Title: Perspective for testing dark energy scenarios with advanced LIGO type gravity wave experiments

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Citation style: Biesiada Marek. (2003). Perspective for testing dark energy scenarios with advanced LIGO type gravity wave experiments. "Acta Physica Polonica B" (Vol. 34, no. 11 (2003), s. 5423-5432).

Vol. 34 (2003) *ACTA PHYSICA POLONICA B* No 11

PERSPECTIVE FOR TESTING DARK ENERGY SCENARIOS WITH ADVANCED LIGO TYPE GRAVITY WAVE EXPERIMENTS*

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(Received October 23, 2003)

Future generation of interferometric gravitational wave detectors is hoped to provide accurate measurements of the final stages of binary inspirals. The sources probed by such experiments are of extragalactic origin and the observed chirp mass distribution carries information about their redshifts. Moreover the luminosity distance is directly observable is such experiments. This creates the possibility to establish a new kind of cosmological tests, supplementary to more standard ones. The paper discusses the utility of gravity wave experiments for testing the dark energy in the Universe, which is one of the most important issues in modern cosmology.

PACS numbers: 04.80.Nn, 04.30.Db, 97.80.–d, 98.80.Es

1. Introduction

Currently two independent observational programs — high redshift supernovae surveys [1] and CMBR small scale anisotropy measurements [2, 3] have brought a new picture of the Universe in the large. The results of these programs interpreted within the FRW models suggest that our Universe is flat (as inferred from the location of acoustic peaks in CMBR power spectrum) and presently accelerates its expansion (as inferred from the SNIa Hubble diagram). Combined with the independent knowledge about the amount of baryons and CDM estimated to be $\Omega_m = 0.3$ [4] it follows that about $\Omega_X = 0.7$ fraction of critical density should be contained in a mysterious component called "dark energy". The most obvious candidate for this smooth component permeating the Universe is the cosmological constant Λ

* Presented at the XXVII International Conference of Theoretical Physics, "Matter to the Deepest", Ustron, Poland, September 15–21, 2003.

representing the energy of the vacuum. Well known fine tuning problems led many people to seek beyond the Λ framework, and the concept of the quintessence had been conceived. Usually the quintessence is described in a phenomenological manner, as a scalar field with an appropriate potential [5]. It turns out, however, that quintessence program also suffers from its own fine tuning problems [6].

Recently the so called Chaplygin gas [7] — a hypothetical component with the equation of state $p = -\frac{A}{\rho_{\rm Ch}}$ was proposed as a challenge to the above mentioned candidates for dark energy. This, also purely phenomenological, entity has interesting connections with string theory [8]. Currently its generalisations admitting the equation of state $p = -\frac{A}{\rho_{\rm Ch}^{\alpha}}$ where $0 \leq \alpha \leq 1$ have been proposed [9].

Another popular line of investigation invokes physical theories with extra spatial dimensions. In these theories our four dimensional Universe is a brane embedded into a higher dimensional space-time and the General Relativity is obtained as an effective theory from a more fundamental one. The standard model matter fields are confined to the brane while gravity propagates in all dimensions. Various cosmological consequences of brane-world scenarios have been discussed in the literature. In the most popular class of such models [10, 11] there exists a certain crossover distance scale $r_{\rm c}$ such that below that scale an observer would measure the usual gravitational force but above the crossover scale the gravity follows 5-dimensional behaviour. Such large scale behaviour would have profound cosmological consequences and some of them have already been discussed in the literature [12–14].

Laser interferometric gravitational wave detectors developed under the projects LIGO, VIRGO and GEO600 are expected to perform a successful direct detection of the gravitational waves. Inspiralling neutron star (NS–NS) binaries are among the most promising astrophysical sources for this class of experiments [15]. They have a remarkable feature that the luminosity distance to a merging binary is a directly observable quantity easy to obtain from the waveforms. This circumstance made it possible to contemplate a possibility of measuring cosmological parameters such like the Hubble constant, or deceleration parameter [16–18]. In particular it was pointed out by Chernoff and Finn [16] how the catalogues of inspiral events can be utilised to make statistical inferences about the Universe. In a similar spirit the possibility to constrain cosmic equation of state from the statistics of inspiral gravitational wave events has been discussed in [19, 20].

In this paper, developing the approach of [19] we shall contemplate the feasibility of constraining the different classes of dark energy scenarios such like Chaplygin gas, brane world and quintessence from the gravitational wave experiments.

2. Cosmological models

2.1. Generalized Chaplygin gas cosmology

Einstein equations for the Friedman–Robertson–Walker model with hydrodynamical energy-momentum tensor $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}$ read:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2(t)}, \tag{1}$$

$$\frac{\ddot{a}(t)}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{2}$$

Let us assume that matter content of the Universe consists of pressure-less gas with energy density ρ_m representing baryonic plus cold dark matter (CDM) and of the generalized Chaplygin gas with the equation of state

$$p_{\mathrm{Ch}} = -\frac{A}{{\rho_{\mathrm{Ch}}}^{\alpha}}, \tag{3}$$

where $0 \leq \alpha \leq 1$, representing the dark energy responsible for the acceleration of the Universe. If one further makes an assumption that these two components do not interact, then the energy conservation equation $\dot{\rho} + 3H(p + \rho) = 0$, where $H = \dot{a}/a$ is the Hubble function, can be integrated separately for matter and Chaplygin gas leading to well known result $\rho_m = \rho_{m,0} a^{-3}$ and $\rho_{\mathrm{Ch}} = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}}$ (see also [9]).

The Friedman equation (1) can be rearranged to the form giving explicitly the Hubble function $H(z) = \dot{a}/a$

$$H(z)^2 = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_{\mathrm{Ch}} \left(A_0 + (1-A_0)(1+z)^{3(1+\alpha)}\right)^{\frac{1}{1+\alpha}} + \Omega_k (1+z)^2\right], \tag{4}$$

where the quantities Ω_i, $i = m, \mathrm{Ch}, k$ represent fractions of critical density currently contained in energy densities of respective components. For the transparency of formulae we have denoted $\Omega_k = -\frac{k}{H_0^2}$ and $A_0 = A/(A+B)$.

Generalised Chaplygin gas models have been intensively studied in the literature [21] and in particular they have been tested against supernovae data. In general there are two approaches to generalised Chaplygin gas models. In the first one Chaplygin gas is invoked as a candidate of dark energy component (alternative to the quintessence) hence one assumes $\Omega_m = 0.3$ and $\Omega_{\mathrm{Ch}} = 0.7$. The second approach is more ambitious: one hopes that Chaplygin gas could account both for dark energy and cold dark matter, hence the respective cosmological model assumes $\Omega_m \approx 0.05$ (baryonic content of the Universe) and $\Omega_{\mathrm{Ch}} \approx 0.95$.

Using available data [2,3] on the position of CMBR peaks Bento *et al.* [22] obtained the following constraints: $0.81 \leq A_0 \leq 0.85$ and $0.2 \leq \alpha \leq 0.6$ at 68% CL in the model with $\Omega_m = 0.05$ and $\Omega_{\rm Ch} = 0.95$ assumed. Using the angular size statistics for extragalactic sources combined with SNIa data it was found in [23] that in the $\Omega_m = 0.3$ and $\Omega_{\rm Ch} = 0.7$ scenario best fitted values of model parameters are $A_0 = 0.83$ and $\alpha = 1.$ respectively.

2.2. Brane-world cosmological model

Brane-world scenarios assume that our four-dimensional spacetime is a brane embedded into 5-dimensional bulk and gravity in 5-dimensions is governed by the 5-dimensional Einstein–Hilbert action. The bulk metric ${}^5g_{AB}$ induces a 4-dimensional metric $g_{\mu\nu}$ on the brane [10, 11]. According to this picture, our 4-dimensional Universe is a surface (a brane) embedded into a higher dimensional bulk space-time in which gravity propagates. As a consequence there exists a certain cross-over scale $r_{\rm c}$ above which an observer will detect higher dimensional effects.

Cosmological models in brane-world scenarios have been widely discussed in the literature [12–14]. In particular the Friedman's equation takes here the following form:

$$H(z)^2 = H_0^2 \left[\left(\sqrt{\Omega_m (1+z)^3 + \Omega_{r_{\rm c}}} + \sqrt{\Omega_{r_{\rm c}}} \right)^2 + \Omega_k (1+z)^2 \right] , \qquad (5)$$

where $\Omega_{r_{\rm c}} = \frac{1}{4 r_{\rm c}^2 H_0^2}$ and $\Omega_k = -\frac{k}{H_0^2}$. It has been shown in [14] that flat brane-world Universe with $\Omega_m = 0.3$ and $r_{\rm c} = 1.4\ H_0^{-1}$ is consistent with current SNIa and CMBR data.

2.3. Quintessence model

For a couple of years the most popular explanation of the accelerating Universe (*i.e.* $\ddot{a} > 0$ and hence $\rho + 3p < 0$) was to assume the existence of a negative pressure $p < -1/3\rho$ component called dark energy. One can heuristically assume that this component, called "quintessence", is described by hydrodynamical energy-momentum tensor with $p = w\rho$ where $-1 < w < -1/3$ [24, 25]. There are many theoretical realisations of the "quintessence" from the oldest idea of dynamical scalar field of Ratra and Peebles [5], its modern versions of slowly rolling down scalar fields tracking the evolution of the scale factor in an appropriate way [26], supersymmetric models [27] up to the ideas associated with large extra dimensions [11].

In quintessential cosmology the Friedman equation reads:

$$H^2(z) = H_0^2 (\Omega_m\ (1+z)^3 + \Omega_Q\ (1+z)^{3(1+w)} + \Omega_k (1+z)^2) , \qquad (6)$$

where by Ω_m and Ω_Q we have denoted present values of relative contributions of clumped matter and quintessence to the critical density.

Confrontation with supernovae and CMBR data led to the constraint $w \leq -0.8$ hence we assume this value of w (with $\Omega_m = 0.3$ and $\Omega_Q = 0.7$) as representative for the class of quintessential cosmological models.

In any case of a cosmological test aimed at determining the geometry of the Universe in the large, one has to calculate some distance measure as a function of redshift. As it is well known (see *e.g.* [28]), one can distinguish three types of distances in cosmology:

(i) proper distance:

$$d_M(z) = \frac{c}{H_0\,|\Omega_k|^{1/2}}\,\mathcal{S}\left(H_0|\Omega_k|^{1/2}\int_0^z \frac{dz'}{H(z')}\right) =: \frac{d_H}{h\,|\Omega_k|^{1/2}}\bar{d}_M(z)\,, \tag{7}$$

(ii) angular diameter distance: $d_A(z) = \frac{1}{1+z} d_M(z)\,,$

(iii) luminosity distance: $d_L(z) = (1+z) d_M(z)\,.$

The $\mathcal{S}(u)$ function is defined as $\mathcal{S}(u) = \sin u$ for $k = +1$, $S(u) = u$ for $k = 0$ and $\mathcal{S}(u) = \sinh u$ for $k = -1$. As usually z denotes the redshift, h denotes the dimensionless Hubble constant *i.e.* $H_0 = h \times 100$ km/s Mpc and $d_H = 3. \times h^{-1}$ Gpc is the Hubble distance (radius of the Hubble horizon). The quantities with an over-bar have been defined by factoring out the dependence on the Hubble constant. For the purpose of concrete calculations we have assumed the dimensionless Hubble constant equal to $h = 0.65$ as suggested by independent observational evidence (*e.g.* the HST project or multiple image quasar systems [29]). The formulae above are the most general ones in the framework of Friedman–Robertson–Walker type cosmology. Further in this paper we will use their versions restricted to flat model $k = 0$ because the flat FRW geometry is strongly supported by CMBR data [2, 3].

3. Redshift distribution of observed events

The gravity wave detector would register only those inspiral events for which the signal-to-noise ratio exceeded certain threshold value ρ_0 [16] which is assumed as $\rho_0 = 8.$ for LIGO-type detectors. An intrinsic chirp mass $\mathcal{M}_0 = \mu^{3/5} M^{2/5}$, with μ and M denoting the reduced and total mass, is the crucial observable quantity describing the inspiralling binary system. The observed chirp mass scales with the redshift $\mathcal{M}(z) = (1+z)\mathcal{M}_0$ and therefore can be used to determine the redshift to the source. Observations of binary pulsars 1913+16 and 1534+12 as well as X-ray observations have strongly indicated that the mass distribution of NS in binaries is sharply peaked around 1.4 $M_\odot$ [16, 17]. Assuming equal mass binary this would mean that one can (in first approximation) take the distribution of intrinsic chirp mass as peaked around 1.2 $M_\odot$. It is a very fortunate circumstance in the context of potential utility of gravitational wave observations. Namely, if one detects an event with a chirp mass significantly exceeding the "canonical" value of 1.2 $M_\odot$ then this excess can be translated into redshift of the source $z = \frac{M_{obs}}{1.2\, M_\odot} - 1$. In fact the sharp mass distribution of NS makes NS-NS binaries the "standard candles" of gravitational wave astronomy. Because the luminosity distance of a merging binary is a direct observable easily read off from the waveforms one has a possibility to determine the distance — redshift relation and hence to estimate certain cosmological parameters [16, 19].

The rate $\dfrac{d\dot{N}(>\rho_0)}{dz}$ at which we observe the inspiral events that originate in the redshift interval $[z,\ z+dz]$ is given by [30]:

$$\begin{aligned}\frac{d\dot{N}(>\rho_0)}{dz} &= \frac{\dot{n}_0}{1+z}\,\eta(z)\,4\pi d_M^2 \frac{d}{dz} d_M(z)\, C_\Theta(x)\\ &= 4\pi\left(\frac{d_H}{h}\right)^3 \frac{\dot{n}_0}{1+z}\,\frac{\tilde{d}_M^2(z)}{H(z)}\, C_\Theta(x)\,,\end{aligned} \tag{8}$$

where $\dot{n}_0$ denotes the local binary coalescing rate per unit comoving volume, $\eta(z)$ factor parametrizes source evolution over the sample and $C_\Theta(x)$ denotes the probability that given detector registers inspiral event at redshift z with $\rho > \rho_0$ (for details see [19, 30]). We assume the detector's characteristics as representative for the second generation of interferometric detectors [19, 20, 30] For the $\dot{n}_0$ one can use "the best guess" for local rate density $\dot{n}_0 \approx 9.9\ \mathrm{h}\ 10^{-8}\ \ \mathrm{Mpc}^{-3}\mathrm{yr}^{-1}$ as inferred from the observed binary pulsar systems that will coalesce in less than a Hubble time [31]. Recent attempts aimed at predicting the coalescing rate from stellar evolution using the population synthesis method [32] gave estimates spread over the range of 2 orders of magnitude. However, the $P(z, > \rho_0)$ statistical observable discussed below is insensitive to the precise value of coalescence rate.

Source evolution over sample is parametrized by the factor $\eta(z)$. We have adopted the functional form of $\eta(z)$ found by Schneider *et al.* [33] from the population synthesis approach and which has also been used by Zhu *et al.* [20] in a similar context.

The method of extracting the cosmological parameters advocated by Finn and Chernoff [16] makes use of the redshift distribution of observed events in a catalogue composed of observations with the signal-to-noise ratio greater than the threshold value ρ_0. Therefore it is important to find this distribution function for different cosmological scenarios. The formula for the expected distribution of observed events in the source redshift can be obtained from the equation (8):

$$\begin{aligned} P(z, > \rho_0) &= \frac{1}{\dot{N}(> \rho_0)} \frac{d\dot{N}(> \rho_0)}{dz} \\ &= \frac{4\pi}{\dot{N}(> \rho_0)} \left(\frac{d_H}{h}\right)^3 \frac{\dot{n}_0}{1+z}\, \eta(z)\, \frac{\bar{d}_M^2(z)}{H(z)}\, C_\Theta(x)\,. \end{aligned} \tag{9}$$

The illustration of numerical computations for the generalised Chaplygin gas models and brane-world cosmologies based on the formulae (9) and (8) is given in figures Figs. 1–3. Similar results for the quintessence models can be found in [19]. Computations consisted in performing sensitivity analysis of $P(z, > \rho_0)$ observable by constructing a grid of models by systematically varying free parameters A_0 and α — in the case of Chaplygin gas models and r_c in brane-world scenarios.

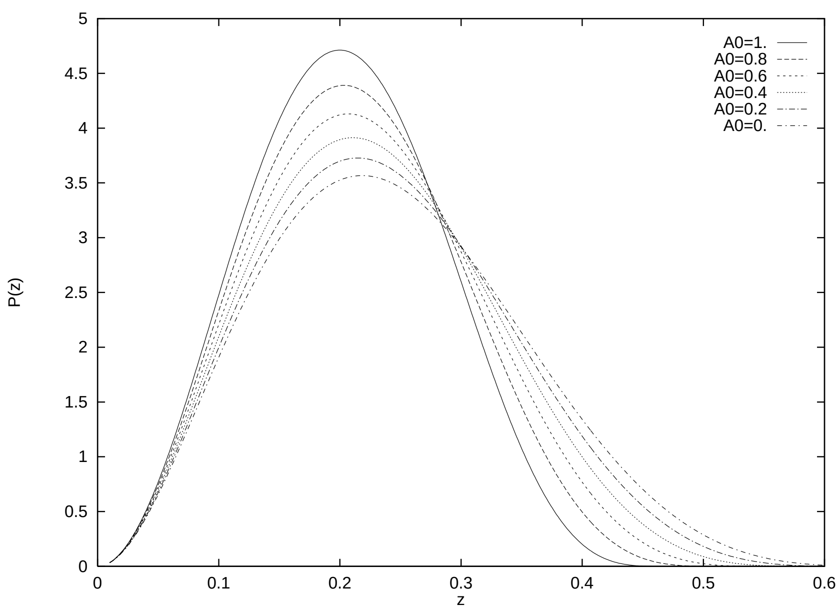

Fig. 1. Sensitivity of the $P(z, > \rho_0)$ distribution function, with respect to A_0 parameter ($\alpha = 0.5$ assumed), in ($\Omega_m = 0.3$, $\Omega_{\mathrm{Ch}} = 0.7$) generalised Chaplygin gas cosmological model.

Figure 1 illustrates sensitivity, with respect to A_0 parameter ($\alpha = 0.5$ assumed), of the $P(z, > \rho_0)$ distribution function in ($\Omega_m = 0.3$, $\Omega_{\mathrm{Ch}} =$ 0.7) generalised Chaplygin gas cosmological model. One can see noticeable

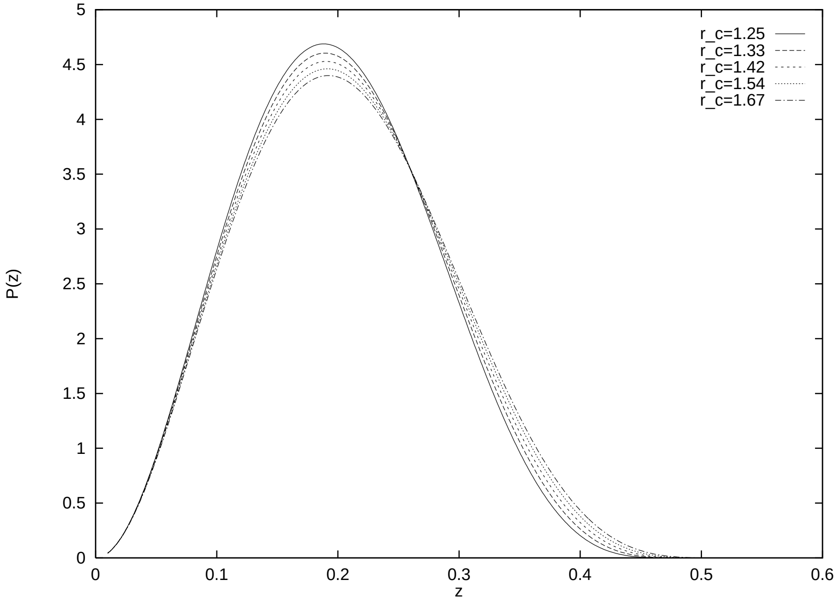

Fig. 2. Sensitivity of the $P(z, > \rho_0)$ distribution function, with respect to the crossover scale $r_{\rm c}$ (in units of the Hubble distance), in the brane-world scenario.

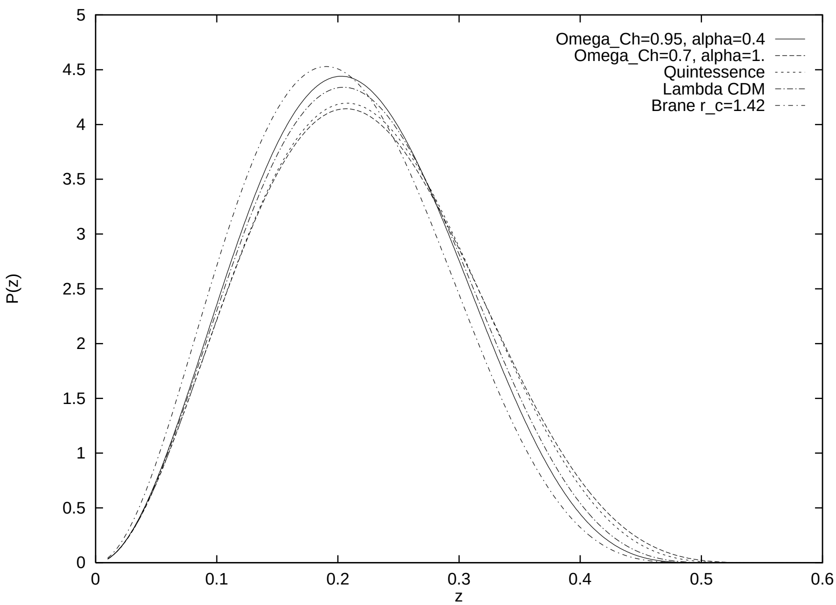

Fig. 3. Redshift distribution of observed events in various dark energy scenarios.

differences in $P(z, > \rho_0)$ distribution functions when A is systematically varied. Similar sensitivity analysis for α revealed a very small effect, hence the respective figure is not reported. The family of curves on Fig. 1 is representative for the full range of α parameter, in the sense that it properly illustrates the emerging trends.

In figure 2 one can see how $P(z, > \rho_0)$ distribution function varies while changing the cross-over scale r_c (in units of the Hubble radius) in braneworld scenarios.

Finally in figure 3 the $P(z, > \rho_0)$ distribution functions for different cosmological scenarios have been plotted collectively. The scenarios comprise: best fitted quintessence model, flat ΛCDM scenario, brane-world scenario (with $r_c = 1.4$) and generalised Chaplygin gas models with $\Omega_{\rm Ch} = 0.3$, $A_0 = 0.83$, $\alpha = 1.$ and $\Omega_{\rm Ch} = 0.05$, $A_0 = 0.83$, $\alpha = 0.4$, respectively.

4. Results and discussion

In the class of generalised Chaplygin gas models the $P(z, > \rho_0)$ observable could be useful for constraining A parameter. Unfortunately, as already mentioned there still exists a strong degeneracy with respect to α. This effect has also been noticed in previous studies using other techniques.

There are also perspectives for constraining the cross-over scale in brane-world scenarios with $P(z, > \rho_0)$ observable, yet this would demand high quality data. Figure 3 shows that there is also a noticeable difference in predicted redshift distribution functions $P(z, > \rho_0)$ for different cosmological scenarios.

At the level of yearly detection rates the difference between scenarios turned out to be negligibly small. However, as we have discussed in [19] the yearly detection rate is very sensitive to source evolution effects and thus it would allow to constrain the freedom in astrophysical input to the $P(z, > \rho_0)$ observable. The redshift distribution $P(z, > \rho_0)$ is in fact inferred from observed chirp mass distribution. Therefore it can in principle be distorted by the intrinsic chirp mass distribution. Theoretical studies of the neutron star formation suggest that masses of nascent neutron stars do not vary much with either mass or composition of the progenitor [16]. Also the mass estimates of observed binary pulsars suggest that there are good reasons to assume a negligible spread of intrinsic chirp mass.

In conclusion one can hope that the catalogues of inspiral events gathered in future gravitational waves experiments can provide helpful information about the nature of the dark energy in the Universe complementary to that obtained by other techniques.

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