BARRIER CROSSING AND TRANSPORT ACTIVATED BY KANGAROO FLUCTUATIONS*

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We study barrier crossing of Brownian particles in a bistable symmetric potential and transport of Brownian particles in spatially periodic structures, driven by both kangaroo fluctuations and thermal equilibrium noise of zero mean values. We consider exponentially and algebraically correlated kangaroo fluctuations. Starting with the full Newton–Langevin equation for the Brownian particle and by introducing scaling as well as dimensionless variables, we show that the equation is very well approximated by overdamped dynamics in which inertial effects can be neglected. We analyze properties of selected macroscopic characteristics of the system such as the mean first passage time (MFPT) of particles from one minimum of the bistable potential to the other and mean stationary velocity of particles moving in a spatially periodic potential. In dependence upon statistics of kangaroo fluctuations and temperature of the system, macroscopic characteristics exhibit distinctive non-monotonic behavior. Accordingly, there exist optimal statistics of fluctuations optimizing macroscopic characteristics.

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1. Introduction

Processes activated by fluctuations and noise play a crucial role in nature. Examples are not only in physics, chemistry or engineering but also in sociology, economy and politics. One of such processes, the noise-assisted escape over a barrier is realized in such diverse phenomena as thermionic emission of electrons from a metal surface, chemical reactions in condensed phases, flux transitions in SQUIDs and transport of molecules in proteins, to mention only a few [1,2]. An archetypal mathematical model is based on an equation

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of motion of a Brownian particle in a bistable potential and driven by thermal non-correlated fluctuations, *i.e.*, by δ-correlated Gaussian white noise. An equivalent description can be presented in terms of a Fokker–Planck equation which determines the time evolution of a probability density for a position of the Brownian particle. Motivated by experiments, theoreticians have started to analyze more realistic models by incorporating a finite noise correlation time or finite bandwidth. A considerable effort has been made to study dynamical systems driven by colored or correlated noise such as exponentially correlated Gaussian or dichotomic processes [2–5]. Unfortunately, systems driven by arbitrary correlated noise are difficult to handle analytically. An example is algebraically (powerly) correlated noise with long time tail [6, 7]. In the paper, we study the influence of such noise on two systems, namely, a bistable system and a ratchet-type system. The former is very well known for a scientific community. The latter has mainly been inspired by biological systems (with the hope to explain transport by protein motors in cells) [8]. The literature on this subject can be found in [9–11].

In Section 2, we formulate the model of noisy dynamics as the Newton equation for a particle in bistable or spatially periodic potentials and driven by two random forces, one describing interaction with surroundings (thermal noise) and the other that mimic nonthermal fluctuations. By introducing specific scaling and dimensionless variables, we demonstrate that the limit of overdamped motion is an extremely good approximation to full dynamics. As nonthermal fluctuations, we consider two classes of kangaroo Markov processes: exponentially and algebraically correlated processes. The former is named the Kubo–Anderson process (Section 3). In Section 4, forward and backward master equations for the two dimensional process in the extended phase space are presented. These are partial integro-differential equations which cannot be solved analytically. Therefore, in Section 5 we apply the simulation method of the Langevin equation to investigate the mean first passage time problem for the bistable system and transport properties of the Brownian particles in spatially periodic structures. Our findings are discussed in Section 6.

2. Model

We study a Brownian motion of particles in a one-dimensional symmetric bistable potential $\hat{V}(\hat{x})$ (Fig. 1) or a spatially periodic potential $\hat{V}(\hat{x}) = \hat{V}(\hat{x}+L)$ of period L (Fig. 2). The dynamics of particles is assumed to be governed by a Newton–Langevin equation of the form

$$M\ddot{\hat{x}} + \gamma\dot{\hat{x}} = -\frac{d\hat{V}(\hat{x})}{d\hat{x}} + \hat{\Gamma}(\hat{t}) + \hat{\xi}(\hat{t})\,, \tag{1}$$

where M denotes mass of the particle and γ is the friction coefficient. The random process $\hat{\Gamma}(\hat{t})$ represents thermal equilibrium fluctuations modelled by δ-correlated Gaussian white noise with the first two moments

$$\langle \hat{\Gamma}(\hat{t}) \rangle = 0\,, \qquad \langle \hat{\Gamma}(\hat{t})\hat{\Gamma}(\hat{u}) \rangle = 2\hat{D}\delta(\hat{t}-\hat{u})\,, \tag{2}$$

where, according to the dissipation-fluctuation theorem, the thermal-noise strength $\hat{D}$ relates to the friction constant and temperature T of the system as follows

$$\hat{D} \equiv \gamma k_{\mathrm{B}} T \tag{3}$$

with k_{B} denoting the Boltzmann constant.

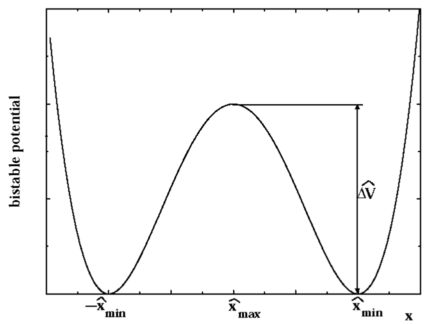

Fig. 1. The bistable potential $\hat{V}(\hat{x})$ of barrier height $\Delta\hat{V}$ and minima at $\hat{x} = \pm\hat{x}_{\mathrm{min}}$.

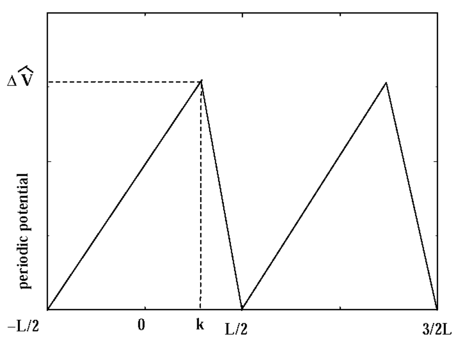

Fig. 2. The piecewise linear periodic potential $\hat{V}(\hat{x})$ of period L, barrier height $\Delta\hat{V}$ and asymmetry k.

The random force $\hat{\xi}(\hat{t})$ represents *zero-mean* nonthermal fluctuations which are modelled here by a *symmetric* kangaroo process (by symmetric process we mean that its probability density $p(\hat{\xi},\hat{t})$ is a symmetric function of $\hat{\xi}$). This process is described briefly in the next section. As usual, we assume that $\hat{\Gamma}(\hat{t})$ is not correlated with $\hat{\xi}(\hat{t})$.

Now, let us introduce dimensionless variables. The symmetric bistable (Fig. 1) or spatially periodic (Fig. 2) potentials $\hat{V}(\hat{x})$ have the barrier height $\Delta\hat{V} = \hat{V}_{\max} - \hat{V}_{\min}$. The bistable potential has two minima at $\hat{x} = \pm\hat{x}_{\min}$ and a maximum at $\hat{x} = \hat{x}_{\max} = 0$. Hence, a characteristic length l_0 is determined by the distance between positions of maximum and minima of the potential, *i.e.*, $l_0 = |\hat{x}_{\min} - \hat{x}_{\max}|$. For the spatially periodic potential, the characteristic length l_0 is determined by the period L of $\hat{V}(\hat{x})$, *i.e.*, $l_0 = L$. To identify a characteristic time τ_0, let us consider a deterministic, overdamped motion of a particle in the potential $\hat{V}(\hat{x})$, namely,

$$\gamma \frac{d\hat{x}}{d\hat{t}} = -\frac{d\hat{V}(\hat{x})}{d\hat{x}}\,. \tag{4}$$

Then we define τ_0 by the relation

$$\gamma \frac{l_0}{\tau_0} = \frac{\Delta\hat{V}}{l_0} \tag{5}$$

and it reads

$$\tau_0 = \frac{\gamma l_0^2}{\Delta\hat{V}}\,. \tag{6}$$

During this time interval, an overdamped particle moves a distance of length l_0 under the influence of the constant force $\Delta\hat{V}/l_0$. Accordingly, the scaling for the position of the Brownian particle is $x = \hat{x}/l_0$ and for time $t = \hat{t}/\tau_0$. In this case, Eq. (1) is transformed into the dimensionless form

$$m\ddot{x} + \dot{x} = f(x) + \Gamma(t) + \xi(t), \quad f(x) = -dV(x)/dx, \tag{7}$$

where

$$m = \frac{M}{\gamma\tau_0} \tag{8}$$

is the dimensionless mass related to inertia of Brownian particles. The rescaled bistable or spatially periodic potentials $V(x) = \hat{V}(\hat{x})/\Delta\hat{V}$ have now the unit-barrier height $\Delta V = V_{\max} - V_{\min} = 1$. Minima of the bistable potential $V(x)$ are located at $x = x_{\min} = \pm 1$ and a maximum at $x = x_{\max} = 0$. The spatially periodic potential $V(x) = V(x+1)$ has a unit period $L = 1$. The dimensionless strength D of rescaled Gaussian white noise $\Gamma(t)$ is measured in units of the barrier height, $D = k_{\mathrm{B}}T/\Delta\hat{V}$. Finally, the rescaled kangaroo noise $\xi(t) = (l_0/\Delta\hat{V})\hat{\xi}(\hat{t})$.

Let us analyze the problem of overdamped dynamics. As the first example, we consider particles in fluid. Jean Perrin in his fundamental experiments in 1908 [12] used particles of radius $R = 10^{-7}$m and of mass $M = 10^{-17}$kg. For fluid being water in room temperature, the viscosity $\eta = 10^{-3}$kg/s m. From the Stokes formula [13], $\gamma = 6\pi\eta R$, one gets for the friction coefficient $\gamma = 2 \times 10^{-9}$kg/s. Assuming a diffusion regime, $\Delta\hat{V} \approx 5k_{\mathrm{B}}T$, room temperatures $T = 300$K of the system and a characteristic length $l_0 = 10^{-5}$m being 100 times greater than the particle radius, we infer that $m = 5 \times 10^{-10} \ll 1$. As the second example, we take into account the kinesin movement along microtubules inside of cells [14]. Microtubules are spatially periodic structures which consist of tubulin heterodimers arranged in rows called protofilaments which, in turn, are oriented nearly parallel to the microtubule axis. A heterodimer is about 8nm long [15] and is composed of two various globular subunits: α-tubulin and β-tubulin. It leads to symmetry breaking of the spatial reflection of the potential $\hat{V}(\hat{x})$ with period $L = 8$nm. The mass of kinesins $m = 6 \times 10^{-22}$kg and their radius $R = 10^{-8}$m. The friction coefficient $\gamma = 2 \times 10^{-8}$ kg/s which is calculated from the Stokes formula with $\eta = 10^{-1}$ kg/ms (it is the effective viscosity coefficient of the medium [15]). If we assume that $\Delta\hat{V} = 5k_{\mathrm{B}}T$ and $T = 310$K then $m = 5 \cdot 10^{-10} \ll 1$. Let us note that the dimensionless mass m at the acceleration term is 10 orders less than the dimensionless friction coefficient 1 at the velocity term. For that reason inertial effects can completely be neglected and the second order differential equation (7) can be approximated by the first order differential equation

$$\dot{x} = f(x) + \Gamma(t) + \xi(t). \tag{9}$$

This is an equation describing overdamped dynamics of Brownian particles and for the above two examples is indeed a very good approximation to the full equation (7). Below, we analyze this simplified model.

3. Kangaroo stochastic process

Nonthermal and nonequilibrium fluctuations can be modelled by the kangaroo process $\xi(t)$. It is a purely discontinuous (Kolmogorov–Feller) stationary stochastic process for which the transition probability per unit time $W(\xi|\xi_0)$ for a flipping from the state ξ_0 into the state ξ factorizes [16], *i.e.*,

$$W(\xi|\xi_0) = Q(\xi)\nu(\xi_0). \tag{10}$$

It means that the system jumps from the state ξ_0 with the frequency $\nu(\xi_0)$. The quantity $\tau(\xi) = 1/\nu(\xi)$ is the mean waiting time in the state ξ. The probability that the process jumps into the state ξ is $Q(\xi)$ and it is normalized over the phase space of $\xi(t)$ to unity. The corresponding Kolomogorov-Feller equation for the probability density $p(\xi,t)$ of this process takes the form [17]

$$\frac{\partial p(\xi,t)}{\partial t} = \mathcal{L}p(\xi,t) = -\nu(\xi)p(\xi,t) + Q(\xi)\int_{-\infty}^{\infty} \nu(\eta)p(\eta,t)d\eta\,, \tag{11}$$

where $\mathcal{L}$ defined by this equation is an infinitesimal (forward) generator of the process $\xi(t)$. The backward operator $\mathcal{L}^+$, acting on an arbitrary function $g(\xi)$, has the form

$$\mathcal{L}^+ g(\xi) = -\nu(\xi)g(\xi) + \nu(\xi)\int_{-\infty}^{\infty} Q(\eta)g(\eta)d\eta\,. \tag{12}$$

It plays a crucial role in first passage time problems.

For the symmetric kangaroo process, which is considered in the paper, its correlation function $F(t)$ is [17]

$$F(t-s) = \langle\xi(t)\xi(s)\rangle = 2\int_0^{\infty} \xi^2 p(\xi)\exp(-\nu(\xi)|t-s|)\,d\xi\,, \tag{13}$$

where $p(\xi) = p(-\xi)$ is a stationary probability distribution of $\xi(t)$ and $\nu(\xi) = \nu(-\xi)$. In this case it is a zero-mean process, $\langle\xi(t)\rangle = 0$.

3.1. Kubo–Anderson fluctuations

The Kubo–Anderson process is a particular case of the kangaroo process when the jumping frequency is constant, $\nu(\xi) = \nu_0$ [17–19]. Then from (13) it follows that the Kubo–Anderson process is exponentially correlated,

$$F(t-s) = \langle\xi(t)\xi(s)\rangle = \langle\xi^2\rangle\exp(-|t-s|/\tau_c) \tag{14}$$

with the correlation time $\tau_c = 1/\nu_0$ and $\langle\xi^2\rangle$ is a mean value of $\xi^2(t)$ over the stationary probability density $p(\xi) = Q(\xi)$, *cf.* (11). We will consider two examples of this noise:

(1) the process $\xi(t)$ is unbounded, defined on $(-\infty,\infty)$ and has the Gaussian stationary distribution,

$$p(\xi) = Q(\xi) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\xi^2/2\sigma^2)\,, \qquad \xi(t)\in(-\infty,\infty)\,. \tag{15}$$

(2) The process $\xi(t)$ is bounded, defined on $[-A, A]$ and has the uniform stationary distribution,

$$p(\xi) = Q(\xi) = \frac{1}{2A}\Theta(x + A)\Theta(A - x)\,, \qquad \xi(t) \in [-A, A]\,. \tag{16}$$

3.2. Algebraically correlated fluctuations

Let us define the kangaroo process for which its correlation function is a power function of time. For the bounded process $\xi(t)$ defined on $[-A, A]$ and uniformly distributed as in (16), its correlation function has the form

$$F(t - s) = \langle \xi(t)\xi(s) \rangle = \frac{1}{A}\int_0^A \xi^2 \exp(-\nu(\xi)|t - s|)\, d\xi \tag{17}$$

We generalize the previous case to the situation when the jumping frequency $\nu(\xi)$ depends powerly on the state, *i.e.*,

$$\nu(\xi) = \nu_0 \left(\frac{|\xi|}{A}\right)^{3\alpha} \tag{18}$$

The distribution $Q(\xi)$ is given by

$$Q(\xi) = \frac{1 + 3\alpha}{2A} \left(\frac{|\xi|}{A}\right)^{3\alpha} \tag{19}$$

and the correlation function takes the form

$$F(t - s) = \langle \xi(t)\xi(s) \rangle = \frac{A^2}{3\alpha}\gamma\left(1/\alpha, \nu_0|t - s|\right) \left(\frac{1}{\nu_0|t - s|}\right)^{1/\alpha}, \tag{20}$$

where $\gamma(a, z)$ is an Euler incomplete Gamma function [20]. As $|t - s| \to \infty$, the function $\gamma\left(1/\alpha, \nu_0|t - s|\right)$ tends to a constant value given by the Euler gamma function $\Gamma(1/\alpha)$. Accordingly, for long time, $\xi(t)$ is algebraically correlated with the exponent $1/\alpha$ exhibiting the long time tail $|t - s|^{-1/\alpha}$. Let us observe that when $\alpha \to 0$ then this process tends to the Kubo–Anderson process: The jumping frequency tends to a constsnt value, $\nu(\xi) \to \nu_0$. Using the relation [20]

$$\gamma(a, x) = a^{-1}x^a \mathrm{e}^{-x}\; {}_1F_1(1, 1 + a, x), \tag{21}$$

where ${}_1F_1(1, 1 + a, x)$ stands for the Kummer (confluent hypergeometric) function, we infer that in (20),

$$\langle \xi(t)\xi(s) \rangle \to \left(\frac{A^2}{3}\right) \exp(-\nu_0|t - s|) \quad \text{as} \quad \alpha \to 0 \tag{22}$$

and is the same as (14) with $\langle \xi^2 \rangle = A^2/3$ for uniformly distributed noise.

4. Master equations

The output process $x(t)$ in (9) is non-Markovian as driven by correlated noise $\xi(t)$. However, the two-dimensional process $\{x(t), \xi(t)\}$ is Markovian and its joint probability density obeys a master equation of the form [16]

$$\begin{aligned}\frac{\partial P(x,\xi,t)}{\partial t} &= -\frac{\partial}{\partial x}[f(x)+\xi]P(x,\xi,t) + D\frac{\partial^2}{\partial x^2}P(x,\xi,t)\\ &\quad -\nu(\xi)P(x,\xi,t) + Q(\xi)\int_{-\infty}^{\infty}\nu(\eta)P(x,\eta,t)d\eta\,. \end{aligned} \tag{23}$$

It is not required to know the probability density $P(x,\xi,t)$ in the extended phase space $\{x(t), \xi(t)\}$. We are rather interested in the probability density $\mathcal{P}(x,t)$ of the process $x(t)$ only. It can be obtained from $P(x,\xi,t)$ by integration it over ξ, *i.e.*,

$$\mathcal{P}(x,t) = \int_{-\infty}^{\infty} P(x,\xi,t)d\xi. \tag{24}$$

Integrating (23) over the noise variable ξ yields the continuity equation for the distribution density, $\mathcal{P}(x,t)$,

$$\frac{\partial \mathcal{P}(x,t)}{\partial t} = -\frac{\partial J(x,t)}{\partial x}, \tag{25}$$

here the probability current $J(x,t)$ of the process $x(t)$ reads

$$J(x,t) = f(x)\mathcal{P}(x,t) - D\frac{\partial \mathcal{P}(x,t)}{\partial x} + \int_{-\infty}^{\infty}\xi P(x,\xi,t)d\xi. \tag{26}$$

The probability current $J(x,t)$ characterizes transport properties of systems because an average velocity of Brownian particles can be expressed by $J(x,t)$ (Sec. 6).

The mean first passage time $T(x)$ of the process $x(t)$ is one of the most important quantity for bistable processes. It is known that the mean first passage time $T(x,\xi)$ of the joint process $\{x(t), \xi(t)\}$ is determined by the backward integro-differential equation

$$\begin{aligned}&[f(x) \;+\; \xi]\frac{\partial}{\partial x}T(x,\xi) + D\frac{\partial^2}{\partial x^2}T(x,\xi)\\ &\quad -\; \nu(\xi)T(x,\xi) + \nu(\xi)\int_{-\infty}^{\infty}Q(\eta)T(x,\eta)d\eta = -1 \end{aligned} \tag{27}$$

with specified boundary conditions. The mean first passage time $T(x)$ of the process $x(t)$ alone can be calculated as follows [21]

$$T(x) = \int_{-\infty}^{\infty} T(x,\xi)p_0(\xi)d\xi\,, \tag{28}$$

where $p_0(\xi)$ is an initial probability density of noise $\xi(t)$. Unfortunately, neither (23) nor (27) can analytically be solved.

5. Simulations of the Langevin equation with kangaroo and Kubo–Anderson noise

The complicated master equations (23) and (27) make the method of numerical simulations the most important tool in the investigation of dynamics of Brownian particles driven by kangaroo and thermal sources of noise. The method of integration the Langevin equation is a standard procedure. First we integrate Eq. (9) over one time step:

$$\begin{aligned} \int_{t_i}^{t_{i+1}} \dot{x}(t)dt &= x(t_{i+1}) - x(t_i) \\ &= \int_{t_i}^{t_{i+1}} f(x(t))dt + \int_{t_i}^{t_{i+1}} \Gamma(t)dt + \int_{t_i}^{t_{i+1}} \xi(t)dt\,. \end{aligned} \tag{29}$$

If $h = t_{i+1} - t_i$ is sufficiently small, we can make following approximations:

(A)

$$\int_{t_i}^{t_{i+1}} f(x(t))dt \approx f(x_i)h\,,$$

where $x_i \equiv x(t_i)$. This is simply a step of Euler integration of ordinary differential equations.

(B)

$$\int_{t_i}^{t_{i+1}} \Gamma(t)dt = W(t_i+h) - W(t_i) = \sqrt{2Dh}N(0,1)\,,$$

where $W(t)$ is the Wiener process and $N(0,1)$ is a Gaussian random variable of zero average and unit standard deviation (we use the fact that the Wiener process is an integral of Gaussian white noise). The number $N(0,1)$ has to be an independent random value in each step (we have used the uniform pseudorandom number generator based on the linear congruential generator with shuffling and then the Box–Muller method to convert it to Gaussian one).

(C)

$$\int_{t_i}^{t_{i+1}} \xi(t)dt \approx \xi(t_i)h\,,$$

provided that h is much smaller than the correlation time of noise $\xi(t)$.

Thus we can write Eq. (29) in the approximate form

$$x_{i+1} = x_i + f(x_i)\cdot h + \sqrt{2Dh}N(0,1) + \xi(t_i)h + O(h^{3/2})\,. \tag{30}$$

The algorithm of integration of the Langevin equation consists of a generation of the trajectory $x(t)$ starting from an initial position $x_0 = x(0)$ (which can be a random variable distributed according to a specified distribution or it can be deterministic). The method of generation of the particular realization of $\xi(t)$ is based on generation of states to which the process jumps and time intervals in which the process stays in a given state. In the case of kangaroo processes those times are dependent on the current state of noise. Therefore first we have to calculate the value ξ of noise as a random variable with distribution (15) or (16), or (19) and then the jumping rate $\nu(\xi)$ which is constant or given by (18). When $\nu(\xi)$ is known the time the process spend in the state ξ is distributed according to

$$P(T) = \nu(\xi)\mathrm{e}^{-T\nu(\xi)} \tag{31}$$

and can be easily generated. To avoid side effects connected with the "bad" properties of a pseudorandom number generator we constructed it in such a way that N particles are simulated at the same time and the loop over particles is inside of the loop over time. Therefore the subsequent calls to the pseudorandom number generator are distributed over different trajectories. This makes the method more stable with respect to quality of pseudorandom number generators. In order to investigate the statistical properties of the generated kangaroo noise we have calculated the correlation function $F(t) = \langle\xi(t)\xi(s)\rangle$. The comparison of $F(t)$ calculated from numerical experiment and analytical formula shows good coincidence for short as well as for long times (see Fig. 3).

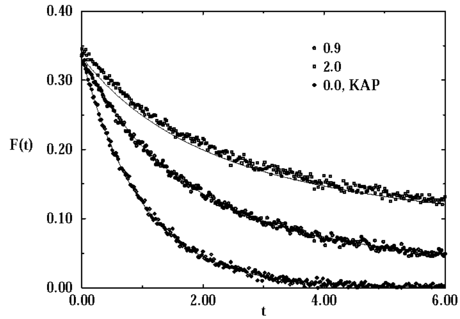

Fig. 3. The correlation function $F(t)$ of the Kubo-Anderson process (KAP) and kagaroo fluctuations with exponents $\alpha = 0.9$ and $\alpha = 2.0$. The amplitude $A = 1$ and the frequency $\nu_0 = 1$.

6. Discussion

In this Section we analyze results of simulations. Two characteristics have been studied, namely, the mean first passage time $T(x)$ and the stationary mean velocity $\langle v \rangle$ of Brownian particles. To explore features of the system we have considered three types of noise:

(i) Exponentially correlated Kubo–Anderson noise with the Gaussian stationary distribution (15).

(ii) Exponentially correlated Kubo–Anderson noise with the uniform stationary distribution (16).

(iii) Algebraically correlated kangaroo noise with the uniform stationary distribution (Sec. 3.2).

6.1. Mean First Passage Time

We considered the particle moving in the bistable potential

$$V(x) = x^4 - 2x^2, \qquad x \in (-\infty, \infty), \tag{32}$$

driven by both thermal and kangaroo sources of noise. For all numerical experiments we have used dimensionless variables and the overdamped equation of motion (9).

To obtain the mean first passage time we integrate Eq. (9) numerically starting from $x(0) = -1$ to $x(T) = +1$ according to the formula (30). The measured time T has to be averaged over a large number of realizations (usually $10^3 - 10^4$). The integration step was taken to be $h = 10^{-3}$ in all simulations. The choice of a comparatively small time step assured that simulations were reliable and precise. However, this resulted in long time of calculations. A typical "run" consisted of 10^8 time steps and we obtained one plot consisting of c.a. 10 points in time of order 24 hours.

MFPT depends on three parameters: jumping frequency ν_0, the strength $D = k_B T/\Delta\hat{V}$ of thermal fluctuations and the variance proportional to σ or A for Gaussian and uniformly distributed noise, respectively. Additionally, in the case of algebraically correlated noise, it depends on the exponent α. For the system driven by Kubo–Anderson noise with the Gaussian stationary distribution (unbounded noise), the dependence of MFPT on frequency ν_0 is depicted in Fig. 4 for three values of a stationary variance $\sigma = \sqrt{\langle\xi^2\rangle}$ and fixed strength of thermal fluctuations (or equivalently temperature of the system). We find that MFPT is a non-monotonic function of the jumping frequency. It decreases as the jumping frequency increases attaining a minimal value at some ν_0 (the extremal point ν_0 shifts to the right for greater values of σ). Next, MFPT grows as $\nu_0 \to \infty$. One can observe that MFPT monotonically diminishes as the variance σ increases. Qualitatively, the same behaviour exhibits MFPT for systems driven by bounded exponentially as well as algebraically correlated noise. We show it in Fig. 5. Inferentially we note that there exists an optimal jumping frequency at which MFPT is the smallest and the activation process is the fastest. Another important feature is related to the fact that exponentially correlated (Kubo–Anderson) bounded noise is "worse" than algebraically correlated kangaroo bounded noise of any non-zero value of the exponent α. In turn, algebraically correlated noise is observed to be a better activator when α is bigger. The larger value of α means the longer tail in the correlation function. Accordingly, long tails make the activation easier.

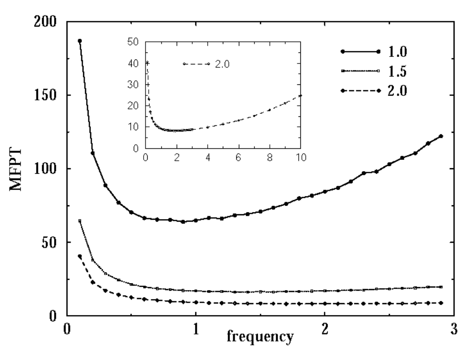

Fig. 4. The depencence of MFPT on the frequency ν_0 in the system activated by Gaussian (unbounded) Kubo-Anderson noise of the variance $\sigma = 1.0$, 1.5 and 2.0 and for fixed temperature $T = 0.001$. The insert shows the same for $\sigma = 2.0$ and for a greater interval of ν_0 showing existence of the extremal value of ν_0 minimizing MFPT.

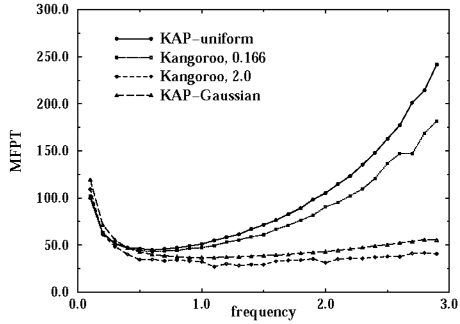

Fig. 5. MFPT against the frequency ν_0 for the system activated by: Gaussian (unbounded) Kubo-Anderson noise (KAP-Gaussian) of the variance $\sigma = 2/\sqrt{3}$; uniform (bounded) Kubo-Anderson noise (KAP-uniform) of the amplitude $A = 2$; kangaroo (bounded) noise with $A = 2$ and $\alpha = 1/6,\ 2$. In all cases, temperature $T = 0.001$.

It seems to be difficult to compare the Gaussian (unbounded) Kubo–Anderson noise with uniform (bounded) noise. The parameters A and σ may be related to each other via, *e.g.*, stationary moments $< \xi^n >$, $n = 1, 2, 3, \ldots$ but it is rather artificial. For bounded noise, $\langle \xi^{2n} \rangle = A^{2n}/(2n+1)$ and for Gaussian noise, $\langle \xi^{2n} \rangle = (2n-1)!!\sigma^{2n}$. However, we can observe the same features of this dependence — the optimal value of the frequency at which MFPT is the smallest.

Next, we focus on the dependence of the escape rate 1/MFPT upon the amplitude A of bounded noise. Details are shown in Fig. 6. The force $f(x) = -dV(x)/dx = -4x^3 + 4x$ has a local minimal value $f(x_0) = -8/3\sqrt{3}$

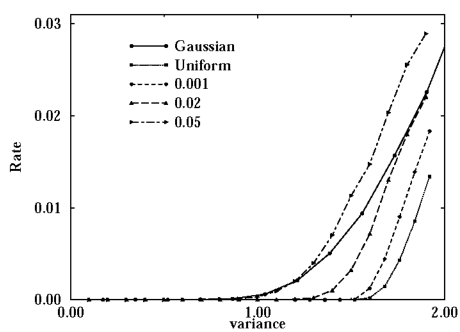

Fig. 6. The escape rate (1/MFPT) as a function of the variance of noise for the system activated by: Gaussian (unbounded) Kubo–Anderson noise of fixed $\nu_0 = 0.4$ and for $T = 0.001$; uniform (bounded) Kubo–Anderson noise with $\nu_0 = 0.4$ and for $T = 0.001$; kangaroo (bounded) noise with fixed $\alpha = 1$ and $\nu_0 = 0.4$ for three various temperatures $T = 0.001,\ 0.02,\ 0.05$.

for $x_0 = -1/\sqrt{3}$. If temperature $T = 0$ and if the amplitude A of noise $\xi(t)$ is smaller than the absolute value of x_0, the crossing over the barrier from the left to the right cannot be realized because $\dot{x} = f(x) + \xi(t) < 0$. The increase of temperature enables weak noise (with the amplitude A below the value x_0) to activate the system because thermal noise is unbounded. In turn, the unbounded Kubo–Anderson process induces the barrier crossing at any temperature of the system. This is the main difference between influence of bounded and unbounded sources of noise. In the former, fluctuations cannot be smaller than some minimal value and there exists a threshold value of the noise amplitude below which there is no activation. In the latter, fluctuations can take an arbitrary large value (with correspondingly low probability) and therefore no threshold exists. We also notice that exponentially correlated Kubo-Anderson noise leads to a smaller activation rate than corresponding algebraically correlated kangaroo noise.

6.2. Transport in spatially periodic structures

We have also performed simulations of transport of Brownian particles in a spatially periodic system driven by algebraically correlated bounded kangaroo noise. The case of exponentially correlated Gaussian (unbounded) Kubo-Anderson noise has been studied elsewhere [18, 19]. The potential we use is piecewise linear (Fig. 2),

$$V(x) = \begin{cases} \frac{1+2x}{1+2k}, & x \in [-1/2, k] \mod 1, \\ \\ \frac{1-2x}{1-2k}, & x \in [k, 1/2] \mod 1, \end{cases} \tag{33}$$

where the parameter k characterizes asymmetry of the potential: if $k = 0$ then it is symmetric. Otherwise, it is asymmetric. Transport properties are determined by the average velocity $\langle v(t) \rangle$ of particles,

$$\langle v(t) \rangle \equiv \left\langle \frac{d\hat{x}}{d\hat{t}} \right\rangle = v_0 \left\langle \frac{dx}{dt} \right\rangle = v_0 \left\langle f(x) \right\rangle = v_0 \int_0^1 f(x) \mathcal{P}(x,t)\, dx, \tag{34}$$

where integration is over a period $L = 1$ of the rescaled potential and the characteristic velocity $v_0 = l_0/\tau_0$. To obtain the above relation, we have utilized Eq. (9) and the fact that mean values of both sources of fluctuations are zero. Using (26), we obtain

$$\langle v(t) \rangle = v_0 \int_0^1 J(x,t)\, dx\,. \tag{35}$$

In the stationary state, $P(x) = \lim_{t\to\infty} P(x,t)$, and $J = \lim_{t\to\infty} J(x,t)$. Then (35) reduces to the form

$$\langle v \rangle = v_0\, J. \tag{36}$$

The dimensionless probability current J depends on the dimensionless parameters, *i.e.*, $J = J(D; \nu_0, A, \alpha; k)$ for bounded noise with the stationary uniform density.

The numerical integration of the equation of motion was carried out according to the algorithm (30). The time required was longer than in the case of MFPT. The number of steps for one set of parameters was usually 3×10^9. Hence, we needed c.a. 50–100 h to obtain one plot. The main purpose of this part of our work is to study the influence of α and ν_0 on transport. A generic graphical representation is the dependence of the probability current on temperature. We depicted this dependence in Fig. 7 for two values of the exponent $\alpha = 1/6$ and $\alpha = 2.0$, and two frequencies $\nu_0 = 1.0$ and $\nu_0 = 2.0$. It is simulated for a fixed amplitude A of kangaroo noise with values between two extremal values of the force $f(x)$. The observed rule is that two factors cause the increase of the current: the increase of α and the decrease of ν_0. Moreover, we plotted the probability current against the frequency for $\alpha = 1/6$ and $\nu_0 = 1$ in the case of zero temperature limit. The current decreases monotonically, approaching zero for an infinitely large frequency ν_0. The limit of $\nu_0 = 0$ corresponds to the adiabatic limit, for which changes of noise are much slower than deterministic relaxation of the system.

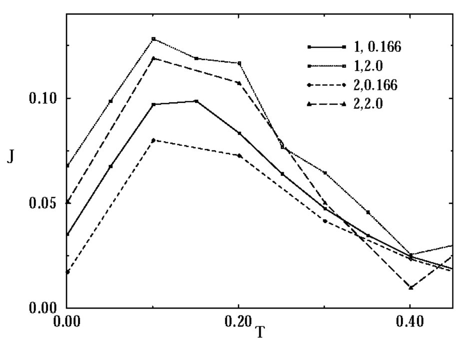

Fig. 7. The probability current versus temperature for the system activated by kangaroo (bounded) noise with $\nu_0 = 1$, $\alpha = 1/6$; $\nu_0 = 2$, $\alpha = 1/6$; $\nu_0 = 1$, $\alpha = 1$; $\nu_0 = 2$, $\alpha = 1$. In all cases, the amplitude $A = 2$.

A general note concerns a direction of particle transport. We considered a region of small-to-intermediate values of frequency. Then the current is positive for a positive asymmetry of the potential with $k > 0$. Conversely, if the potential has a negative asymmetry $k < 0$, the resulting current assumes

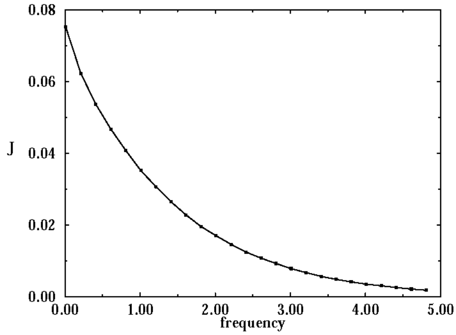

Fig. 8. The probability current as a function of the frequency ν_0 for the system activated by kangaroo (bounded) noise with $\alpha = 1/6$, $L = 2$ and fixed $T = 0.001$.

negative values. If $k = 0$ then $J = 0$. As follows from [18,19], for large values of frequency (or small correlation time) of Gaussian (unbounded) Kubo-Anderson noise the current is negative. The explanation of this current-reversal phenomenon is presented in [18].

Finally, our simulations lead to the conclusion that long tails (bigger α) in noise correlation affect advantageously on transport in spatially periodic structures.

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