LABORATORY EQUIPMENT AND TECHNIQUE

ON THE ENERGIES SPREAD OF THE GAUSSIAN DISTRIBUTED BEAM IN NUCLEAR SCATTERING EXPERIMENTS

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This paper presents an analysis of the energy spread of the products of a nuclear reaction in thin targets for a Gaussian — distributed beam of incident particles.

1. *Introduction*

It is well known that beams of particles from cyclotrons have a Gaussian energy distribution. Scattering on a target causes the incident beam to become more spread out in energy. On the other hand, the improved accuracy of measurements in recent years makes it indispensible to have exact knowledge of the energy spectrum of the outgoing beam.

In this paper the employment of certain assumptions leads to the derivation of simple formulae which make it possible to calculate the resulting energy spectrum of the outgoing beam.

2. *Monoenergetic incident beam*

We consider the reaction $A(a, b)B$, as depicted in Fig. 1 for two positions of the target. The following assumptions are made:

I. The target is so thin that the particle's loss of energy is a linear function of its range.

II. In the considered range of energy loss we have $dE_b/dE_a \cong \text{const} \overset{\text{df}}{=} \eta$, where $E_b = f(E_a, \theta)$ is given by the well-known formula

$$E_b^{1/2} = (m_B+m_a)^{-1}\,[\cos\theta\,(m_a m_b E_a)^{1/2} \pm \pm\{m_a m_b E_a \cos^2\theta + (m_B+m_b)[m_B Q + (m_B - m_a)E_a]\}]. \tag{1}$$

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On the basis of these assumptions it is easily found that for transmission

$$\Delta E_b(x) \cong \Delta_a \eta x + \Delta_b(1-x)$$

and for reflection

$$\Delta E_b(x) \cong \Delta_a \eta x + \Delta_b x$$

where Δ_a is the energy loss of particle a over the distance S_1, Δ_b is the energy loss of particle b over the distance S_2 (Fig. 1) and $x = OP/S_1$ is the depth parameter of reaction.

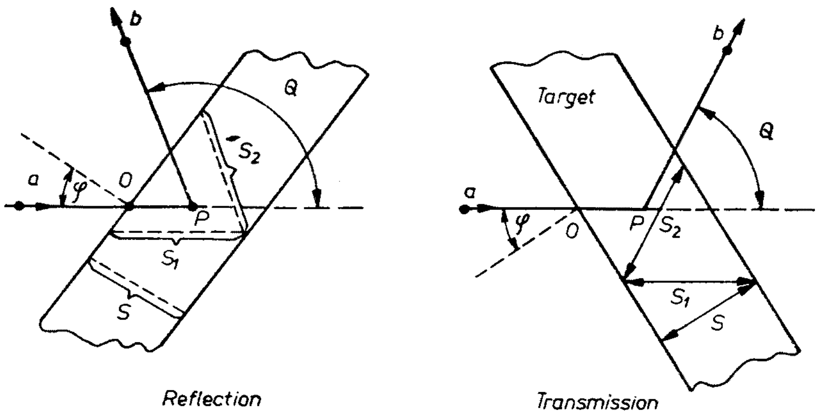

Fig. 1. Geometry of reaction: a — incident beam, b — outgoing beam, P — point of reaction, φ — angle between target and beam a, θ — angle between outgoing and indicent beam, S — normal target thickness; $S_1 = S/\cos\varphi$, $S_2 = S/\cos(\theta-\varphi)$ for transmission and $S_2 = S/\cos(\theta+\varphi)$ for reflection. x is the depth parameter for the reaction, equal to OP/S_1

Since for probabilities relation $P(\Delta E_b)d(\Delta E_b) = p(x)dx$ holds, and since $p(x) \equiv 1$, the spread in the energy of reaction products is given by

$$P(\Delta E_b) = [d(\Delta E_b)/dx]^{-1}.$$

This given for transmission

$$P(\Delta E_b) = (d-c)^{-1} \neq f(x)$$

and for reflection

$$P(\Delta E_b) = (d+c)^{-1}$$

where d and c are the maximum and minimum values of the two quantities Δ_b and $\eta\Delta_a$.

For $d = c$ in the transmission process we get the well-known formula for zero energy spread (this was derived by Cohen (1959) and Nagib (1960) who carried out a precise analysis of the spread in energy for a monoenergetic beam)

$$\cos(\theta-\varphi)/\cos\varphi = I_b/\eta I_a.$$

Here, I_a and I_b are the characteristic ionization values for the particles a and b for a given target material.

3. *Gaussian incident beam*

Now, $P(E_a) = G(E_a)$ is by definition a Gaussian distribution with the parameters E_a^0 (mean energy) and G_a (variance of the distribution). From the statistical relation $P(E_a)dE_a = P'(E_b)dE_b$, by virtue of assumption II, we obtain $P'(E_b) = G(E_b)$, with the parameters $\sigma_b = \sigma_a \eta$ and $E_b^0 = f(E_a^0, \theta)$ given by Eq. (1).

The resultant spread due to the Gaussian distribution of the beam from the accelerator and the linear spread arising in the target for a monoenergetic beam will thus be for transmission

$$P(E_b) = \frac{1}{d-c} \int_{E_b+c}^{E_b+d} G(E_b')dE_b'.$$

After transformation of the coordinates this yields

$$P(z) = \frac{1}{d-c} \int_{z+z_1}^{z+z_2} G(z')dz'$$

where

$$z = (E_b - E_b^0)\sigma_b^{-1},\ z_1 = c\sigma_b^{-1},\ z_2 = d\sigma_b^{-1}$$

and $G(z')$ = Gauss function with $\sigma = 1$.

Similarly, in the reflection case we get

$$P(z) = \frac{1}{d+c} \int_{z}^{z+z_1+z_2} G(z')dz'.$$

Taking the transformation $z_i = z - z_s$, where $z_s = -\frac{z_1+z_2}{2}$, we have

$$P(z_i) = \frac{1}{d+c} \int_{z_i-p}^{z_i+p} G(z')dz'$$

both for reflection (R) and transmission (T) with p equal $\frac{z_1+z_2}{2}$ and $\frac{z_2-z_1}{2}$ correspondingly.

$P(z_i)$ is symmetrical with respect to $z_i = 0$ $(z = z_s)$. The centers of the two curves are shifted by the amount $z_s = \frac{-z_1+z_2}{2}$ which is equal to the energy loss in the reaction which takes place at $x = \frac{1}{2}$.

From the condition for z_0 (dimensionless and equal to half the value at half maximum), $P(z_i = z_0) = \frac{1}{2}P(z_i = 0)$, we get

$$\int_0^p G(z')dz' = \int_x^{2p+x} G(z')dz'$$

where $z_0 = x + p$. With this formula we can calculate the z_0 *versus* p curve (Fig. 2).

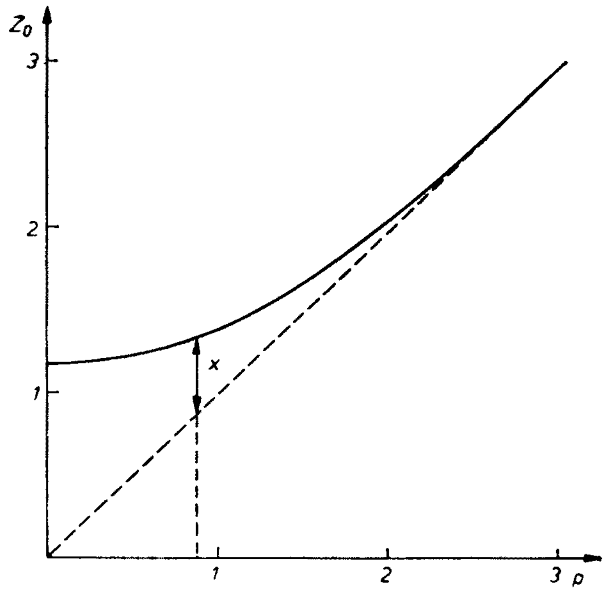

Fig. 2. The half value at half maximum of resultant energy spread curve in $\sigma_p = 33.1$ keV units as a function of parameter of the "effective" target thickness

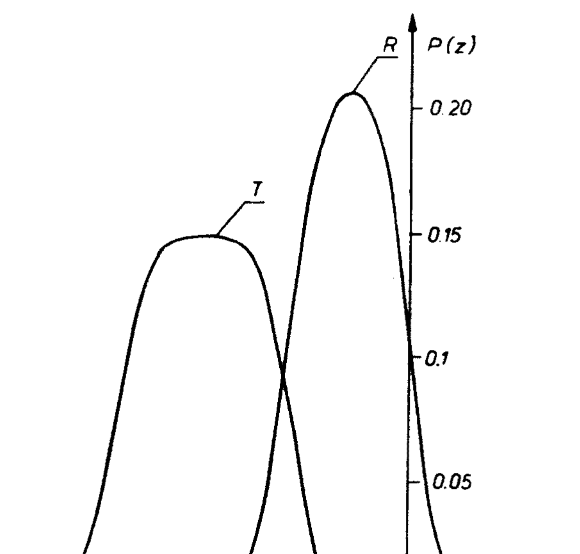

Fig. 3. Theoretical energy spread curves for outgoing beam for (d, p) reaction on ^{12}C with $Q = 2.72$ MeV, in Cracow cyclotron. Target thickness (S) absorbs 100 keV of the deuteron energy. Curve R is for the reflection case ($\theta = 145°$ and $\varphi = 45°$) and curve T is for the transmission case ($\theta = 145°$ and $\varphi = 65°$). Z is given in $\sigma = 33.1$ keV units

The half-width of the resultant spread in energy of the particles b is

$$d_b' = 2z_0(p)\sigma_b = \frac{z_0(p)d_b}{\sqrt{2\ln 2}} = \frac{z_0(p)\eta d_a}{\sqrt{2\ln 2}}$$

where d_a and d_b are the half-widths of the Gaussian distributions for the particles a and b, respectively. The curve for $p = 0$ is the primary Gaussian curve with $z_0 = \sqrt{2\ln 2}$. For growing p the curve remain symmetrical, but become more spread out (less for transmission, more for reflection).

By way of illustration Fig. 3 presents the energy spread curves for the (d, p) reaction on ^{12}C with $Q = +2.72$ MeV, $E_d^0 = 12.7$ MeV and $d_d = 120$ keV (data for the Cracow cyclotron). From Eq. (1) we have $E_p^0 = 10.585$ MeV and $\eta = 0.65$. $\sigma_p = 60$ keV $\cdot\ \eta/\sqrt{2\ln 2}$ $= 33.1$ keV is the unit for z defined before. The target thickness (S) is such that 100 keV of a deuteron's energy is absorbed during its passage perpendicularly through the target. Curve R of Fig. 3 is the curve for reflection, with $\theta = 145°$ and $\varphi = 45°$; whereas curve T is the curve for transmission, with $\theta = 145°$ and $\varphi = 65°$. For both curves the half values at half maximum (z_0) are nearly equal to p defined previously (curve R: $p = 2.38$ and $z_0 \cong 2.40$; curve T: $p = 3.40$ and $z_0 \cong 3.40$). The shifts of the centers of the curves are equal to z_s (-2.38 for curve R and -8.02 for curve T).

4. *Discussion*

The assumptions of linear energy absorption and stability of the derivative η in the region of the resultant spread in energy are satisfied well for the 'thin' targets used in practice. Some doubts may arise, however, about the assumption that the cross-section of the reaction is constant in the region of the resultant spread in energy of the particles a. Also neglected here is the effect of the spread in energy of products which arises in the nuclear reaction itself. Both of these effects depend on the specific type of reaction and can be treated similarly as has been done here.

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