We have to compute the gauge field

$$\pi(\alpha) = \sum_i a^i \left[D, b^i\right] . \tag{9}$$

Simple calculation leads to

$$[D, b^i] =$$
$$\begin{pmatrix} \not{\partial} b_1^i & \gamma_5 \otimes \left(M_{12} b_2^i - b_1^i M_{12}\right) & \gamma_5 \otimes \left(M_{13} b_3^i - b_1^i M_{13}\right) & \dots \\ \gamma_5 \otimes \left(M_{21} b_1^i - b_2^i M_{21}\right) & \gamma_5 \otimes \not{\partial} b_2^i & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_5 \otimes \left(M_{41} b_1^i - b_4^i M_{41}\right) & \dots & \dots & \not{\partial} b_4^i \end{pmatrix} . \tag{10}$$

So that

$$\pi(\alpha) = \begin{pmatrix} A_1 & \gamma_5 \otimes \phi_{12} & \gamma_5 \otimes \phi_{13} & \gamma_5 \otimes \phi_{14} \\ \gamma_5 \otimes \phi_{21} & A_2 & \gamma_5 \otimes \phi_{23} & \gamma_5 \otimes \phi_{24} \\ \gamma_5 \otimes \phi_{31} & \gamma_5 \otimes \phi_{32} & A_3 & \gamma_5 \otimes \phi_{34} \\ \gamma_5 \otimes \phi_{41} & \gamma_5 \otimes \phi_{42} & \gamma_5 \otimes \phi_{43} & A_4 \end{pmatrix} , \tag{11}$$

where

$$A_m = \sum_i a_m^i \not{\partial} b_m^i \tag{12}$$

and

$$\phi_{mn} = \sum_i a_m^i \left(M_{mn} b_n^i - b_m^i M_{mn}\right) . \tag{13}$$

Further, we have to calculate

$$\pi(d\alpha) = [D, \alpha] = \sum_i \left[D a^i\right] \left[D, b^i\right] \tag{14}$$

and

$$\pi(\Theta) = \pi\left(d\alpha + \alpha^2\right) . \tag{15}$$

This leads to

$$\pi(\Theta)_{mm} = \tfrac{1}{2}\gamma^{\mu\nu}F^{m}_{\mu\nu} + \sum_{p\neq m} |K_{mp}|^2|\phi_{mp} + M_{mp}|^2 - Y_m - X'_{mm}\,, \quad (16)$$

where

$$X'_{mm} = \sum_i a^i_m \partial\!\!\!/^2 b^i_m\,, \quad (17)$$

$$Y_m = \sum_{p\neq m}\sum_i a^i_m |K_{mp}|^2|M_{mp}|^2 b^i_m\,, \quad 18)$$

$$F^m_{\mu\nu} = \partial_\mu A^m_\nu - \partial_\nu A^m_\mu + \left[A^m_\mu, A^m_\nu\right]\,. \quad (19)$$

Here, we have "generalized", following [8], the matrices M_{ij}

$$M_{ij} \to K_{ij} \otimes M_{ij}\,. \quad (20)$$

Now, K_{ij} describes the mixing among families and M_{ij} describes the vacuum expectation values of the Higgs sector. The off-diagonal elements are given by

$$\begin{aligned}
&\pi(\Theta)_{mn} = \\
&-\gamma_5 K_{mn}\left(\partial\!\!\!/\phi_{mn} + A_m(\phi_{mn} + M_{mn}) - (\phi_{mn} + M_{mn})A_n\right) - X_{mn} \\
&+ \sum_{p\neq m,n} K_{mp}K_{pn}\left((\phi_{mp} + M_{mp})(\phi_{pn} + M_{pn}) - M_{mp}M_{pn}\right)\,, \quad (21)
\end{aligned}$$

where

$$X_{mn} = \sum_i \sum_{p\neq m,n} K_{mp}K_{pn}\left(M_{mp}M_{pn}b^i_n - b^i_m M_{mp}M_{pn}\right)\,.$$

This completes the bosonic sector of the model:

$$\begin{aligned}
&\mathcal{L}_{YM} = \\
&-\sum_{m=1}^{m=4} \mathrm{Tr}\Bigg(\tfrac{1}{4}F^m_{\mu\nu}F^{m\,\mu\nu} - \tfrac{1}{2}\left|\sum_{p\neq m}\left(|K_{mp}|^2|\phi_{mp} + M_{mp}|^2 - Y_m\right) - X'_{mp}\right|^2 \\
&+ \tfrac{1}{2}\sum_{p\neq m} |K_{mp}|^2|\partial_\mu(\phi_{mp} + M_{mp}) + A^m_\mu(\phi_{mp} + M_{mp}) - (\phi_{mp} + M_{mp})A^p_\mu|^2 \\
&- \tfrac{1}{2}\sum_{n\neq m}\sum_{p\neq m,n} \left||K_{mp}|^2\left((\phi_{mp} + M_{mp})(\phi_{pn} + M_{pn}) - M_{mp}M_{pn}\right) - X_{mn}\right|^2\Bigg)\,.
\end{aligned} \quad (22)$$

3.1. The SU(2)× U(1) model

To obtain the SU(2)× U(1) electroweak unification model we should consider the algebra of the form $M_{2\times 2} \oplus M_{1\times 1}$, where $M_{i\times i}$ are the $i \times i$ matrices over the ring of complex valued function on spacetime. This aim can be also achieved by considering the extension of spacetime of the form $S \times \{1, 2\}$, the product of the ordinary spacetime S by a two-point set [5, 6].

We choose the following free Dirac operator [8]

$$D = \begin{pmatrix} \partial\!\!\!/ \otimes 1 & \gamma_5 \otimes M_{12} \\ \gamma_5 \otimes M_{21} & \partial\!\!\!/ \otimes 1 \end{pmatrix}, \tag{23}$$

with the mass matrix of the form

$$M_{12} = \begin{pmatrix} 0 \\ \mu \end{pmatrix} = S\,. \tag{24}$$

By repeating the above calculation we get

$$\pi\left(\alpha\right) = \begin{pmatrix} A^J_{1I} & H^J \\ H_I & A_2 \end{pmatrix}, \tag{25}$$

$$H = \mu \sum_i a^i_1 \left(S b^i_2 - b^i_2 S\right), \tag{26}$$

$$a_1 \varepsilon M_{2\times 2}\left(C^\infty\left(M\right)\right), \quad a_2 \varepsilon M_{1\times 1} = C^\infty\left(M\right). \tag{27}$$

In addition, we will demand that

$$\mathrm{Tr}\, A_1 = \mathrm{Tr}\, A_2 = A_2 \tag{28}$$

in order to reduce the gauge group from U(2)×U(1) to SU(2) × U(1).

The auxiliary fields take the form

$$X_{12} = X_{21} = 0\,, \quad Y_2 = \mu^2\,, \tag{29}$$

$$Y_1 = \mu^2 \sum_i a^i_1 T b^i_1\,, \tag{30}$$

$$T = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{31}$$

and can be easily get rid off. Finally, we get [8]

$$
\begin{aligned}
&\mathcal{I}_{YM} = \\
&\tfrac{1}{4}\left(\left(F^1_{\mu\nu}\right)^I_J\left(F^{1\mu\nu}\right)^J_I + F^2_{\mu\nu}F^{2\mu\nu}\right) \\
&+\tfrac{1}{2}TrKK^\dagger|\partial_\mu\left(H^I+H^I_0\right)+A^{1I}_{\mu J}\left(H^J+H^J_0\right)-\left(H^I+H^I_0\right)A^2_\mu|^2 \\
&-\tfrac{1}{2}\left(Tr\left(KK^\dagger\right)^2-\left(TrKK^\dagger\right)^2\right)\left(\left(H^I+H^I_0\right)\left(H^\dagger_I+H^\dagger_{0I}\right)-\mu^2\right)^2 .
\end{aligned}
\tag{32}
$$

The fermionic sector has the form

$$
\begin{aligned}
\mathcal{L}_f =&\bar{l}_L\left(D+\pi\left(A\right)\right)l_L+\bar{e}_R\left(\not\partial+A_2\right)e_R \\
&+\bar{l}_L\left(H+H_0\right)e_RK+\bar{e}_R\left(H^\dagger+H^\dagger_0\right)l_LK^\dagger .
\end{aligned}
\tag{33}
$$

To get a realistic model we have to include the strong interaction. "Unfortunately" the colour gauge group is unbroken. This makes the things more complicated because unbroken gauge symmetries are not in the spirit of the noncommutative geometry approach. To this end, we have to extend the gauge group by the SU(3)$\times$ U(1) factor and identify the two U(1) factors (sort of charge quantization condition can be deduced from this [4]).

The left-right symmetric model can be also constructed [8, 9]. The complications connected with the SU$(3)_{colour}$ factor suggest that grand unified models are more natural then the "partial unification" in the noncommutative framework.

3.2. Grand unification

The formalism discussed here formalism can be easily applied to grand unification. If one considers the algebra

$$
C^\infty\left(S\right)\otimes\left(M_{5\times5}\left(\boldsymbol{C}\right)\oplus M_{5\times5}\left(\boldsymbol{C}\right)\oplus M_{1\times1}\left(\boldsymbol{R}\right)\right) , \tag{34}
$$

where $\boldsymbol{C}$ and $\boldsymbol{R}$ denote the complex and real numbers, and demands the permutation symmetry between the two $M_{5\times5}$ terms, one gets

$$
\pi\left(\rho\right)=\begin{pmatrix} A & \Sigma & H \\ \Sigma & A & H \\ H^* & H^* & 0 \end{pmatrix} . \tag{35}
$$

Here, H is a complex scalar field and Σ a 5×5 self adjoint scalar field. One have to force the condition Tr $A=0$ on the gauge field A in order to reduce the gauge group from U(5) to SU(5). One can find such values for the mass matrices

$$M_{12} = M_{21} = \Sigma_0 \,, \tag{36}$$

and

$$M_{13} = M_{23} = H_0 \tag{37}$$

that interesting although phenomenologically unacceptable SU(5) GUT models are "produced" [8]. The more natural choice of $M_{1\times 1}(\boldsymbol{C}) = \boldsymbol{C}$ instead of $M_{1\times 1}(\boldsymbol{R}) = \boldsymbol{R}$ in (34) leads to noncommutative analogues of the "flipped unification" models. Such models might result in a phenomenologically acceptable model. In the seminal paper [10], it was shown that the SO(10) GUT is also possible in the noncommutative framework! One have to consider the algebra

$$P_+ \mathrm{Cliff}(\mathrm{SO}\,(10))\, P_+ \oplus \boldsymbol{R}\,, \tag{38}$$

where

$$P_+ = \tfrac{1}{2}\,(1 + \Gamma_{11}) \tag{39}$$

as the factor that extent the algebra of function on spacetime.

3.3. Nonlinear Higgs mechanism

Here we would like to point out that the noncommutative generalization of gauge theory may predict a nonlinearly realized spontaneous symmetry breaking, known under the acronym BESS (breaking electroweak sector strongly) [11–13]. Our main argument for BESS can be stated as follows. The noncommutative version of the standard model predicts the required form of the Higgs sector but fermion masses (Yukawa couplings) and the number of generation, N_G, are free parameters. There must be at least two generations but why not, say, 127? It is natural to suppose that N_G is big or even unlimited and that the fermion masses emerge as a result of interaction and the spacetime structure. We see only the lightest fermions because the energy at our disposal is not high enough. The Higgs particle has not yet been discovered. Does it really exist as a physical particle? We will show that it can be thought of in the limit $m_H \to \infty$. The main argument against BESS is that such models are nonrenormalizable. Noncommutative geometry says that our notion of spacetime is only an approximation (an effective electromagnetic spacetime). The correct description is in terms of algebras. Should we not give up the requirement of renormalizability? BESS models can certainly lead to physical prediction [14]. General relativity provide us with analogous arguments. Following the rules described above, we can construct the Lagrangian of the Standard Model [6, 15]

$$\mathcal{L}_{YM} = \int \{ \tfrac{1}{4} N_G \left(F^1_{\mu\nu} F^{1\mu\nu} + F^2_{\mu\nu} F^{2\mu\nu} + F^c_{\mu\nu} F^{c\mu\nu} \right) + \tfrac{1}{2} \operatorname{Tr} \left(M M^\dagger \right) |\partial H + A_1 H - H^\dagger A_2|^2 - \tfrac{1}{2} \left(\operatorname{Tr} \left(M M^\dagger \right)^2 - \left(Tr M M^\dagger \right)^2 \right) \left(H H^\dagger - 1 \right)^2 \} d^4x \,. \quad (40)$$

The fermionic action is given by

$$\mathcal{L}_f = \langle \psi | D + \pi(\rho) | \psi \rangle = \int \left(\bar{\psi}_L \bar{D} \psi_L + \bar{\psi}_R \bar{D} \psi_R + \bar{\psi}_L H \psi_R + \bar{\psi}_R H^\dagger \psi_L \right) d^4x \,, \quad (41)$$

where we have included the diagonal part of $\pi(\rho)$ term into $\bar{D}$.

Let us look closer at the full Lagrangian, $\mathcal{L} = L_{YM} + L_f$. It has the standard form except for the N_G factor in front of the gauge field kinetic terms that comes from the trace over generations. The analogous term in $\mathcal{L}_f$ give the sum over generations. We know that there are only three light generations of fermions but is that all? We should count all generations in $\mathcal{L}$! This means that the coefficient in front of the $F_{\mu\nu}F^{\mu\nu}$ terms should depend on N_G and, in fact, give us information about the total numbers of generations because it is absent from the fermionic part! This is not true. The orthodox normalization is correct. We should normalize the Dixmier trace [3, 4] that leads to (4, 5) so that the coefficient N_G disappears. The simplest and most natural solution is to normalize Tr so that $\operatorname{Tr} Id_{N_G} = 1$ [8]. This ensures also that Tr_ω is always finite. There is a natural inner product on the algebra of complex square matrices given by $\operatorname{Tr}(AB^\dagger)$. If one applies the Cauchy–Schwarz inequality to this inner product, one gets

$$\operatorname{Tr} \left(M M^\dagger \right)^2 \leq \left(\operatorname{Tr} M M^\dagger \right)^2 . \quad (42)$$

We cannot ensure the correct sign of the Higgs mass term without the above normalization. The normalization of the trace Tr leads to

$$\operatorname{Tr} \left(M M^\dagger \right)^2 \leq N_G \left(\operatorname{Tr} M M^\dagger \right)^2 . \quad (43)$$

This means that for a big N_G the coefficient $K = \operatorname{Tr} \left(M M^\dagger \right)^2 - \left(\operatorname{Tr} M M^\dagger \right)^2$ may be very large. In fact, it is possible that $K \to \infty$ if the number of heavy generations is unlimited. This force the condition $H H^\dagger = 1$ in the

Lagrangian and removes the Higgs particle from the spectrum! If we are going to interpret the Yukawa coupling in the standard way then we are not allowed to rescale arbitrarily the Higgs field and the limiting case leads to

$$m_H = \sqrt{2\frac{\mathrm{Tr}\left(MM^\dagger\right)^2 - \left(\mathrm{Tr}\, MM^\dagger\right)^2}{\mathrm{Tr}\left(MM^\dagger\right)}} \to \infty \tag{44}$$

as should be expected. The fermionic masses are generated in such a (non-linear) model by means of Yukawa couplings in a way analogous to that of the standard model [11–13]. The fermionic part of the Lagrangian given by Eq. (41) has the required form!

4. Final remarks

We have reviewed recent development in the noncommutative particle physics. As we wanted to reduce the mathematical apparatus to the minimum to make it accessible non-mathematically oriented physicists, we have neglected the mathematical subtleties and the spacetime structure. The interested reader is referred to [3, 4].

The complete understanding of the noncommutative particle physics is impossible without quantization. Up to know, we are able to get more or less interesting classical Lagrangian that can be quantized in the usual way. But it may not be the correct way of doing noncommutative physics! Toy model considerations suggest that certain relations among physical variables predicted by the classical Lagrangians are spoiled by quantum correction. To get the Lagrangian, we have to get rid of the "auxiliary fields" using equations of motion. Is it possible in a quantum theory? If not, we should consider the possibility of condensation of the bosonic sector along the lines considered in [16]. In general we should expect relation among vev's of the scalar and vector fields because in the noncommutative framework thy are related.

There is also the question of possible extra terms that are not allowed or vanish in the orthodox approach [17]. Such terms, if found, may result in unexpected physical consequences.

I greatly enjoyed the hospitality extended to me during a stay at the Faculty for Physics at the University of Bielefeld, where the final version of the talk was discussed and written down.

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