LETTERS TO THE EDITOR

TWO-BODY RESONANCE APPROXIMATION OF A THREE-PARTICLE DECAY

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An expansion of the Veneziano amplitude for a three-particle decay into two body resonances is studied. It is shown that summing resonances from various channels does not contradict the duality and that the series converges reasonably fast.

One of the popular phenomenological descriptions of a three-body state is the expansion into a series of all possible two-body resonances in all allowed combinations. Since many years, this has been a standard way of describing the Dalitz plot distributions both in experimental and theoretical investigations. For instance, this type of expansion has been recently used in the Illinois partial wave analysis program (cf. Ascoli et al. 1970, Ascoli and Wyld 1975) and in theoretical analyses of three body final states (cf. Amado 1975).

A theoretical objection is often raised that a simultaneous expansion of a three body state into all allowed two-body subsystems is in contradiction with the concept of duality. This is by analogy with the scattering process where the summation of the direct and exchanged resonances simultaneously leads to double counting of some contributions (cf. Dolen, Horn and Schmid 1968). It has been pointed out, however, that a dual three-body decay amplitude can be represented as a sum of two-body subsystems in various combinations (cf. Białas, Turnau and Zalewski 1970), in contrast to the two-body scattering

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amplitude. This unexpected property of the Veneziano amplitude is a direct consequence of mathematical properties of the Euler beta function, established in the last century (see Whittaker and Watson 1963). The question crucial for practical applications is whether the decay-channel expansion of the Veneziano amplitude converges fast enough. In other words, we ask ourselves if by taking just a few first terms of the expansion, i. e. just a few low-mass resonances one can obtain a good approximation of the whole amplitude.

The purpose of the present note is to study the convergence of the two-body resonance expansion of a three-body decay amplitude on the example of the process

$$M \to m+m+m, \tag{1}$$

where M is an arbitrary mass and m is the pion mass. Up to some kinematical factors which do not influence the convergence, the Veneziano amplitude for this process can be written as a combination of the Euler beta functions which can be represented as a sum of resonant contributions (see Białas, Turnau and Zalewski 1970, Whittaker and Watson 1963)

$$B(1-\alpha_s, 1-\alpha_t) = \sum_{n=0}^{N} R_n \left(\frac{1}{n+1-\alpha_s} + \frac{1}{n+1-\alpha_t} \right) \quad \text{for } N \to \infty. \tag{2}$$

Here α_s and α_t are the Regge trajectories in two two-body subchannels and the coefficients are

$$\begin{aligned} R_0 &= 1, \\ R_n &= \frac{(-1)^n}{n!} \frac{\Gamma(n+2-\alpha_s-\alpha_t)}{\Gamma(2-\alpha_s-\alpha_t)} \\ &= \frac{(-1)^n}{n!} (2-\alpha_s-\alpha_t)(3-\alpha_s-\alpha_t) \ldots (n+1-\alpha_s-\alpha_t). \end{aligned} \tag{3}$$

We take a reasonable parametrization for the trajectories (Lovelace 1968)

$$\alpha_s = 0.483+0.885s+i\ 0.28\sqrt{s-4m^2} \tag{4}$$

and similarly for α_t. Variables $s = s_{12}$ and $t = s_{13}$ denote here the effective masses squared of subsystems 1 and 2, and of 1 and 3. The resonance positions are then at the values of s and t of $s_n, t_n = 0.583,\ 1.713,\ 2.843,\ 3.973$ etc.

The region of convergence of series (2) is

$$|\alpha_s+\alpha_t| > 1 \tag{5}$$

and covers the whole Dalitz plot. Therefore expansion (2) can be applied to the decay channel.

We have compared the values of the real parts, imaginary parts, absolute values and phases of the B function (2) and the approximants for various masses M and various numbers of terms N both on two-dimensional Dalitz plots and on one-dimensional projections. For technical reasons we present here only the one dimensional projections for $N = 0, 1, 2$ and 3 and for $M \leqslant 3.5$ GeV. These are shown in Fig. 1.

We see from the figures that expansion (2) converges fairly fast. In a region $M \leqslant M_{max}$ it is sufficient to take only the resonances with $s, t \leqslant M^2_{max}$. Then only the high-mass tail of the last resonance deviates significantly from the B-function, usually to about

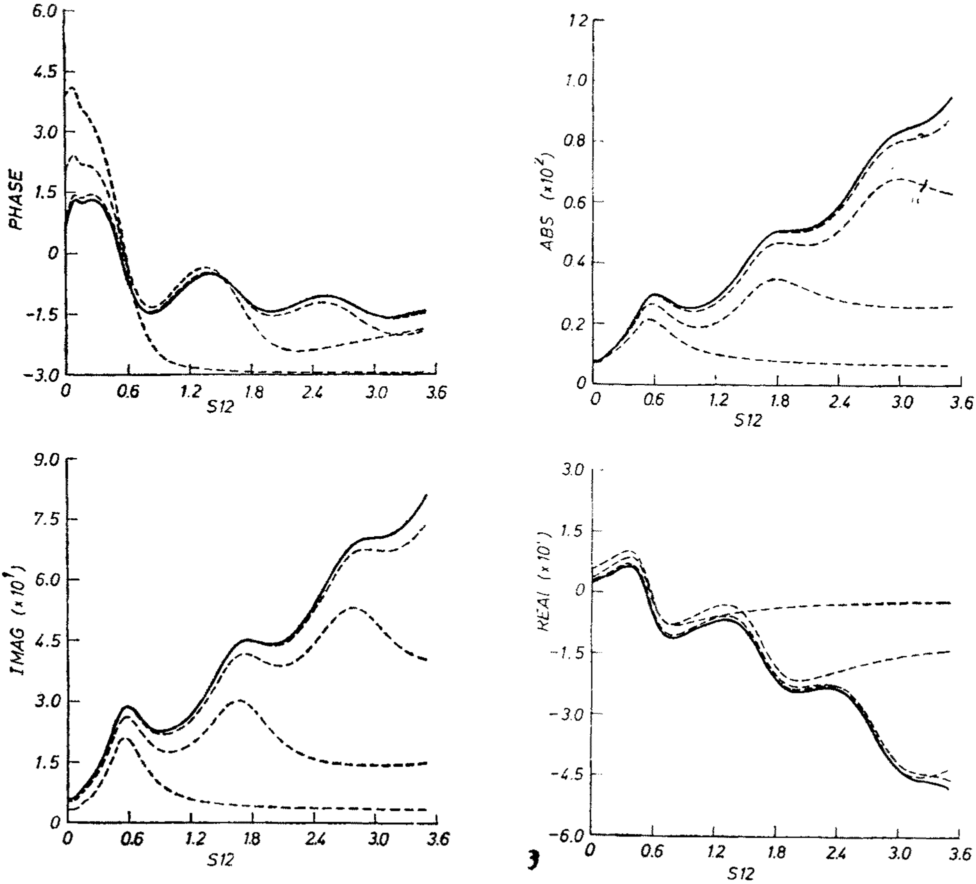

Fig. 1. One dimensional projections of the absolute value phase, real part and imaginary part of the Veneziano function for a three-particle decay. The projected region is $s_{12}, s_{13} < 3.5$ GeV. The continuous curve is the Veneziano function, the broken lines are approximants with one, two, three and four resonances

20–30 percent. By taking one more resonance one obtains an approximation which is good up to a few percent.

We can conclude that the usual method of describing a three-particle state in terms of two-body resonances is compatible with the concept of duality and that there is no double counting involved. In fact, the Veneziano function is very well approximated by a sum of just a few first terms.

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