

## You have downloaded a document from RE-BUŚ <br> repository of the University of Silesia in Katowice

Title: Quark model predictions for the decay distributions of strange baryons produced in the reactions $/ 2+1 / 2+$ to $1 / \mathrm{x}+3 / 2+$

Author: Marek Zrałek

Citation style: Zrałek Marek. (1971). Quark model predictions for the decay distributions of strange baryons produced in the reactions $/ 2+1 / 2+$ to $1 / x+3 / 2+$. "Acta Physica Polonica, B" ( No. 2/3 (1971), s. 251-258).


Uznanie autorstwa - Licencja ta pozwala na kopiowanie, zmienianie, rozprowadzanie, przedstawianie i wykonywanie utworu jedynie pod warunkiem oznaczenia autorstwa.

# QUARK MODEL PREDICTIONS FOR THE DECAY DISTRIBUTIONS OF STRANGE BARYONS PRODUCED IN THE REACTIONS 

$\frac{1}{2}+\frac{1}{2}+\rightarrow \frac{1}{2}+\frac{3+}{2}$

By M. Zrafek<br>Institute of Physics, Silesian University, Katowice*

(Received August 17, 1970)


#### Abstract

The quark model is used to predict the joint decay distributions of pairs of strange baryons produced in the reactions $\frac{1}{2}+\frac{1}{2}+\rightarrow \frac{1}{2}+\frac{3}{2}+$ with polarized target. For reactions with particles polarized perpendicularly to the reaction plane it is possible to make absolute predictions using data obtained from reactions with unpolarized particles. Relations between the statistical tensors provide a method of evaluating the additivity angle for the $3 / 2^{+}$isobar, if the polarization of the target in the reaction plane is not zero.


## 1. Introduction

The additive quark model is useful for the investigation decay distributions of particles in two body reactions (cf. Ref. [1-8]).

In this paper we present the predictions of this model for the decay distributions or particles produced in the reactions

$$
\begin{equation*}
B B \rightarrow B B^{*} \tag{1}
\end{equation*}
$$

with polarized target. Here $B$ is a $1 / 2^{+}$baryon and $B^{*}$ is a $3 / 2^{+}$resonance. Up to now a similar analysis has not yet been performed, because in all processes (1), in which the decays of the baryons $B$ and $B^{*}$ conserve parity, there are only two measurable independent statistical tensors components $T_{0}^{2}$ and $T_{2}^{2}$. In this case the quark model predicts only that the component $T_{2}^{2}$ is independent of the target polarization.

If, however, in the examined reactions (1) the $1 / 2^{+}$baryon decays without conserving parity (as is the case for the weak decays of strange baryons), we can measure all the tensors $T_{M_{1} M_{2}}^{J_{1} J_{2}}$ with $J_{1}$ and $J_{2}$ odd (cf.e.g. [10]). Then for reactions (1) with unpolarized target only 11 independent tensor components are not equal zero. All the tensors with $J=3$ vanish ( $c f$. Ref. [10]). In the case of the polarized target there are 35 measurable independent tensor components expressed by six independent complex scalar amplitudes.

We have found 35 linear relations between these tensors and tensors measurable in the reactions with unpolarized particles. All the components $T_{M_{1} M_{3}}^{J_{3} J_{2}}$ with $M_{1}+M_{2}$ even for re-

[^0]actions with polarized target can be calculated if we know the tensors for unpolarized target. Therefore for reactions (1) with particles polarized perpendicularly to the reaction plane it is possible to predict decay distributions using data obtained from reactions with unpolarized particles.

The tensor components $T_{M_{1} M_{2}}^{J_{S} J_{2}}$ with $M_{1}+M_{2}$ odd are proportional to the polarization in the reaction plane $P$. They can also be expressed by tensor components measureble in the reactions without polarization. In this case the 16 obtained relations depend still on one parameter, namely on the additivity angle $\psi$ for the $3 / 2^{+}$isobar (cf.e.g. [5]). These relations may be used to evaluate the additivity angle and, on the other hand, to check the quark model predictions. All these relations expressing the tensors with polarized target by the tensors with unpolarized target are presented in Sections 2 and 3 . Besides, there are also 6 relations between different tensor components with polarized target.

In the Appendix we give the expressions for the statistical tensors in terms of the quark quark amplitudes.

## 2. Relations between statistical tensors with $M_{1}+M_{\mathbf{2}}$ even

The method of deriving of the quark model formulae for statistical tensors was already discussed $e . g$. in Refs [3], [5], [9], (for a review of properties of statistical tensors $c f$. Ref. [11]. Therefore we give here only the final results, i.e. the relations between statistical tensors for processes (1). All calculations are done in the transversity frame, i. e. with the spin projection on the normal to the reaction plane. The density matrix for the initial baryon can be written in the form

$$
\varrho=\frac{1}{2}\left(\begin{array}{cc}
1+P_{n} & -i P e^{-i a}  \tag{2}\\
i P e^{i a} & 1-P_{n}
\end{array}\right) .
$$

Here $P_{n}$ is the target polarization along the normal to the scattering plane, $P$ is the polarization in the scattering plane, and $\alpha$ is the angle between the beam direction and the polarization vector in the scattering plane.

For the tensor components $T_{M_{1} M_{2}}^{J_{3} M_{2}}$ with $M_{1}+M_{2}$ even one obtains the following linear relations

$$
\begin{gather*}
T_{11}^{12}\left(P_{n}\right)=T_{11}^{12}\left(P_{n}=0\right)+\sqrt{\frac{3}{5}} T_{11}^{11}\left(P_{n}=0\right) \cdot P_{n},  \tag{3}\\
T_{1-1}^{12}\left(P_{n}\right)=T_{1-1}^{12}\left(P_{n}=0\right)+\sqrt{\frac{3}{5}} T_{1-1}^{11}\left(P_{n}=0\right) \cdot P_{n},  \tag{4}\\
T_{11}^{11}\left(P_{n}\right)=T_{11}^{11}\left(P_{n}=0\right)+\frac{1}{\sqrt{15}} T_{11}^{12}\left(P_{n}=0\right) \cdot P_{n},  \tag{5}\\
T_{1-1}^{11}\left(P_{n}\right)=T_{1-1}^{11}\left(P_{n}=0\right)+\frac{1}{\sqrt{15}} T_{1-1}^{12}\left(P_{n}=0\right) \cdot P_{n},  \tag{6}\\
T_{02}^{12}\left(P_{n}\right)=T_{02}^{12}\left(P_{n}=0\right),  \tag{7}\\
T_{02}^{02}\left(P_{n}\right)=T_{02}^{02}\left(P_{n}=0\right), \tag{8}
\end{gather*}
$$

$$
\begin{gather*}
T_{00}^{12}\left(P_{n}\right)=T_{00}^{12}\left(P_{n}=0\right)+\frac{2}{\sqrt{5}} T_{00}^{11}\left(P_{n}=0\right) \cdot P_{n},  \tag{9}\\
T_{00}^{11}\left(P_{n}\right)=T_{00}^{11}\left(P_{n}=0\right)+\left(\frac{\sqrt{5}}{3} T_{00}^{10}\left(P_{n}=0\right)+\frac{2}{3 \sqrt{5}} T_{00}^{12}\left(P_{n}=0\right)\right) P_{n}, \\
T_{00}^{10}\left(P_{n}\right)=T_{00}^{10}\left(P_{n}=0\right)+\frac{1}{\sqrt{5}} T_{00}^{11}\left(P_{n}=0\right) \cdot P_{n},  \tag{11}\\
T_{00}^{02}\left(P_{n}\right)=T_{00}^{02}\left(P_{n}=0\right)+\frac{2}{\sqrt{5}} T_{00}^{01}\left(P_{n}=0\right) \cdot P_{n},  \tag{1}\\
T_{00}^{01}\left(P_{n}\right)=T_{00}^{01}\left(P_{n}=0\right)+\left(\frac{5}{6 \sqrt{10}}+\frac{2}{3 \sqrt{5}} T_{00}^{02}\left(P_{n}=0\right)\right) P_{n},  \tag{13}\\
T_{13}^{13}\left(P_{n}\right)=T_{13}^{13}\left(P_{n}=0\right)=0,  \tag{14}\\
T_{11}^{13}\left(P_{n}\right)=2 \sqrt{\frac{2}{5}} T_{11}^{12}\left(P_{n}=0\right) \cdot P_{n},  \tag{15}\\
T_{1-1}^{13}\left(P_{n}\right)=2 \sqrt{\frac{2}{5}} T_{1-1}^{12}\left(P_{n}=0\right) \cdot P_{n},  \tag{16}\\
T_{1-3}^{13}\left(P_{n}\right)=T_{1-3}^{13}\left(P_{n}=0\right)=0,  \tag{17}\\
T_{02}^{13}\left(P_{n}\right)=T_{02}^{12}\left(P_{n}=0\right) \cdot P_{n},  \tag{18}\\
T_{00}^{03}\left(P_{n}\right)=T_{02}^{02}\left(P_{n}=0\right) \cdot P_{n},  \tag{19}\\
T_{00}^{13}\left(P_{n}\right)=\frac{3}{\sqrt{5}} T_{00}^{12}\left(P_{n}=0\right) \cdot P_{n},  \tag{20}\\
T_{00}^{03}\left(P_{n}\right)=\frac{3}{\sqrt{5}} T_{00}^{02}\left(P_{n}=0\right) \cdot P_{n} . \tag{21}
\end{gather*}
$$

These relations are independent of the polarization in the scattering plane $P$.
From these relations we see that all the tensor components determining the decay distributions with the target polarized perpendiculary to the reaction plane are expressed by the tensors $T^{J_{1} J_{2}}\left(P_{n}=0\right)$ measured without polarized target. Because $T_{2}^{2}=\sqrt{2} T_{02}^{02}$, we see from Eq. (8) that $T_{2}^{2}$ is independent of the target polarization. It is the only interesting result of the quark model for reactions (1), if only the even - $M$ tensor components are measured. Because all the tensor components with $M_{1}+M_{2}$ odd are proportional to the polarization in the reaction plane $P$ (see the Appendix) we see from Eqs (3)-(21) that for processes (1) without polarization ( $P_{n}=P=0$ ) only 11 tensor components are not equal zero. All the tensors $T_{M_{1} M_{2}}^{J, 3}\left(P_{n}=P=0\right)$ vanish (cf. Ref. [10]). Besides there are no linear relations bet-
ween tensors $T_{M_{1} M_{2}}^{J_{1} J_{2}}\left(P_{n}=P=0\right)$. There are still two relations of this type expressing a combination of tensor components with $P_{n} \neq 0$ by tensor components with $P_{n}=0$

$$
\begin{align*}
T_{11}^{11}\left(P_{n}\right)-\frac{1}{2 \sqrt{6}} T_{11}^{13}\left(P_{n}\right) & =T_{11}^{11}\left(P_{n}=0\right)  \tag{22}\\
T_{1-1}^{11}\left(P_{n}\right)-\frac{1}{2 \sqrt{6}} T_{1-1}^{1}\left(P_{n}\right) & =T_{1-1}^{11}\left(P_{n}=0\right) \tag{23}
\end{align*}
$$

## 3. Relations between statistical tensors with $M_{1}+M_{2}$ odd

For the tensor components $T_{M_{1} M_{2}}^{y_{1} J_{2}}$ with $M_{1}+M_{2}$ odd one obtains the following linear relations

$$
\begin{gather*}
T_{12}^{12}(P)=-i \sqrt{\frac{3}{5}} T_{11}^{11}\left(P_{n}=0\right) P e^{i(\alpha+\psi)},  \tag{24}\\
T_{10}^{12}(P)=i \frac{1}{2} \sqrt{\frac{6}{5}} P\left(T_{11}^{12}\left(P_{n}=0\right) e^{-i(\alpha+\psi)}+T_{1-1}^{12}\left(P_{n}=0\right) e^{i(\alpha+\psi)}\right)  \tag{25}\\
T_{1-2}^{12}(P)=-i \sqrt{\frac{3}{5}} T_{1-1}^{11}\left(P_{n}=0\right) P e^{-i(\alpha+\psi)},  \tag{26}\\
T_{10}^{11}(P)=i \frac{1}{\sqrt{30}} P\left(T_{11}^{12}\left(P_{n}=0\right) e^{-i(\alpha+\psi)}+T_{1-1}^{12}\left(P_{n}=0\right) e^{i(\alpha+\psi)}\right)  \tag{27}\\
T_{10}^{10}(P)=i \frac{1}{\sqrt{10}} P\left(T_{1-1}^{11}\left(P_{n}=0\right) e^{i(\alpha+\psi)}+T_{11}^{11}\left(P_{n}=0\right) e^{-i(\alpha+\psi)}\right) \\
T_{01}^{12}(P)=-i \sqrt{\frac{3}{10}} T_{00}^{11}\left(P_{n}=0\right) e^{i(\alpha+\psi)}
\end{gather*}
$$

$$
\begin{equation*}
T_{00}^{11}(P)=i \frac{1}{3 \sqrt{10}} P\left(T_{00}^{12}\left(P_{n}=0\right)-5 T_{00}^{10}\left(P_{n}=0\right)\right) e^{i(\alpha+\psi)}+i \frac{1}{\sqrt{15}} P T_{02}^{12}\left(P_{n}=0\right) e^{-i(\alpha+\psi)} \tag{30}
\end{equation*}
$$

$$
\begin{equation*}
T_{01}^{02}(P)=-i \sqrt{\frac{3}{10}} T_{00}^{01}\left(P_{n}=0\right) e^{i(\alpha+\psi)} \tag{31}
\end{equation*}
$$

$$
\begin{equation*}
T_{01}^{01}(P)=i P\left(\frac{1}{3 \sqrt{10}} T_{00}^{02}\left(P_{n}=0\right)-\frac{\sqrt{5}}{12}\right) e^{i(\alpha+\psi)}+i \frac{1}{\sqrt{15}} P T_{02}^{02}\left(P_{n}=0\right) e^{-i(\alpha+\psi)} \tag{32}
\end{equation*}
$$

$$
\begin{equation*}
T_{12}^{13}(P)=-i P T_{11}^{12}\left(P_{n}=0\right) e^{i(\alpha+\psi)} \tag{33}
\end{equation*}
$$

$$
\begin{equation*}
T_{10}^{13}(P)=-i P \sqrt{\frac{3}{10}}\left(T_{1-1}^{12}\left(P_{n}=0\right) e^{i(\alpha+\psi)}+T_{11}^{12}\left(P_{n}=0\right) e^{-i(\alpha+\psi)}\right) \tag{34}
\end{equation*}
$$

$$
\begin{equation*}
T_{1-2}^{13}(P)=-i P T_{1-1}^{12}\left(P_{n}=0\right) e^{-i(\alpha+\psi)} \tag{35}
\end{equation*}
$$

$$
\begin{equation*}
T_{03}^{13}(P)=-i P \sqrt{\frac{3}{2}} T_{02}^{12}\left(P_{n}=0\right) e^{i(\sigma+\psi)} \tag{36}
\end{equation*}
$$

$$
\begin{gather*}
T_{03}^{03}(P)=-i P \sqrt{\frac{3}{2}} T_{02}^{02}\left(P_{n}=0\right) e^{i(\alpha+\psi)},  \tag{37}\\
T_{01}^{13}(P)=-i P\left(\sqrt{\frac{3}{5}} T_{00}^{12}\left(P_{n}=0\right) e^{i(\alpha+\psi)}+\frac{1}{\sqrt{10}} T_{02}^{12\left(P_{n}=0\right) e^{-i(\alpha+\psi)}}\right),  \tag{38}\\
T_{01}^{03}(P)=-i P\left(\sqrt{\frac{3}{5}} T_{00}^{02}\left(P_{n}=0\right) e^{i(\alpha+\psi)}+\frac{1}{\sqrt{10}} T_{02}^{02}\left(P_{n}=0\right) e^{-i(\alpha+\psi)}\right) . \tag{39}
\end{gather*}
$$

They, in turn, do not depend on $P_{n}$.
As seen from these formulae it is possible to determine all the tensors $T^{J_{1} J_{3}}(P \neq 0)$ if we know $T^{J_{1} J_{2}}(P=0)$ and the additivity angle $\psi$.

Relations (24)-(39) enable one to determine the additivity angle and, besides, may serve as a check of the additive quark model.

We have still two relations between tensor components with $P \neq 0$ involving the additivity angle

$$
\begin{align*}
T_{10}^{13}(P) & =\frac{1}{2} \sqrt{\frac{6}{5}}\left(T_{1-2}^{13}(P) e^{2 i(\alpha+\psi)}+T_{12}^{13}(P) e^{-2 i(\alpha+\psi)},\right.  \tag{40}\\
T_{10}^{10}(P) & =-\frac{1}{\sqrt{6}}\left(T_{1-2}^{12}(P) e^{2 i(\alpha+\psi)}+T_{12}^{12}(P) e^{-2 i(\alpha+\psi)}\right) . \tag{41}
\end{align*}
$$

Further there is still a relation independent of $P$ or $\psi$

$$
\begin{equation*}
3 T_{10}^{11}=T_{10}^{12}=-T_{10}^{13} \tag{42}
\end{equation*}
$$

but all three components appearing here are zero unless $P \neq 0$.
Finally, the quark model predicts that the following two complex components ( $=0$ for $P=0$ ) have the same phases:

$$
\begin{equation*}
\text { phase of } T_{01}^{12}=\text { phase of } T_{01}^{02} . \tag{43}
\end{equation*}
$$

This exhausts the list of quark model predictions for our reaction (1).
The author would like to thank Dr A. Kotanski for suggesting the problem and many helpful discussions.

## APPENDIX

Using the similar method to that described in Ref. [5] we obtain for the statistical tensor components with $M_{1}+M_{2}$ even

$$
\begin{align*}
& T_{11}^{12}=-\frac{1}{4} \sqrt{\frac{3}{2}}\left(f_{7} f_{2}^{*}+f_{3} f_{0}^{*}+\left(f_{7} f_{2}^{*}-f_{3} f_{0}^{*}\right) P_{n}\right),  \tag{A.1}\\
& T_{1-1}^{12}=-\frac{1}{4} \sqrt{\frac{3}{2}}\left(f_{7} f_{4}^{*}+f_{5} f_{0}^{*}+\left(f_{5} f_{0}^{*}-f_{7} f_{4}^{*}\right) P_{n}\right) . \tag{A.2}
\end{align*}
$$

$$
\begin{align*}
& T_{11}^{11}=\frac{1}{4 \sqrt{10}}\left(5\left(f_{3} f_{0}^{*}-f_{7} f_{2}^{*}\right)-\left(f_{3} f_{0}^{*}+f_{7} f_{2}^{*}\right) P_{n}\right) .  \tag{A.3}\\
& T_{1-1}^{11}=-\frac{1}{4 \sqrt{10}}\left(5\left(f_{5} f_{0}^{*}-f_{7} f_{4}^{*}\right)+\left(f_{7} f_{4}^{*}+f_{5} f_{0}^{*}\right) P_{n}\right),  \tag{A.4}\\
& T_{02}^{12}=\frac{\sqrt{3}}{4}\left(f_{4} f_{2}^{*}-f_{3} f_{5}^{*}\right),  \tag{A.5}\\
& T_{02}^{02}=\frac{\sqrt{3}}{4}\left(f_{3} f_{5}^{*}+f_{4} f_{2}^{*}\right),  \tag{A.6}\\
& T_{00}^{12}=\frac{1}{4 \sqrt{2}}\left(\left|f_{2}\right|^{2}-\left|f_{3}\right|^{2}+\left|f_{4}\right|^{2}-\left|f_{5}\right|^{2}+\left|f_{7}\right|^{2}-\left|f_{0}\right|^{2}\right)+\frac{1}{2 \sqrt{2}}\left(\left|f_{2}\right|^{2}+\left|f_{3}\right|^{2}-\left|f_{4}\right|^{2}-\left|f_{5}\right|^{2}\right) P_{n},  \tag{A.7}\\
& T_{00}^{11}=\frac{5}{4 \sqrt{10}}\left(\left|f_{2}\right|^{2}+\left|f_{3}\right|^{2}-\left|f_{4}\right|^{2}-\left|f_{5}\right|^{2}\right)+\frac{1}{\sqrt{10}}\left(\left|f_{2}\right|^{2}-\left|f_{3}\right|^{2}+\left|f_{4}\right|^{2}-\left|f_{5}\right|^{2}-\right. \\
& \left.-\frac{1}{4}\left|f_{7}\right|^{2}+\frac{1}{4}\left|f_{0}\right|^{2}\right) P_{n}, \\
& T_{00}^{10}=\frac{1}{2 \sqrt{2}}\left(\left.\left|f_{21}^{\prime 2}-\left|f_{3}\right|^{2}+\left|f_{4}\right|^{2}-\left|f_{5}\right|^{2}-\frac{1}{2}\right| f_{7}\right|^{2}+\frac{1}{2}\left|f_{0}\right|^{2}\right) \div \\
& +\frac{1}{4 \sqrt{2}}\left(\left|f_{2}\right|^{2}+\left|f_{3}\right|^{2}-\left|f_{4}\right|^{2}-\left|f_{5}\right|^{2}\right) P_{n},  \tag{A.9}\\
& T_{00}^{02}=\frac{1}{4 \sqrt{2}}\left(\left|f_{2}\right|^{2}+\left|f_{3}\right|^{2}+\left|f_{4}\right|^{2}+\left|f_{5}\right|^{2}-\left|f_{7}\right|^{2}-\left|f_{0}\right|^{2}\right)+ \\
& +\frac{1}{2 \sqrt{2}}\left(\left|f_{2}\right|^{2}-\left|f_{3}\right|^{2}-\left|f_{4}\right|^{2}+\left|f_{5}\right|^{2}\right) P_{n},  \tag{A.10}\\
& T_{00}^{01}=\frac{5}{4 \sqrt{10}}\left(!f_{2}^{\prime 2}-\left|f_{3}\right|^{2}-\left|f_{4}\right|^{2}+\mid f_{5}^{\prime 2}\right)+ \\
& +\frac{1}{\sqrt{10}}\left(\left|f_{2}\right|^{2}+\left|f_{\mathbf{3}}\right|^{2}+\left|f_{4}\right|^{2}+\left|f_{5}\right|^{2}+\frac{1}{4}\left|f_{7}\right|^{2}+\frac{1}{4}\left|f_{0}\right|^{2}\right) P_{n},  \tag{A.11}\\
& T_{13}^{13}=0,  \tag{A.12}\\
& T_{11}^{\mathbf{3}}=-\frac{1}{2} \sqrt{\frac{3}{5}}\left(f_{3} f_{0}^{*}+f_{\mathbf{7}} f_{2}^{*}\right) P_{n} .  \tag{A.13}\\
& T_{1-1}^{13}=-\frac{1}{2} \sqrt{\frac{3}{5}}\left(f_{7} f_{4}^{*}+f_{5} f_{0}^{*}\right) P_{n},  \tag{A.14}\\
& T_{1-3}^{13}=0,  \tag{A.15}\\
& T_{02}^{13}=\frac{\sqrt{3}}{4}\left(f_{4} f_{2}^{*}-f_{3} f_{5}^{*}\right) P_{n}, \tag{A.16}
\end{align*}
$$

$$
\begin{gather*}
T_{00}^{13}=\frac{3}{4 \sqrt{10}}\left(\left|f_{2}\right|^{2}-\left|f_{3}\right|^{2}+\left|f_{4}\right|^{2}-\left|f_{5}\right|^{2}+\left|f_{7}\right|^{2}-\left|f_{0}\right|^{2}\right) P_{n}  \tag{A.17}\\
T_{02}^{03}=\frac{\sqrt{3}}{4}\left(f_{3} f_{5}^{*}+f_{4} f_{2}^{*}\right) P_{n},  \tag{A.18}\\
T_{00}^{03}=\frac{3}{4 \sqrt{10}}\left(\left|f_{2}\right|^{2}+\left|f_{3}\right|^{2}+\left|f_{4}\right|^{2}+\left|f_{5}\right|^{2}-\left|f_{7}\right|^{2}-\left|f_{0}\right|^{2}\right) P_{n} \tag{A.19}
\end{gather*}
$$

For the statistical tensor components with $M_{1}+M_{2}$ odd one obtains

$$
\begin{align*}
& T_{12}^{12}=-i \frac{1}{4} \sqrt{\frac{3}{2}}\left(f_{3} f_{0}^{*}-f_{7} f_{2}^{*}\right) P e^{i(\alpha+\psi)},  \tag{A.20}\\
& T_{10}^{12}=-i \frac{3}{8 \sqrt{5}} P\left(\left(f_{3} f_{0}^{*}+f_{7} f_{2}^{*}\right) e^{-i(\alpha+\psi)}+\left(f_{7} f_{4}^{*}+f_{5} f_{0}^{*}\right) e^{i(\alpha+\psi)}\right),  \tag{A.21}\\
& T_{1-2}^{12}=-i \frac{1}{4} \sqrt{\frac{3}{2}}\left(f_{7} f_{\mathbf{4}}^{*}-f_{5} f_{\mathbf{0}}^{*}\right) P e^{-i(\alpha+\psi)},  \tag{A.22}\\
& T_{10}^{11}=-i \frac{1}{8 \sqrt{5}} P\left(\left(f_{3} f_{0}^{*}+f_{7} f_{2}^{*}\right) e^{-i(\alpha+\psi)}+\left(f_{7} f_{4}^{*}+f_{5} f_{0}^{*}\right) e^{i(\alpha+\psi)}\right),  \tag{A.23}\\
& T_{10}^{10}=-i \frac{1}{8} P\left(\left(f_{5} f_{0}^{*}-f_{7} f_{4}^{*}\right) e^{i(\alpha+\psi)}+\left(f_{7} f_{2}^{*}-f_{3} f_{0}^{*}\right) e^{-i(\alpha+\psi)}\right),  \tag{A.24}\\
& T_{01}^{12}=-i \frac{\sqrt{3}}{8} P\left(\left|f_{2}\right|^{2}+\left|f_{3}\right|^{2}-\left|f_{4}\right|^{2}-\left|f_{5}\right|^{2}\right) e^{i(\alpha+\varphi)},  \tag{A.25}\\
& T_{01}^{11}=i \frac{1}{4 \sqrt{5}} P\left(\left(\frac{3}{2}\left(-\left|f_{2}\right|^{2}+\left|f_{3}\right|^{2}-\left|f_{4}\right|^{2}+\left|f_{5}\right|^{2}\right)+\left|f_{7}\right|^{2}-\left|f_{0}\right|^{2}\right) e^{i(\alpha+\psi)}+\right. \\
& \left.+\left(f_{4} f_{2}^{*}-f_{3} f_{5}^{*}\right) e^{-i(\boldsymbol{\alpha}+\varphi)}\right),  \tag{A.26}\\
& T_{01}^{02}=-i \frac{\sqrt{3}}{8}\left(\left|f_{2}\right|^{2}-\left|f_{3}\right|^{2}-\left|f_{4}\right|^{2}+\left|f_{5}\right|^{2}\right) P e^{i(a+\psi)},  \tag{A.27}\\
& T_{01}^{01}=-i \frac{1}{4 \sqrt{5}} P\left(\left(\frac{3}{2}\left(i,\left.f_{2}\right|^{2}+\left|f_{3}\right|^{2}+\left|f_{4}\right|^{2}+\left|f_{5}\right|^{2}\right) \div\left|f_{7}^{\prime 2}+\left|f_{0}\right|^{2}\right) e^{i(\alpha+\varphi)-}\right.\right. \\
& \left.-\left(f_{3} f_{5}^{*}+f_{4} f_{2}^{*}\right) e^{-i(\alpha+\varphi)}\right),  \tag{A.28}\\
& T_{12}^{13}=i \frac{1}{4} \sqrt{\frac{3}{2}} P\left(f_{3} f_{0}^{*}+f_{7} f_{2}^{*}\right) e^{i(\alpha+\psi)},  \tag{A.29}\\
& T_{10}^{13}=i \frac{3}{8 \sqrt{5}} P\left(\left(f_{5} f_{0}^{*}+f_{7} f_{4}^{*}\right) e^{i(\alpha+\varphi)}+\left(f_{7} f_{2}^{*}+f_{3} f_{0}^{*}\right) e^{-i(\alpha+\varphi)}\right), \tag{A.30}
\end{align*}
$$

$$
\begin{gather*}
T_{1-2}^{13}=i \frac{1}{4} \sqrt{\frac{3}{2}} P\left(f_{7} f_{4}^{*}+f_{5} f_{0}^{*}\right) e^{-i(\alpha+\psi)},  \tag{A.31}\\
T_{03}^{13}=i \frac{3}{4 \sqrt{2}}\left(f_{3} f_{5}^{*}-f_{4} f_{2}^{*}\right) P e^{i(\alpha+\psi)},  \tag{A.32}\\
T_{03}^{03}=-i \frac{3}{4 \sqrt{2}}\left(f_{3} f_{5}^{*}+f_{4} f_{2}^{*}\right) P e^{i(\alpha+\psi)},  \tag{A.33}\\
T_{01}^{13}=i \frac{1}{4} \sqrt{\frac{3}{10}} P\left(\left(-\left|f_{2}\right|^{2}+\left|f_{3}\right|^{2}-\left|f_{4}\right|^{2}+\left|f_{5}\right|^{2}-\left|f_{7}\right|^{2}+\left|f_{0}\right|^{2}\right) e^{i(\alpha+\psi)}+\right. \\
 \tag{A.34}\\
\left.+\left(f_{3} f_{5}^{*}-f_{4} f_{2}^{*}\right) e^{-i(\alpha+\psi)}\right) \\
T_{01}^{03}=-i \frac{1}{4} \sqrt{\frac{3}{10}} P\left(\left|f_{2}\right|^{2}+\left|f_{3}\right|^{2}+\left|f_{4}\right|^{2}+\left|f_{5}\right|^{2}-\left|f_{7}\right|^{2}-\left|f_{0}\right|^{2}\right) e^{i(\alpha+\psi)}+  \tag{A.35}\\
\\
\left.+\left(f_{3} f_{5}^{*}+f_{4} f_{2}^{*}\right) e^{-i(\alpha+\psi)}\right) .
\end{gather*}
$$

Here $f_{1}, f_{2}, f_{3}, f_{4}, f_{5}, f_{0}$ and $f_{7}$ are linear combinations scalar amplitudes, and $\psi$ is the additivity angle for the $3 / 2^{+}$isobar.

Some of the formulae in this paper were simplified by the normalization condition

$$
\begin{equation*}
\left(\left|f_{2}\right|^{2}+\left|f_{3}\right|^{2}+\left|f_{4}\right|^{2}+\left|f_{5}\right|^{2}+\frac{1}{2}\left|f_{7}\right|^{2}+\frac{1}{2}\left|f_{0}\right|^{2}\right)+\frac{P_{n}}{2}\left(\left|f_{2}\right|^{2}-\left|f_{3}\right|^{2}-\left|f_{4}\right|^{2}+\left|f_{5}\right|^{2}\right)=1 \tag{A.36}
\end{equation*}
$$

## REFERENCES

[1] A. Białas, K. Zalewski, Phys. Letters, 26B, 180 (1968).
[2] A. Białas, K. Zalewski, Nuclear Phys., B6, 465 (1968).
[3] A. Kotański, K. Zalewski, Nuclear Phys., B13, 119 (1969).
[4] Bonn-Durham-Nijmegen-Paris (E. P.)-Strasbourg-Torino Collaboration, Phys. Letters, 28B, 72 (1968).
[5] A. Kotański, K. Zalewski, Nuclear Phys., B15, 242 (1970).
[6] Aachen-Berlin-CERN Collaboration, Nuclear Phys., B8, 485 (1968).
[7] Bruxelles-CERN Collaboration, Nuovo Cimento, 56A, 397 (1969).
[8] H. Friedman, R. R. Ross, Phys. Rev. Letters, 22, 152 (1969).
[9] A. Kotański, K. Zalewski, Nuclear Phys., B12, 72 (1969).
[10] A. Białas, A. Kotański, Nuclear Phys., B19, 667 (1970).
[11] A. Kotański, K. Zalewski, Nuclear Phys., B4, 559 (1968); B20, 236 (1970).


[^0]:    * Address: Instytut Fizyki, Uniwersytet Slaski, Katowice, Bankowa 12, Poland.

