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Citation style: Lesiak M., Zipper Wiktor, Czakański Jacek. (1985). A potential model of fusion with transmission coefficients calculated by the matrix method. "Acta Physica Polonica. B" (1985, no. 8, s. 775-783).

Vol. B16 (1985) ACTA PHYSICA POLONICA No 8

A POTENTIAL MODEL OF FUSION WITH TRANSMISSION COEFFICIENTS CALCULATED BY THE MATRIX METHOD*

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(*Received December 6, 1984*)

A barrier penetration model of heavy-ions fusion is presented. To calculate the transmission coefficients through any one-dimensional barrier of nucleus-nucleus real potential a matrix method is used. The parameters of the model are the critical radius and the parameters of nuclear interaction. The model is tested on several cases of fusion, i.e. $\alpha + {}^{40,44}Ca$, ${}^{12}C + {}^{12}C$, ${}^{16}O + {}^{16}O$ and ${}^{12}C + {}^{24}Mg$ and it is found to reproduce the data quite well.

PACS numbers: 25.70.Ij

1. *Introduction*

The interaction potential $V(r, l)$ of two colliding nuclei in the sudden-approximation is given as a sum of nuclear V_x, Coulomb V_c and centrifugal terms, i.e.

$$V(r, l) = V_x(r) + V_c(r) + \frac{\hbar^2 l(l+1)}{2\mu r^2}. \tag{1}$$

In the potential model of fusion it is assumed that the incoming particle penetrates the potential barrier and the fusion occurs, when it reaches some critical distance R_{cr} [1, 2]. In our model the energy dissipation of the relative motion at distances $R > R_{cr}$ is neglected. At the distance $R = R_{cr}$ the kinetic energy of the ions is then decreased abruptly. The fusion cross section can be written as follows [2]:

$$\sigma_{fus}(E) = \frac{\pi}{k^2} \sum_{l=0}^{\infty} [1+(-1)^{l+2I}\delta/(2I+1)]\,(2l+1)T_l(E), \tag{2}$$

where k is the wave number of incoming particle, and $\delta = 0$ ($\delta = 1$) for nonidentical (identical) nuclei, I is the spin of the incident particle, $T_l(E)$'s are the transmission coefficients through the potential barrier at R_{cr} for a given angular momentum l.

* This work was supported by the Institute of Experimental Physics of the Warsaw University under contract MR. I. 5.12.04.

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The crucial point in Eq. (2) is the calculation of the transmission coefficients $T_l(E)$'s. There are several approaches to calculate T_l's: f.i. the semiclassical method [3], the parabolic approximation [1, 4], the double barrier method [2] and the parabolic approximation with a Coulomb tail [5]. These approaches to calculate T_l's give relatively simple expressions for the fusion cross section but the approximations introduced influence the physical interpretation of the results significantly. As a consequence of these approximate calculations of the transmission coefficients one gets usually wrong information about the fusion process, even if one knows the interaction potential. In the present paper we propose a matrix method for the calculation of transmission coefficients for any shape of the static nuclear potential barrier.

2. *The matrix method of T_l's calculation*

Let us take two points on the r-axis, namely R_{cr} and R_{cut} (see Fig. 1). We assume that the fusion appears for $r < R_{cr}$. R_{cut} is the distance beyond which the nuclear potential V_N becomes negligible. The potential curve for a given angular momentum l is divided

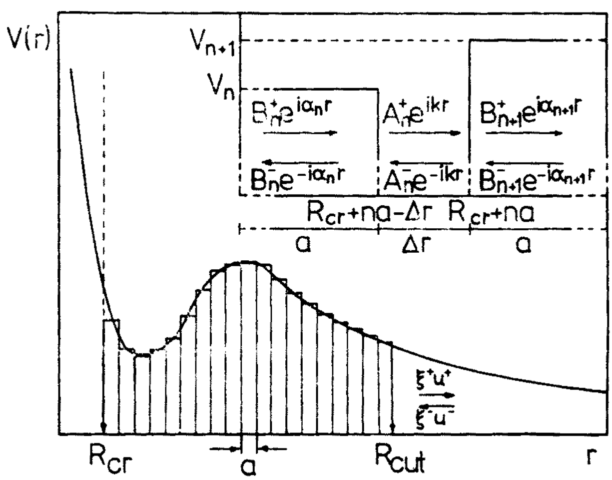

Fig. 1. The shape of the potential barrier for a given angular momentum l. The form of the wave functions in all intervals is defined in the inset

in the region of $R_{cr} \leqslant r \leqslant R_{cut}$ into N equal parts. Each part forms a rectangular barrier of height $V_n = V(R_{cr}+(n-\frac{1}{2})a)$, $(n = 1, 2, ..., N)$ and width $a = (R_{cut}-R_{cr})N$. The wave going to the right (left) is labelled by "+" ("−"). We assume $V(r) = 0$ in the intervals $[R_{cr}-\Delta r, R_{cr})$, $[R_{cr}+a-\Delta r, R_{cr}+a)$, ..., $[R_{cr}+Na-\Delta r, R_{cr}+Na)$, (without points R_{cr}, $R_{cr}+a$, ..., $R_{cr}+Na$). The corresponding wave functions have the following forms:

$$\psi = A^+e^{ikr}+A^-e^{-ikr}, \quad \psi = A_1^+e^{ikr}+A_1^-e^{-ikr}, \ldots, \quad \psi = A_N^+e^{ikr}+A_N^-e^{-ikr}.$$

On the other hand in the intervals $[R_{cr}+(n-1)a,\ R_{cr}+na-\Delta r)$ (with points R_{cr}, $R_{cr}+na$). the corresponding wave functions have the forms $\psi = B_n^+ e^{i\alpha_n r}+B_n^- e^{-i\alpha_n r}$ ($n = 1, 2, \ldots, N$) For $r \geqslant R_{cut}$, $\psi = \xi^+u^+ + \xi^-u^-$, where $u^\pm = G \pm iF$ and G, F are the Coulomb wave functions [5].

The wave function and its derivative must be continuous at the points R_{cr}, $R_{cr}+a-\Delta r$, $R_{cr}+a$, ..., $R_{cr}+Na-\Delta r$, $R_{cr}+Na$. If we eliminate all A_n and B_n ($n = 1, 2 \ldots, N$) from the equations of continuity, we have for $\Delta r \to 0$:

$$\begin{pmatrix} A^+ e^{ikR_{cr}} \\ A^- e^{-ikR_{cr}} \end{pmatrix} = \left(\prod_{n=1}^{N\to\infty} \hat{T}(\alpha_n) \right) \hat{C} \begin{pmatrix} \xi^+ \\ \xi^- \end{pmatrix}, \tag{3}$$

where the matrix elements of $\hat{T}(\alpha_n)$ and $\hat{C}$ are the following:

$$t_{11}(\alpha_n) = t_{22}^*(\alpha_n) = \cos \alpha_n a - \tfrac{1}{2}\, i \left(\frac{\alpha_n}{k} + \frac{k}{\alpha_n} \right) \sin \alpha_n a, \tag{4a}$$

$$t_{21}(\alpha_n) = t_{12}^*(\alpha_n) = \tfrac{1}{2}\, i \left(\frac{\alpha_n}{k} - \frac{k}{\alpha_n} \right) \sin \alpha_n a, \tag{4b}$$

$$c_{11} = c_{22}^* = \tfrac{1}{2} \left[u^+(\varrho_{cut}) - i u^{+\prime}(\varrho_{cut}) \right], \tag{5a}$$

$$c_{21} = c_{12}^* = \tfrac{1}{2} \left[u^+(\varrho_{cut}) + i u^{+\prime}(\varrho_{cut}) \right], \tag{5b}$$

with

$$k = (2\mu E/\hbar^2)^{1/2}, \tag{6a}$$

$$\alpha_n = [2\mu(E-V_n)/\hbar^2]^{1/2}, \tag{6b}$$

$$\varrho_{cut} = k \cdot R_{cut}, \tag{7a}$$

$$u^{\pm\prime} = \left[\frac{du^\pm}{d\varrho} \right]_{\varrho=\varrho_{cut}}. \tag{7b}$$

If $E = V_n$, the term $\dfrac{k}{\alpha_n} \sin \alpha_n a$ which appears in Eq. (4) has the value ka and the singularity disappears at the turning points.

If $N \to \infty$ i.e. $a \to 0$, the transmissions coefficient through the potential barrier for the wave going to the right has the form:

$$T^+ = \left[\left| \frac{\xi^+}{A^+} \right|^2 \right]_{\xi^-=0} \tag{8a}$$

and for the wave going to the left:

$$T^- = \left[\left| \frac{A^-}{\xi^-} \right|^2 \right]_{A^+=0}. \tag{8b}$$

One can show, that $T^+ = T^-$, because the Wronskian $W(G, F) = 1$. Thus the transmission coefficients do not depend on the direction of the incoming wave. The formula (3) will be used in calculations of the fusion cross sections from equation (2).

3. *Applications and discussion of the results*

We have analyzed the experimental fusion cross sections for the systems $\alpha+^{40,44}$Ca, ^{12}C+^{12}C, ^{12}C+^{24}Mg and ^{16}O+^{16}O. For the cases $\alpha+^{40,44}$Ca we have assumed that the nuclear potential V_x is equal to the real part of the optical model potential which reproduces the elastic scattering very well [6] i.e.:

$$V_x(r) = -U_0 f^2(r, d, b),$$

where

$$U_0 = U_1 + U_2 E_\alpha,$$

and

$$f(r, d, b) = \left[1+\exp\left(\frac{r-dA_{\mathrm{T}}^{1/3}}{b}\right)\right]^{-1}.$$

E_α is the energy of α-particle in the laboratory system.

For the other cases we have used the Satchler folding model potential [7, 18]. The Coulomb potential was taken to be that of an uniformly charged sphere of radius R_{coul}. We have calculated the fusion cross sections as a function of energy with transmission coefficients based on the matrix method. The critical values of R_{cr} used in the calculations are given in Table I. The width barrier parameter a was taken to be $a = 0.05$ fm.

The results of our calculations are presented as a solid line in Fig. 2. For comparison we present here also the results based on the Hill-Wheeler parabolic approximation (dashed line) for the $\alpha+^{40,44}$Ca systems. One can see, that the present approach gives a much better fit to the experimental data [10] than the Hill-Wheeler method.

As a second case, we have fitted the experimental data of Kovar et al. for the ^{12}C+^{12}C system [11] — the results are presented in Fig. 3. One can see, that the experimental points are well reproduced up to an energy of 22 MeV. In Fig. 3 the data of Parks et al. [12] and Nambodiri [13] are also presented. The points of this last experiment are placed near the curve of present model. The Hill-Wheeler method reproduces well only the points of the experiment made by Nambodiri.

The curves given by our method and by the parabolic approximation are very similar for the ^{12}C+^{24}Mg [14] (Fig. 4) and ^{16}O+^{16}O [15, 17] (Fig. 5) systems. Both describe the experiment very well. The bad fits of the fusion excitation function obtained by the Hill--Wheeler method can be seen on the graph of the $\alpha+^{40}$Ca reaction (Fig. 6a). The parabola used in this method is a bad approximation for the potential barrier curve; these two curves agree only at the vertex area. If we consider then the reaction ^{12}C+^{24}Mg, we see from Fig. 4 that both curves lie very close to one another, so the parabola is a good approximation for the potential barrier in this case (Fig. 6b).

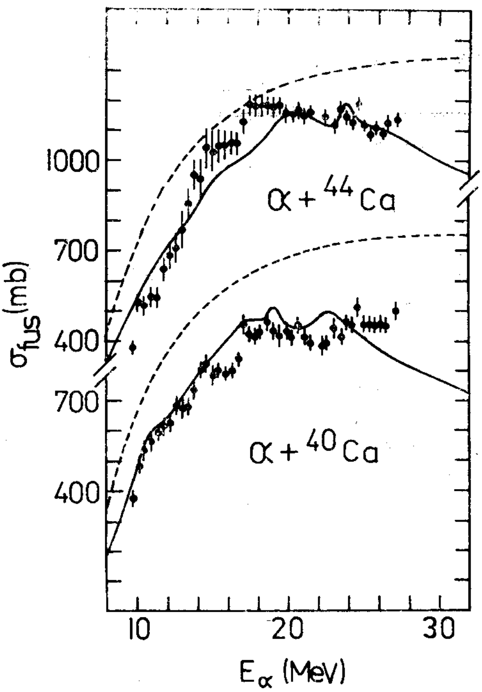

Fig. 2. Fusion cross sections for $\alpha+{}^{40,44}Ca$ from Ref. [10]. The solid line is the result of present model. The dashed curve gives the Hill-Wheeler approximation

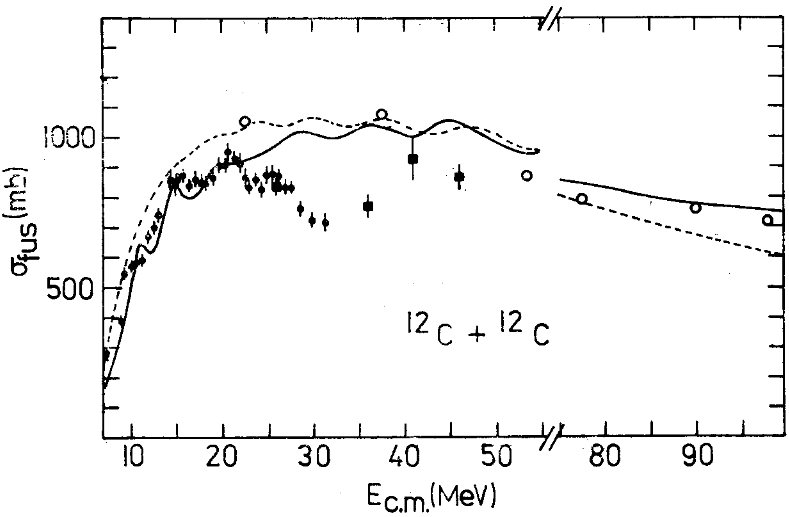

Fig. 3. Fusion cross sections for ${}^{12}C+{}^{12}C$. Data (●) are from Ref. [11], (■) from Ref. [12], (○) from Ref. [13]. The curves have the same meaning as in Fig. 2

TABLE I

Potentials used in the calculation. $R_{cr} = r_{cr}(A_T^{1/2}+A_I^{1/3})$. For $\alpha+^{40,44}$Ca: $R_{cr} = r_{cr}A_T^{1/3}$. For details see text

Reaction	Potential	R_{cut} (fm)	R_{cr} (fm)	r_{cr} (fm)
$\alpha+^{40}$Ca	U_1 = 198.6 MeV U_2 = −0.334 d = 1.37 fm b = 1.29 fm R_{coul} = 4.45 fm	12.015	1.81±0.05	0.53±0.02
$\alpha+^{44}$Ca	U_1 = 171.8 MeV U_2 = −0.146 d = 1.42 fm b = 1.25 fm R_{coul} = 4.58 fm	12.015	2.21±0.05	0.63±0.02
^{12}C+^{12}C	Satchler-fold. R_{coul} = 5.95 fm	11.625	3.48±0.05	0.76±0.01
^{12}C+^{24}Mg	Satchler-fold. R_{coul} = 6.73 fm	15.025	4.93±0.05	0.95±0.01
^{16}O+^{16}O	Satchler-fold. R_{coul} = 6.55 fm	13.025	5.53±0.05	1.10±0.01

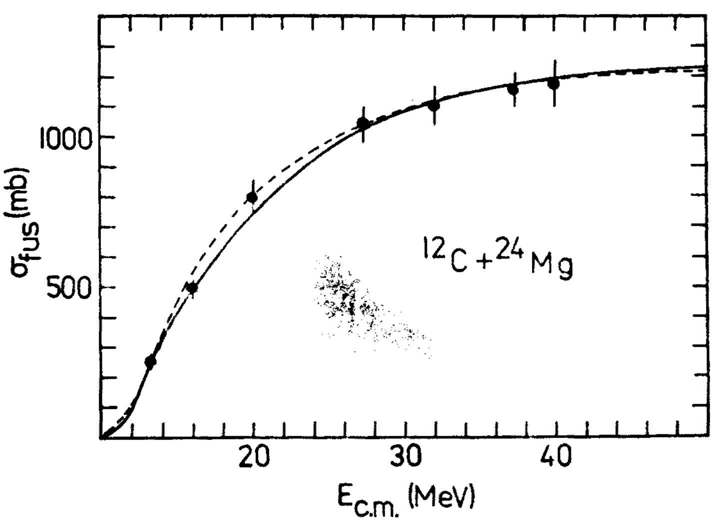

Fig. 4. Fusion cross sections for ^{12}C+^{24}Mg. Data were taken from Ref. [14]. The curves have the same meaning as in Fig. 2

Summarizing, it follows from the present considerations, that the parabolic potential approximation can be used only if the shape of the potential barrier is close to a parabola. In all other cases, one should be cautious with the interpretation of the results obtained by the use of the Hill-Wheeler method, even if one gets a good fit for the fusion excitation function since it can lead to a wrong physical interpretation. It follows from the analysis of $^{12}C+^{24}Mg$ and $^{16}O+^{16}O$ reactions, that the critical distances in the present calculations are close to one fm.

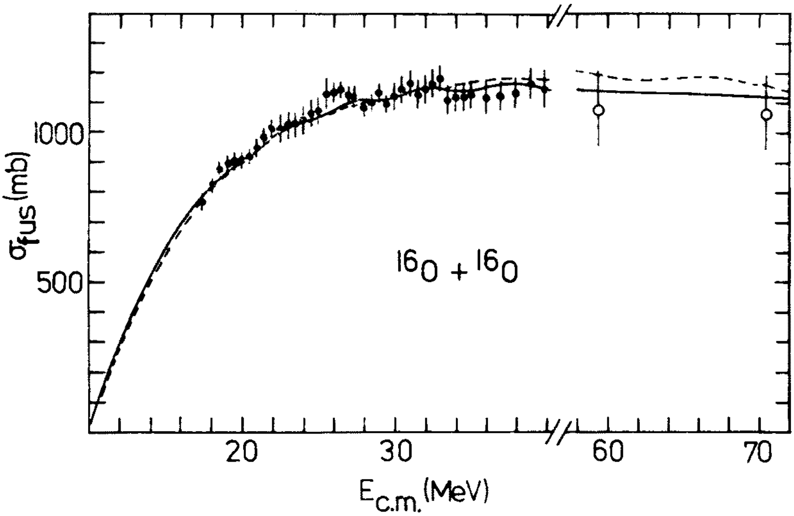

Fig. 5. Fusion cross sections for $^{16}O+^{16}O$. Data (●) are from Ref. [15] and (○) from Ref. [17]. The curves have the same meaning as in Fig. 2

Galin et al. [8] propose the value $R_{cr} = 1.0 \pm 0.07$ fm, which is equal approximately to the sum of two half nuclear matter density radii. A similar observation is contained in the work of Natowitz et al. [9]. One should notice that our R_{cr} is not defined by the radius of the half density, but that it defines the distance for which the assumed nuclear potential is meaningful for the reproduction of both elastic scattering and fusion processes [16, 19]. Hence our critical radius is a free parameter and it is impossible at present to calculate it correctly.

In any case, however, the critical distance R_{cr} is determined by the shape of the potential of the colliding ions, their internal structure and, in some cases, by shell effects. The most popular approach to study the fusion process has been proposed by Glas and Mosel [1]. However, a consequence of a simple barrier penetration model with the inverted parabola approximation is that the barrier parameters obtained from the Hill-Wheeler transmission probability are not realistic.

The main advantage of the present model is that the transmission coefficients are calculated for any shape of a realistic potential barrier. Using this method we have been able to reproduce the experimental fusion data of the strongly nonsymmetric system

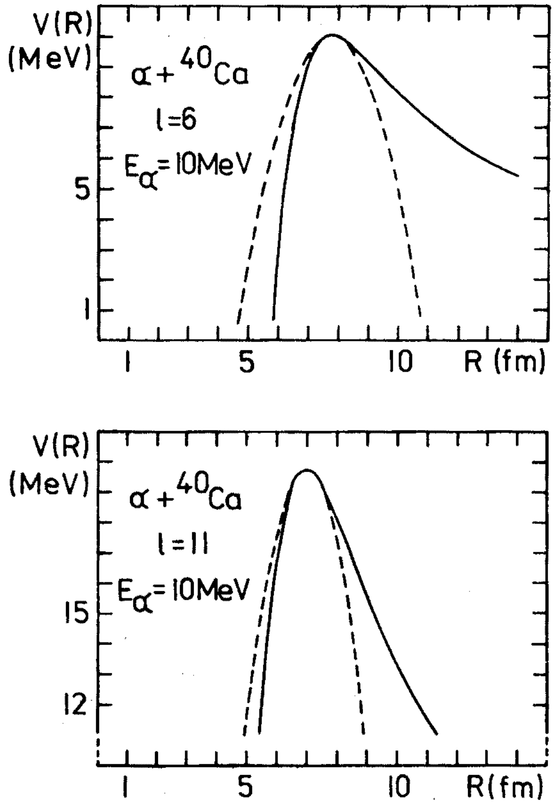

Fig. 6a

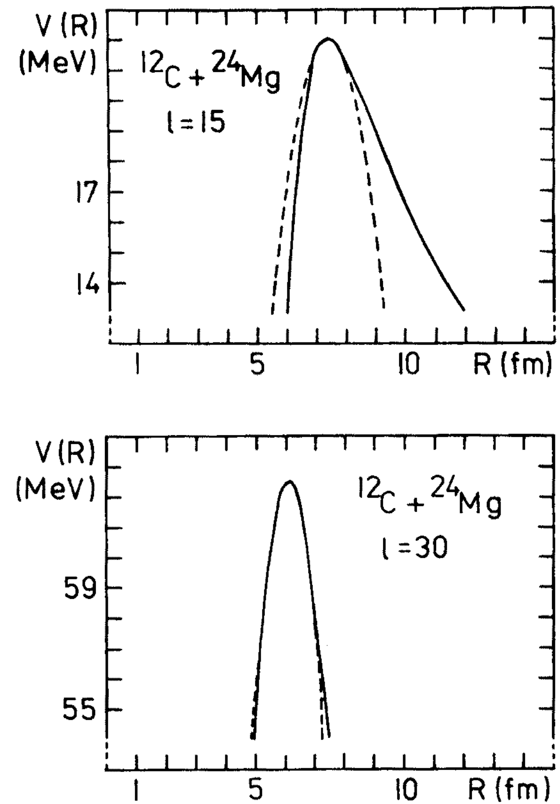

Fig. 6b

Fig. 6a, b. The interaction potential $V(r)$, for the $\alpha+{}^{40}Ca$ and ${}^{12}C+{}^{24}Mg$ systems in comparison to the method of parabolic simulation at the top of potential barrier

$\alpha + {}^{40,44}Ca$ as well as the symmetric ${}^{12}C + {}^{12}C$ and ${}^{16}O + {}^{16}O$ systems. Our model is valid over a broad energy range and works with any realistic nucleus-nucleus interaction. It takes into account the exact form of the Coulomb potential and the centrifugal barrier and it uses a general matrix formalism for calculating the transmission coefficient through a realistic potential barrier.

In our calculations we have used the real potential of a Saxon-Woods optical model potential, and the Satchler folding potential. These potentials are known to reproduce satisfactorily the elastic scattering data [16, 18] and in the present paper we have shown, that they fit also the energy behaviour of the fusion cross section.

The authors are deeply indebted to G.R. Satchler for providing the numerical values of our potential.

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