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MAGNETIC SUSCEPTIBILITIES IN MESOSCOPIC CYLINDERS*

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We calculate and discuss orbital magnetic susceptibilities in mesoscopic cylinders made of a normal metal or a semiconductor for different shapes of the Fermi surfaces and for different circumferences of the cylinders.

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It is well known that paramagnetic or diamagnetic persistent currents, run by coherent electrons, can be induced in mesoscopic systems by a static magnetic field [1]. Persistent currents in mesoscopic systems have been the subject of many investigations [1–3]. For recent reviews on persistent currents in normal metals see Ref. [4].

In this paper we present some model calculations of orbital magnetic susceptibilities in thinwalled mesoscopic cylinders made of a normal metal or a semiconductor. We assume that electrons interact via the magnetostatic (current–current) interaction taken here in the self-consistent mean field approximation. As a result the current is calculated in a self-consistent way and one obtains spontaneous current solutions at low temperatures [3]. Both the current and the susceptibility are enhanced by the presence of the interactions. The magnitude of persistent current and hence of the susceptibility depends also strongly on the coherence of currents from different channels, *i.e.* on the shape of the Fermi Surface (FS). We show below that strong magnetic response can be obtained for systems with the FS exhibiting some departure from the spherical shape.

It is well known [5] that mesoscopic systems can exhibit strong orbital magnetism for temperature $T \leq \Delta_0/2\pi^2 k_{\mathrm{B}} \equiv T'$ ($\Delta_0 = hv_{\mathrm{F}}/L_x$ is the

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"inverse time of flight" or in other words, the quantum size energy gap for an electron going around the circumference of the cylinder, L_x is the circumference of the cylinder). The orbital magnetic susceptibility of the electron gas can be then much larger than the Landau value and can have either sign [5].

In this paper we neglect spin because the orbital quantum numbers close to the Fermi energy are large, and the inclusion of spin does not change qualitatively the considered phenomena. Systems with spin have been analysed in Ref. [6].

In the system of cylindrical geometry with the magnetic field applied along the cylinder axis, the total flux ϕ which drives the current I is the sum of the external flux ϕ_e and the flux ϕ_I coming from the current itself

$$\phi = \phi_e + \phi_I, \quad \phi_I = \mathcal{L} I \,, \tag{1}$$

where $\mathcal{L}$ is the inductance coefficient, for a long cylinder $\mathcal{L} = \mu_0 \pi R^2 / L_z$, R is the radius of the cylinder.

In normal metal cylinders of mesoscopic size coherent and normal electrons coexist. The persistent current response is given by coherent electrons and it is reduced by the presence of disorder and fluctuations of different kinds. In our recent papers [3, 7] we derived the formula for persistent currents as a function of temperature and magnetic flux in the presence of fluctuations of the total momentum and disorder,

$$\begin{aligned} I(\phi,\gamma,T) &= M_r^{(1-1/u)} \sum_{m/1}^{M_z} \sum_{q/1}^{\infty} \frac{4I_0(m)}{\pi} \\ &\quad \times \left(\frac{L_x}{2\gamma} + \frac{2\pi^2 k_{\mathrm{B}} T}{\Delta_0}\right) \frac{\exp\left[-q\left(\frac{L_x}{\gamma} + \frac{2\pi^2 k_{\mathrm{B}} T}{\Delta_0(1-\rho)}\right)\right]}{1-\exp\left[-q\left(\frac{L_x}{\gamma} + \frac{4\pi^2 k_{\mathrm{B}} T}{\Delta_0(1-\rho)}\right)\right]} \\ &\quad \times \cos(q k_{F_x}(m) L_x) \sin\left(2\pi q \frac{\phi}{\phi_0}\right) , \end{aligned} \tag{2}$$

where $1/\gamma$ is a disorder parameter, related to the mean free path l_e by the formula [7],

$$l_e \cong a(1/\gamma)^{-4/3} \left[3.5a \ln\left(\frac{L_x}{a}\right)\right]^{-4/3} , \tag{3}$$

ρ $(0 \le \rho \le 1)$ is the relative fluctuation parameter, ρ is proportional to the number of normal (non-coherent) electrons in the system; $I_0(m) = e\hbar k_{F_x}(m)/(m_e L_x)$; M_z is the number of channels in the k_z direction, M_r is the number of channels in the y direction, a is the lattice constant.

Persistent currents given by Eq. (2) can be paramagnetic or diamagnetic. In this paper we discuss only the paramagnetic current solutions.

In order to discuss the influence of ϕ_I on the coherent properties of mesoscopic cylinders we calculate the energy gap at the FS for electrons going around the circumference of the cylinder (and for $\phi < \phi_0/2$):

$$\Delta_{\mathrm{F}} \equiv E_{\alpha_{\mathrm{F}}+1,m} - E_{\alpha_{\mathrm{F}},m} = \Delta_0 \left(1 - 2\frac{\phi_e}{\phi_0} + 2\frac{\mathcal{L}\,|I|}{\phi_0}\right) , \tag{4}$$

where $E_{\alpha,m} = \hbar^2[(\alpha_m - \phi/\phi_0)^2 + k_z^2(m)R^2)]/(2m_e R^2)$. Δ_{F} contains a term

$$\Delta_d \equiv \Delta_0 \frac{\mathcal{L}\,|I|}{\phi_0} , \tag{5}$$

Δ_d is the dynamic part of the energy gap. Δ_d strongly depends on the relative fluctuation ρ, what is presented in Fig. 1.

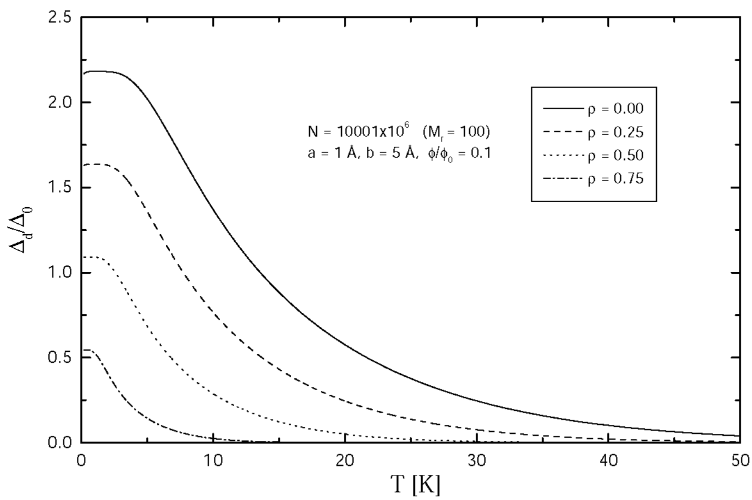

Fig. 1. The dynamic energy gap Δ_d as a function of temperature T for different values of the relative fluctuaction ρ.

Thus on the top of the quantum size energy gap there appears in the system the dynamic gap which comes from the magnetostatic interaction and which increases the coherence in the sample.

The total current depends on the correlation of currents from different channels which is related to the shape of the FS. In the follwing we study the magnetic response of the normal metal for different shapes of the FS. To simulate different shapes of the 2D FS [3, 8] we used the relation

$$k_{\mathrm{F}}^u = |k_{\mathrm{F}_x}|^u + k_{\mathrm{F}_z}^u , \tag{6}$$

where u is a real number,

$$k_{\mathrm{F}_x}(m) = k_{\mathrm{F}}\left[1 - \left(\frac{k_{\mathrm{F}_z}(m)}{k_{\mathrm{F}}}\right)^u\right]^{(1/u)}, \quad k_{\mathrm{F}_z}(m) = \frac{m\pi}{L_z},$$

m is a positive integer.

For $u = 2$ we get the semicircular FS, for $u \geq 12$ we get the nearly rectangular FS, and for $2 < u < 12$ we get the FS intermediate between semicircular and rectangular, see Fig. 2. For $u = 2$ the currents from different channels add almost without correlation and the total current is the smallest. The correlation and the magnitude of the current increases with increasing u, *i.e.* with increasing curvature of the FS.

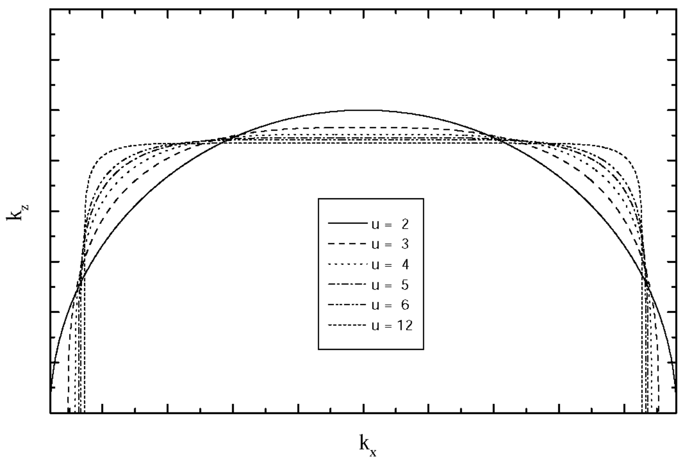

Fig. 2. Shapes of the Fermi surfaces for different values of parameter u.

$M_r^{(1-1/u)}$ is introduced here to simulate the shape of the FS in the k_y direction. In order to calculate the current I in real 3D systems exactly (*i.e.*, where k_{F_x} depends as on k_{F_z} as on k_{F_y}), very fast computers would be necessary.

Eqs. (1) and (2) form the two self-consistent equations for the current. Using them we can calculate the susceptibility $\chi(T)$:

$$\chi = \frac{\partial M}{\partial H_e}, \tag{7}$$

where the magnetization M is given by the formula

$$M = \frac{I}{L_z}, \tag{8}$$

$$H_e = \frac{\phi_e}{\mu_0 \pi R^2}\,, \tag{9}$$

H_e is the external magnetic field. Inserting Eqs. (8) and (9) into (7) and making use of Eq. (1) we obtain the final formula for the susceptibility in interacting system:

$$\chi = \frac{\chi_0}{1-\chi_0}\,, \tag{10}$$

where

$$\chi_0 = \mathcal{L}\frac{\partial I(\phi,\gamma,T)}{\partial \phi}\,. \tag{11}$$

We can see from Eq. (10) that if $\chi_0 \to 1$ we get an instability towards an ordered state.

In Figs. 3 and 4 the susceptibilities as functions of temperature T for different FS (different u) and different circumferences L_x of the cylinder are presented. We see that at a given temperature T the magnetic response increases with increasing curvature of the FS and with decreasing L_x. The crossover temperature T_c from para- to ferromagnetic state also increases with increasing curvature of the FS (increasing u) and decreasing circumference L_x.

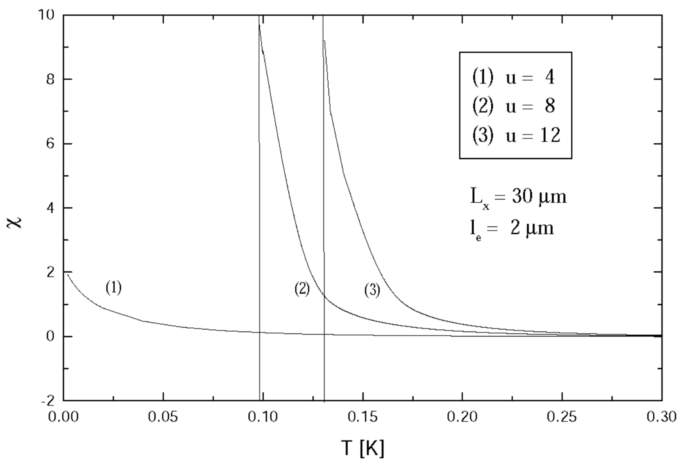

Fig. 3. Susceptibility χ as a function of temperature T for different shapes of the Fermi surfaces (different u) and $l_e = 2$ μm, $M_r = 120$.

Now let us compare the orbital magnetism of a mesoscopic system interacting via the magnetostatic interaction with the magnetism of a set of spins coupled by the exchange interaction. Let us assume we have N_s spins. In a paramagnetic state

$$M_H = \chi_1 H_e\,, \tag{12}$$

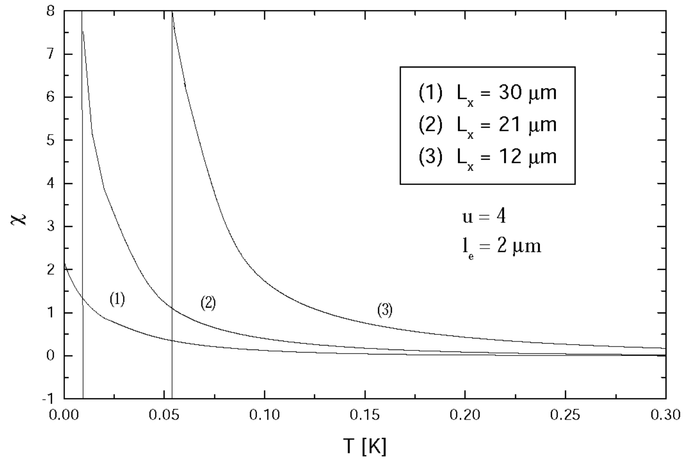

Fig. 4. Susceptibility χ as a function of temperature T for different circumferences L_x of the cylinder and $l_e = 2$ μm, $M_r = 120$.

where M_H is the magnetisation, and

$$\chi_1 = \frac{N_s \mu_1^2}{k_{\rm B} T} \equiv \frac{C}{T}\,, \tag{13}$$

$\mu_1 = M_H/N_s$ is the magnetic moment.

In the case of a spin ferromagnet, according to Weiss theory, there is a local field coming from spin interactions λM_H, and hence

$$M_H = \chi_1(H_e + \lambda M_H) \equiv \chi H_e\,, \tag{14}$$

where $\lambda = 2J/N_s$, J is the interaction constant.

From Eq. (14) we find the formula for the magnetic susceptibility

$$\chi = \frac{\chi_1}{1 - \lambda \chi_1}\,, \tag{15}$$

If $N_s \gg 1$ then λ is very small. According to Eq. (13)

$$\chi = \frac{C}{T - C\lambda} \equiv \frac{C}{T - T_{\rm CW}}\,, \tag{16}$$

where $T_{\rm CW}$ is the Curie–Weiss temperature, *i.e.* the temperature at which a phase transition from the paramagnetic to the ferromagnetic state occurs.

Summarizing, in the presented paper we have discussed the orbital magnetic susceptibility in a mesoscopic cylinder made of a normal metal or a semiconductor. We have investigated the dependence of the magnetic response on the shapes of the Fermi surfaces and on the circumferences of the cylinders. The crossover temperature T_c from para- to ferromagnetic state increases with increasing curvature of the FS (increasing u) and decreasing circumference L_x.

The formula for the orbital magnetic susceptibility can be compared with a susceptibility of a set of interacting spins. In these two cases we observe a different behavior in the external magnetic field. We can also notice that the "orbital ferromagnetism" is a phenomenon opposite to superconductivity where $H_e + M_H = 0$ and hence $\chi = -1$, *i.e.* full expulsion of the magnetic flux from the sample occurs.

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