# GEOMETRIC PHASE OF OPEN TWO-LEVEL SYSTEMS*

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Geometric phase of open quantum systems is reviewed. An emphasis is given on specific features of the geometric phase which can serve as an indicator of type and strength of interaction between two-level system (qubit) and its bosonic environment. We study three examples: *(i)* a single qubit dephasingly coupled to the environment, *(ii)* a qubit being a part of quantum register, and *(iii)* a neutrino interacting with matter and environment.

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## 1. Introduction

Geometric phase is an example of an 'obvious' features of quantum mechanics which could remain overlooked by almost two generations of physicists [1]. Its role in various branches of quantum physics is hard to overestimate. The range of applications includes (among other) molecular systems, quantum Hall effects and field theory. For a complete and modern review the book [2] is recommended. In recent years the concept of the geometric phase has entered one more field of applicability: quantum information processing. The so-called holonomic quantum computation [3] has been discovered and recognized as a promising direction in quantum information retrieval due to their potential fault tolerance.

Our review is devoted to the concept of the geometric phase gained by evolving *open* quantum systems. Our primary aim is to convince the reader that the geometric phase, as a global feature of quantum evolution, is capable to reflect the information on the character of interaction between an open system and its environment. The paper is organized as follows: we start

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with a short introduction of a highly non-trivial concept of the geometric phase in *non-unitary* evolution of quantum systems in *mixed* states. Next, we recall the model of pure decoherence of a two-level system and present the exact reduced dynamics for the sytem. In the remaining part we shortly review three systems examplifying usefulness of geometric phases: *(i)* a qubit weakly coupled to the bosonic bath; *(ii)* a simplified quantum register and *(iii)* the oscillating neutrino interacting with environment.

## 2. Geometric phase for open quantum systems

The origin and development of the notion of geometric phase and some related geometric concepts of quantum mechanics are summarized in Ref. [1]. Here we are going to pass directly to the problem of the notion of geometric phase for open quantum systems, or as will be seen below, what the geometric phase *may be*, as there is no unique concept of the geometric phase for mixed states under arbitrary nonunitary dynamics. The first attempt to construct geometric phase has been proposed by Uhlmann. His proposal is of purely mathematical character, and despite its elegance, suffers from unsuffiently clear experimental meaning [4]. Other approaches are based on quantum trajectories [5], quantum interferometry [6] and the state purification (kinematic approach) [7]. In our investigations we have decided to use the kinematic approach. The idea seems to be simple: instead of considering mixed states of a quantum system (here we think always about qubits) one can construct its purification, *i.e.* the open system is embeded into a larger *closed* quantum system (described in terms of pure states only) by attaching the ancillary finite dimensional system such that the reduced dynamics reproduce the original open qubit dynamics (notice: the embeding is not into real bath, but into a fictitious system). Having pure state dynamics of the qubit–ancilla system in terms of a wave function one can introduce a meaningful concept of a phase with respect to this wave function. Such a construction is far from being trivial as there is an infinite family of possible purifications and one needs to choose 'the best one'. It can be achieved by a proper choice of the geometric concept of the *parallel transport* of purified states [7]. The GP constructed in Ref. [7] exhibits primary features: *(i)* it is purification-independent, *(ii)* gauge invariant and *(iii)* reduces to the standard definition in the limit of an unitary evolution, *i.e.* in the limit of closed qubit system. Probably the main advantage of studying this phase is its *measurability* via carefully prepared interferometric experiments (theoretically) proposed in [6, 7] and recently successfully performed in Ref. [8]. Our choice is thus determined by its potential experimental implementation. Here we recall the construction of the GP based on the method presented in Ref. [7]. First, we rewrite the density matrix for the qubit in the spectral-decomposition form

$$\rho(t) = \sum_{i=1}^{2} \lambda_i(t)\,|w_i(t)\rangle\langle w_i(t)|\,, \tag{1}$$

where $\lambda_i(t)$ and $|w_i(t)\rangle$ are the eigenvalues and eigenvectors of the density matrix $\rho(t)$ calculated at the time instant $t$. The GP $\Phi(t)$ corresponding to such an evolution is defined by the relation [7]

$$\Phi(t) = \mathrm{Arg}\left[\sum_{i=1}^{2}[\lambda_i(0)\lambda_i(t)]^{1/2}\langle w_i(0)|w_i(t)\rangle \exp\left(-\int_0^t \langle w_i(s)|\dot{w}_i(s)\rangle ds\right)\right], \tag{2}$$

where $\mathrm{Arg}\,[z]$ denotes the principal argument of the complex number $z$, $\langle w_i|w_j\rangle$ is a scalar product and the dot indicates the derivative with respect to time $s$.

## 3. Pure decoherence of a qubit

We study a qubit $Q$ (an arbitrary two-level quantum system) coupled to its environment. We assume a pure dephasing interaction between the qubit and the bosonic environment and we ignore the energy decay of the qubit. This assumption is reasonable in the case when the phase coherence decays much faster than the energy (the fastest relaxation process is pure dephasing). When the process of energy dissipation can be neglected and the only process which is responsible for the 'openness' is the *pure decoherence* the synonym of the term *dephasing* is also used in papers. With no energy dissipation there is still an irreversible information loss in the quantum system [9]. We model such a system by the Hamiltonian ($\hbar = 1$)

$$H = H_Q \otimes \mathcal{I}_B + \mathcal{I}_Q \otimes H_B + H_I\,, \tag{3}$$

where $\mathcal{I}_Q$ and $\mathcal{I}_B$ are identity operators in corresponding Hilbert spaces of the qubit $Q$ and the environment $B$, respectively. Let us denote the qubit canonical basis by $\{|1\rangle, |-1\rangle\}$. The qubit Hamiltonian $H_Q$ represented in this basis reads

$$H_Q = \varepsilon_+|1\rangle\langle 1| + \varepsilon_-|-1\rangle\langle -1|\,, \tag{4}$$

where $\varepsilon_\pm$ are the qubit energy levels. Let us notice that the level separation is a simplest tunable parameter of the model as it is related *e.g.* to a magnetic field acting on the spin system: if $\varepsilon_+ = -\varepsilon_- = \varepsilon$ then $H_Q = \varepsilon S^z$ is the spin Hamiltonian, where $S^z = |1\rangle\langle 1| - |-1\rangle\langle -1|$ and $\varepsilon$ is proportional to the amplitude of the magnetic field. The environment is modeled as a one-dimensional bosononic field and is described by the Hamiltonian $H_B$ of the form

$$H_B = \int_0^\infty d\omega\, h(\omega) a^\dagger(\omega) a(\omega)\,, \tag{5}$$

where the real-valued spectral function $h(\omega)$ depends on specific features of the environment. The operators $a^\dagger(\omega)$ and $a(\omega)$ are the creation and annihilation boson operators, respectively. The qubit-environment interaction, in general, can be assumed to be asymmetric

$$\begin{aligned} H_I &= |1\rangle\langle 1| \otimes H_+ + |-1\rangle\langle -1| \otimes H_-\,, \\ H_\pm &= \pm \int_0^\infty d\omega \left[ g_\pm^*(\omega) a(\omega) + g_\pm(\omega) a^\dagger(\omega) \right]. \end{aligned} \tag{6}$$

The van Hove operators $H_\pm$ depend on the coupling functions $g_\pm(\omega)$ (the star $^*$ denotes the complex conjugate). The Hamiltonian (3) can be reformulated in the following form

$$H = |1\rangle\langle 1| \otimes H_1 + |-1\rangle\langle -1| \otimes H_{-1}\,, \tag{7}$$

$$H_{1/-1} = H_B + H_\pm + \varepsilon_\pm \mathcal{I}_B\,. \tag{8}$$

Hamiltonians of the similar structure like (7) have been studied in the context of a quantum kicked rotator [10], chaotic dynamics of a periodically driven superconducting single electron transistor [11], the Josephson flux qubit [12] and quantum dots [13]. The model may also serve as a component of a simple quantum register [9]. Moreover, it contains, as particular cases, the widely used van Hove model [14] (for $g_+(\omega) = g_-(\omega)$) and the Friedrichs model [15] (for either $g_+(\omega) = 0$ or $g_-(\omega) = 0$). The generalized spin-boson model (7) has been applied to analyze the electron-transfer reactions [16] and the interconversion of electronic and vibrational energy [17].

The model (3)–(6) is exactly solvable in the sense that the exact density matrix of the qubit can be obtained for a wide class of initial states of the total system.

### *3.1. Dynamics of the total system*

Here we present a derivation of the reduced dynamics for the simplest case of symmetric (van Hove type) coupling *i.e.*

$$g_+(\omega) \equiv g_-(\omega) =: g(\omega) h(\omega)\,. \tag{9}$$

The case of symmetric coupling has been studied in various context in Refs. [18, 19, 20, 21]. The discussion of a more general model is present in [22].

Let us notice that in the canonical basis, the Hamiltonian (7) is a block-diagonal $2 \times 2$ matrix reading

$$H = \mathrm{diag}[H_1, H_{-1}]\,. \tag{10}$$

This form is convenient because we can directly apply results of Refs. [22,23] and solve the Schrödinger equation with the Hamiltonian (3) since the block-diagonal structure remains preserved for an exponential of the block-diagonal matrices. Let us specify an initial state of the total system assuming a product state, namely

$$|\Psi(0)\rangle = (b_1|1\rangle + b_{-1}|-1\rangle) \otimes |R\rangle\,, \tag{11}$$

where $b_1$ and $b_{-1}$ fully determine the qubit initial state and $|R\rangle$ is the initial state of the environment.

Time evolution of the state (11) is governed by [23]

$$\begin{aligned} |\Psi(t)\rangle &= b_1 e^{-i\Lambda_1(t)}|1\rangle \otimes D(g_t - g)e^{-iH_B t}|R\rangle \\ &\quad + b_{-1} e^{-i\Lambda_{-1}(t)}|-1\rangle \otimes D(g - g_t)e^{-iH_B t}|R\rangle\,, \end{aligned} \tag{12}$$

where the phases $\Lambda_1(t)$ and $\Lambda_2(t)$ have the form

$$\begin{aligned} \Lambda_1(t) &= \varepsilon_+ t - \int_0^\infty d\omega |g(\omega)|^2 \left\{h(\omega)t - \sin[h(\omega)t]\right\}\,, \\ \Lambda_{-1}(t) &= \varepsilon_- t - \int_0^\infty d\omega |g(\omega)|^2 \left\{h(\omega)t - \sin[h(\omega)t]\right\}\,. \end{aligned} \tag{13}$$

For any function $f$, the notation $f_t$ means

$$f_t(\omega) = e^{-ih(\omega)t} f(\omega)\,. \tag{14}$$

The displacement operator $D(f)$ reads [24]

$$D(f) = \exp\left\{ \int_0^\infty d\omega \left[ f(\omega)a^\dagger(\omega) - f^*(\omega)a(\omega) \right] \right\} \tag{15}$$

for an arbitrary square-integrable function $f$.

The explicit form of the wave function (12) of the total system allows to obtain a full information on the system. The corresponding density matrix $\varrho(t)$ of the total (isolated) system has the form

$$\varrho(t) = |\Psi(t)\rangle\langle\Psi(t)|\,. \tag{16}$$

### 3.2. Reduced dynamics

We do not need to know full information on the total system: qubit + environment. Rather the dynamics of the qubit only, an object of a potential experiment influenced by the environment, is crucial. The qubit dynamics can be obtained for the initial states given by Eq. (11) or, more generally, for a larger class of states defined by relation

$$\varrho(0) = \sum_{i,j=1,-1} p_{ij} |i\rangle\langle j| \otimes |R\rangle\langle R| \,, \tag{17}$$

where $p_{ij}$ are non-negative parameters. The reduced state $\rho(t)$ for the qubit only is obtained as a (partial with respect to the environment) trace of the full $Q+B$ state, and can be expressed in the form

$$\begin{aligned} \rho(t) &= \mathrm{Tr}_B \left\{ e^{-iHt} \varrho(0) e^{iHt} \right\} \\ &= \sum_{i,j=1,-1} p_{ij} |i\rangle\langle j| \otimes \mathrm{Tr}_B \left( e^{-iH_i t} |R\rangle\langle R| e^{iH_j t} \right) \\ &= \sum_{i,j=1,-1} p_{ij} c_{ji}(t) |i\rangle\langle j| \,, \end{aligned} \tag{18}$$

where $\mathrm{Tr}_B$ denotes partial tracing over the environment degrees of freedom, the environment operators $H_i (i=1,-1)$ are given by Eq. (8) and the functions $c_{ji}(t)$ are defined by the relation

$$c_{ji}(t) = \left\langle e^{-iH_j t} R | e^{-iH_i t} R \right\rangle = \langle \psi_j(t) | \psi_i(t) \rangle \ , \tag{19}$$

*i.e.* $c_{ji}(t)$ are determined by the scalar product of the environmental wave functions $|\psi_j(t)\rangle$ and $|\psi_i(t)\rangle$.

### 3.3. Explicit results

The pure decoherence model is simple enough to provide a very rare bonus: one can obtain explicit results for the quantum system affected by an environment, at least for certain classes of initial prepation. It follows from Eq. (18) that the qubit reduced dynamics can exactly be constructed, provided one is able to evaluate the corresponding scalar products in Eq. (19). It is possible at least for two classes of initial states of the environment: for the vacuum state $|R\rangle = |\Omega\rangle$ and the coherent states $|R\rangle = D(z)|\Omega\rangle$, where $D$ is the displacement operator (15). A coherent state of the bosonic field is determined by a complex function $z(\omega)$ (by analogy to the one mode cases, where standard coherent states are parameterized by a complex number).

We assume the initial coherent state in the form

$$|R\rangle = D(z)|\Omega\rangle = \exp\left\{\int_0^\infty d\omega\left[z(\omega)a^\dagger(\omega) - z^*(\omega)a(\omega)\right]\right\}|\Omega\rangle \tag{20}$$

for an arbitrary square-integrable function $z(\omega)$. The vacuum state is a particular case of the coherent state for $z(\omega) = 0$. For the coherent initial states of the environment, Eq. (12) takes the form

$$|\Psi(t)\rangle = b_1|1\rangle \otimes |\psi_1(t)\rangle + b_{-1}|-1\rangle \otimes |\psi_{-1}(t)\rangle\,, \tag{21}$$

where

$$\begin{aligned} |\psi_1(t)\rangle &= e^{-i\Lambda_2(t)}D(g_t - g + z_t)\,|\Omega\rangle\,, \\ |\psi_{-1}(t)\rangle &= e^{-i\Lambda_{-2}(t)}D(g - g_t + z_t)\,|\Omega\rangle\,, \end{aligned} \tag{22}$$

and the relation

$$D(g)D(f) = e^{i\mathrm{Im}\langle g|f\rangle}D(g+f) \tag{23}$$

has been applied, $\mathrm{Im}\langle g|f\rangle$ is the imaginary part of the scalar product of two functions $g$ and $f$ defined as

$$\langle g|f\rangle = \int_0^\infty d\omega g(\omega)f^*(\omega)\,. \tag{24}$$

The phases $\Lambda_2(t)$ and $\Lambda_{-2}(t)$ read

$$\begin{aligned} \Lambda_2(t) &= \Lambda_1(t) - \mathrm{Im}\langle g_t - g|z_t\rangle\,, \\ \Lambda_{-2}(t) &= \Lambda_{-1}(t) - \mathrm{Im}\langle g - g_t|z_t\rangle\,. \end{aligned} \tag{25}$$

It is convenient and very popular to present an initial qubit state $|\theta, \phi\rangle$ in relation to a vector on the Bloch sphere, *i.e.* to parameterize the state by two angles, namely,

$$|\theta, \phi\rangle = \cos(\theta/2)|1\rangle + e^{i\phi}\sin(\theta/2)|-1\rangle\,, \tag{26}$$

where $\theta$ and $\phi$ are the polar and azimuthal angles, respectively. This parameterization corresponds to $b_1 = \cos(\theta/2)$ and $b_{-1} = e^{i\phi}\sin(\theta/2)$ in Eq. (11). The initial state $\rho(0)$ for the reduced qubit dynamics takes then the matrix form

$$\rho(0) = \begin{pmatrix} \cos^2(\theta/2) & (1/2)\sin\theta e^{-i\phi} \\ (1/2)\sin\theta e^{i\phi} & \sin^2(\theta/2) \end{pmatrix}. \tag{27}$$

From Eq. (18) one gets the time evolution of the reduced density matrix $\rho(t)$ in the form

$$\rho(t) = \begin{pmatrix} \cos^2(\theta/2) & (1/2)A(t)\sin\theta e^{-i\phi} \\ (1/2)A^*(t)\sin\theta e^{i\phi} & \sin^2(\theta/2) \end{pmatrix} . \tag{28}$$

Let us notice that this formula can be used as a definition of the *dephasing channel*, where the influence of the infinite bosonic environment is represented by the relaxation function

$$A(t) = w(t)e^{-i\Phi(t)} , \qquad A(0) = 1 . \tag{29}$$

The *damping part* reads

$$w(t) = \langle \Omega | D(2g_t - 2g^-) | \Omega \rangle = e^{-r(t)} , \tag{30}$$

with the decoherence function

$$r(t) = 4 \int_0^\infty d\omega |g(\omega)|^2 \left\{1 - \cos[h(\omega)t]\right\} . \tag{31}$$

The second part which can be recognized is the *phase part*

$$\begin{aligned} \Phi(t) &= \Lambda_1(t) - \Lambda_{-1}(t) - \mathrm{Im}\left[\langle g_t - g | z_t \rangle \right. \\ &\quad \left. + \langle g_t - g | z_t \rangle + \langle g_t - g - z_t | g_t - g + z_t \rangle \right] . \end{aligned} \tag{32}$$

One can observe that a coherent initial state does not affect the damping part but modifies a phase part of the reduced statistical operator.

In a general case, the functions $g$ and $\alpha$ are complex functions of a real variable. For further clarity, we simplify the problem and assume from now on that they are *real functions*. In this case, the total phase reads

$$\Phi(t) = (\varepsilon_+ - \varepsilon_-)t + \Phi_z(t) , \tag{33}$$

where the last part

$$\Phi_z(t) = 4 \int_0^\infty d\omega g(\omega) z(\omega) \sin[h(\omega)t] \tag{34}$$

is the phase related to the initial coherent state. For the initial vacuum state $z(\omega) = 0$ and this contribution is clearly absent.

### 3.4. Exemplum

Pure decoherence model is simple enough to obtain results in analytically closed form. We consider here a simplest case when the environment is formed by one-dimensional bosonic field prepared initially in its ground state, *i.e.* for $z(\omega) \equiv 0$. The coupling between qubit and the field is conventionally given in terms of the spectral function

$$g^2(\omega) = \alpha\, \omega^{\mu-1} \exp(-\omega/\omega_c)\,, \tag{35}$$

where $\alpha > 0$ is the qubit-environment coupling constant, $\omega_c$ is a cut-off frequency and $\mu > -1$ is the "ohmicity" parameter: the case $-1 < \mu < 0$ corresponds to the sub-ohmic, $\mu = 0$ to the ohmic and $\mu > 0$ to super-ohmic environments, respectively. As a result one gets explicit formula for the damping

$$r(t) = 4\mathcal{L}(\alpha, \mu, t)\,, \tag{36}$$

where

$$\mathcal{L}(\alpha, \mu, t) = \alpha \Gamma(\mu) \omega_c^{\mu} \left\{ 1 - \frac{\cos\left[\mu \arctan(\omega_{\rm c} t)\right]}{(1+\omega_{\rm c}^2 t^2)^{\mu/2}} \right\} \tag{37}$$

and $\Gamma(z)$ is the Euler gamma function.

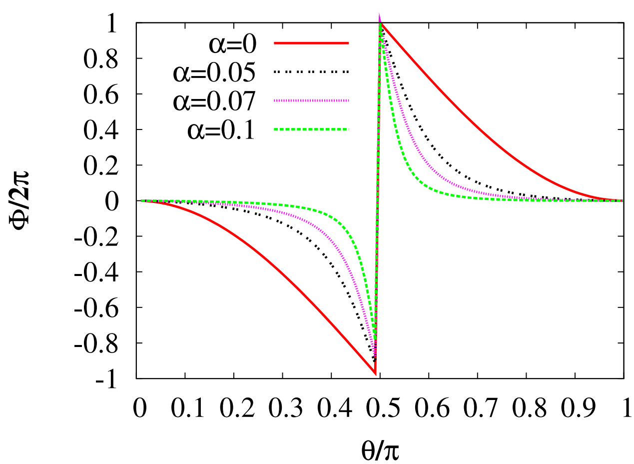


Fig. 1. (Color online) Geometric phase of a qubit dephasingly coupled to the environment in the initial vacuum state for selected values of the qubit-environment coupling parameter $\alpha$ calculated for a quasi-cyclic evolution with time $t = 2\pi$ (in unit of $\omega_{\rm c}$). The remaining parameters are: $\varepsilon_+ = -\varepsilon_- = \varepsilon = 1, \mu = 0.05$.

We derived all necessary formulas necessary for calculation of the geometric phase. The most important component of the calculation procedure is the reduced density matrix (28). From Eq. (28) one can obtain its spectral decomposition (1) and next, via the relation (2), the geometric phase. In Fig. 1 we present the geometric phase calculated after *quasi-cyclic* evolution (such that in the absence of dephasing $\alpha = 0$ it reduces to the cyclic one [1]). Let us notice a very peculiar feature of the geometric phase in dephasing environments: for the initial state (27) at $\theta = \pi/2$ the phase remains unaffected by the environment. It is due to a high symmetry of the considered model, as discussed in Ref. [25]. This property is a natural halmark for dephasing character of the qubit-environment coupling and vanishes for a general type of interaction [26].

## 4. Weak coupling: beyond pure dephasing

Pure decoherence is a very iluminaiting and motivating model. Unfortunately, it is far from being general. Any open system exchanges energy with its environment and neglecting the corresponding heat flow is justified only at certain time scales. There are clearly systems which properties can be effectively measured after the time long enough to system-environment energy transfer to occur. It will not be surprising that for such systems the geometric phase can also serve as a very convenient tool for detailed investigations of various features.

It is well known that, contrary to the classical open systems, there is no general method for constructing the (microscopically consistent) reduced dynamics [21]. The existing methods are limited to a relatively narrow class of models and its parameters. The main obstructions and limitations for effective investigations are related to the system-environment coupling strength or the need of uncorrelated system-environment initial preparation (*i.e.* justifying the Born approximation and the requirement of complete positivity of the reduced dynamics [27]). To summarize, those who want to be rigorous are left with only a few possibilities.

The most general tools of building rigorous reduced dynamics from microsopic Hamiltonians are the perturbative methods. The one with a clear and well established mathematical background is owned to Davies [27]. Having acceptable models of the reduced dynamics one can pose a natural question: does the geometric phase indicate a significance of the abberancy of the system from the 'pure decoherence approximation'? The answer is affirmative. In Ref. [26] we studied the qubit *weakly* coupled to the environment. It has been assumed that the Davies approximation holds true. It allows to construct the suitable kinetic equation whose solution is a desired reduced dynamics. Unfortunately, the complexity of the problem forces to apply numerical analysis. One of the results reported in our paper [26] is that the geometric phase (2) 'feels' if there is a symmetry breaking term (related to the intra-qubit tunneling) responsible for a qubit-bath energy transfer. What remarkable, this impact can be inferred just from measuring the phase for *a single* qubit-environment initial preparation, *i.e.* the measurement of the phase of a single initial preparation of the system can tell us if the energy transfer (the heat flow) between qubit and its bath is significant.

## 5. Many qubit quantum registers

Quantum register is a multi-qubit device designed for a storage and retrieval of quantum information [28]. In general, the register is a highly complicated system with many components of non-negligible interactions. Quantum information is related to the configuration of that system. By the 'configuration' we mean the state of its components rather than the spatial structure. Any modifications require to perform quantum operations applied to certain components without *uncontrolled* effect on the remaining part. The effect can be fully avoided only in the theory. As a result, one can consider the selected one qubit surrounded by other qubits forming a kind of environment.

Studying dynamics of realistic many component quantum systems requires highly sophisticated techniques. In Ref. [29], we studied a simplest approach which serve in many branches of solid state physics as a primary tool: the mean-field approach. We have limited our studies to the case when the mean field approach becomes exact: the *global* coupling between qubits forming an infinite register [29]. In this limit all components of the register interact with another with the same strength. The more compact the device the more sensible the approximation is. Again, the geometric phase has been shown to indicate both the type and the strength of coupling, at least for the studied class of models. Moreover, the geometric phase becomes *stabilized* if, except the mean-field, the qubit is affected by the thermal type environment.

## 6. Oscillating neutrinos

The applicability of geometric phase in studying various properties of open quantum systems goes beyond quantum information processing. One of recent examples is reported in our paper [30] and is concerning the very 'urgent' and unsolved problem in physics of neutrinos [31]: are this particles of Dirac or Majorana type? In other words, are they or they are not their own anti-particles?

If we limit our consideration to two types of oscillating neutrinos (typically to $\mu$ and $e$ neutrinos [31]) one is left with an effective two-level system. One can further attempt to take into account a (very weak) interaction between neutrinos and the ordinary matter and one arrives at the problem of *dissipative neutrino oscillations* expressed in terms of simple phenomenological models [32]. Even in the absence of the dissipation, the geometric phase of oscillating neutrino is worth to be studied [33]. The details are beyond the scope of this short review. Here, we want to emphasize only one of the features: the geometric phase, provided that the dissipation is sufficiently general (with an off-diagonal contribution to the Kossakowski matrix [27]), is different for Dirac and Majorana type neutrinos [30]. This result opens a new perspectives for the neutrino physics and is limited 'only' by technical difficulties in measuring the geometric phase of neutrinos.

## 7. Summary

Geometric phase is a quantity which containes information about a total evolution of quantum system. This feature originates from its *global* character. Its relation to the notion of the *paralell transport* attributes the phase with a deep geometric and topological meaning [1]. In this review we presented that this feature can be used for investigating the way, 'how the open system is open indeed'. In other words, our aim was to convince and attact the reader to think about geometric phase as a highly efficient tool in studying properties of open quantum systems. There are more ways than one to skin a cat, but we hope that the selection of examples included into this paper helped us to accomplish our goal.

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